

Homework 2 (Written Problems)

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1 Problem 1

DFA for no xzxy

5-tuple for the DFA is:

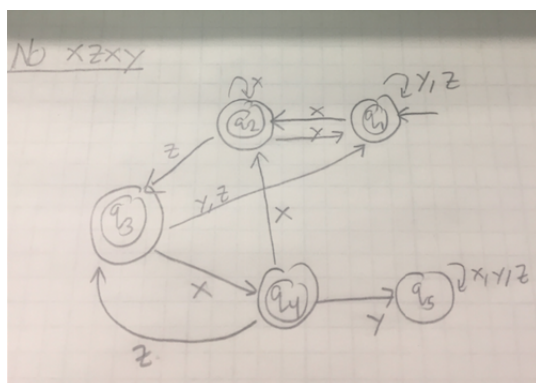
$$Q = \{q_1, q_2, q_3, q_4, q_5\}$$

$$\Sigma = \{x, y, z\}$$

δ	x	y	z
q_1	q_2	q_1	q_1
q_2	q_2	q_1	q_3
q_3	q_4	q_1	q_1
q_4	q_2	q_5	q_3
q_5	q_5	q_5	q_5

$$q_0 = q_1$$

$$F = \{q_1, q_2, q_3, q_4\}$$



DFA for z odd y even

6-tuple for the DFA is:

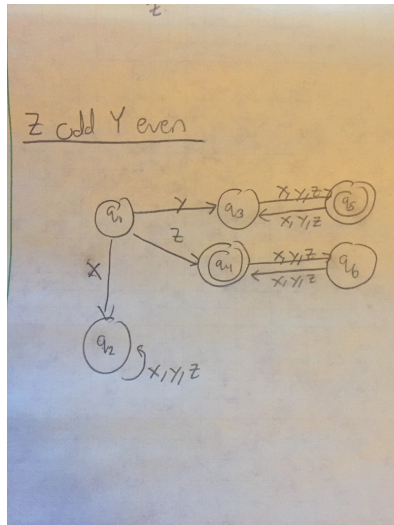
$$Q = \{q_1, q_2, q_3, q_4, q_5, q_6\}$$

$$\Sigma = \{x, y, z\}$$

δ	x	y	z
q_1	q_2	q_3	q_4
q_2	q_2	q_2	q_2
q_3	q_5	q_5	q_5
q_4	q_6	q_6	q_6
q_5	q_3	q_3	q_3
q_6	q_4	q_4	q_4

$$q_0 = q_1$$

$$F = \{q_4, q_5\}$$



2 Problem 2

- A: $D1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$
 B: $D2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$
 C: $D3 = (Q_3, \Sigma, \delta_3, q_3, F_3)$

Minor(A,B,C):

$$x \in A, x \notin B || C$$

$$x \in B, x \notin A||C$$

$$x \in C, x \notin A||B$$

$$x \notin A||B||C$$

We have:

$$D = (Q, \Sigma, \delta, q_0, F)$$

Σ is the same

$$Q = Q_1 \times Q_2 \times Q_3$$

$$q = (q_1, q_2, q_3)$$

$$\delta((a, b, c), x) = (\delta_1(a, x), \delta_2(b, x), \delta_3(c, x))$$

$$\text{for all } a \in Q_1, b \in Q_2, c \in Q_3$$

$$F = F_1 \times (Q_2 \setminus F_2) \times (Q_3 \setminus F_3) \cup (Q_1 \setminus F_1) \times F_2 \times (Q_3 \setminus F_3) \cup (Q_1 \setminus F_1 \times (Q_2 \setminus F_2) \times F_3 \cup (Q_1 \setminus F_1) \times (Q_2 \setminus F_2) \times (Q_3 \setminus F_3))$$

3 Problem 3

Every string in A^* can be constructed from A

$$A = \{w \in \{0,1\}^* \mid |\text{numberOf}_0(w) - \text{numberOf}_1(w)| = 1\}$$

$$A = \{000, 0, 1, 001, 100, 110, 011, 101, \dots\}$$

Fact: A is not regular, so there is no DFA for A and we need to show that A^* is regular.

$$A^* = \{w = w_1, w_2, \dots, w_n \mid w_i \in A \forall n \geq 0\}$$

$$\{0,1\} \subseteq A \rightarrow \{0,1\}^* \subseteq A^*$$

$$\Sigma^* \subseteq A^* \subseteq \Sigma^*$$

$$A^* = \Sigma^*$$

4 Problem 3

$$A \subseteq \{0,1\}^*$$

$$A \subseteq \{\epsilon, 0, 1, 00, \dots, 1111\}$$

$$|x| \leq 4$$

$$|A| \leq 31$$

Number of possible languages for A:

$$1 + 2 + 4 + 8 + 16 = 31 < 2^{31} = 2147483648$$

Number of possible regex of size at most 10:

$$(\cup)^* + 10! < 7^{10} = 282475249$$

There are not enough regex to account for all the languages based on the pigeonhole principle.