INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

17 September 2025

Subject CM2 – Economic Modelling
Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 The table below shows the cumulative incurred claims for a portfolio of general insurance policies. The ultimate loss ratio is expected to be in line with the 2021 accident year and claims are assumed to be fully developed by the end of development year 3.

Accident year	*Development year*				*Earned premiums*
	0	*1*	*2*	*3*	
2021	67	84	101	113	128
2022	70	89	120		145
2023	133	180			216
2024	300				468

(i) Calculate the total reserve required to meet the outstanding claims using the Bornhuetter–Ferguson method. [7]

(ii) Explain why an insurance regulator may view the use of the Bornhuetter–Ferguson method as more appropriate than the basic chain ladder method in this case. [2]

[Total 9]

2 The number of claims on a type of insurance policy follows a Poisson process with parameter $\lambda = 1$. The insurer models individual claim amounts using an exponential distribution with a mean of \$1,000. Premiums are set with a loading factor of 25%.

(i) State Lundberg's inequality, defining all terms you use. [2]

(ii) Calculate the adjustment coefficient. [4]

The insurer's initial wealth is \$5,000.

(iii) Calculate an upper bound for the insurer's probability of ultimate ruin using Lundberg's inequality. [1]

The insurer is concerned about the administrative time taken to process claims and is proposing to pay a fixed lump sum equal to \$1,000 in respect of each claim, regardless of the actual loss incurred.

(iv) Explain what impact the insurer's proposal would have on the probability of ultimate ruin. [2]

[Total 9]

3 Consider a single period multifactor model of security returns where:

$$R_i = \alpha_i + \sum_{j=1}^{K} \beta_{ij} I_j + \varepsilon_i$$

where:

- R_i is the return on security i
- α_i and β_{ij} are security-specific constants
- ε_i is a cross-sectionally independent random component that is independent of all I_j
- I_j is the cross-sectionally independent rate of change in factor j.

(i) Derive an expression for C_{ij}, the covariance between the returns on two securities i and j, using the model above. [4]

Consider a portfolio of N securities, with equal weights held in each security.

(ii) Discuss the implications of the expression in part (i) on the diversification of the portfolio. [3]

Now consider a single index model where:

$$R_i = \alpha_i + \beta_i R_M + \varepsilon_i$$

where:

- R_i is the return on security i
- α_i and β_i are security specific constants
- R_M is the return on the market
- ε_i is a random component that is independent of R_M.

The returns of two securities, X and Y, are fitted to the model over a 1-year period.

The expected return on the market is 7.5% p.a. and the standard deviation of the return on the market is 1.0%.

The fitted model has the following attributes:

	X	Y
α_i	α_X	12%
β_i	1.5	β_Y
Var[ε_i]	0.0006	0.0001
$E[R_i]$	12.3%	$E[R_Y]$

(iii) Calculate α_X and β_Y assuming that the covariance of returns on X and Y is -0.000075. [4]

(iv) Show how a portfolio of these two securities could be used to achieve diversification. [4]

[Total 15]

4 (i) State four of the assumptions underlying the Black–Scholes option pricing model. [2]

An investor holds a European call option, c, on a share. The option expires in 3 years and the strike price is \$60. The underlying share price is currently \$75 and its volatility is 15%. The risk-free force of interest is 4% p.a.

(ii) Calculate the price of the option using the Black–Scholes model. [3]

An analyst states that if the share price were to instantaneously increase to \$80 then the price of the option would change to \$27.21.

(iii) Estimate the delta of the option, Δ_c, using your answer from part (i) and the analyst's information above. [2]

The investor creates the following portfolio:

- long one of the call options described above
- short one European put option, p, on the same underlying share and with the same strike price and expiry date
- short one underlying share.

(iv) Show, using put–call parity, that $\Delta_c - \Delta_p = 1$. [3]

(v) Justify why the investor may have created the portfolio described above. [2]

(vi) Discuss how and why Δ_c will change if the share price increases. [2]

[Total 14]

5 A company issues a 1-year zero-coupon bond that pays \$6 on maturity. The probability of default on this bond is estimated to be 20%. In case of default, the maturity payment is assumed to follow a discrete uniform distribution U(\$0,\$5) taking values \$0, \$1, \$2, …, \$5 each with probability 1/6.

(i) Calculate, for the payment on maturity:

(a) the downside semi-variance.

(b) the shortfall probability below \$4.

(c) the expected shortfall below \$4.

[6]

(ii) Comment on how each of the answers to part (i) (a), (b) and (c) would change if the probability of default increased to 30%. You do not need to carry out any further calculations. [1]

[Total 7]

6 (i) List the four axioms used to derive the expected utility theorem. [2]

An actuary uses a quadratic utility function of the form:

$$U(w) = w - dw^2$$

(ii) Determine the range of values for the constant d where $U(w)$ satisfies the following conditions:

(a) non-satiation

(b) diminishing marginal utility of wealth.

[2]

The actuary's current wealth is \$100,000 and their current utility is 75,000.

(iii) Calculate the value of d in the actuary's utility function. [1]

The actuary is planning their wedding. The cost of the wedding will be \$15,000 and the actuary has identified four possible outcomes as shown in the table:

Outcome	*Description*	*Impact*	*Probability*
1	The wedding takes place without incident	No additional cost	70%
2	An incident occurs and the actuary will have to pay damages	Additional cost of \$2,000	15%
3	Either party cancels the wedding	No additional cost, but no refund of original cost	5%
4	The wedding venue is changed at short notice	Additional cost of \$5,000	10%

(iv) Calculate the actuary's expected utility assuming that their wealth before paying for the wedding is \$100,000 and there are no other income or expenses. [3]

An insurance company offers wedding insurance that will:

- cover the cost of any damages up to \$10,000.
- pay the policyholder half of the cost of the wedding if either party cancels.
- cover all additional costs in relation to a replacement venue if the wedding venue is changed at short notice.

(v) Calculate the maximum premium that the actuary would be willing to pay for this insurance. [3]

[Total 11]

7 At a small casino, there are N customers playing a game. In each round of the game, each customer bets \$1 on one of two outcomes and if the customer chooses the correct outcome, they win the round. You may assume for the rest of the question that each customer:

- makes their choice randomly.
- has a chance of winning of 0.5 every time a round is played.
- has infinite wealth.

When a customer wins they receive \$2, which includes the return of their initial bet. When they lose, the casino keeps the \$1 bet. The casino is considered bankrupt if its wealth ever reaches \$0 and bankruptcy is an absorbing state.

The casino's initial wealth is \$5. Assume initially that $N = 1$.

(i) Write down the probability that the casino is bankrupt after two rounds of the game. [1]

(ii) Calculate the probability that the casino is bankrupt after five rounds of the game. [2]

Now suppose that $N = 5$ for each round of the game. The casino's initial wealth is still \$5.

(iii) (a) Determine the distribution of the casino's wealth, i.e. the possible outcomes and their probabilities:

- after one round of the game.
- after two rounds of the game.

(b) Hence, calculate the probability that the casino becomes bankrupt during the first two rounds of the game.

[5]

(iv) Explain why the probability the casino is bankrupt after five rounds would be higher when $N = 5$ than the equivalent probability when $N = 1$. [2]

(v) Discuss, with reference to your answers to parts (i) to (iv) inclusive, how the principles of ruin theory for insurers are relevant to this casino. [2]

[Total 12]

8 (i) Define the following forms of the efficient markets hypothesis:

(a) weak form

(b) semi-strong form

(c) strong form.

[3]

Consider the following scenarios:

1. A company announces unexpected positive earnings and its stock price immediately rises as a result.
2. A board member of a pharmaceutical company learns about a pending approval for a new drug but is unable to make abnormal profits because the stock price already reflects the expected approval.
3. An investor conducts fundamental analysis by carefully studying company financial statements, earnings reports and industry news. By identifying undervalued stocks they can earn abnormal returns.

(ii) Explain for each scenario if it is consistent with:

(a) the strong form efficient market hypothesis.

(b) the semi-strong form efficient market hypothesis.

[6]
[Total 9]

9 Consider an asset with value Y_i at time i. The value Y_i can either increase by 20% or reduce by 10% over a single time step based on a probability measure R, where the probability of an up step is 75% and the probability of a down step is 25%. Let F_i denote the filtration of the process Y_i at time i.

The 'Tower Property' is:

$$E_R(Y_2|F_0) = E_R(E_R(Y_2|F_1)|F_0)$$

(i) Demonstrate that the 'Tower Property' holds for Y_i. [4]

(ii) State briefly why the 'Tower Property' would be useful when pricing a derivative on the underlying asset Y_i under a risk-neutral probability measure.

[1]
[Total 5]

10 A portfolio consists of Asset S and Asset T as shown below:

	Investment in asset	*Daily volatility*
Asset S	\$500	1.5%
Asset T	\$700	1.3%

The correlation coefficient between the returns on S and T is 0.5. You may assume that asset returns are normally distributed.

(i) Calculate the following:

(a) the 9-day Value at Risk (VaR) for each of asset S and T

(b) the 9-day VaR for the portfolio.

[6]

[**Hint:** You may assume that n-day VaR = 1-day VaR × $n^{0.5}$.]

(ii) Analyse what your answers to part (i) show about the impact of diversification on VaR. [3]

[Total 9]

END OF PAPER