

HW2

Thomas Le 913081873

Armand Nasser 912679383

Problem 1

1.1

$\text{Var}(X - Y) = \text{Var}[X + (-Y)] = \text{Var}(X) + \text{Var}(Y) - 2\text{Cov}(X, Y)$ (mailing tube (3.73))

Let X and Y be independent random variables. Then, $E(XY) = E(X) * E(Y)$.

By definition, $\text{Cov}(U, V) = E(UV) - E(U) * E(V)$ (mailing tube (3.72)).

Thus, in this problem, $\text{Cov}(X, Y) = E(XY) - E(X) * E(Y)$
 $= E(XY) - E(XY)$
 $= 0$

Thus, when X and Y are independent, then $\text{Var}(X - Y) = \text{Var}(X) + \text{Var}(Y)$

1.2

L_i is the the number of passengers on the bus after the bus leaves the i th stop.

B_i is the number of passengers that board the bus on the i th stop

$L_1 \text{ support} = \{0, 1, 2\}$

$E(L_1) = 0(0.5) + 1(0.4) + 2(0.1) = 0.6$

$L_2 \text{ support} = \{0, 1, 2, 3, 4\}$

$P(L_2 = 1) = 0.4096$

- $P(B_1 = 0 \text{ and } B_2 = 1) = 0.5 * 0.4 = 0.2$
- $P(B_1 = 1 \text{ and } 0 \text{ leaves and } B_1 = 0) = 0.4 * 0.8 * 0.5 = 0.16$
- $P(B_1 = 1 \text{ and } 1 \text{ leaves and } B_2 = 1) = 0.4 * 0.2 * 0.4 = 0.032$
- $P(B_1 = 2 \text{ and only } 1 \text{ leaves and } B_2 = 0) = 0.1 * 2(0.2*0.8) * 0.5 = 0.016$
- $P(B_1 = 2 \text{ and } 2 \text{ leaves and } B_2 = 1) = 0.1 * 0.2*0.2 * 0.4 = 0.0016$

$P(L_2 = 2) = 0.2344$

- $P(B_1 = 0 \text{ and } B_2 = 2) = 0.5 * 0.1 = 0.05$
- $P(B_1 = 1 \text{ and } 0 \text{ leaves and } B_2 = 1) = 0.4 * 0.8 * 0.4 = 0.128$
- $P(B_1 = 1 \text{ and } 1 \text{ leaves and } B_2 = 2) = 0.4 * 0.2 * 0.1 = 0.008$
- $P(B_1 = 2 \text{ and } 0 \text{ leaves and } B_2 = 0) = 0.1 * 0.8*0.8 * 0.5 = 0.032$
- $P(B_1 = 2 \text{ and only } 1 \text{ leaves and } B_2 = 1) = 0.1 * 2*(0.2*0.8) * 0.4 = 0.016$
- $P(B_1 = 2 \text{ and } 2 \text{ leaves and } B_2 = 2) = 0.1 * 0.2*0.2 * 0.1 = 0.0004$

$P(L_2 = 3) = 0.0708$

- $P(B_1 = 1 \text{ and } 0 \text{ leaves and } B_2 = 2) = 0.4 * 0.8 * 0.1 = 0.032$
- $P(B_1 = 2 \text{ and } 0 \text{ leaves and } B_2 = 1) = 0.1 * 0.8 * 0.8 * 0.4 = 0.0256$

$- P(B1 = 2 \text{ and } 1 \text{ leaves and } B2 = 2) = 0.1 * 2 * (0.2 * 0.8) * 0.1 = 0.0032$
 $P(L2 = 4) = 0.0064$
 $- P(B1 = 2 \text{ and } 0 \text{ leaves and } B2 = 2) = 0.1 * 0.8 * 0.8 * 0.1 = 0.0064$
 $E(L2) = 1 * 0.4096 + 2 * 0.2344 + 3 * 0.0708 + 4 * 0.0064 = 1.1164$
 $L2 - L1 \text{ support} = \{-2, -1, 0, 1, 2\}$
 $P(L2 - L1 = -2) = 0.002$
 $- P(B1 = 2 \text{ and } 2 \text{ leaves and } B2 = 0) = 0.1 * 0.2^2 * 0.5 = 0.002$
 $P(L2 - L1 = -1) = 0.072$
 $- P(B1 = 1 \text{ and } 1 \text{ leaves and } B2 = 0) = 0.4 * 0.2 * 0.5 = 0.04$
 $- P(B1 = 2 \text{ and } 2 \text{ leaves and } B2 = 1) = 0.1 * 2 * 0.2^2 * 0.4 = 0.0032$
 $P(L2 - L1 = 1) = 0.232$
 $- P(B1 = 0 \text{ and } B2 = 1) = 0.5 * 0.4 = 0.2$
 $- P(B1 = 1 \text{ and } 1 \text{ leaves and } B2 = 1) = 0.4 * 0.2 * 0.4 = 0.032$
 $P(L2 - L1 = 2) = 0.0884$
 $- P(B1 = 0 \text{ and } B2 = 2) = 0.5 * 0.1 = 0.05$
 $- P(B1 = 1 \text{ and } 0 \text{ leaves and } B2 = 2) = 0.4 * 0.8 * 0.1 = 0.032$
 $- P(B1 = 2 \text{ and } 0 \text{ leaves and } B2 = 2) = 0.1 * 0.8^2 * 0.1 = 0.0064$
 $E(L2 - L1) = -2 * 0.002 + -1 * 0.072 + 1 * 0.232 + 2 * 0.0884 = 0.3328$
 $E(L2^2) = 1^2 * 0.4096 + 2^2 * 0.2344 + 3^2 * 0.0708 + 4^2 * 0.0064 = 2.0868$
 $Var(L2) = E(L2^2) - (E(L2))^2 = 2.0868 - 1.1164^2 = 0.840451$
 $E(L1^2) = 0(0.5) + 1^2 * (0.4) + 2^2 * (0.1) = 0.8$
 $Var(L1) = E(L1^2) - (E(L1))^2 = 0.8 - 0.6^2 = 0.44$
 $E((L2 - L1)^2) = -2^2 * 0.002 + -1^2 * 0.072 + 1^2 * 0.232 + 2^2 * 0.0884 = 0.5056$
 $Var(L2 - L1) = E((L2-L1)^2) - (E(L2-L1))^2 = 0.5056 - 0.3328^2 = 0.3948$
 $Var(X) - Var(Y) = Var(L2) - Var(L1) = 0.840451 - 0.44 = 0.400451$

Problem 2

Let $E(X)$ be the expected value of the amount of money to win the game.
 The game ends when $r = 3$ consecutive heads are tossed or there are $s = 6$ total tosses.

$$E(X) = 1P(1) + 2P(2) + 3P(3) + 4P(4) + 5P(5) + 6P(6)$$

$P(1) = 0$ # because require 3 consec heads to stop
 $P(2) = 0$
 $P(3) = (1/2)^3$ # HHH

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P(4) = (1/2)^4      # THHH
P(5) = 2 * (1/2)^5  # TTTHH or HTTHH
P(6) = 1 - P(3 or 4 or 5)
      = 1 - 0.25
      = 0.75

=> E(X) = 3*0.125 + 4*0.0625 + 5*0.0625 + 6*0.75
      = $5.44

```

Problem 3

Problem 4

4.1

The values the random variable X can take are $X = \{2, 3, 4, 5, 6, 7, 8\}$

e.g the rolls can be: $\{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

4.2

The PMF is:

X	2	3	4	5	6	7	8
P(x)	1/16	2/16	3/16	4/16	3/16	2/16	1/16

4.3

The Expected value of X is:

$$E(x) = x \cdot f(x) = 2 \cdot (1/16) + 3 \cdot (2/16) + 4 \cdot (3/16) + 5 \cdot (4/16) + 6 \cdot (3/16) + 7 \cdot (2/16) + 8 \cdot (1/16) = 80/16$$

Problem 5

5.1

The PMF of x:

$$P(x = X) = nCx \cdot p^x \cdot (1-p)^{n-x}$$

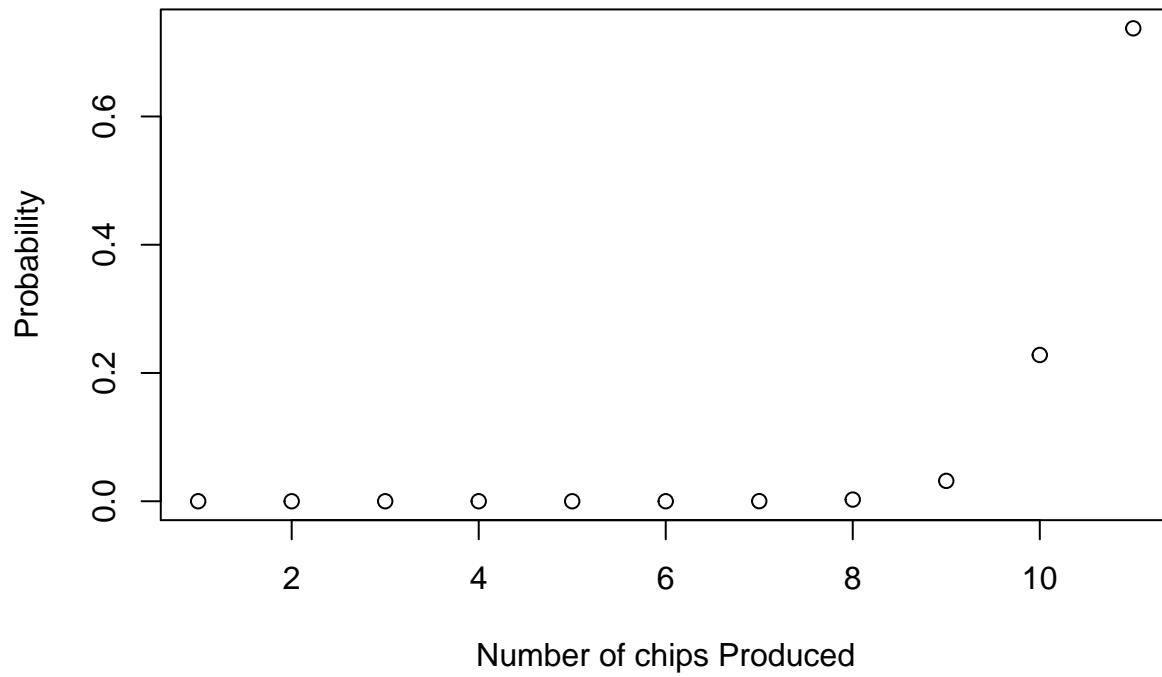
$$P = 0.97, n = 10$$

```

X <- 0:10
plot(dbinom(X,10,0.97), col="black", main = "Binomial Distribution", xlab = "Number of chips Produced",

```

Binomial Distribution



5.2

The rate of failure $p' = 1 - 0.97 = 0.03$

```
1-pbinom(1,10,0.03)
```

```
## [1] 0.03450656
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