

hw1

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Problem 2

Looking to solve:

- $P(c = 0 \mid X1 = X2)$
- $P(c = 1 \mid X1 = X2)$
- $P(c = 2 \mid X1 = X2)$

0 Collisions

$$P(c = 0 \mid X1 = X2) = P(c = 0 \mid X1 = X2 = 1) + P(c = 0 \mid X1 = X2 = 2)$$

$$\begin{aligned} P(c = 0 \mid X1 = X2 = 1) &= P(w/2 \text{ active nodes, only 1 sends for epoch 1}) * [P(\text{no activate and the one a} \\ &= 2p(1-p) * [(1-q)(1-p) + q2p(1-p)] \\ &= 0.24192 \end{aligned}$$

$$\begin{aligned} P(c = 0 \mid X1 = X2 = 2) &= P(w/2 \text{ active nodes, neither send, epoch 1}) * P(w/2 \text{ active nodes, neither send} \\ &= (1-p)(1-p) * (1-p)(1-p) \\ &= 0.1296 \end{aligned}$$

$$\Rightarrow P(c = 0 \mid X1 = X2) = 0.24192 + 0.1296 = 0.37152$$

1 Collision

$$P(c = 1 \mid X1 = X2) = P(c = 1 \mid X1 = X2 = 1) + P(c = 1 \mid X1 = X2 = 2)$$

collision in second epoch

$$\begin{aligned} P(c = 1 \mid X1 = X2 = 1) &= P(\text{only 1 node sends in epoch 1}) * P(\text{both send} \mid \text{node activates}) \\ &= 2p(1-p) * p*p*q \\ &= 0.06144 \end{aligned}$$

collision in 1st epoch, results in $X1 \neq 1$, so we discount that

collision in 1st epoch

$$\begin{aligned} P(c = 1 \mid X1 = X2 = 2) &= P(\text{both send}) * P(\text{neither send}) \\ &= p*p * (1-p)*(1-p) \\ &= 0.0576 \end{aligned}$$

$$\Rightarrow P(c = 1 \mid X1 = X2) = 0.06144 + 0.0576 = 0.11844$$

2 Collisions

$$P(c = 2 \mid X1 = X2) = P(c = 2 \mid X1 = X2 = 1) + P(c = 2 \mid X1 = X2 = 2)$$

$$P(c = 2 \mid X1 = X2 = 1) = 0$$

$$\begin{aligned} P(c = 2 \mid X1 = X2 = 2) &= P(\text{both send in epoch 1}) * P(\text{both send in epoch 2}) \\ &= p*p * p*p \\ &= 0.0256 \end{aligned}$$

$$\Rightarrow P(c = 2 \mid X1 = X2) = 0 + 0.0256 = 0.0256$$

Summary

- $P(c = 0 \mid X1 = X2) = 0.24192 + 0.1296 = 0.37152$
- $P(c = 1 \mid X1 = X2) = 0.06144 + 0.0576 = 0.11844$
- $P(c = 2 \mid X1 = X2) = 0 + 0.0256 = 0.0256$