Homework 5

Armand Nasseri

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1 Problem 1: Undecidability

a) Assume A is decidable. Then there is a decider M_A for A. Then we will build a decider for $Halt_{TM}$ as follows.

On input $< M, w >$:

create a TM M_1: on input x, run M on w, if halts, accept. else reject.

run M_A on $< M_1 >$, if accepts, then accept. else reject.

since M_A is the decider, so this is a valid reduction and M_A accepts $< M_1 >$ iff M_1 halts on all input (including ε) iff M halts on w.

So, $Halt_TM$ is decidable, but in class it is shown to be undecidable. Therefore, this is a contradiction and A is undecidable.

b) We first must show that A is recognizable by building the recognizer as follows:

on input $< M >$: run M on ε, if halts then accept.

So, we can conclude that A is recognizable. If $\overline{A}$ is recognizable, then that implies that A is decidable, which is a contradiction to part A which is undecidable. Therefore, $\overline{A}$ is not recognizable.

c) Rices Theorem:
Condition 1: $L(M_1) = L(M_2)$, there are two possible cases,

- they are Σ^*, then both of them in L.
- they are not Σ^*, then both of them are not in L.

So, if $L(M_1) = L(M_2)$ then $< M_1 >\in L$ iff $< M_2 >\in L$

Condition 2: Pick two TMs where $L(M) = \emptyset$ and $L(M_1) = \Sigma^*$. Then $M_1 \in L$ but $M \notin L$.

d) A_2, A_3, A_5

2 Problem 2: Vertex Cover

a) Let A be the algorithm for problem Vertex Cover. Algorithm B:

def B($< G >$):

n = $|V|$

for k = 1 to n:

run A on $< G, k >$

if yes, return k.

The number of times calling A is at most n.

b) Let C be the algorithm to find the actual smallest Vertex Cover:

def C($< G >$):

VC = $\{\}$

for v = 1 to n:

if $B(G - v) < B(G)$

$VC = VC \cup \{v\}$

$G = G - v$

return VC.

The number of times calling B is at most $2n = O(n)$.