# INSTITUTE AND FACULTY OF ACTUARIES

# EXAMINATION

8 April 2025

## Subject CM1 – Actuarial Mathematics for Modelling Core Practices

## Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

**1** Calculate $A_{[70]:\overline{3}|}$.

Basis:
Mortality: AM92 Select
Interest: 5% p.a. effective

[3]

**2** An investor purchases an 8-year, fixed-interest bond at issue. The bond pays half-yearly coupons in arrears at a rate of 3% p.a. and will be redeemed at 104%.

The investor is subject to income tax at 20% and capital gains tax at 25%.

The bond is priced to achieve a net effective real yield of 6.5% p.a.

Calculate, showing all working, the price per $100 of the bond at issue. [5]

**3** An annuity provides a benefit of £20,000 p.a. payable annually in advance throughout the life of the policyholder aged 50 exact.

Calculate, showing all working:

(a) the expected present value of the benefits.

(b) the variance of the present value of the benefits.

Basis:
Mortality: AM92 Select
Interest: 4% p.a. effective

[4]

**4** The force of interest per annum at time $t$ years is given by:

$$\delta(t) = \begin{cases} 0.05 & 0 < t \le 6 \\ 0.026 + 0.004t & 6 < t \le 12 \end{cases}$$

£1,500 is invested at $t = 1$ year for a period of 5 years.

(i) Calculate, showing all working, the accumulated value of this investment at the end of the 5-year period. [1]

(ii) Calculate, showing all working, $i^{(12)}$, the annual nominal rate of interest convertible monthly, earned on the investment.

You should express your answer as a percentage rounded to the nearest three decimal places. [2]

A continuous payment stream is received at a rate of $e^{0.02t}$ units per annum between $t = 2$ and $t = 6$.

(iii) Calculate, showing all working, the accumulated value of the payment stream at $t = 10$. [5]

[Total 8]

**5** An annuity of £30,000 p.a. is payable monthly in arrears for a period of 10 years. The annuity is deferred for exactly 5 years.

Calculate, showing all working and using appropriate annuity functions, the present value of the deferred annuity, assuming that the following interest rates apply for the 15-year period:

- 4% p.a. effective for the first 8 years
- 6% p.a. nominal convertible monthly for the remaining 7 years.

[7]

**6** A life office issues a unit-linked endowment policy with a term of 3 years to a life aged 55 exact.

Details of the policy are as follows:

- The premium is £1,800 payable annually in advance.
- The death benefit, payable at the end of the year of death and after the Annual Management Charge (AMC) has been deducted, is the greater of £2,500 and the bid value of the units.
- The maturity value is equal to the bid value of the units.
- The unit fund cashflows (all figures in £) are:

| | *Year* | | |
|---|---|---|---|
| | *1* | *2* | *3* |
| Unit fund at start of year | 0.00 | 1,730.44 | 3,535.72 |
| Allocated premium | 1,710.00 | 1,710.00 | 1,710.00 |
| Bid-offer spread | −51.30 | −51.30 | −51.30 |
| Interest | 116.11 | 237.24 | 363.61 |
| Unit fund at end of year (before AMC) | 1,774.81 | 3,626.38 | 5,558.03 |
| AMC | −44.37 | −90.66 | −138.95 |
| Unit fund at end of year (after AMC) | 1,730.44 | 3,535.72 | 5,419.08 |

Calculate, showing all working:

(a) the profit signature.

(b) the expected present value of the profits.

Pricing basis:
Mortality: AM92 Ultimate
Rate of interest earned on non-unit cash flows: 5% p.a. effective
Expenses: £200 at the start of each policy year
Risk discount rate: 10% p.a. effective

[7]

**7** A 20-year loan of \$250,000 is repaid by level monthly repayments in arrears. The effective rate of interest applicable to the loan is 6% p.a. effective for the first 5 years and then 7.5% p.a. effective for the rest of the term.

(i) Calculate, showing all working and using appropriate annuity factors, the monthly repayment. [4]

(ii) Calculate, showing all working and using appropriate annuity factors, the capital and interest elements of the 61st instalment. [3]

(iii) Calculate, showing all working, the total interest paid on the loan. [2]

[Total 9]

**8** A special joint whole of life policy is issued to a male life aged 54 exact and a female life aged 51 exact. The policy pays £20,000 immediately on the second death.

Level premiums are payable monthly in advance ceasing after 10 years, or on the first death if earlier.

If both lives survive for 15 years from policy inception, a lump sum benefit equal to a full year of premiums is immediately payable.

Calculate, showing all working, the monthly premium payable.

Basis:
Interest rate: 4% p.a. effective
Mortality: PMA92C20 for the male and PFA92C20 for the female
Expenses: None

[9]

**9** A fixed-interest bond pays annual coupons in arrears of 6% p.a. and is redeemable at 103% in 3 years' time.

The following forward rates apply:

| | *Time, t (years)* | | |
|---|---|---|---|
| | *0* | *1* | *2* |
| $f_t$ (%) | 2.2 | 2.4 | 2.7 |

$f_t$ is the 1-year discrete forward rate of interest over the year from time $t$ to time $t + 1$.

(i) Calculate, showing all working:

(a) the price per \$100 nominal of the bond.

(b) the 3-year par yield.

(c) the gross redemption yield of the bond, using linear interpolation.

[7]

(ii) Explain how your answers to parts (i)(b) and (i)(c) would be affected, if at all, by an increase in the coupon rate. [3]

[Total 10]

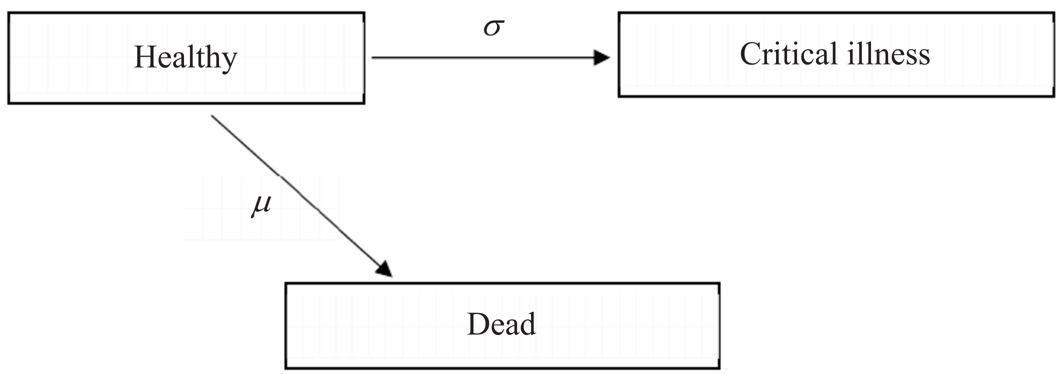
**10** A life insurance company issues 30-year term critical illness policies to lives aged 35 exact.

The benefit is a lump sum payable immediately on diagnosis of a critical illness as follows:

- No critical illness benefit is payable in the first 2 years of the policy.
- £100,000 if the critical illness is diagnosed between 2 and 10 years after issue.
- £150,000 if the critical illness is diagnosed between 10 and 30 years after issue.

Level premiums are payable continuously throughout the term from the date of issue and cease immediately on a claim.

The insurance company uses the following three-state continuous-time Markov model to calculate the premium and reserves for the policy.




(i) Calculate, showing all working, the annual premium payable continuously. [8]

(ii) Calculate, showing all working, the prospective gross premium reserve after 11 years. [3]

Basis:
Mortality: $\mu = 0.005$
Critical illness: $\sigma = 0.002$
Force of interest: 6% p.a.

[Total 11]

**11** On 1 January 2019 a life insurance company issued increasing whole life assurance policies to single lives aged 55 exact.

The death benefit, payable at the end of the year of death was £50,000 in the first policy year, thereafter increasing at the beginning of each subsequent policy year at a rate of 1.92308% p.a. compound.

The level annual premiums are payable annually in advance throughout the policyholder's life.

(i) Show that the premium is approximately £1,463. [3]

On 1 January 2024 there are 500 policies still in force. During 2024, 6 policyholders die.

(ii) Calculate, showing all working, the mortality profit or loss for the calendar year 2024. [7]

(iii) Comment on your answer to part (ii). [2]

Basis:
Mortality: AM92 Ultimate
Interest: 6% p.a. effective
Expenses: None

[Total 12]

**12** A life insurance company issues a 25-year with profit endowment assurance policy. The benefit is the basic sum assured plus any attaching bonus and is payable at the end of the year of death or on survival to the end of the term.

Level premiums are payable monthly in advance ceasing after 25 years or on the death of the policyholder if earlier.

Simple reversionary bonuses vest at the end of each policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

The company issues the policy to lives aged 40 exact. The basic sum assured is $250,000.

(i) Calculate, showing all working, the monthly gross premium for this policy. [12]

For the first 15 years of the policy, the actual bonuses declared were equal to those assumed when calculating the gross premium.

(ii) Write down the equation for the gross premium prospective reserve at the end of the 15th policy year. You are not required to perform any calculations. [3]

Basis:
Mortality: AM92 Ultimate
Interest rate: 6% p.a.
Commission:
Initial: 15% of the total premiums payable in the first policy year
Renewal: 2.5% of the second and subsequent monthly premiums
Expenses:
Renewal: $80 at the start of each policy year, excluding the first
Bonus: Simple bonus rate of 1% of basic sum assured

[Total 15]

## END OF PAPER

Institute and Faculty of Actuaries

# EXAMINERS' REPORT

## CM1 Actuarial Mathematics for Modelling

## Core Principles

## Paper A

April 2025

**Introduction**

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

For some candidates, this may be their first attempt at a professional qualification examination. The Examiners expect all candidates to have a good level of knowledge and understanding of the topics and therefore candidates should revise thoroughly to prepare for closed-book and in-person examinations. In our experience, candidates that are insufficiently prepared are not successful because of lack of knowledge, time management issues and/or because they do not properly answer the questions.

Many candidates rely on past exam papers and examiner reports in preparing for exams. Great caution must be exercised in doing so because each exam question is unique. As with all professional examinations, it is insufficient to repeat points of principle, formula or other textbook works. The examinations are designed to test "higher order" thinking including candidates' ability to apply their knowledge to the facts presented in detail, synthesise and analyse their findings, and present conclusions or advice. Successful candidates concentrate on answering the questions asked rather than repeating their knowledge without application.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
June 2025

## A. General comments on the *aims of this subject and how it is marked*

CM1 provides a grounding in the principles of modelling as applied to actuarial work focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations, but candidates are not penalised for this. However, candidates may not be awarded full marks where excessive rounding has been used or where insufficient working is shown.

Although the solutions show full actuarial notation, candidates were generally expected to use standard keystrokes in their solutions.

Candidates should pay attention to any instructions included in the questions. Failure to do so can lead to fewer marks being awarded. In particular, where the instruction, "showing all working" is included and the candidate shows little or no working, then the candidate will be awarded very few marks even if the final answer is correct.

Where a question specifies a method to use (e.g. determine the present value of income using annuity functions) then, if a candidate uses a different method, the candidate will not be awarded full marks, indeed, the candidate might even be awarded no marks.

Candidates are advised to familiarise themselves with the meaning of the command verbs (e.g. state, determine, calculate). These identify what needs to be included in answers in order to be awarded full marks.

## B. Comments on *candidate performance in this diet of the examination*

The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those candidates.

As in previous exam diets, there appeared to be a large number of insufficiently prepared candidates who had underestimated the quantity of study required for the subject.

Where candidates made numerical errors, the examiners awarded marks for the correct method used and also for the parts of the calculation that were correct. However, many candidates often did not show enough of their working to fully benefit from this.

## C. Pass Mark

The Pass Mark for this exam was 59.
1376 presented themselves and 548 passed.

## Solutions for Subject CM1A – April 2025

### Q1

$$A_{[70]:\overline{3}|} = q_{[70]} \times v + p_{[70]} \times q_{[70]+1} \times v^2 + {}_2p_{[70]} \times q_{72} \times v^3 + {}_3p_{[70]} \times v^3$$

or

$$A_{[70]:\overline{3}|} = q_{[70]} \times v + p_{[70]} \times q_{[70]+1} \times v^2 + {}_2p_{[70]} \times v^3$$

or

$$A_{[70]:\overline{3}|} = \frac{d_{[70]} \times v + d_{[70]+1} \times v^2 + d_{72} \times v^3 + l_{[70]} \times v^3}{l_{[70]}}$$

or

$$A_{[70]:\overline{3}|} = \frac{d_{[70]} \times v + d_{[70]+1} \times v^2 + l_{72} \times v^3}{l_{[70]}}$$

[1½]

$q_{[70]} = 0.016582$

$q_{[70]+1} = 0.024441$

$q_{72} = 0.030718$

$${}_2p_{[70]} = \frac{l_{72}}{l_{[70]}} = \frac{7,637.6208}{7,960.9776} = 0.959382275$$

$l_{73} = 7,403.0084$

$${}_3p_{[70]} = \frac{l_{73}}{l_{[70]}} = \frac{7,403.0084}{7,960.9776} = 0.929911975$$

Lookups [1]

Therefore, $A_{[70]:\overline{3}|}$ = 0.01579238 + 0.02180111 + 0.8287505 = 0.8663440

[½]

**[Total 3]**

| ***Commentary:*** |
|---|
| *An acceptable alternative was to calculate an annuity for the term and use a premium conversion formula to derive the endowment assurance factor.*<br><br>*A large proportion of candidates failed to recognise the need to work from first principles. Where a question uses interest rates other than those listed in the Formulae and Tables for Examinations, it is probable that the question requires candidates to derive the required factor from first principles. For this question using tabulated assurance or annuity rates did not result in candidates scoring any marks.*<br><br>*Common errors included:*<br>• *Incorrect treatment of select mortality, especially in year 3.*<br>• *Arithmetic slips.* |

- *Calculating a term assurance factor rather than the required endowment assurance factor. Candidates are reminded to read questions carefully to understand what is required.*

**Q2**

$i^{(2)} = 2(1.065^{0.5} - 1) = 6.397674\%$ [½]

Carry out the capital gains test:

$(1-t_1)\times\dfrac{D}{R} = 0.8\times\dfrac{3}{104} = 2.3076923\%$ [½]

$2.3076923\% < i^{(2)}$ [½]

So, there is a capital gain. [½]

$P = 0.8\times 3\times a_{\overline{8|}}^{(2)} + 104v^8 - 0.25\left(104-P\right)v^8$, evaluated at i=6.5% [2]

$a_{\overline{8|}}^{(2)} = \dfrac{1-1.065^{-8}}{0.06397674} = 6.18613557$ [½]

$P = \dfrac{61.976758}{0.8489422} = 73.0047$ i.e. \$73.00 [½]

**[Total 5]**

| ***Commentary:*** |
|---|
| *Well answered.*<br><br>*In order to gain full credit, candidates must include all the specified steps in the capital gains test.* |

**Q3**

(a)

$$\text{EPV} = E\left[20,000\,\ddot{a}_{\overline{K_{[50]}+1|}}\right]$$

$$= 20,000\times\ddot{a}_{[50]}$$

Where $\ddot{a}_{[50]} = 17.454$

EPV = £349,080 [1½]

(b)

$$Var\left(20,000\times\ddot{a}_{\overline{K_{[50]+1}|}}\right) = 20,000^2\times Var\left(\ddot{a}_{\overline{K_{[50]+1}|}}\right)$$

$$= 20,000^2 \times Var\left[\frac{1-v^{K_{[50]}+1}}{d}\right]$$

$$= 20,000^2 \times \frac{1}{d^2} \times \left({}^2A_{[50]} - \left(A_{[50]}\right)^2\right)$$

[1½]

Where $d = 0.038462$ and ${}^2A_{[50]} = 0.13017$ $\quad A_{[50]} = 0.32868$ [½]

Therefore, the variance is £$^2$ 5,986,509,335 (= £77,373$^2$) [½]

**[Total 4]**

| ***Commentary:*** |
|---|
| *Part (a)*<br>*Generally, well answered.*<br><br>*Part (b)*<br>*Generally, poorly answered.*<br>*Common errors included:*<br>• *Using an incorrect formula, that was not in terms of assurance factors.*<br>• *Using the sum assured and/or d, rather than the squared values of those terms.* |

**Q4**

(i)

$$1,500\exp\left(\int_1^6 0.05dt\right) = 1,500\exp\left(0.05\times(6-1)\right) = £1,926.038125$$

[1]

(ii)

$$1,500\times\left(1+\frac{i^{(12)}}{12}\right)^{12\times5} = 1,926.038125 \Rightarrow i^{(12)} = 0.050104311$$

i.e. 5.010% pa convertible monthly

or

$$\left(1+\frac{i^{(12)}}{12}\right)^{12} = \exp(0.05) \Rightarrow i^{(12)} = 0.050104311$$ = 5.010% pa convertible

monthly [2]

(iii)

Accumulated value at t=6

$$\int_2^6 \exp(0.02t)\times\exp\int_t^6 0.05ds dt$$

$$=\int_2^6 \exp(0.02t)\times\exp(0.05\times(6-t))dt$$

$$=\int_2^6 \exp(0.02t)\times\exp(0.3)\times\exp(-0.05t)dt$$

$$=\exp(0.3)\int_2^6 \exp(-0.03t)dt=\exp(0.3)\times\left[\frac{\exp(-0.03t)}{-0.03}\right]_2^6$$ [2]

$$=\exp(0.3)\times\left(\frac{\exp(-0.18)-\exp(-0.06)}{-0.03}\right)=\exp(0.3)\times 3.549810739=£4.791743291$$ [½] + [½]

The accumulation factor from time t = 6 to time t = 10 is given by:

$$\exp\left(\int_6^{10} 0.026+0.004t dt\right)=\exp\left[0.026t+0.002t^2\right]_6^{10}$$ [½] + [½]

$$=\exp[(0.26+0.2)-(0.156+0.072)]=\exp(0.232)$$ [½]

accumulated value at time t = 10 is

3.549810739 x exp(0.3) x exp(0.232) = £6,042962 [½]

**[Total 8]**

| ***Commentary:*** |
|---|
| *Approximately half of candidates performed poorly in parts (i) and (ii), showing a complete lack of understanding of integrating even simple continuous payments.*<br><br>*Part (iii) covered a more complex scenario and was poorly answered. Candidates showed poor understanding of continuous payment streams. Answers were also often difficult to follow due to lack of explanatory text and/or missing steps.*<br><br>*This is a syllabus item that clearly requires more study for the majority of candidates.* |

## Q5

It is given that $i_1 = 4\%$ and $i_2^{(12)} = 6\%$. Working with annual time units, we have:

$$PV=30{,}000\times\left[a_{\overline{3}|i_1}^{(12)}+a_{\overline{7}|i_2}^{(12)}v_1^3\right]\times v_1^5$$ [3]

$$30{,}000\times\left[\frac{1-v_1^3}{i_1^{(12)}}+\frac{1-v_2^7}{i_2^{(12)}}v_1^3\right]\times v_1^5$$

$i_1^{(12)}$, $i_2$ come from:

$$\left(1+\frac{i_1^{(12)}}{12}\right)^{12} = 1 + i_1 \qquad \Rightarrow \qquad i_1^{(12)} = \left[\sqrt[12]{1.04} - 1\right] \times 12 = 0.03928487739$$

[1]

$$\left(1+\frac{i_2^{(12)}}{12}\right)^{12} = 1 + i_2 \qquad \Rightarrow \qquad 1 + i_2 = \left(1+\frac{0.06}{12}\right)^{12} = 1.005^{12}$$

[½]

Therefore:

$$\text{PV} = 30{,}000 \times \left[\frac{1-v_1^3}{i_1^{(12)}} + \frac{1-v_2^7}{i_2^{(12)}} v_1^3\right] \times v_1^5$$

$$PV = 30{,}000 \times \left[\frac{1-1.04^{-3}}{0.03928487739} + \frac{1-1.005^{-7\times 12}}{0.06} 1.04^{-3}\right] \times 1.04^{-5}$$

$$= 30{,}000 \times (2.825607427 + 5.704420203 \times 0.8889963587) \times 0.82193$$

$$= £194{,}718.22$$

[2½]

**[Total 7]**

| ***Commentary:*** |
|---|
| *Generally, well answered.*<br><br>*Common errors included:*<br>• *Not converting the provided interest rates correctly.*<br>• *Using incorrect time periods for the annuity calculations.*<br>• *Using an incorrect interest rate when deferring a payment.*<br><br>*Alternative solutions based on monthly time units were accepted.* |

**Q6**

(a) The cash flows into and out of the non-unit fund over the duration of the contract are:

| Year | 1 | 2 | 3 |
|---|---|---|---|
| Unallocated premium | 1800-1710<br>=90.00 | 90.00 | 90.00 |
| Bid-offer spread | 51.30 | 51.30 | 51.30 |
| Expenses | -200.00 | -200.00 | -200.00 |

| | | | |
|---|---|---|---|
| Interest | 0.05 x (90+51.30-200)<br>=-2.94 | -2.94 | -2.94 |
| Management charge | 44.37 | 90.66 | 138.95 |
| Additional death cost | (2500-1730.44) x 0.004469<br>=-3.44<br>Where $q_{55} = 0.004469$ | 0.00<br>Since Unit fund at end of year (after MC)>2500 | 0.00<br>Since Unit fund at end of year (after MC)>2500 |
| Profit vector | -20.71 | +29.02 | +77.31 |

Then, the profit signature is given by:

| Year | 1 | 2 | 3 |
|---|---|---|---|
| Profit vector | -20.71 | +29.02 | +77.31 |
| In-force probability | 1.000000 | 0.995531 | 0.990528 |
| Profit signature | -£20.71 | +£28.89 | +£76.58 |

unallocated premium [½]
interest [1]
B/O spread, expenses and mgt charge being brought in from question correctly [½]
additional death benefit figures [1½]
profit vector [½]
in force probs [1]
profit signature figures [½]
**[5½]**

b)

Using a risk discount rate of 10% per annum, the present value of future profits is given by:

$-20.71v_{10\%} + 28.89v_{10\%}^2 + 76.58v_{10\%}^3 = £62.58$ [1½]

**[Total 7]**

| ***Commentary:*** |
|---|
| *This was an easy unit-linked question; however, the majority of candidates appear to have been surprised by the question and answered it poorly. Candidates are reminded that any part of the syllabus can be examined in either of the A and B papers.*<br><br>*Some candidates who produced good attempts lost marks through confusing the profit signature with the profit vector and vice versa.*<br><br>*For full credit, candidates must show their workings. It is not sufficient to provide a table of figures without a full explanation of the calculations behind the figures.* |

**Q7**

(i)

X= initial monthly instalment

$250,000 = 12Xa^{(12)}_{\overline{5}|6\%} + 12X \times v^{5}_{6\%} \times a^{(12)}_{\overline{15}|7.5\%}$ [2]

$i = 6\% \Rightarrow i^{(12)} = 5.8411\%\ (table) \qquad (5.8410607\%,\ calc)$

$i = 7.5\% \Rightarrow i^{(12)} = 7.2539028\%\ (calc)$

$a^{(12)}_{\overline{5}|6\%} = 4.326985\ (calc) \quad \text{or} \quad \left(\dfrac{0.06}{0.058411}\right) \times 4.2124 = 4.32699\ (table)$

$v^{5}_{6\%} = 0.74726\ (table) \quad \text{or} \quad 0.747258173\ (calc)$

$a^{(12)}_{\overline{15}|7.5\%} = 9.126590\ (calc)$ [1]

$250,000 = 12X \times 4.326985 + 12X \times 0.74726 \times 9.126590$

$250,000 = X \times 133.762851$

$X = \$1,868.979309$ [1]

(ii)

capital outstanding after 60th instalment has been paid

$12 \times 1,868.979309 \times a^{(12)}_{\overline{15}|7.5\%} = 12 \times 1,868.979309 \times 9.126590 = \$204,688.897807$ [1½]

Interest component: $204,688.897807 \times \dfrac{0.072539028}{12} = \$1,237.327812$ [1]

Capital component: $1\ 868.979309 - 1,237.327812 = \$631.651497$ [½]

(iii)

$12 \times 1,868.979309 \times 20 - 250,000 = 448,555.034268 - 250,000 = \$198,555.034268$ [2]

**[Total 9]**

| ***Commentary:*** |
|---|
| *Part (i) was generally very well answered.*<br><br>*Part (ii) was generally well answered.*<br>*Common errors included:*<br>• *Using an incorrect term for calculating the capital outstanding.*<br>• *Applying an incorrect interest rate to derive the interest element.*<br><br>*Part (iii) was generally well answered.* |

**Q8**

Premiums:

$$12P\ddot{a}^{(12)}_{54:51:\overline{10}|} = 98.28399P$$

$$\ddot{a}^{(12)}_{54:51:\overline{10}|} = \ddot{a}^{(12)}_{54:51} - \left(v^{10} \times \frac{l_{64:61}}{l_{54:51}}\right) \times \ddot{a}^{(12)}_{64:61}$$

$$= 16.285667 - \left(1.04^{-10} \times \frac{9,696.990 \times 9,828.163}{9,914.580 \times 9,947.452}\right) \times 12.400667$$

$$= 8.190333$$

$$\ddot{a}^{(12)}_{54:51} = \ddot{a}_{54:51} - \frac{11}{24} = 16.744 - \frac{11}{24} = 16.285667$$

$$\ddot{a}^{(12)}_{64:61} = \ddot{a}_{64:61} - \frac{11}{24} = 12.859 - \frac{11}{24} = 12.400667$$

[3]

Death benefit:

$$EPV = 20,000 \times \bar{A}_{\overline{54:51}} = 20,000 \times 0.226436 = 4,528.713793$$

$$\bar{A}_{\overline{54:51}} = (1+i)^{0.5} \times A_{\overline{54:51}} = 1.04^{0.5} \times 0.222038 = 0.226436$$

$$A_{\overline{54:51}} = 1 - \frac{i}{1+i} \times \ddot{a}_{\overline{54:51}} = 1 - \frac{0.04}{1.04} \times 20.227 = 0.222038$$

$$\ddot{a}_{\overline{54:51}} = \ddot{a}_{54} + \ddot{a}_{51} - \ddot{a}_{54:51} = 17.680 + 19.291 - 16.744 = 20.227$$

[3]

<u>Alternative</u>:

$$EPV = 20,000 \times \bar{A}_{\overline{54:51}} = 20,000 \times (\bar{A}_{54} + \bar{A}_{51} - \bar{A}_{54:51})$$

$$\bar{A}_{54} = 1 - \delta\bar{a}_{54}$$

$$\bar{a}_{54} = \ddot{a}_{54} - \frac{1}{2}$$

Survival bonus:

$$EPV = 12P \times {}_{15}p_{54}^{m} \times {}_{15}p_{51}^{f} \times v^{15}$$

$$= 12P \times \frac{9658.285}{9947.452} \times \frac{9346.970}{9914.580} \times 1.04^{-15} = 6.099101P$$ [2]

EPV(Benefits)=EPV(Premiums)

$\Rightarrow 4{,}528.713793 + 6.099191P = 98.28399P$

$P = 49.12642$

$P = £49.13$ [1]

**[Total 9]**

| ***Commentary:*** |
|---|
| *Common errors included:*<br>• *Incorrect formulae for the monthly premium adjustment.*<br>• *Using a monthly premium when calculating the survival bonus.*<br>• *Ignoring the claims acceleration factor.*<br>• *Ignoring interest when calculating the survival bonus (i.e. ignoring the $v^{15}$ term).*<br><br>*There are several methods of calculating the value of death benefits. Any correct alternative approach to that given above were given full credit.* |

**Q9**

(i)(a)

$$P = \frac{6}{1.022} + \frac{6}{1.022 \times 1.024} + \frac{109}{1.022 \times 1.024 \times 1.027}$$ [1½]

=113.0197873

=$113.02 [½]

(i)(b)

$$100 = \frac{C}{1.022} + \frac{C}{1.022 \times 1.024} + \frac{100 + C}{1.022 \times 1.024 \times 1.027}$$ [1½]

$C$ = 2.4291266% [½]

(i)(c )

$113.02 = 6a_{\overline{3|}}^{i\%} + 103v^{3}$, where i is the gross redemption yield. [1]

$i = 2.0\% \Rightarrow RHS = 114.363$

$i = 2.5\% \Rightarrow RHS = 112.782$

[1]

Using interpolation,

$$i \approx 0.025 - (0.025 - 0.02) \times \frac{113.02 - 112.782}{114.363 - 112.782} = 2.42\%$$

[1]

(ii)

The par yield is a measure of the relationship between the term of a bond and the yield.
The par yield does not depend on the coupon rate. [½]
Therefore, the par yield would remain unchanged. [½]
The GRY is a weighted average of the forward rates, weighted by cashflow at that duration. A higher coupon rate increases the weight applied to the earlier cashflows. [1]
The forward rates at shorter durations are lower, so the GRY will reduce. [1]

**[Total 10]**

| ***Commentary:*** |
| --- |
| *Part (i) was generally well answered. Although many candidates did not demonstrate understanding of the concept of a par yield. Candidates should note that in order to gain full credit the linear interpolation in (i)(c) needed to be shown in full.*<br><br>*Part (ii) was generally answered poorly.*<br>*The question is asked in the context of the term structure of interest rates (spot rates and forward rates) and so the explanation should be given in the same context.*<br>*Most candidates referred to an increase in price. This statement, although true, did not address the question asked and gained no credit.* |

**Q10**

(i)

$${}_t p_{35}^{HH} = \exp\left[-\int_0^t (\mu + \sigma) ds\right] = \exp\left[-\int_0^t (0.005 + 0.002) ds\right] = \exp(-0.007t) = {}_t p_{35}^{\overline{HH}}$$

[1]

Let P= annual premium

EPV of premiums =

$$P\int_0^{30} e^{-\delta t} \times {}_t p_{35}^{HH}\, dt = P\int_0^{30} e^{-0.06t} \times e^{-0.007t} dt$$

$$= P\int_0^{30} e^{-0.067t} dt = P \times \left[\frac{e^{-0.067t}}{-0.067}\right]_0^{30} = P \text{ x } 12.92554$$

[2]

EPV of benefits =

$$100{,}000\int_{2}^{10} e^{-\delta t}\times {}_tp_{35}^{HH}\times\sigma\, dt+150{,}000\int_{10}^{30} e^{-\delta t}\times {}_tp_{35}^{HH}\times\sigma\, dt$$

$$=100{,}000\int_{2}^{10} e^{-0.06t}\times e^{-0.007t}\times 0.002\; dt+150{,}000\int_{10}^{30} e^{-0.06t}\times e^{-0.007t}\times 0.002\; dt$$ [3]

$$=\frac{200}{0.067}\left[e^{-2\text{x}0.067}-e^{-10\text{x}0.067}\right]+\frac{300}{0.067}\left[e^{-10\text{x}0.067}-e^{-30\text{x}0.067}\right]$$ [1]

$$=1083.228+1691.283\;=\;2774.511$$

$$\Rightarrow P=\frac{2774.511}{12.92554}=£214.65\text{ pa}$$ [1]

(ii)

$${}_{11}V=150{,}000\int_{0}^{19} e^{-0.06t}\times e^{-0.007t}\times 0.002\;\; dt\;\text{ - }P\int_{0}^{19} e^{-0.067\,t}dt$$ [1]

$$=300\int_{0}^{19} e^{-0.067t}\; dt\;\text{ - }\;214.65\int_{0}^{19} e^{-0.067t}dt$$ [1]

$$= (300-214.65)\text{ x }\frac{1}{0.067}\left[e^{-0\text{x}0.067}-e^{-19\text{x}0.067}\right]$$

$= £917.17$ [1]

**[Total 11]**

| ***Commentary:*** |
|---|
| *Generally, very poorly answered.*<br><br>*In part (i) many candidates demonstrated their lack of understanding of continuous time Markov models.*<br><br>*Part (ii)*<br>*A common error was using incorrect boundaries for the integration. For a prospective reserve we are looking towards the future, starting from now (t=0). So, we need to integrate from zero to the end of the term.*<br><br>*The question contained an ambiguity in the conditions under which premiums cease to be paid. Candidates were not penalised where they interpreted the payment term differently from the solution above.* |

**Q11**

(i)

The sum assured increases by 1.92308% (= e) so the present value of benefits is given by:

$$\frac{50,000\times q_{55}}{1+i}+\frac{50,000\times(1+e)\times p_{55}\times q_{56}}{(1+i)^2}+\frac{50,000\times(1+e)^2\times {}_2p_{55}\times q_{57}}{(1+i)^3}+....$$

$$=\frac{50,000}{1+e}\times\left(q_{55}\times\frac{1+e}{1+i}\ +p_{55}\times q_{56}\times\left(\frac{1+e}{1+i}\right)^2\ +\ {}_2p_{55}\times q_{57}\times\left(\frac{1+e}{1+i}\right)^3+...\right)$$

$$=\frac{50,000}{1+e}\times A_{55}^{j\%}$$

Premium is therefore given by:

$$P=\frac{50,000\times(1.0192308)^{-1}\times A_{55}^{4\%}}{\ddot{a}_{55}^{6\%}}$$ [1½]

Where $\ddot{a}_{55}^{6\%}=13.057$ $\quad A_{55}^{4\%}=0.38950$ [½]

$$j\%=\frac{1.06}{1.0192308}-1\approx 4\%$$ [½]

Therefore $P=\dfrac{19,107.54659}{13.057}$ = £1,463.39 which is approximately equal to £1,463. [½]

(ii)

Calculate the reserve at t=6 (at the end of 2024)

$$V_{t=6}=50,000\times(1.0192308)^5\times A_{61}^{4\%}-1,463\times\ddot{a}_{61}^{6\%}$$ [2]

Where $\ddot{a}_{61}^{6\%}=11.638$ $\quad A_{61}^{4\%}=0.47041$ [½]

Therefore, the reserve is £8,844.37 [½]

The death strain at risk is given by:

$DSAR=50,000\times(1.0192308)^5-V_{t=6}=£\,46,151.83$ [1½]

The expected death strain is given by:

$EDS=500\times q_{60}\times DSAR$ where $q_{60}=0.008022$, hence EDS = £185,115 [1½]

The actual death strain is given by:

$ADS=6\times DSAR$, hence ADS = £ 276,911.00 [½]

Mortality profit is equal to EDS – ADS,

Mortality profit is thus -91,796.00 i.e. a loss of £91,796 [½]

(iii)

On the death of a policyholder the company will release the reserve held but will pay out the sum assured. The difference is the death strain. Here the sum assured (the amount the company must pay out) is greater than the reserve held – so the company experiences a mortality loss on unexpected deaths. [1]
The company expected 4.011 deaths but actually experienced 6 deaths. [½]

As there were more deaths than expected and the DSAR is positive the company has made a loss.
or

As there were more deaths than expected and this is an assurance benefit (so benefit is payable on death) the company has made a loss. [½]

**[Total 12]**

| ***Commentary:*** |
|---|
| *Part (i) generally well answered.*<br><br>*Part (ii)*<br>*Common errors included:*<br>• *Calculating the reserve for the wrong policy year.*<br>• *Using an incorrect sum assured in the reserve and/or the death strain at risk calculations.*<br>• *Using a mortality rate that was inconsistent with the policy year of the reserve.*<br><br>*Part (iii) was poorly answered. Candidates generally failed to explain why a change from the expected mortality would cause a mortality profit or loss* |

**Q12**

(i)

<u>EPV of premium:</u>

<u>P=monthly premium</u>

$$12P\ddot{a}^{(12)}_{40:\overline{25|}} = 12P\left(\ddot{a}_{40:\overline{25|}} - \frac{11}{24}\left(1 - v^{25} \times {}_{25}p_{40}\right)\right)$$ [1]

$$= 12P\left(13.288 - \frac{11}{24}\left(1 - 0.232999 \times 0.894988\right)\right) = 12.9252434 \times 12P = 155.1029208P$$ [½]

$\ddot{a}_{40:\overline{25|}} = 13.288$

$l_{40} = 9856.2863$

$l_{65} = 8821.2612$

lookups [1]

<u>EPV of benefit</u>

$$= 247,500 A_{40:\overline{25|}} + 2,500 \left(IA\right)^1_{40:\overline{25|}} + \underbrace{\left(312,500 - 247,500\right)}_{65,000} A_{40:\overline{25|}}^{\;\;\;1}$$ [3]

$A_{40:\overline{25|}} = 0.24787$ [½]

$A_{40:\overline{25|}}^{\;\;\;1}$ calculated above

$$\left(IA\right)^1_{40:\overline{25|}} = \left(IA\right)_{40} - v^{25} \times {}_{25}p_{40} \left[\left(IA\right)_{65} + 25A_{65}\right]$$ [1]

$$= 3.85435 - 0.208531055[15.5541] = 0.61083712$$ [1]

$\left(IA\right)_{40} = 3.85435$

$\left(IA\right)_{65} = 5.50985$

$A_{65} = 0.40177$

lookups [1]

EPV benefits $= 247,500 \times 0.24787 + 2,500 \times 0.61083712 + 65,000 \times 0.208531055$

$= 76,429.436364312$

_Alternative_:

$$247,500 A_{40:\overline{25|}} + 2,500 \left(IA\right)_{40:\overline{25|}} + 2,500 A_{40:\overline{25|}}^{\;\;\;1}$$

$$\left(250,000 - 2,500\right) A^1_{40:\overline{25|}} + 2,500 \left(IA\right)^1_{40:\overline{25|}} + 312,500 A_{40:\overline{25|}}^{\;\;\;1}$$

$$247,500 A^1_{40:\overline{25|}} + 2,500 \left(IA\right)^1_{40:\overline{25|}} + 312,500 A_{40:\overline{25|}}^{\;\;\;1}$$

Where

$$A^1_{40:\overline{25|}} = A_{40:\overline{25|}} - v^{25} \times {}_{25}p_{40} = 0.24787 - 0.208531 = 0.039338945$$

$$\left(IA\right)^1_{40:\overline{25|}} = \left(IA\right)_{40} - v^{25} \times {}_{25}p_{40} \left[\left(IA\right)_{65} + 25A_{65} - 25\right] = 5.8241135$$

EPV of Expenses and Commissions

$$0.15 \times 12P + 0.025P\left(12\ddot{a}^{(12)}_{40:\overline{25|}} - 1\right) + 80\left(\ddot{a}_{40:\overline{25|}} - 1\right)$$ [2½]

$$= 0.15 \times 12P + 0.025P\left(155.1029208 - 1\right) + 80\left(13.288 - 1\right)$$

$$= 5.65257302 \times P + 983.04$$

Therefore,

$$P \times 155.1029208 = 76,429.436364312 + 5.65257302 \times P + 983.04$$

$$P \times 149.4503478 = 77,412.476364312 \Rightarrow P = \$517.98123944$$ [½]

(ii)

$$ {}_{15}V = 285,000A^{1}_{55:\overline{10|}} + 2,500\left(IA\right)^{1}_{55:\overline{10|}} + 312,500A_{55:\overset{\;\;1}{\overline{10|}}} $$

$$ +0.025 \times 12P\left(\ddot{a}^{(12)}_{55:\overline{10|}}\right) + 80\ddot{a}_{55:\overline{10|}} - 12P\ddot{a}^{(12)}_{55:\overline{10|}} $$

[3]

**[Total 15]**

| ***Commentary:*** |
|---|
| *In part (i), many candidates struggled to calculate correctly the expected present value of benefits and the expected present value of expenses. Candidates' use of actuarial notation was often not precise enough, in particular confusion between increasing term assurance functions and increasing endowment assurance functions.*<br>*In the EPV of benefits, the benefit payable on survival was often incorrect.*<br>*In the EPV of expenses, the renewal commission and renewal expenses were often incorrect.*<br>*Attention to detail is essential in questions such as this where a large quantity of information is provided.*<br><br>*In part (ii) common errors included: -*<br>• *Incorrectly calculating the sum assured at the required date.*<br>• *Omitting the premium component.* |

**[Paper Total 100]**

# END OF EXAMINERS' REPORT

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