INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

September 2025

Subject CM1 – Actuarial Mathematics for Modelling Core Practices

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 A loan is repaid by a decreasing annuity payable annually in arrears for 30 years.

The repayment at the end of the first year is £50,000 and subsequent repayments decrease by £500 each year.

The repayments are calculated using a rate of interest of 4.5% p.a. effective. Calculate, showing all working and using annuity functions, the amount of the loan. [4]

2 Calculate, showing all working, $a^{(4)}_{85.5\,:\,\overline{0.75}|}$.

Basis:
Interest rate: 9% p.a. effective
Mortality: PMA92C20

You should assume a constant force of mortality between integer ages. [5]

3 A life insurance company issues a with profit whole life policy to a life aged 50 exact. The benefit is payable at the end of the year of death and is equal to the basic sum assured of \$150,000 plus any attaching bonus.

Level premiums are paid annually in advance for 25 years or until the death of the policyholder, if earlier.

Simple reversionary bonuses are added at the start of each policy year (including the first).

The curtate future lifetime of a life aged x exact is denoted as K_x.

(i) Write down the gross future loss random variable for this policy at outset in terms of the curtate future lifetime of the policyholder. Allow for all elements of the pricing basis. [4]

Pricing basis:

Mortality:	AM92 Select
Interest rate:	6% p.a. effective
Initial expense:	\$250
Renewal expense:	\$75 at the start of each policy year excluding the first year
Bonus:	Simple bonus rate of 1% of the basic sum assured

The gross retrospective reserve and gross prospective reserves are calculated using the pricing basis above.

The gross annual premium for this policy is calculated using the pricing basis above with a profit loading of 5% of the first year's premium.

(ii) Without carrying out any calculations, discuss if the gross retrospective reserve and gross prospective reserves for this policy would be equal. [3]

[Total 7]

4 An individual holds two bonds, A and B, both with a term of 15 years.

Bond A pays coupons of \$350 at the end of every 5-year period and a redemption payment of \$1,000.

Bond B is a zero-coupon bond with a redemption payment of \$2,000.

The effective interest rate is 6% p.a.

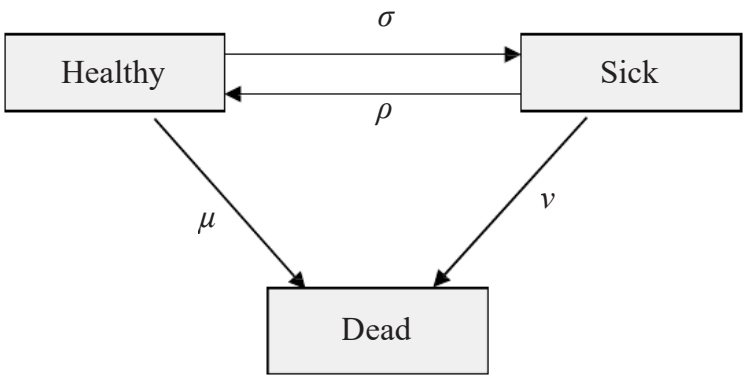
(i) Calculate, showing all working, the discounted mean term of the bond holding. [5]

(ii) Calculate, using your answer to part (i), the volatility of the bond holding. [1]

(iii) Estimate, using your answer to part (ii), the present value of the bond holding if the effective interest rate changes to 6.5% p.a. Show all working. [2]

[Total 8]

5 A three-state transition model is shown in the following diagram:



Calculate, showing all working, the present value of the benefits for:

(i) a life aged 50 exact who has just entered the sick state and is paid £10,000 p.a. payable continuously throughout this period of sickness subject to a maximum period of 5 years. [4]

(ii) a healthy life aged 35 exact who will be paid an annuity certain on reaching age 65 provided they remain healthy throughout the intervening period. The annuity is £15,000 p.a. payable quarterly in advance for 10 years. [4]

Basis:
Force of interest: 6% p.a.
The transition probabilities are constant at all ages:
$\mu = 3\%$ p.a.
$\sigma = 5\%$ p.a.
$\rho = 2\%$ p.a.
$\nu = 5.5\%$ p.a.

[Total 8]

6 The employees of a large company are assumed to experience decrements in line with the following multiple decrement table.

Forces of decrement are assumed to be constant over single years of age.

Age (x)	*Active lives* $(al)_x$	*Retirement* $(ad)^r x$	*Withdrawals* $(ad)^w x$	*Deaths* $(ad)^d x$
16	10,000	0	750	5
17	9,245	0	600	5
18	8,640	0	500	5
...	...	...	...	...
39	3,150	0	30	7
40	3,113	0	20	8
...	...	...	...	...
59	2,200	0	10	10
60	2,180	50	8	12
61	2,110	60	7	15
62	2,028	85	5	18
63	1,920	100	4	20
64	1,796	200	3	25
65	1,568	1,532	1	35

(i) Calculate, showing all working, the dependent probability that an employee who joins the company on their 18th birthday will retire from active status on or after their 61st birthday. [1]

(ii) Calculate, showing all working, the independent probability that an employee aged exactly 17 will withdraw before their 18th birthday. [3]

For employees aged 63 and over, the company pays a death benefit of \$20,000 if the employee dies while in service. The benefit is paid at the end of the year of death.

A new employee joins the company at exact age 40.

(iii) Calculate, showing all working, the expected present value of the death benefit for this new employee on joining the company. Assume an effective rate of interest of 5% p.a. [3]

[Total 7]

7 A company wishes to invest in a new project. Lump sum investments are required at the start of each of the first 3 years of the project of \$2.2 million, \$2.4 million and \$4.1 million, respectively.

Exactly 2 years from the start of the project, the company expects to begin receiving a continuous income of \$2.3 million p.a. The income is expected to continue for 20 years at which point the project will end with no residual value.

(i) Calculate, showing all working, the discounted payback period at an effective interest rate of 12% p.a. Give your answer in years, rounded to the nearest three decimal places. [5]

The company can borrow money to finance the project at the same effective rate of interest as that used to value the project (i.e. 12% p.a.). Any sums borrowed will be repaid as soon as funds become available.

After the repayment of all sums borrowed, the company will retain all income received and earn interest on the retained income at an effective rate of 7.5% p.a.

(ii) Calculate, showing all working, the expected accumulated profit at the end of the project. [3]

[Total 8]

8 A 3-year unit-linked endowment assurance contract is issued to lives aged 60 exact. Policyholders pay a level premium of £360 p.a. in advance.

The profit signature, in £s, for this contract has been calculated as (88, −11, −46) using the following assumptions:

$q_{60} = 0.0083$

$q_{61} = 0.0091$

Surrenders: none.

(i) Calculate, showing all working, the profit vector. [3]

The policy is amended to include a surrender option. On surrender the bid value of the unit fund is payable, less a surrender penalty of 50% of the annual premium. All other policy features remain the same.

It is assumed that at the end of the first year and at the end of the second year 20% of policies then in force are surrendered.

(ii) Calculate, showing all working, the revised profit signature. [7]

(iii) Calculate, showing all working, the change in the net present value of the policy as a result of the policy amendment. Assume an annual effective risk discount rate of 6%. [2]

[Total 12]

9 A male and female both aged 50 exact purchase a deferred last survivor annuity by means of a single premium on 1 January 2020.

The annuity of £10,000 p.a., which is payable quarterly in advance, commences from age 60 if at least one of the lives is alive at that age.

Calculate, showing all working, the prospective reserve on 1 January 2024 assuming:

(i) only the female is alive at that date. [3]

(ii) both lives are alive at that date. [6]

Basis:
Mortality: PMA92C20 and PFA92C20
Interest rate: 4% p.a. effective
Expenses: None

[Total 9]

10 A share pays annual dividends and has just paid a dividend of £0.50. The next two dividends will increase by 3% p.a. compound. Thereafter, dividends will increase by 5% p.a. compound.

Assume an effective rate of interest of 7% p.a.

(i) Calculate, showing all working, the present value of the expected dividend stream for a portfolio of 200 shares held in perpetuity. [5]

An investor buys 200 of these shares for £24.90 each. The investor holds the shares for exactly 3 years, receives the dividends and sells the shares for £26.15 each immediately after the third dividend has been paid. The value of the inflation index over the investment period is shown in the table below:

Time (years)	*Index*
0	123
1	118
2	121
3	128

(ii) Calculate, using interpolation and showing all working, the annual real rate of return achieved by the investor. [6]

[Total 11]

11 The force of interest is a function of time, and at any time t (measured in years) is given by the formula:

$$\delta(t) = \begin{cases} 0.02 & 0 \le t < 5 \\ 0.06 & 5 \le t \end{cases}$$

(i) Derive, and simplify as far as possible, expressions for $v(t)$ where $v(t)$ is the present value at time $t = 0$ of a unit sum of money due at time t.

You should consider separately the cases $0 \le t < 5$ and $t \ge 5$. [5]

A continuous payment stream is received at a rate of $10e^{0.02t}$ between $t = 4$ and $t = 6$.

(ii) Calculate, showing all working, the present value of the payment stream at time $t = 0$. [4]

[Total 9]

12 On 1 January 2016 a life insurance company issues the following policies to male lives aged 50 exact:

- A 10-year pure endowment assurance with sum assured \$100,000. Premiums are payable annually in advance throughout the policy term or until earlier death
- A 10-year single premium temporary annuity with annual benefit of \$15,000 payable annually in arrears.

At the beginning of 2024 the following number of policies are in force in each portfolio:

Pure endowment assurance portfolio: 856
Temporary annuity portfolio: 540

During 2024 there are six deaths in the pure endowment assurance portfolio and two in the temporary annuity portfolio.

Calculate, showing all working, the total mortality profit for 2024.

Basis:
Mortality: AM92 Ultimate
Interest rate: 6% p.a. effective

Ignore any expenses.

[12]

END OF PAPER