Homework 2 (Written Problems)

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1 Problem 1

DFA for no xzxy

5-tuple for the DFA is:

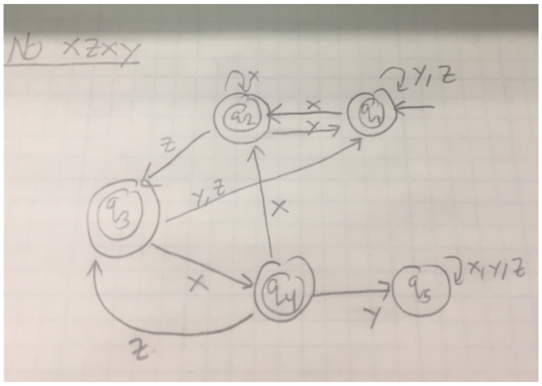
$Q = \{q_1, q_2, q_3, q_4, q_5\}$

$\Sigma = \{x, y, z\}$

δ	x	y	z
q_1	q_2	q_1	q_1
q_2	q_2	q_1	q_3
q_3	q_4	q_1	q_1
q_4	q_2	q_5	q_3
q_5	q_5	q_5	q_5

$q_0 = q_1$

$F = \{q_1, q_2, q_3, q_4\}$



DFA for z odd y even

6-tuple for the DFA is:

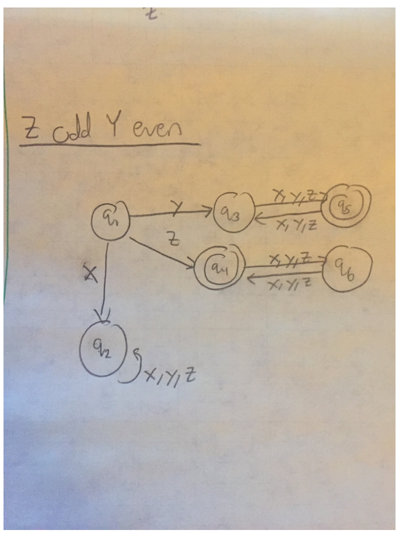
$Q = \{q_1, q_2, q_3, q_4, q_5, q_6\}$

$\Sigma = \{x, y, z\}$

δ	x	y	z
q_1	q_2	q_3	q_4
q_2	q_2	q_2	q_2
q_3	q_5	q_5	q_5
q_4	q_6	q_6	q_6
q_5	q_3	q_3	q_3
q_6	q_4	q_4	q_4

$q_0 = q_1$

$F = \{q_4, q_5\}$



2 Problem 2

Because A, B, C are regular, then there are DFAs for each of them. Let D_1 be DFA for A, D_2 be DFA for B and D_3 be DFA for C which are defined as follow,

A: $D_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$
B: $D_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$
C: $D_3 = (Q_3, \Sigma, \delta_3, q_3, F_3)$

If $w \in Minor(A, B, C)$, then it will be in one of the four cases:

$x \in A, x \notin B \cup C$

$x \in B, x \notin A \cup C$

$x \in C, x \notin A \cup B$

$x \notin A \cup B \cup C$

We define DFA for $Minor(A, B, C)$ as follow:

$D = (Q, \Sigma, \delta, q_0, F)$

Σ is the same alphabet.

$Q = Q_1 \times Q_2 \times Q_3$

$q_0 = (q_1, q_2, q_3)$

$\delta((a, b, c), x) = (\delta_1(a, x), \delta_2(b, x), \delta_3(c, x)) \quad \forall (a, b, c) \in Q_1 \times Q_2 \times Q_3$ and $x \in \Sigma$.

$F = (F_1 \times (Q_2 \setminus F_2) \times (Q_3 \setminus F_3)) \cup (Q_1 \setminus F_1) \times F_2 \times (Q_3 \setminus F_3) \cup (Q_1 \setminus F_1 \times (Q_2 \setminus F_2) \times F_3 \cup (Q_1 \setminus F_1) \times (Q_2 \setminus F_2) \times (Q_3 \setminus F_3)$

Now, let $w \in Minor(A, B, C)$, w will be in one of four cases above. Without loss of generality, assume $w \in A$ but $w \notin B \cup C$. Suppose D_1, D_2, D_3 runs on w ending up respectively at q_{1n}, q_{2n}, q_{3n}. Then, $q_{1n} \in F_1$, $q_{2n} \notin F_2$, and $q_{2n} \notin F_3$. So, $(q_{1n}, q_{2n}, q_{3n}) \in F_1 \times (Q_2 \setminus F_2) \times (Q_3 \setminus F_3)$. Therefore, D accepts w

3 Problem 3

Every string in A* can be constructed from A

$A = \{w \in \{0{,}1\}^*| \; |n_0(w) - n_1(w)| = 1\}$

Here are some strings in A,
$A = \{0, 1, 001, 100, 110, 011, 101...\}$

We need to show that A* is regular.

$A^* = \{w = w_1, w_2, ...w_n | w_i \in A \; \forall n >= 0\}$

Because $\{0, 1\} \subseteq A$, it follows that $\{0, 1\}^* \subseteq$A*

$\Sigma^* \subseteq A* \subseteq \Sigma*$

We have $A^* = \Sigma^*$. Clearly, Σ^* is regular because we can construct DFA for it or Regex $(0|1)*$. So, A^* is regular.

4 Problem 4

$A \subseteq \{0,1\}*$

$A \subseteq \{\epsilon, 0, 1, 00, ...1111\}$

$|x| <= 4$

$|A| <= 31$

Number of possible languages for A:

$1 + 2 + 4 + 8 + 16 = 31 -> 2^{31} = 2147483648$

Number of possible regex of size at most 10:

$(\ ) \ * + 1 \ 0 \ 1 -> 8^{10} = 1073741824$

There are not enough regex to account for all the languages based on the pigeonhole principle. There exists some language A that is not represented by any regex.