INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

13 September 2021 (am)

Subject CM1 – Actuarial Mathematics

Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 A 10-year unit linked contract has the following profit signature before any non-unit reserves are created:

Year	1	2	3	4	5	6	7	8	9	10
	+1	–1	+1	+1	+1	–1	0	–1	+1	+1

Non-unit reserves are set up to zeroise the negative cashflows.

Determine the revised profit signature, ignoring interest and mortality. [3]

2 Calculate, showing all working

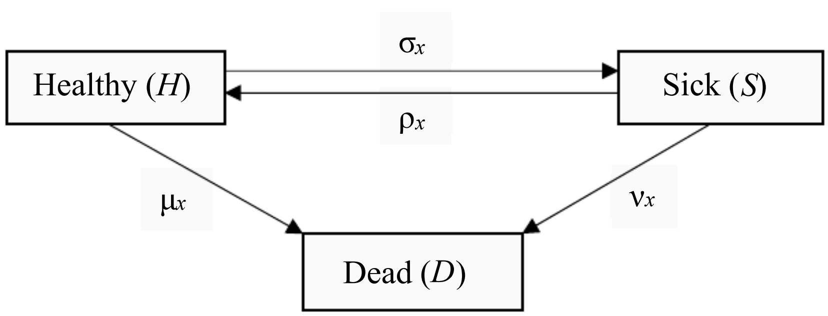
(a) $\mu_{55:60}$

(b) ${}_5 p_{55:60}$

(c) ${}_2 q^{\;1}_{60:60}$

Assume lives are independent with regards to mortality and that both lives are subject to the PFA92C20 mortality table. [4]

3 A life insurance company uses the following multiple state model to price its sickness policies.



Describe, in words, what each of the following integral expressions represents. You may assume the time periods are measured in years.

(a) $2{,}000 \times \int_0^{20} e^{-\delta t} \times {}_t p_{40}^{HS}\, dt$

(b) $1{,}000 \times \int_0^{20} e^{-\delta t} \times {}_t p_{40}^{\overline{HH}}\, dt$

(c) $20{,}000 \times \int_0^{20} e^{-\delta t} \times {}_t p_{40}^{HS} \times \nu_{40+t} dt$

[6]

4 A company has agreed to build and operate a ferry service for a regional government.

The company will invest \$10 million at the outset, and a further \$8 million after 1 year.

The ferry will then come into operation and the company will receive payments at the end of each year, the first payment occurring at the end of the second year of the project.

The amount of payment at the end of the second year will be \$4 million, increasing by \$0.5 million in each of the subsequent years until the annual payment is \$7 million, after which the payments will reduce by \$1 million each year. When the payments have reduced to zero, the company's involvement in the project will end.

Calculate the net present value of the project at a rate of interest of 6% p.a. effective.
Note: You should show your working and determine the present value of income using annuity functions.

[6]

5 An equity is expected to pay its first dividend in exactly 2 years' time. It is assumed that this dividend will be \$0.20 per share.

Subsequent annual dividends are assumed to grow at 6% p.a. compound for the following 10 years, and at 3% p.a. compound in perpetuity thereafter.

Calculate, showing all working, the price of the share to the nearest \$0.01, that would give an effective rate of return of 7% p.a. [7]

6 A bond is issued at time $t = 0$ at a price of \$107.60 per \$100 nominal. The bond pays coupons of 6% p.a., annually in arrears, and will be redeemed at par in 3 years' time.

The 2-year par yield at time $t = 0$ is 6.5% p.a. The 1-year forward rate of interest at time $t = 1$ year is 4.5% p.a. effective.

[7]

Calculate, showing all working and assuming no arbitrage, the implied 1-year, 2-year and 3-year annual effective spot rates.

7 A life insurance company issues a reversionary annuity policy to a male and female, both aged exactly 65.

The annuity of $30,000 p.a., payable monthly in arrears, commences on the first death, and payments cease on the death of the second life, or on the 15th anniversary of the policy inception if earlier.

Calculate, showing all working, the single premium for the policy.

Basis:

Mortality:	PMA92C20 for the male life and PFA92C20 for the female life The lives are independent with respect to mortality
Interest:	4% p.a.
Expenses:	Initial: $250 incurred at the outset Renewal: 3% of each annuity payment

[10]

8 The force of interest, $\delta(t)$, is a function of time, and at any time, t, measured in years, is given by the formula:

$$\delta(t)=\begin{cases}0.06+0.02t & 0\le t\le 4\\ 0.08-0.01t & t>4\end{cases}$$

$A(0,t)$ the accumulation at time t of a unit of money invested at time 0, can be written as:

$$A(0,t)=\begin{cases}e^{\boldsymbol{a}+\boldsymbol{b}t+\boldsymbol{c}t^2} & 0\le t\le 4\\ e^{\boldsymbol{f}+\boldsymbol{g}t+\boldsymbol{h}t^2} & t>4\end{cases}$$

(i) Determine the values of ***a***, ***b***, ***c***, ***f***, ***g*** and ***h***. [6]

A sum of $600 is invested at $t = 3$ and a further sum of $900 is invested at $t = 9$.

(ii) Calculate, showing all working, the accumulated amount at $t = 13$. [4]

(iii) Calculate, showing all working, the yield of the investment described in part (ii) expressed as an effective rate of interest per month to the nearest 0.1% [3]

(iv) Comment on your answer to part (iii). [1]

[Total 14]

9 A life insurance company issues 30-year pure endowment assurance policies to a group of lives aged exactly 30. Each policy provides a sum assured of $50,000 payable on survival to the end of the term. Premiums on the policy are payable annually in advance for 30 years or until earlier death.

There were two deaths during the 25th policy year and the number of policies in force at the end of that year was 315. There were no exits other than death during the year.

(i) Calculate, showing all working, the mortality profit or loss arising in the 25th policy year. [5]

(ii) Comment on your result obtained in part (i).

Basis:

Mortality: AM92 Ultimate

Interest: 4% p.a. effective

Expenses: None [3]

[Total 8]

10 The table below is an extract from a multiple decrement table that is currently used to model the deaths and withdrawals of employees working for a large company in the hospitality industry. No decrements occur other than by death or withdrawal.

Age (x)	*Number of employees* $(al)_x$	*Number of deaths* $(ad)_x^d$	*Number of withdrawals* $(ad)_x^w$
47	50,000	390	1,500

Recent experience has resulted in an estimate that, at all ages:

- the annual independent force of mortality for employees is now 60% of that implied by the q_x rates in the ELT15 (Females) table.
- the annual independent probability of withdrawal for employees is now 250% of that used to construct the above table.

(i) Calculate, showing all working, the revised independent forces of mortality and withdrawal, each to six significant figures, for age 47. You should state any assumptions that you make. [7]

(ii) Construct the revised multiple decrement table, showing your results to two decimal places. [5]

(iii) Identify any concerns with the use of this revised multiple decrement table to model the future deaths and withdrawals of employees of the company. [3]

[Total 15]

11 A life insurance company issues a 15-year with profit endowment assurance policy to a life aged 50 exact. Premiums are payable monthly in advance for 15 years or until earlier death. The sum assured is payable at the end of the year of death or at the end of the term if earlier.

(i) Demonstrate that the basic sum assured a policyholder can purchase for a premium of \$500 per month is approximately \$93,000 (to the nearest \$1,000). [8]

Pricing basis:

Mortality:	AM92 Ultimate
Interest:	6% p.a. effective
Reversionary bonus:	1.9231% p.a. compound, vesting at the end of each year (i.e. The death benefit does not include the bonus relating to the policy year of death).
Initial expenses:	60% of the annual premium, incurred at policy commencement
Renewal expenses:	4% of the annual premium, incurred annually from the start of the second year onwards

Assume that the policyholder purchased a basic sum assured of \$93,000 with the premium of \$500 per month.

(ii) Demonstrate that the annual effective rate of return that a policyholder will earn on this contract, if they survive to the end of the 15 years, is at least 0.434% p.a. [3]

(iii) Explain why the contract may be attractive to policyholders in spite of the low level of minimum rate of return given in part (ii). [3]

During each of the first 5 years of the contract, the office declared compound reversionary bonuses of 5% p.a.

(iv) Calculate, showing all working, the prospective gross premium reserve at the end of the fifth year of the contract, using the basis given in part (i). [6]
[Total 20]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

September 2021

Subject CM1 – Actuarial Mathematics
Core Principles
Paper A

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
December 2021

A. General comments on the *aims of this subject and how it is marked*

CM1 provides a grounding in the principles of modelling as applied to actuarial work - focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations but candidates are not penalised for this. However, candidates may lose marks where excessive rounding has been used or where insufficient working is shown.

Although the solutions show full actuarial notation, candidates were generally expected to use standard keystrokes in their solutions.

Candidates should pay attention to any instructions included in questions; failure to do so can lead to fewer marks being awarded.

In particular, where the instruction, "*showing all working*" is included and the candidate shows little or no working, then the candidate will be awarded very few marks.

Where a question specifies a method to use (e.g. *determine the present value of income using annuity functions*) then when a candidate uses a different method the candidate will not be awarded full marks.

Candidates are advised to familiarise themselves with the meaning of the command verbs (e.g. state, determine, calculate). These identify what needs to be included in answers in order to be awarded full marks.

B. Comments on *candidate performance in this diet of the examination.*

The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those scripts.

A large number of candidates appeared to be inadequately prepared, in terms of not having sufficiently covered the entire breadth of the subject. We would advise candidates not to underestimate the quantity of study required for this subject.

Candidates should be aware that the questions cannot be answered using knowledge alone and well prepared candidates will demonstrate application of their knowledge to the questions asked.

Where candidates made numerical errors, the examiners awarded marks for the correct method used and also for the parts of the calculation that were correct. However, many candidates often did not show enough of their working to fully benefit from this.

The Examiners felt that the "open book" nature of the online exam led some candidates to rely on their notes much more than if the exam had been "closed book". The Examiners strongly recommend that candidates prepare for online exams just as thoroughly as they would do if the exam were of the traditional "closed book" format. Candidates are recommended to use their notes only as a tool to check or confirm answers where necessary, rather than as a source for looking up the answers.

C. Pass Mark

The Pass Mark for this exam was 53.
1344 presented themselves and 533 passed.

Whilst the paper was of a similar standard to previous sittings there were questions within this paper that covered areas of the syllabus which hadn't been examined recently. It was apparent that many candidates were not adequately prepared for this and therefore the examiners took this into consideration when setting the pass mark.

Candidates need to be aware that examiners can ask questions from across the breadth of the syllabus and that they will be asked to apply their knowledge in different situations. Candidates are advised to access as much learning material as available to them and not rely on past papers alone when preparing to sit the examination.

Solutions for Subject CM1 Paper A– September 2021

Q1

t	1	2	3	4	5	6	7	8	9	10
	+1	-1	+1	+1	+1	-1	0	-1	+1	+1

t=10 and 9: adjusted cashflows = +1 and +1

t=8: zeroise cashflow, adjusted cashflow = 0, need reserves @ t=7 of +1

t=7: adjusted cashflow =0 -1 = -1 (after reserve),
Zeroise cashflow, adjusted cashflow = 0, need reserves @ t=6 of +1

t=6: adjusted cashflow = -1-1= -2 (after reserve),
Zeroise cashflow, adjusted cashflow = 0, need reserves @ t=5 of +2

t=5: adjusted cashflow = 1-2=-1 (after reserve),
Zeroise cashflow, adjusted cashflow = 0, need reserves @ t=4 of +1

t=4: adjusted cashflow = 1-1 = 0 (after reserve),
STOP THIS RUN OF ZEROISING

t=3: cashflow = +1

t=2: zeroise cashflow, adjusted cashflow = 0, need reserves @ t=1 of +1

t=1: adjusted cashflow = +1-1=0 (after reserve)

Reserve

${}_tV$	1	2	3	4	5	6	7	8	9	10
	+1	0	0	+1	+2	+1	+1	0	0	0

[1]

Revised profit signature is

t	1	2	3	4	5	6	7	8	9	10
	0	0	+1	0	0	0	0	0	+1	+1

[2]

Generally well answered.

Candidates needed to include some explanation on the derivation of the revised profit signature to gain full marks since the command word was "determine".

Q2

(a)

$$\mu_{55:60} = \mu_{55} + \mu_{60} = 0.000976 + 0.001918 = 0.002894$$ [1]

(b)

$${}_5p_{55:60} = \frac{l_{60}}{l_{55}} \times \frac{l_{65}}{l_{60}} = \frac{l_{65}}{l_{55}} = \frac{9703.708}{9917.623} = 0.97843082$$

[1]

(c)

$${}_2q^{\,1}_{60:60} = \frac{1}{2} \times \left({}_2q_{60:60}\right) = \frac{1}{2} \times \left(1 - {}_2p_{60:60}\right) = \frac{1}{2} \times \left(1 - {}_2p_{60} \times {}_2p_{60}\right)$$

[1]

$$= \frac{1}{2} \times \left(1 - \left(\frac{l_{62}}{l_{60}}\right)^2\right) = \frac{1}{2} \times \left(1 - \left(\frac{9804.173}{9848.431}\right)^2\right) = 0.004483816$$

[1]

[Total 4]

Well answered.

Q3

In all parts we are calculating expected present values [½]
and the life is healthy at t = 0 (x = 40) [½]

(a)

2,000 per annum payable continuously [½]
when the life is in the sick state [½]
for the first and any subsequent bouts of sickness [½]

All payments cease at time 20 (age 60) [½]

(b)
1,000 per annum payable continuously [½]
while the life remains in the healthy state [½]
The contract ceases when the life leaves the healthy state for the first time or at time 20 (or when life attains age 60) [½]

(c)
20,000 lump sum payable immediately [½]
on death from the sick state [½]
if death occurs before time 20 (age 60) [½]

[Total 6]

Few candidates scored full marks. Most candidates omitted important elements in their descriptions such as the initial states of the life (healthy), annuity payments being made continuously, lump sums being paid immediately, whether return to the healthy state was permitted.

A number of candidates were unable to demonstrate they understood the difference between an annuity type benefit and a lump sum benefit payable immediately when a transition occurs.

Q4

Working in \$m

(a) PV of outgo $= 10 + 8v = 17.5472$ [1]

(b) PV of income

$$= v\left(0.5\left(Ia\right)_{\overline{6}|} + 3.5a_{\overline{6}|}\right) + v^7\left(8a_{\overline{7}|} - \left(Ia\right)_{\overline{7}|}\right)$$

[3]

$$= v\left(0.5\times16.3767 + 3.5\times4.9173\right) + v^7\left(8\times5.5824 - 21.0321\right)$$

$$= 23.9612 + 15.7134 = 39.6746$$ [1½]

NPV = 39.6746 – 17.5472 =\$22.1274m [½]

Generally well answered.

This question specified that candidates should use annuity functions; those that did not were heavily penalised.

Some candidates were unable to determine as to when annuity cashflows started. This led them to include erroneous discount factors.

Q5

The price is given by the present value of the expected future dividend payments:

$$P = v^2 D_0\left(1+1.06v+1.06^2v^2+...+1.06^{10}v^{10}\left(1+1.03v+1.03^2v^2+...\right)\right) \text{ @ } 7\%\ p.a.$$

$$= v^2 D_0\left(\ddot{a}_{\overline{10}|}^{j\%} + 1.06^{10}v^{10}\ddot{a}_{\overline{\infty}|}^{i^*\%}\right)$$

[3]

where $\frac{1.06}{1.07} = \frac{1}{1+j} \Rightarrow j = 0.94340\%$ and $\frac{1.03}{1.07} = \frac{1}{1+i^*} \Rightarrow i^* = 3.88350\%$ [1]

and

$D_0 = \$0.20$ and $\ddot{a}_{\overline{10}|}^{i'} = 9.58975$ and $\ddot{a}_{\overline{\infty}|}^{i*} = 26.75$ [2]

$$P = \frac{0.20}{1.1449}\left(9.58975 + 0.910376 \times 26.75\right) = 5.929$$

[½]

Therefore, the required price is \$5.93 [½]

Many candidates failed to demonstrate how to translate a description of cashflows into an actuarial equation that accurately valued those cashflows.

A common error was leaving out the second (6%) growth rate.

A number of candidates missed out on marks by not giving their final answer to 2 decimal places.

Q6

Let i_t = annual effective spot rate for t-year period

$f_{t,1}$ = annual effective one-year forward rate at time t

107.60 = $6(v_{i_1} + v_{i_2}^2 + v_{i_3}^3) + 100v_{i_3}^3$ (1) [1]

100 = $6.5(v_{i_1} + v_{i_2}^2) + 100v_{i_2}^2$ (2) [1]

$(1+i_2)^2 = (1+i_1)\times(1+f_{1,1}) = (1+i_1)\times 1.045$ (3) [1]

From (3), $v_{i_2}^2 = v_{i_1} \times \frac{1}{1.045}$

$\Rightarrow$ from (2), 100 = $6.5\times v_{i_1} + (6.5+100)\times v_{i_1} \times \frac{1}{1.045}$

$\Rightarrow$ 100 = $v_{i_1} \times (6.5 + \frac{6.5}{1.045} + \frac{100}{1.045})$

$\Rightarrow i_1 = 8.41388\%$ p.a. [2]

And from (3), $i_2 = 6.43895\%$ p.a. [1]

And from (1), 107.60 = $6\times(v_{i_1} + v_{i_2}^2) + 106\times v_{i_3}^3 \Rightarrow i_3 = 3.08345\%$ p.a. [1]

Poorly answered.

Many candidates appeared to not understand the relationship between spot rates and forward rates, nor the meaning of par yield. This led many candidates to setting up incorrect formulae.

Many candidates also calculated the gross redemption yield for the 3-year fixed interest bond. This provided no additional information to solve the question and therefore no marks for awarded for this.

Q7 $EPV\ \ Expenses\ = 30,000 \times 0.03 \times Z + 250$ [1]

Where Z= EPV of \$1 pa of the reversionary annuities

$= a^{(12)\,m}_{65:\overline{15|}} + a^{(12)\,f}_{65:\overline{15|}} - 2a^{(12)\,mf}_{65:65:\overline{15|}}$ [2]

$$a^{(12)m}_{65:\overline{15|}} = a^{(12)m}_{65} - v^{15} \times \frac{l^m_{80}}{l^m_{65}} \times a^{(12)m}_{80} = \left(\ddot{a}^m_{65} - \frac{13}{24}\right) - v^{15} \times \frac{l^m_{80}}{l^m_{65}} \times \left(\ddot{a}^m_{80} - \frac{13}{24}\right)$$

$$= \left(13.666 - \frac{13}{24}\right) - v^{15} \times \frac{6,953.536}{9,647.797} \times \left(7.506 - \frac{13}{24}\right) = 10.3372$$

$$a^{(12)f}_{65:\overline{15|}} = a^{(12)f}_{65} - v^{15} \times \frac{l^f_{80}}{l^f_{65}} \times a^{(12)f}_{80} = \left(\ddot{a}^f_{65} - \frac{13}{24}\right) - v^{15} \times \frac{l^f_{80}}{l^f_{65}} \times \left(\ddot{a}^f_{80} - \frac{13}{24}\right)$$

$$= \left(14.871 - \frac{13}{24}\right) - v^{15} \times \frac{7,724.737}{9,703.708} \times \left(8.989 - \frac{13}{24}\right) = 10.5954$$

$$a^{(12)mf}_{65:65:\overline{15|}} = a^{(12)mf}_{65:65} - v^{15} \times \frac{l^m_{80}}{l^m_{65}} \times \frac{l^f_{80}}{l^f_{65}} \times a^{(12)mf}_{80:80} = \left(\ddot{a}^{mf}_{65:65} - \frac{13}{24}\right) - v^{15} \times \frac{l^m_{80}}{l^m_{65}} \times \frac{l^f_{80}}{l^f_{65}} \times \left(\ddot{a}^{mf}_{80:80} - \frac{13}{24}\right)$$

$$= \ = \left(11.958 - \frac{13}{24}\right) - v^{15} \times \frac{l^m_{80}}{l^m_{65}} \times \frac{l^f_{80}}{l^f_{65}} \times \left(5.857 - \frac{13}{24}\right) = 9.7230$$ [5½]

$\Rightarrow\ Z = 10.3372\ +\ 10.5954\ -\ 2 \times 9.7230\ = 1.4867$ [½]

Single Premium = EPV Expenses + EPV annuity

$30,000 \times 1.03 \times 1.4867 + 250 = 46,189.13$

Single Premium is \$46,189 [1]

Poorly answered.

Many candidates appeared to not understand how to value a reversionary annuity.

Several candidates missed part of the annuity payment details when valuing the benefit. For example, assuming the benefit was payable in advance, or annually, or assuming the benefit was payable for the whole of the policyholder's life.

One common error was using an incorrect adjustment to the tabulated annuity rates to arrive at the monthly in arrears annuity rate.

Q8

(i)

If $0 < t \le 4$ then accumulation to time t is

$$e^{\int_0^t \delta(t)dt} = e^{\int_0^t (0.06+0.02t)\,dt} = e^{(0.06t+0.01t^2)}$$ [1]

If $4 < t$ then accumulation to time t is

$$e^{\left[\int_0^4 (0.06+0.02t)dt + \int_4^t (0.08-0.01t)dt\right]}$$ [1]

$$= e^{\left(0.06\times 4+0.01\times 4^2\right)}.e^{\left[0.08t-0.005t^2\right]_4^t} = e^{0.4}.e^{\left[0.08t-0.005t^2-(0.32-0.08)\right]} = e^{\left(0.16+0.08t-0.005t^2\right)}$$ [1]

So:

a = 0, [½]

b = 0.06 [½]

c = 0.01 [½]

f = 0.16 [½]

g = 0.08 [½]

h= -0.005 [½]

(ii)

Accumulated amount at t=13 is:

$$600\times\frac{A(0,13)}{A(0,3)} + 900\times\frac{A(0,13)}{A(0,9)}$$ [2]

$$= 600\times\left(\frac{e^{0.355}}{e^{0.270}}\right) + 900\times\left(\frac{e^{0.355}}{e^{0.475}}\right)$$ [1]

= 653.23 + 798.23 = £1,451.46 [1]

(iii)

Let j% per monthly yield

Then $600(1+j)^{120} + 900(1+j)^{48} = 1,451.46$ [1]

It can be seen that j = 0% gives a left-hand side value of 1,500 which implies that j must be slightly negative [½]

Try j = -0.1% and left-hand side value is 1,389.92 (which is further from 1451.46 than 1500) [1]

Hence, answer is 0.0 % per month [½]

(iv)
The force of interest is negative for all times after time 8 [½]
So the 900 decreases in value from the time of investment onwards. Whereas the 600 increases for a period and then decreases [½]
The overall effect is that the accumulated value of the investments returns roughly to the original amounts invested, hence the overall yield is approximately 0.0% per annum [½]
[Marks available 1½, maximum 1]

Part (i) Generally well answered. Candidates should remember that the command verb "Determine" means that candidates will only be awarded full marks if they made clear their reasoning. Listing the numeric answers is not sufficient to gain full marks.

Part (ii) Many candidates appeared to be unfamiliar with how the accumulation factors derived in part (i) could be used to answer part (ii), and so used a valid alternative but more time-consuming approach.

A common error was to use the accumulation factors derive in part (i) but not to apply it from time zero.

Part (iii) Poorly answered. A common error was to calculate the effective interest rate per annum instead of per month.

Part (iv) Poorly answered.

Q9
(i)

EPV Premiums $P\ddot{a}^{4\%}_{30:\overline{30}|} = 50,000 \times v^{30}\dfrac{l_{60}}{l_{30}}$

$$P \times 17.756 = 50,000 \times 0.30832 \times \frac{9,287.2164}{9,925.2094} = 14,425.00 \Rightarrow P = 812.40$$
[½]

Reserve: ${}_{25}V = 50,000 \times v^5 \times \dfrac{l_{60}}{l_{55}} - 812.40 \times \ddot{a}_{55:\overline{5}|}$ [½]

$$= 50,000 \times 0.821927 \times \frac{9,287.2164}{9,557.8179} - 812.40 \times 4.585 = 36,207.97$$
[½]

Mortality profit

$$DSAR = 0 - 36,207.97 = -36,207.97$$
[1]

$$E(death) = (315+2) \times q_{54} = 317 \times 0.003976 = 1.2604$$
[1]

$$EDS = 1.2604 \times (-36,207.97) = -45,636.24$$
[½]

$ADS = 2 \times (-36,207.97) = -72,415.95$ [½]

$EDS - ADS = 26,779.71$

Mortality profit is $26,780. [½]

(ii)

A death leads to a release of reserve which contributes to profit [1]

With a Pure Endowment no death benefit is paid [1]

So higher than expected deaths leads to higher than expected profit [½]

The company expected approximately 1.3 deaths whereas 2 deaths actually occurred

So, mortality was heavier than expected [1]

Thus higher mortality led to a mortality profit [½]

[Marks available 4, maximum 3]

Well answered.

Common errors included using a mortality rate when calculating the number of expected deaths for an age that did not correspond to the reserve calculated and including a sum assured when calculating the death sum at risk.

In part (ii) well prepared candidates covered the importance of the release of the reserve

Q10

(i)

(Denoting the revised rates with a ' suffix)

If we assume forces of decrement are constant over individual years of age [½]

and that independent and dependent forces of decrement are equal, [½]

Then we can use: -

$$\mu_{47}^{w} = \frac{(ad)_{47}^{w}}{(ad)_{47}^{d} + (ad)_{47}^{w}} \times \left(-\ln(ap)_{47}\right) = \frac{1,500}{390 + 1,500} \times \left(-\ln(0.9622)\right) = 0.0305817$$ [2]

$$q_{47}^{w} = \left(1 - e^{-\mu_{47}^{w}}\right) = \left(1 - e^{-0.0305817}\right) = 0.0301188$$ [1]

The revised independent probability of withdrawal, q'^{w}_{47}, is therefore

$2.5 \times 0.0301188 = 0.0752970$ [½]

And

$$\mu_{47}^{'w} = -\ln\left(1 - q_{47}^{'w}\right) = 0.0782827$$ [1]

The force of mortality at age 47 implied by the ELT15 Females rates will be:

$$-\ln\left(1 - q_{47}^{ELT15(F)}\right) = -\ln(1 - 0.00219) = 0.00219240$$ [1]

and the revised independent force of mortality,

$$\mu_{47}^{'d} = 0.6 \times 0.00219240 = 0.00131544$$ [½]

(ii)
We have

$$(aq)_{47}^{'d} = \frac{\mu_{47}^{'d}}{\mu_{47}^{'d} + \mu_{47}^{'w}}\left(1 - e^{-\left(\mu_{47}^{'d} + \mu_{47}^{'w}\right)}\right)$$ [1]

$$= \frac{0.00131544}{0.00131544 + 0.0782827}\left(1 - e^{-(0.00131544+0.0782827)}\right) = 0.00126445$$ [1]

$$(aq)_{47}^{'w} = \frac{\mu_{47}^{'w}}{\mu_{47}^{'d} + \mu_{47}^{'w}}\left(1 - e^{-\left(\mu_{47}^{'d} + \mu_{47}^{'w}\right)}\right)$$

$$= \frac{0.078287}{0.00131544 + 0.0782827}\left(1 - e^{-(0.00131544+0.0782827)}\right) = 0.0752482$$ [1]

Thus

Age (x)	Number of employees $(al)_x$	Number of deaths $(ad)_x^d$	Number of withdrawals $(ad)_x^w$
47	50,000.00	63.22	3,762.41

[2]

(iii)
Concerns with the use of the revised multiple decrement table:

The population in the future may be different to the past population on which the analysis was conducted [1]

No allowance is made for future improvements in mortality or company initiatives that may reduce withdrawals [1]

There may be group events (such as workplace fire, outbreak of Covid) which may occur in the future but did not occur in the investigation period [1]

The investigation estimates may not be accurate [½]

The assumptions underlying the calculations in part (i) may not be valid [½]

The new table is significantly different from the original table [½]

Other valid points on why using the past to model the future may not be valid [½]

[Marks available 5, maximum 3]

[Total 15]

Part (i) Very poorly answered.

Most candidates did not appear to appreciate the difference between dependent and independent decrement rates, nor the difference between forces and rates of decrement. Most candidates missed out steps when deriving the required results. Many candidates lost out on marks as they did not quote the results to 6 significant figures.

Part (ii) Many candidates made a good attempt, even if they had struggled in part (i).

Part (iii) Poorly answered.

Q11

(i)

$$6{,}000\ddot{a}^{(12)}_{50:\overline{15}|6\%} = 0.6\times(6{,}000) + 0.04\times(6{,}000)\times\left(\ddot{a}_{50:\overline{15}|6\%} - 1\right)$$

$$+\frac{1}{1.019231}\times S\times A^{1}_{50:\overline{15}|j\%} + S\times\frac{l_{65}}{l_{50}}\times v^{15}_{j\%}\text{with } \frac{1.019231}{1.06} = \frac{1}{1+j} \Rightarrow j = 0.04$$

$$\text{with } \frac{1.019231}{1.06} = \frac{1}{1+j} \Rightarrow j = 0.04$$

[5]

$$\ddot{a}_{50:\overline{15}|6\%} = 10.038; \quad \frac{l_{65}}{l_{50}} = \frac{8{,}821.2612}{9{,}712.0728}; \quad A_{50:\overline{15}|4\%} = 0.56719;$$

$$A^{1}_{50:\overline{15}|4\%} = A_{50:\overline{15}|4\%} - \frac{l_{65}}{l_{50}}v^{15}_{4\%} = 0.06286$$ [2½]

$$\ddot{a}^{(12)}_{50:\overline{15}|6\%} = \ddot{a}_{50:\overline{15}|6\%} - \frac{11}{24}\left(1 - \frac{l_{65}}{l_{50}}v^{15}_{6\%}\right) = 9.753$$

$$\Rightarrow 6000\times 9.753 = 3{,}600 + 0.04\times 6{,}000\times 9.038 + S\left(0.06167 + 0.50433\right)$$

$$\Rightarrow S = 93{,}199$$

\$93,199 rounded to the nearest \$1,000 is \$93,000. [½]

(ii)

Require $93{,}000 = 6{,}000\ddot{s}^{(12)}_{\overline{15}|@i}$ [1]

Now

$$\ddot{s}^{(12)}_{\overline{15}|@\,0.434\%} = \frac{(1.00434)^{15} - 1}{d^{(12)}}$$

$$\text{where } \left(1 - \frac{d^{(12)}}{12}\right)^{-12} = 1.00434 \Rightarrow d^{(12)} = 0.004329828$$

$$\ddot{s}^{(12)}_{\overline{15}|@\,0.434\%} = 15.5007$$ [1]

$$6{,}000\ddot{s}^{(12)}_{\overline{15}|@\,0.434\%} = 6{,}000\times 15.50071$$

$$= 93{,}004.2 \approx 93{,}000$$ [½]

Therefore rate of return at least 0.434% per annum [½]

(iii)

Policyholder would expect a higher maturity benefit than basic sum assured of \$93,000 [½]
as they would expect bonuses to be declared every year [1]
A higher maturity benefit will give a higher return [½]

In addition to the survival benefit the policyholder will receive death cover during the term of the contact which is not measured by the return calculated in part (ii) but has value to the policyholder [1]

The policy may be obligatory, for example to back a mortgage [½]

This policy may be cheaper than others on the market [½]

Although 0.434% is low, it is guaranteed over 15 years and this guarantee might be important, for example for those planning to retire at 65 [½]

[Marks available 4½, maximum 3]

(iv)

$$\text{SA} = 93,000(1.05)^5 = 118,694$$ [1]

$${}_5V = \frac{118,694}{1.019231} A^1_{55:\overline{10}|4\%} + 118,694 v^{10}_{4\%} \frac{l_{65}}{l_{55}} + 0.04 \times 6,000 \ddot{a}_{55:\overline{10}|6\%} - 6,000 \ddot{a}^{(12)}_{55:\overline{10}|6\%}$$

[3]

$$\ddot{a}_{55:\overline{10}|6\%} = 7.610; \quad \frac{l_{65}}{l_{55}} = \frac{8,821.2612}{9,557.8179}; \quad A_{55:\overline{10}|4\%} = 0.68388;$$

$$A^1_{55:\overline{10}|4\%} = A_{55:\overline{10}|4\%} - \frac{l_{65}}{l_{55}} v^{10}_{4\%} = 0.06037$$

$$\ddot{a}^{(12)}_{55:\overline{10}|6\%} = \ddot{a}_{55:\overline{10}|6\%} - \frac{11}{24}\left(1 - \frac{l_{65}}{l_{55}} v^{10}_{6\%}\right) = 7.388$$

[1]

$${}_5V = 7,031.18 + 74,006.18 + 1826.40 - 44,327.25 = 38,536.51$$

Reserve is \$38,537 [1]

[Total 20]

Part (i) Generally well answered.

Common errors centred on how the benefits were valued, with many candidates not splitting the benefits into those payable on death and those on survival, in order to be able to apply the bonus adjustment only to the benefit on death. Another common error was valuing the renewal expenses using a monthly annuity rate.

Part (ii) Poorly answered.

Many candidates attempted to include mortality in their calculations. Candidates did not seem able to switch between the mathematics of finance and life contingencies within one question.

Part (iii) Many candidates failed to consider the nature of the benefits provided by a with profit endowment assurance and instead concentrated on the size of the premium.

Part (iv) Common errors included using a sum assured that ignored historic bonuses, using expense elements taken from the initial equation of value (e.g., including a deduction from the renewal expense annuity, or including an initial expense).

[Paper Total 100]

END OF EXAMINERS' REPORT