En pulsation :

$$u(\omega,T) = \frac{\hbar}{\pi^2 c^3} \frac{\omega^{3}}{e^{\hbar\omega/kT}-1} \quad \text{à maximiser :}$$

$$\frac{\partial u}{\partial \omega} = \frac{3\cancel{\hbar}}{\cancel{\pi^2 c^3}} \frac{\cancel{\omega^2}}{e^{\hbar\omega/kT}-1} + \frac{\cancel{\hbar}\omega^{\cancel{3}}}{\cancel{\pi^2 c^3}} \left(\frac{-e^{\hbar\omega/kT}}{\left(e^{\hbar\omega/kT}-1\right)^2} \cdot \frac{\hbar}{kT} \right) = 0$$

$$\times \left(e^{\hbar\omega/kT}-1\right)^2 \quad 3e^{\hbar\omega/kT} - 3 - \omega e^{\hbar\omega/kT} \frac{\hbar}{kT} = 0.$$

$$x_\omega = \frac{\hbar\omega}{kT} \rightarrow e^{x_\omega} - 1 - \frac{x_\omega e^{x_\omega}}{3} = 0 \Rightarrow \boxed{1 - e^{-x_\omega} - \frac{x_\omega}{3} = 0}$$

+ calculette.

ie $x_\omega = 2,821 \rightarrow \boxed{\omega_m = 2,821 \frac{kT}{\hbar}}$

en longueur d'onde : $u(\lambda,T)\,d\lambda = -u(\nu,T)\,d\nu$ et $c = \lambda\nu$

ie $u(\lambda,T) = \frac{8\pi hc}{\lambda^{5}} \frac{1}{e^{hc/\lambda kT}-1}$ ie $d\nu = -c\frac{d\lambda}{\lambda^2}$

$$\text{et } \frac{\partial u}{\partial \lambda} = \frac{-\cancel{40}^{5}\,\cancel{h\pi c}}{\lambda^{6}} \frac{1}{e^{hc/\lambda kT}-1} + \frac{\cancel{8\pi hc}}{\cancel{\lambda^5}} \left(\frac{+\frac{hc}{\lambda^2 kT} e^{hc/\lambda kT}}{\left(e^{hc/\lambda kT}-1\right)^2} \right) = 0$$

$$\Rightarrow \frac{-5}{\lambda} \frac{1}{e^{hc/\lambda kT}-1} + \frac{hc}{\lambda^2 kT} \frac{e^{hc/\lambda kT}}{\left(e^{hc/\lambda kT}-1\right)^2} = 0$$

$$x = \frac{hc}{\lambda kT} \quad \Rightarrow \frac{-5}{\lambda}\left(e^x - 1\right) + \frac{1}{\lambda} x e^x = 0$$

$$\Rightarrow \frac{\pm 5}{\lambda} + \frac{5}{\lambda} e^{-x} + \frac{x}{\lambda} = 0 \Rightarrow \boxed{e^{-x} + \frac{x}{5} - 1 = 0}$$

+ calculette

$$\Rightarrow x = 4,965 \text{ . et } \boxed{\lambda_m = \frac{hc}{4,965\, k_B T}}$$