hw1 problem 2

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Problem 2

Looking to solve:

- P(c = 0 | X1 = X2)
- P(c = 1 | X1 = X2)
- P(c = 2 | X1 = X2)

0 Collisions

```
P(c = 0 | X1 = X2) = P(c = 0 | X1 = X2 = 1) + P(c = 0 | X1 = X2 = 2)

P(c = 0 | X1 = X2 = 1 ) = P(w/ 2 active nodes, only 1 sends for epoch 1)
                            * [P(no activate and the one active does not send) + P(activate and X2=1) ]
                        = 2p(1-p) * [(1-q)(1-p) + q2p(1-p)]
                        = 0.24192

P(c = 0 | X1 = X2 = 2) = P(w/ 2 active nodes, neither send, epoch 1)
                            * P(w/ 2 active nodes, neither send, epoch 2)
                        = (1-p)(1-p) * (1-p)(1-p)
                        = 0.1296

=> P(c = 0 | X1 = X2) = 0.24192 + 0.1296 = 0.37152
```

1 Collision

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P(c = 1 | X1 = X2) = P(c = 1 | X1 = X2 = 1) + P(c = 1 | X1 = X2 = 2)

# collision in second epoch
P(c = 1 | X1 = X2 = 1) = P(only 1 node sends in epoch 1) * P(both send | node activates)
                       = 2p(1-p) * p*p*q
                       = 0.06144

# collision in 1st epoch, results in X1 != 1, so we discount that

# collision in 1st epoch
P(c = 1 | X1 = X2 = 2) = P(both send) * P(neither send)
                       = p*p * (1-p)*(1-p)
                       = 0.0576

=> P(c = 1 | X1 = X2) = 0.06144 + 0.0576 = 0.11844
```

2 Collisions

```
P(c = 2 | X1 = X2) = P(c = 2 | X1 = X2 = 1) + P(c = 20| X1 = X2 = 2)

P(c = 2 | X1 = X2 = 1) = 0

P(c = 2 | X1 = X2 = 2) = P(both send in epoch 1) * P(both send in epoch 2)
                       = p*p * p*p
                       = 0.0256

=> P(c = 2 | X1 = X2) = 0 + 0.0256 = 0.0256
```

Summary

- $\mathrm{P}(\mathrm{c} = 0 \mid \mathrm{X1} = \mathrm{X2}) = 0.24192 + 0.1296 = 0.37152$
- $\mathrm{P}(\mathrm{c} = 1 \mid \mathrm{X1} = \mathrm{X2}) = 0.06144 + 0.0576 = 0.11844$
- $\mathrm{P}(\mathrm{c} = 2 \mid \mathrm{X1} = \mathrm{X2}) = 0 + 0.0256 = 0.0256$