HW2

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Problem 1

1.1

Var(X - Y) = Var[X + (-Y)] = Var(X) + Var(Y) - 2Cov(X, Y) (mailing tube (3.73))

Let X and Y be independent random variables. Then, E(XY) = E(X) * E(Y).

By definition, Cov(U, V) = E(UV) - E(U) * E(V) (mailing tube (3.72)).

Thus, in this problem, Cov(X, Y) = E(XY) - E(X) * E(Y)
= E(XY) - E(XY)
= 0

Thus, when X and Y are independent, then Var(X - Y) = Var(X) + Var(Y)

1.2

Li is the the number of passengers on the bus after the bus leaves the ith stop.
Bi is the number of passengers that board the bus on the ith stop

L1 support = {0, 1, 2}

=> E(L1) = 0(0.5) + 1(0.4) + 2(0.1) = 0.6

L2 support = {0, 1, 2, 3, 4}

P(L2 = 1) = 0.4096
- P(B1 = 0 and B2 = 1) = 0.5 * 0.4 = 0.2
- P(B1 = 1 and 0 leaves and B1 = 0) = 0.4 * 0.8 * 0.5 = 0.16
- P(B1 = 1 and 1 leaves and B2 = 1) = 0.4 * 0.2 * 0.4 = 0.032
- P(B1 = 2 and only 1 leaves and B2 = 0) = 0.1 * 2(0.2*0.8) * 0.5 = 0.016
- P(B1 = 2 and 2 leaves and B2 = 1) = 0.1 * 0.2*0.2 * 0.4 = 0.0016

P(L2 = 2) = 0.2344
- P(B1 = 0 and B2 = 2) = 0.5 * 0.1 = 0.05
- P(B1 = 1 and 0 leaves and B2 = 1) = 0.4 * 0.8 * 0.4 = 0.128
- P(B1 = 1 and 1 leaves and B2 = 2) = 0.4 * 0.2 * 0.1 = 0.008
- P(B1 = 2 and 0 leaves and B2 = 0) = 0.1 * 0.8*0.8 * 0.5 = 0.032
- P(B1 = 2 and only 1 leaves and B2 = 1) = 0.1 * 2*(0.2*0.8) * 0.4 = 0.016
- P(B1 = 2 and 2 leaves and B2 = 2) = 0.1 * 0.2*0.2 * 0.1 = 0.0004

P(L2 = 3) = 0.0708
- P(B1 = 1 and 0 leaves and B2 = 2) = 0.4 * 0.8 * 0.1 = 0.032
- P(B1 = 2 and 0 leaves and B2 = 1) = 0.1 * 0.8 * 0.8 * 0.4 = 0.0256

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  - P(B1 = 2 and 1 leaves and B2 = 2) = 0.1 * 2*(0.2 * 0.8) * 0.1 = 0.0032

P(L2 = 4) = 0.0064
  - P(B1 = 2 and 0 leaves and B2 = 2) = 0.1 * 0.8 * 0.8 * 0.1 = 0.0064

=> E(L2) = 1 * 0.4096 + 2 * 0.2344 + 3 * 0.0708 + 4 * 0.0064 = 1.1164

L2 - L1 support = {-2, -1, 0, 1, 2}

P(L2 - L1 = -2) = 0.002
  - P(B1 = 2 and 2 leaves and B2 = 0) = 0.1 * 0.2^2 * 0.5 = 0.002

P(L2 - L1 = -1) = 0.072
  - P(B1 = 1 and 1 leaves and B2 = 0) = 0.4 * 0.2 * 0.5 = 0.04
  - P(B1 = 2 and 2 leaves and B2 = 1) = 0.1 * 2 * 0.2^2 * 0.4 = 0.0032

P(L2 - L1 = 1) = 0.232
  - P(B1 = 0 and B2 = 1) = 0.5 * 0.4 = 0.2
  - P(B1 = 1 and 1 leaves and B2 = 1) = 0.4 * 0.2 * 0.4 = 0.032

P(L2 - L1 = 2) = 0.0884
  - P(B1 = 0 and B2 = 2) = 0.5 * 0.1 = 0.05
  - P(B1 = 1 and 0 leaves and B2 = 2) = 0.4 * 0.8 * 0.1 = 0.032
  - P(B1 = 2 and 0 leaves and B2 = 2) = 0.1 * 0.8^2 * 0.1 = 0.0064

=> E(L2 - L1) = -2 * 0.002 + -1 * 0.072 + 1 * 0.232 + 2 * 0.0884 = 0.3328

E(L2^2) = 1^2 * 0.4096 + 2^2 * 0.2344 + 3^2 * 0.0708 + 4^2 * 0.0064 = 2.0868

=> Var(L2) = E(L2^2) - (E(L2))^2 = 2.0868 - 1.1164^2 = 0.840451

E(L1^2) = 0(0.5) + 1^2 * (0.4) + 2^2 * (0.1) = 0.8

=> Var(L1) = E(L1^2) - (E(L1))^2 = 0.8 - 0.6^2 = 0.44

E((L2 - L1)^2 = -2^2 * 0.002 + -1^2 * 0.072 + 1^2 * 0.232 + 2^2 * 0.0884 = 0.5056

=> Var(L2 - L1) = E((L2-L1)^2) - (E(L2-L1))^2 = 0.5056 - 0.3328^2 = 0.3948

=> Var(X) - Var(Y) = Var(L2) - Var(L1) = 0.840451 - 0.44 = 0.400451
```

Problem 2

```
Let E(X) be the expected value of the amount of money to win the game.
The game ends when r = 3 consecutive heads are tossed or there are s = 6 total tosses.


E(X) = 1P(1) + 2P(2) + 3P(3) + 4P(4) + 5P(5) + 6P(6)

P(1) = 0            # because require 3 consec heads to stop
P(2) = 0
P(3) = (1/2)^3      # HHH
```

```
P(4) = (1/2)^4      # THHHH
P(5) = 2 * (1/2)^5  # TTHHH or HTHHH
P(6) = 1 - P(3 or 4 or 5)
     = 1 - 0.25
     = 0.75

=> E(X) = 3*0.125 + 4*0.0625 + 5*0.0625 + 6*0.75
        = $5.44
```

Problem 3

Problem 4

4.1

The values the random variable X can take are X = {2, 3, 4, 5, 6, 7, 8}

e.g the rolls can be: {(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)}

4.2

The PMF is:

X	2	3	4	5	6	7	8
P(x)	1/16	2/16	3/16	4/16	3/16	2/16	1/16

4.3

The Expected value of X is:

```
E(x) = x*f(x) = 2*(1/16) + 3*(2/16) + 4*(3/16) + 5*(4/16) + 6(3/16) + 7*(2/16) + 8*(1/16)
= 80/16
```

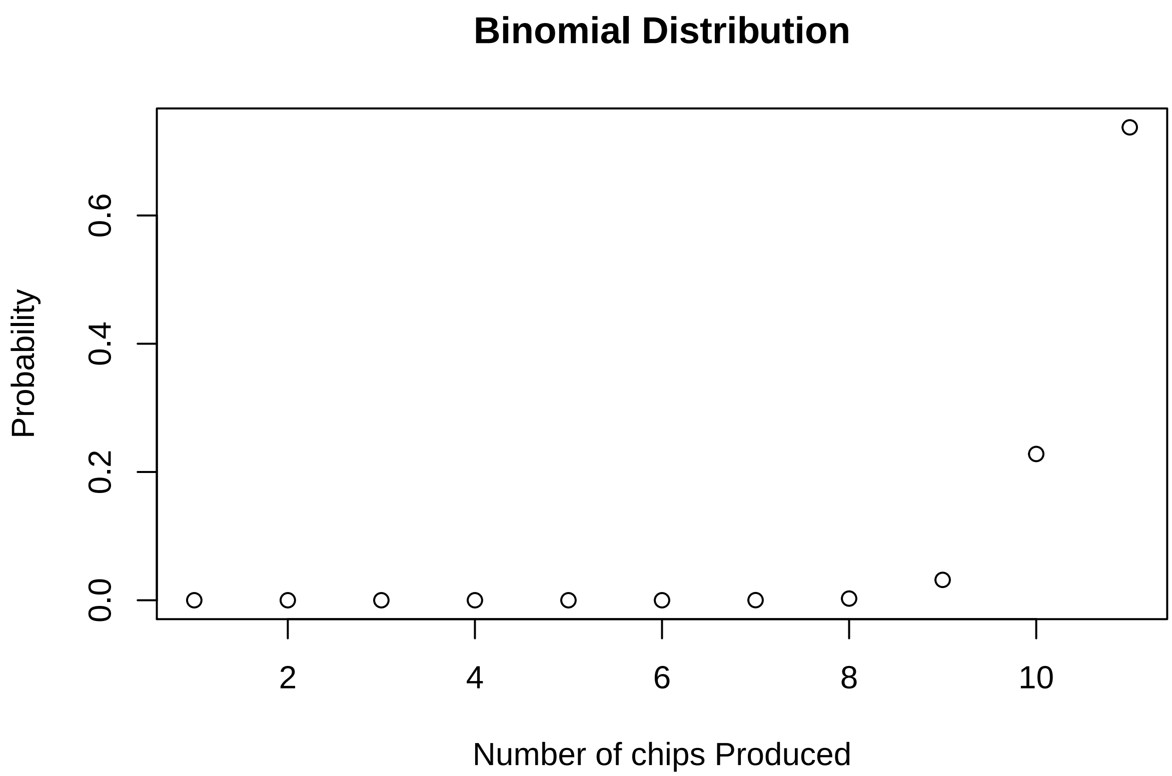
Problem 5

5.1

```
The PMF of x:
    P(x = X) = nCx * p^x * (1-p)^n-x
    P = 0.97, n = 10
```

```
X <- 0:10

plot(dbinom(X,10,0.97), col="black", main = "Binomial Distribution", xlab = "Number of chips Produced",
```



5.2

The rate of failure p' = 1 - 0.97 = 0.03

```
1-pbinom(1,10,0.03)
```

```
## [1] 0.03450656
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