

MATH 2420 Notes

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1 Differential equations intro

- What are diff eqs
 - Needed to describe natural processes
 - $y'(x) = \cos(x)$ is an example
 - Solving diff eq = find unknown function
 - * Diff eq if it has derivatives in it
 - $y = \sin(x) + C$ has infinitely many solutions
 - Initial value problem
 - * Given diff eq, find solution satisfying additional constraints
 - * Ex: $y' = y$; $y = Ce^x$; $y(0) = 2$; $C = 2$
 - Focus on explicit eqs
 - Separation of variables
 - * $M(y)\frac{dy}{dx} + N(X) = 0$
 - * $M(y)dy = -N(x)dx$ (*dx)
 - * Final, integrate
 - Example:

Object is falling at rate of mg ,
friction(γ) is pushing against it.

$$ma = mg - \gamma v$$

$$a = \frac{dv}{dt}$$

m, g, γ all const

$$v = \int v(t)dt$$

$$m = 1, g = 9.8 = 10, \gamma = 5$$

$$\frac{dv}{dt} = 10 - 5v$$

$$dV = (10 - 5v)dt$$

$$\int \frac{dv}{(10-5v)} = \int dt$$

$$\frac{-1}{5} \ln(10 - 5v) = t + C$$

$$v = 2 + c * e^{-5t}$$
 - Tricks:

1. Change of variables:

$$\frac{dy}{dx} = \cos(y - x)$$

Change y: $z = y - x$

$$\frac{dz}{dx} = \frac{dy}{dx} - 1$$

$$\frac{dz}{dx} + 1 = \cos(z)$$

Using eq: $1 - \cos(\alpha) = 2\sin^2(\frac{\alpha}{2})$

$$dz = (\cos(z) - 1)dx$$

$$\int \frac{dz}{\cos(z)-1} = \int dx$$

$$\int \frac{dz}{2\sin^2(\frac{z}{2})} = x + C$$

$$\cot(\frac{z}{2}) = x + C$$

$$\cot(\frac{y-x}{2}) = x + C$$

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