INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

17 September 2024 (am)

Subject CM2 – Economic Modelling
Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 An insurer holds an asset with an annual return, R, where R has probability density function $f(r)$. The insurer assumes that R follows a Normal(μ,σ^2) distribution.

Consider the 95% Value at Risk (VaR) where 95% VaR $= -t$ if $P(R < t) = 5\%$.

(i) Derive an expression for the 95% VaR for R in terms of μ and σ^2. [2]

The insurer defines, for any percentage level, Tail VaR as $E[-R|R < -\text{VaR}]$.

(ii) Show that the 95% Tail VaR for R can be expressed as:

$$\frac{\sigma}{5\%} f_{0,1}(-1.64485) - \mu$$

[4]

[**Hint:** If $f_{0,1}(r)$ is the probability density function of the Standard Normal Distribution then $r f_{0,1}(r) = -\frac{d}{dr} f_{0,1}(r)$.]

Assume now that $\mu = 0.04$ and $\sigma^2 = 0.006$.

(iii) Calculate for R:

(a) the 95% VaR.

(b) the 95% Tail VaR.

[3]

(iv) Set out the benefits and limitations of the VaR and Tail VaR metrics for monitoring risk. [2]

[Total 11]

2 A credit analyst has been tasked with assessing the credit risk of the following two companies:

- Company A has a market value of assets of \$60 million and debt of \$50 million with maturity of 2 years.
- Company B has a market value of assets of \$80 million and debt of \$50 million with maturity of 3 years.

The continuously compounded risk-free rate of return is 3% p.a. Both companies have asset volatility of 25%.

(i) Estimate the risk-neutral probability of default of each company before its debt matures using the Merton model. [5]

Company A has issued 100,000 non-dividend paying shares. The current arbitrage-free prices of options on the shares, with maturity in 2 years' time and a strike price of \$100, are as follows:

- put option = \$35.20
- call option = \$65.60.

(ii) Calculate the value of Company A's debt per \$100 nominal. [3]

The total value of Company A increases by \$1 million.

(iii) Determine the approximate change in:

(a) the share price.

(b) the debt value per \$100 nominal.

[4]

(iv) Comment on the difference between your answers to parts (iii)(a) and (iii)(b). [2]

[Total 14]

3 Consider a portfolio of ten travel insurance policies that will pay out in the event of a holiday being cancelled or finishing early due to circumstances outside of the policyholder's control. Each policy has been taken out by a different family, but they are all planning holidays to the same location at the same time.

(i) Explain what difficulties an insurer might face when writing policies to cover this risk. [3]

(ii) Suggest two changes to the portfolio or the policy terms that would mitigate the issues identified in part (i). [2]

[Total 5]

4 A non-dividend paying stock has a current price of \$100. In any 1-month period, the price of the stock is expected to either increase by 15% or decrease by 10%. The risk-free force of interest is 6% p.a.

(i) Calculate the risk-neutral probability of an up-step in a 1-month period. [1]

Consider a 2-month European call option on the stock with a strike price of \$125.

(ii) Calculate the value of the option at time $t = 0$ using a binomial tree. [3]

(iii) Construct a replicating portfolio for the option at the start and end of the first month. [5]

(iv) Demonstrate that the value of the replicating portfolio at time $t = 0$ is the same as the option value calculated in part (ii). [1]

[Total 10]

5 An investor makes decisions using a quadratic utility function of the form $U(w) = bw + cw^2$.

(i) Determine the absolute and relative risk aversion for this utility function in terms of b and c. [3]

The investor currently has wealth of \$150 and utility of 6,750.

The investor is offered a gamble that results in a profit of \$25 with probability p and a loss of \$25 with probability $(1-p)$. They will only accept the gamble if $p \geq 0.54167$.

(ii) Explain what this implies about the investor's risk aversion. [1]

(iii) Determine values for b and c. [6]

(iv) Determine the maximum wealth for which the function $U(w)$ satisfies the principle of non-satiation. [2]

The investor accepts the gamble and wins. They now have wealth of \$175. They are offered the same gamble again: a profit of \$25 with probability p and a loss of \$25 with probability $(1-p)$.

(v) Comment, with reasons, on whether the minimum value for p at which the investor will accept the gamble is now higher or lower than 0.54167. You are not expected to perform any further calculations. [4]

[Total 16]

6 Consider a mean-variance portfolio model for two securities, A and B, with returns S_A and S_B, expected returns E_A and E_B and variance of returns V_A and V_B. You are told that $E_B = 4E_A$ and $V_B = 4V_A$.

The correlation between the returns on the two securities is ρ.

(i) Determine, in terms of E_A, the expected return on the minimum variance portfolio containing x_A units of security A and x_B units of security B if:

(a) $\rho = 0$

(b) $\rho = 1$.

[5]

(ii) Calculate the variance of the return on the minimum variance portfolio in part (i)(b). [2]

(iii) Comment on the risk and expected return of the portfolio in part (i)(b). [2]

[Total 9]

7 Consider a process $\{N(t)\}_{t>0}$ denoting the number of claims *N* an insurer experiences up to time *t*. One of the requirements for this process to be Poisson is to assume that, when $s < t$, the number of claims in the time interval $(s, t]$ is independent of the number of claims up to time s.

(i) Explain in your own words what this assumption implies about the number of claims that the insurer experiences. [1]

(ii) Discuss if it is reasonable for the insurer to make this assumption. [3]

The insurer wants to provide a 5-year policy for a single premium at time $t = 0$ of \$100. An unlimited number of claims are allowed and each claim will be for \$40. You should assume that the insurer has no assets before writing this policy and issues no other policies.

The insurer assumes that the number of claims by time *t* will follow a process $N(t)$, where $N(t)$ follows a Poisson distribution with parameter λt, where $\lambda = 0.2$.

(iii) Calculate the probability of ruin for the insurer by time $t = 5$. [3]

The insurer is considering extending the policy for longer than 5 years but with no more premium income.

(iv) Explain what will happen to the probability of ruin for $t > 5$. You are not expected to perform any further calculations. [3]

[Total 10]

8 Consider a market in which the following corporate bonds exist, both issued by the same company:

- a 1-year zero-coupon corporate bond, currently priced at $85.61 per $100 nominal
- a 2-year zero-coupon corporate bond, currently priced at $76.91 per $100 nominal.

The risk-free force of interest is 5% p.a.

A bank estimates that the loss for each of the corporate bonds if they default will be 50% of the nominal value.

(i) Estimate the probability of default for:

(a) the 1-year corporate bond.

(b) the 2-year corporate bond.

[3]

(ii) Calculate the yield on:

(a) the 1-year corporate bond.

(b) the 2-year corporate bond.

[2]

(iii) Explain why the yields in part (ii) are intuitively consistent with the default probabilities in part (i). [3]

[Total 8]

9 An insurer uses the following utility function for wealth of w for some $x > 0$:

$$U(w) = \begin{cases} 0 & \text{for } w < x \\ \left(\dfrac{w}{100}\right)^{0.5} & \text{for } w \geq x \end{cases}$$

(i) Suggest why the insurer may use a utility function with a discontinuity at $w = x$. [1]

(ii) Explain if this utility function satisfies the principles of:

(a) risk aversion.

(b) non-satiation.

[3]

The insurer has current wealth of \$100 and x = \$90.

The insurer is considering issuing one policy. The probability of a claim arising on the policy is 25% and the size of the claim would always be \$50. Only one claim is allowed.

(iii) Show that the insurer would need to charge a premium of at least \$40 (to the nearest \$) to give a higher expected utility than the insurer's current utility. [4]

(iv) Explain why it may not be practical to charge this premium. [2]

[Total 10]

10 Consider a non-dividend-paying stock whose current price is \$10,000. An investor buys a put option on the stock for a premium of \$307. The strike price of the option is \$9,500 and the time to expiry is 1 year. The risk-free force of interest is 6% p.a.

The Black–Scholes option pricing formula is assumed to hold.

(i) Calculate the price of a call option with the same time to maturity and strike price as the put option. [2]

Now consider a second call option on the stock with the same time to maturity but a strike price of \$10,500. An investor estimates the volatility of the stock to be 18.9%.

(ii) Calculate the price of the second call option. [3]

(iii) Explain how, and why, the value of the second call option would change if:

(a) the share price increases.

(b) the risk-free force of interest increases.

[2]

[Total 7]

END OF PAPER

Institute and Faculty of Actuaries

EXAMINERS' REPORT

CM2 - Economic Modelling
Core Principles
Paper A

September 2024

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

For some candidates, this may be their first attempt at answering an examination using open books and online. The Examiners expect all candidates to have a good level of knowledge and understanding of the topics and therefore candidates should not be overly dependent on open book materials. In our experience, candidates that spend too long researching answers in their materials will not be successful either because of time management issues or because they do not properly answer the questions.

Many candidates rely on past exam papers and examiner reports. Great caution must be exercised in doing so because each exam question is unique. As with all professional examinations, it is insufficient to repeat points of principle, formula or other text book works. The examinations are designed to test "higher order" thinking including candidates' ability to apply their knowledge to the facts presented in detail, synthesise and analyse their findings, and present conclusions or advice. Successful candidates concentrate on answering the questions asked rather than repeating their knowledge without application.

Candidates should note that from the April 2025 exam session, all examinations will continue to be delivered virtually and will have online proctoring. Exams will be closed book and closed web. The ability to refer to past examiner reports and past papers during the exam is not permitted. Candidates attempting to do so will be in breach of the Assessment Regulations and subject to inappropriate conduct investigations. Further details of the new exams can be found on the IFOA website.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
December 2024

A. General comments on the *aims of this subject and how it is marked*

The aim of subject CM2 is to develop the necessary skills to construct asset liability models, value financial derivatives and calculate reserves for insurance or guarantees. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.

The marking approach for CM2 is flexible in the sense that different answers to those shown in the solution can earn marks if they are relevant and appropriate. Marks for the methodology are also awarded including marks for using the right method even if an error in an earlier part of the question prevents the final answer from being correct. The marking focusses on rewarding candidates' understanding of the concepts, including their ability to articulate arguments clearly.

B. Comments on *candidate performance in this diet of the examination*

This exam was sat online and most questions focussed on applied calculations and analysis of the results. Some of the questions required students to apply concepts in the Syllabus to scenarios they might not have seen before and the stronger students scored highly here. Average marks were roughly in line with the historic norm for the subject and the pass mark was also within a normal range for CM2.

This exam included less algebra than some previous CM2 papers, but there was still evidence that some students found algebra tricky when answering questions in Word. Students should note that rearranging and solving algebra on screen can sometimes be hard if you are used to using pen and paper, so this is a worthwhile skill to practise before the exams. It's also worth saying that using the equation editor in Word to set out formulae is not necessary, your workings just need to be clear enough for the examiner to follow them.

Q2 proved to be the most challenging question on the paper. This question required candidates to work with the Merton model to derive default probabilities and describe how the impact share and debt values. Most candidates completed the early calculations correctly but struggled with the later analysis which required a deeper understanding of the Merton model.

C. Pass Mark

The Pass Mark for this exam was 60.
1,426 presented themselves and 661 passed.

Solutions for Subject CM2A - September 2024

Q1

(i)

$P\ (Z < \frac{t-\mu}{\sigma}) = 5\%$ [½]

$\frac{t-\mu}{\sigma} = -1.64485$ [½]

$VaR = -t = 1.64485\sigma - \mu$ [1]

(ii)

$\text{TailVaR} = \frac{-1}{5\%}\int_{-\infty}^{-VaR} rf(r)dr$ [½]

We change the variable using $r = \mu + \sigma z$ so $f(r)dr = f(\mu + \sigma z)\sigma dz =$

$\frac{-1}{5\%}\int_{-\infty}^{\frac{-VaR-\mu}{\sigma}}(\mu + \sigma z)\sigma f(\mu+\sigma z)dz$ [½]

But $\sigma f(\mu+\sigma z) = f_{0,1}(z)$ so

$$= \frac{-1}{5\%}\int_{-\infty}^{\frac{-VaR-\mu}{\sigma}}(\mu + \sigma z)\, f_{0,1}\,(z)dz$$ [½]

$$= \frac{-1}{5\%}\left(\mu\int_{-\infty}^{\frac{-VaR-\mu}{\sigma}} f_{0,1}\,(z)dz + \sigma\int_{-\infty}^{\frac{-VaR-\mu}{\sigma}} zf_{0,1}\,(z)dz\right)$$ [½]

Then applying the hint to $zf_{0,1}\,(z)$

$$= \frac{-1}{5\%}\left(\mu\Phi\left(\frac{-VaR-\mu}{\sigma}\right) - \sigma\int_{-\infty}^{\frac{-VaR-\mu}{\sigma}}\frac{d}{dz}\ f_{0,1}\,(z)dz\right)$$ [½]

$$= \frac{-1}{5\%}\left(\mu\Phi\left(\frac{-VaR-\mu}{\sigma}\right) - \sigma\, f_{0,1}\,\left(\frac{-VaR-\mu}{\sigma}\right)\right)$$ [½]

Then using part (i)

$$= \frac{-1}{5\%}\left(\mu\Phi(-1.64485) - \sigma\, f_{0,1}\,(-1.64485)\right)$$ [½]

$$= \frac{-1}{5\%}\left(0.05\mu - \sigma\, f_{0,1}\,(-1.64485)\right)$$

$$= \frac{\sigma}{5\%}\ f_{0,1}\,(-1.64485) - \mu$$ [½]

(iii)

a. $VaR = 1.64485\sqrt{60} - 4 = 8.742\%$ [1]

b. $TailVaR = \frac{\sigma}{5\%}f_{0,1}(-1.64485) - \mu$ [1]

$$= \frac{\sqrt{60}}{5\%}\frac{1}{\sqrt{2\pi}}\exp(-0.5 \times (-1.64485)^2) - 4$$

$$= 11.98\%$$

[1]

(iv)
Value at Risk does not give an indication of the size of the loss in the tail of the distribution, only the likelihood. [½]
The availability of data to determine the tail of the distribution may limit usefulness of Value at Risk. [½]
TailVar is useful for monitoring exposure to risk because the expected underperformance relative to a benchmark is easy to understand. [½]
TailVar gives one a single figure representing the average of the worst x% of losses, but does not reveal how these losses are distributed. [½]

[Total 11]

Commentary:
Part (i) was generally well answered, and the examiners were lenient with whether VaR should be expressed as t or -t. *Part (ii) required students to work with the integral definition of TailVar and change the variable using the hint provided. This proved to be the poorest answered question part in the paper, with many candidates either trying to work with an expected value definition of TailVar or not setting out their workings clearly.* *Parts (iii) and (iv) were generally answered well, though in (iii) some candidates were only able to calculate the VaR and not the TailVaR.*

Q2

(i)
Under the Merton model the risk neutral probability of default can be calculated as

N($-d_2$) where N is the normal distribution and

$$d_2 = \frac{ln\left(\frac{V}{D}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

[1]

For Company A

$$d_2 = \frac{ln\left(\frac{V}{D}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{ln\left(\frac{60}{50}\right) + \left(0.03 - \frac{0.25^2}{2}\right)\times 2}{0.25\sqrt{2}} = 0.5086$$

[1]

Prob(default) = N(-0.5086)= 0.3055=30.55%

[1]

For Company B

$$d_2 = \frac{ln\left(\frac{V}{D}\right) + \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} = \frac{ln\left(\frac{80}{50}\right) + \left(0.03 - \frac{0.25^2}{2}\right)\times 3}{0.25\sqrt{3}} = 1.0768$$

[1]

Prob(default) = N(-1.0768)=0.1408=14.08% [1]

(ii)
Under put-call parity the share price is:
$S_t = c + Ke^{-rT} - p = 65.6 + 100e^{-0.06} - 35.2 = 124.58$ [1]
$Total\ shares = 124.58 \times 100{,}000 = 12{,}458{,}000$ [½]
$Total\ Debt = 60{,}000{,}000 - 12{,}458{,}000 = 47{,}542{,}000$ [½]
This is \$95.08 per \$100 nominal [1]

(iii)
The sensitivity of the share price to a change in the company's gross value is dS_t/dV_t . [1]
If we regard S_t as a call option on the asset value V_t (current value \$60m, strike \$50m) then this is the Greek delta. [1]
From the Black-Scholes formula and the volatility above we find that
d1 = 0.8622 [1]
Delta = N(d1) = 0.8057 [1]
So a \$1m increase in asset value will give a \$805,700 increase in the total share value [½]
and a \$194,300 increase in total bond value. [½]
This is an increase of \$8.06 in the share price [½]
and an increase of \$0.39 in the bond price per \$100 nominal. [½]
[Maximum 4]

(iv)
The total value of the company's assets is not significantly larger than the nominal value of the bonds [½]
This means that an increase in the company's assets improves the likelihood that the full nominal amount will be paid to debtholders on maturity [½]
The bond price therefore has some sensitivity to changes in the value of the company [½]
Therefore the improvement in the value of shares is smaller than the increase in the value of the company's assets. [½]
[Total 14]

Commentary:
Part (i) of this question was answered well by most candidates. *In part (ii) many candidates were able to calculate the share price but failed to calculate the total share and debt values.* *Parts (iii) and (iv) were not answered well, with many candidates failing to recall Delta as N(d1) or making general comments in part (iv) that were not specific to the scenario in the question.*

Q3

(i)

High correlation between risks means little pooling advantage [1]
Small number of policies, only ten [1]
Risk is potentially quite large depending on nature of holidays [1]
Moral hazard – holidaymakers might be more likely to behave in way that increases risk due to having insurance [1]
Adverse selection – more likely to take out insurance if they already know that there is risk associated with their holiday. [1]

[Maximum 3]

(ii)

Tighten definition of payable event [1]
Diversify policies between different destinations [1]
Diversify policies between different times of year [1]
Introduce policy excess. [1]

[Maximum 2]

[Total 5]

Commentary:
This question was answered well by most candidates. Some repeated the same points multiple times in part (i) – correlation of risk is a key issue here but there are also other issues that the insurer needs to consider.

Q4

(i)

u = 1.15, d = 0.9, r = .06/12

risk-neutral probability q = (e^r – d)/ (u-d) = (exp(.06/12)- .9)/ (1.15-0.9) = (1.005013-.9)/.25 = .42005 [1]

(ii)

$$V_1(1) = exp(-r) * [q_1(1) * c_2(1) + (1 - q_1(1)) * c_2(2)]$$
$$= exp(-.06/12) * [.42005 * 7.25 + (1 - .42005) * 0] = 3.030174$$ [1]

$$V_1(2) = exp(-.06/12) * [.42005 * 0 + (1 - .42005) * 0] = 0$$ [1]
$$V_0 = exp(-.06/12) * [.42005 * 3.030124 + (1 - .42005) * 0] = \$1.2664$$ [1]

(iii)

Given that an up-movement occurs over the first time period, a portfolio of θ_{11} shares and ψ_{11} cash set up at time 1 will replicate the value of the call option at time 2 if:

$132.25 * \theta_{11} + \psi_{11} * exp(.06/12) = 7.25$ *and* $103.5 * \theta_{11} + \psi_{11} * exp(.06/12) = 0$ [1]

$\theta_{11} = (7.25 - 0)/\,28.75 = 0.252174$

ψ_{11} = -25.96983

Thus, the value of the replicating portfolio and hence the option at (1,1) is

$\theta_{11} * 115 + \psi_{11} = 0.252174 * 115 - 25.96983 = 3.030165$ [1]

Given that a down-movement occurs over the first time period, a portfolio of θ_{12} shares and ψ_{12} cash set up at time 1 will replicate the value of call option at time 2 if:

$103.5 * \theta_{12} + \psi_{12} * exp\,(.06/12) = 0$ *and*

$81 * \theta_{12} + \psi_{12} * exp\,(.06/12) = 0$

$\theta_{12} = 0$

$\psi_{12} = 0$ [1]

Thus, the value of the replicating portfolio and hence the option at (1,2) is

$0 * 90 + 0 = 0$

Finally, a portfolio of θ_{01} shares and ψ_{01} cash set up at time 0 will replicate the value of the call option at time 1 if:

$\theta_{01} * 115 + \psi_{01} * exp\,(.06/12) = 3.030165$ and

$\theta_{01} * 90 + \psi_{01} * exp\,(.06/12) = 0$ [1]

Solving these two equations we get

$\theta_{01} * 25 = 3.030165$

$\theta_{01} = 0.121207$

$\psi_{01} = -10.8542$ [1]

(iv)

The value of the replicating portfolio at time 0 is given by

$\theta_{01} * 100 + \psi_{01} = .121207 * 100 + -10.8542 = \$1.2665$ [1]

[Total 10]

Commentary:
Parts (i) and (ii) of this question were straightforward and most candidates scored well. Some used a monthly interest rate and scored zero for part (i) but it was still possible to score full marks in the rest of the question following through the incorrect value of q. *Stronger candidates performed well in part (iii) but many failed to attempt it. Part (iv) was generally answered well by those who had attempted part (iii).*

Q5

(i)

$U(w) = bw + cw^2$

$U'(w) = b + 2cw$ [½]

$U''(w) = 2c$ [½]

$A(w) = \dfrac{-U''(w)}{U'(w)} = \dfrac{-2c}{b + 2cw}$ [1]

$R(w) = wA(w) = \dfrac{-2cw}{b + 2cw}$ [1]

(ii)

This shows that the investor would reject a fair gamble [½]
Which implies the investor is risk averse [½]

(iii)

The investor's current wealth is $150 and utility is 6,750, therefore:

$U(w) = 150b + 150^2c = 6750$ [1]

The minimum p at which they will accept the gamble will satisfy the equation where current utility is equal to the expected utility of the gamble, [1]
i.e.:

$6750 = pU(175) + (1 - p)U(125)$ [1]
$= 0.54167(175b + 175^2c) + 0.45833(125b + 125^2c)$ [1]
Solving simultaneous equations gives c=-0.1 [1]
And b=60 [1]

(iv)

To satisfy the principle of non-satiation $U'(w) > 0$ [1]
$60 - 0.2w > 0$
$w < 300$ [1]

(v)

The utility function exhibits increasing absolute risk aversion [1]
Therefore at a higher level of wealth they are more risk averse [1]
And so need a higher probability of a win to accept the gamble [1]
So we will need p > 0.54167 [1]

[Total 16]

Commentary:

This question caused little difficulty for most candidates, though there were some algebraic slips in part (iii). A significant number of candidates reached the wrong conclusion in part (v) about whether p should increase or reduce.

Q6

(i)

a. From the core reading:

The minimum variance can easily be shown to occur when:

6. $x_A = \dfrac{V_B - C_{AB}}{V_A - 2C_{AB} + V_B}$. [½]

where x_A is the proportion invested in security A.

$$x_A = \frac{4V_A}{V_A + 4V_A}$$

$x_A = 0.8$ [½]

$x_B = 1 - x_A = 0.2$ [½]

$E_P = x_A E_A + x_B E_B = 0.8E_A + 0.2 * 4E_A = 1.6E_A$ [½]

b. $C_{AB} = \sqrt{V_A V_B} = \sqrt{V_A * 4V_A} = 2V_A$ [1]

$$x_A = \frac{4V_A - 2V_A}{V_A - 4V_A + 4V_A}$$

$x_A = 2$ [½]

$x_B = -1$ [½]

$E_P = x_A E_A + x_B E_B = 2E_A - 4E_A = -2E_A$ [1]

(ii)

$V_P = x_A^2 V_A + x_B^2 V_B + 2x_A x_B C_{AB}$ [1]

$= 4V_A + 4V_A - 8V_A = 0$ [1]

(iii)

The variance is zero therefore the portfolio is risk-free. [1]

Either the expected return on the portfolio is negative, or E_A is negative, both of which feel like unusual scenarios. [1]

[Total 9]

Commentary:

In part (i) most candidates calculated the portfolio correctly when rho = 0 but failed to calculate it when rho =1.

Parts (ii) and (iii) were generally answered well, and some credit was given in part (iii) for answers that do not match the model solution but were consistent with the candidate's incorrect answers to parts (i) and (ii). Some candidates struggled with part (i) then did not attempts parts (ii) or (iii), but there were marks that could be picked up in both these parts even without an answer to part (i).

Q7

(i)
This property means that, in a given interval, the number of claims is unaffected by the number of claims in the past. [1]
This is also known as the process being 'memoryless'.

(ii)
On the one hand, using a Poisson process leads to tractable mathematics in the field of ruin theory [½]
And on the surface it appears intuitive e.g. we would not expect the number of claims e.g. ten years ago to affect the number of claims now [½]
However, it is easy to imagine scenarios in which the assumption of independence breaks down [½]
e.g. during a mass claims event, it is highly likely that there will be more claims next week given there were a large number of claims this week [1]
This means the assumption may in practice be questionable [½]

(iii)
The insurer will be ruined if they receive three or more claims. [½]
So the probability is 1 - P(N(5)=0) - P(N(5)=1) - P(N(5)=2) [½]
= 1 – 0.368 – 0.368 – 0.184 [1]
= 1 – 0.920 [½]
= 0.080 [½]

(iv)
The probability of ruin will increase with time [½]
because there is more time for claims to be received [½]
but no more premium income. [½]
The probability of ruin will tend towards 1 [½]
It will never quite hit 1 in finite time [½]
Though the ultimate probability of ruin will be 1 [½]

[Total 10]

Commentary:
This question was mostly answered well. In part (ii) some answers were too theoretical to score full marks and needed more of a link to the real world situation of an insurer. Some candidates used a continuous approximation to the distribution of claims in part (iii) but this was not needed given that the discrete calculation is fairly simple when we are dealing with small numbers of claims. *In part (iv) most candidates identified that the lack of future premium income is key to considering how the probability of ruin will change.*

Q8

(i)

(a)

The current value of the bond is (100 – L) * exp(-0.05), where L is the expected loss due to default.

Therefore 85.61 = (100 – L) * exp(-0.05) [½]

So L = $10 [½]

But L = $100 x Prob of default x Loss given default

If the loss given default is 50% then the probability of default must be 20%. [½]

(b)

The current value of the bond is (100 – L) * exp(-2 * 0.05), where L is the expected loss due to default.

Therefore 76.91 = (100 – L) * exp(-2 * 0.05) [½]

So L = $15 [½]

But L = $100 x Prob of default x Loss given default

If the loss given default is 50% then the probability of default must be 30%. [½]

(ii)

(a)

Yield = ln(100/85.61) = 15.5% [1]

(b)

Yield = ln(100/76.91)/2 = 13.1% [1]

(iii)

The probability of the two year bond defaulting is higher than the one-year bond [½]

But the probability of default in year 2 is actually lower [½]

So the yield from t=1 to t=2 should be lower than the yield from t=0 to t=1 [1]

Giving a lower overall yield on the two year bond than the one year bond [1]

[Total 8]

Commentary:
Parts (i) and (ii) were generally answered well, with most candidates calculating correct probabilities and yields. *In part (iii) many candidates made a simple remark that higher risk = higher yield but failed to relate their answer to the scenario and how the probability of default differs between year 1 and year 2.*

Q9

(i)
There might be regulatory rules requiring the insurer to hold capital of at least x [½]
So that any level of wealth below x is equally bad for the insurer [½]

(ii)
(a)
This function satisfies risk-aversion for w>x [½]
But not for w<x [½]
Because for w<x u'(w)=0 [1]
(b)
It satisfies non-satiation for w>x [½]
Because u'(w) > 0 [½]

(iii)
Current utility = (100 / 100) ^ 0.5 = 1 [1]
For a premium of 39:
E(U) = 0.75 * U(139) + 0.25 * U(139 – 50) [½]
= 0.75 * (139/100)^0.5 + 0.25 * 0
= 0.8842 [1]
For a premium of 40:
E(U) = 0.75 * U(140) + 0.25 * U(140 – 50) [½]
= 0.75 * (140/100)^0.5 + 0.25 * (90/100)^0.5
= 1.1246 [1]

(iv)
The expected loss on the policy is 25% * $50 = $12.50 [½]
So the premium is over 3x larger than the expected loss [½]
This might work for a very risk-averse customer [½]
But it is not likely that anyone would pay this premium [½]
[Total 10]

Commentary:
Parts (i) and (ii) of this question were mostly answered well, though in part (ii) some candidates only considered the situation when w>x which was not enough to score full marks. *In part (iii) many candidates completed calculations for a premium of $40 but only the stronger candidates also considered a premium of $39 to show that $40 is the lowest that fits the criteria.*

Q10

(i)

The put-call parity relationship states that:

$c_t + Ke^{-r(T-t)} = p_t + S_t$

Substituting the values given:

$c_t + 9{,}500 * e^{-.06} = 307 + 10{,}000$ [1]

$c_t + 8946.763 = 10{,}307$

$c_t = \$1{,}360.24$ [1]

(ii)

The price of the $10,500 call option is $c_t \ = \ s_t\Phi(d_1) - \ Ke^{-r(T-t)} \ * \Phi(d_2)$

$= \ 10000 * \Phi(d_1) - \ 10500 * e^{-0.06} \ * \Phi(d_2)$ [1]

$= \ 10000 * \Phi(0.1538) - \ 10500 * e^{-0.06} \ * \Phi(-0.0352)$

$= 10000 * 0.561134 - 10500 * e^{-0.06} \ * 0.485978$ [1]

$= \$805.73$ [1]

(iii)

(a)

The value of the option would increase, because the option allows us to buy the share for a fixed value so an increase in the share price increases the intrinsic value of the option. [1]

(b)

The value of the option would increase, because holding the option and cash instead of the share is more valuable when interest rates are higher. [1]

[Total 7]

Commentary:
This was the bme missed oest-answered question on the paper, with many candidates scoring full marks. Sout on marks in part (iii) for explaining how the call option value would change but not explaining why.

[Paper Total 100]

END OF EXAMINERS' REPORT

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