INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

26 April 2023 (am)

Subject CM2 – Financial Engineering and Loss Reserving Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 A singer has the following utility function:

$$U(w) = \ln(w) \quad (w > 0).$$

(i) Show that this utility function is iso-elastic. [3]

The singer measures their wealth as the present value of their future earnings. They are concerned about possible future damage to their voice and consider the following scenarios:

Scenario	*Probability (%)*	*Present value of earnings ($)*
No damage	94.9	1,500,000
Slight damage	5.0	1,300,000
Severe damage	0.1	10,000

The singer is considering buying insurance to cover their loss of earnings if their voice is damaged. The insurance will pay a lump sum equal to the reduction in the present value of earnings compared to the 'no damage' scenario.

(ii) Calculate the singer's expected loss when compared to no damage. [1]

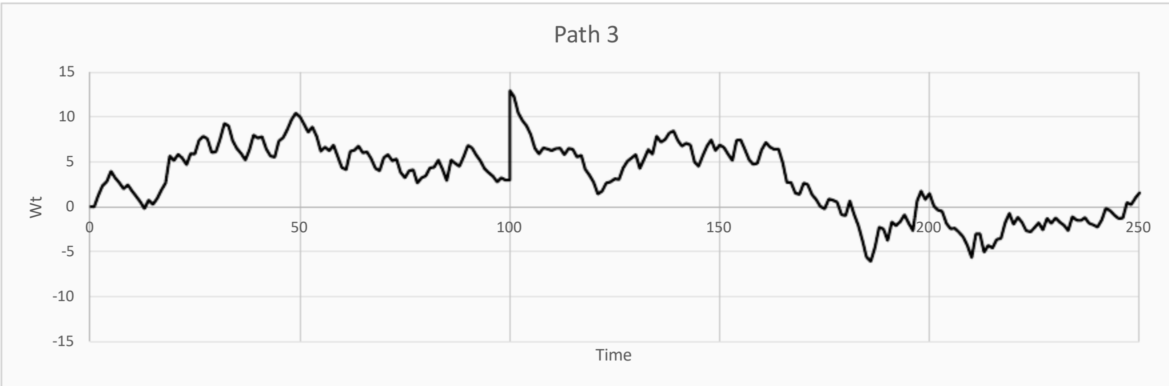
(iii) Calculate the maximum premium the singer would be willing to pay for this insurance based on their utility function. [3]

(iv) Discuss briefly what your answers to parts (ii) and (iii) suggest about the singer's appetite to risk. [3]

(v) Suggest two reasons why an insurer may not want to cover this risk. [5]
[Total 15]

2 (i) State three potential uses for stochastic security price models. For each use, explain why the model should be stochastic. [3]

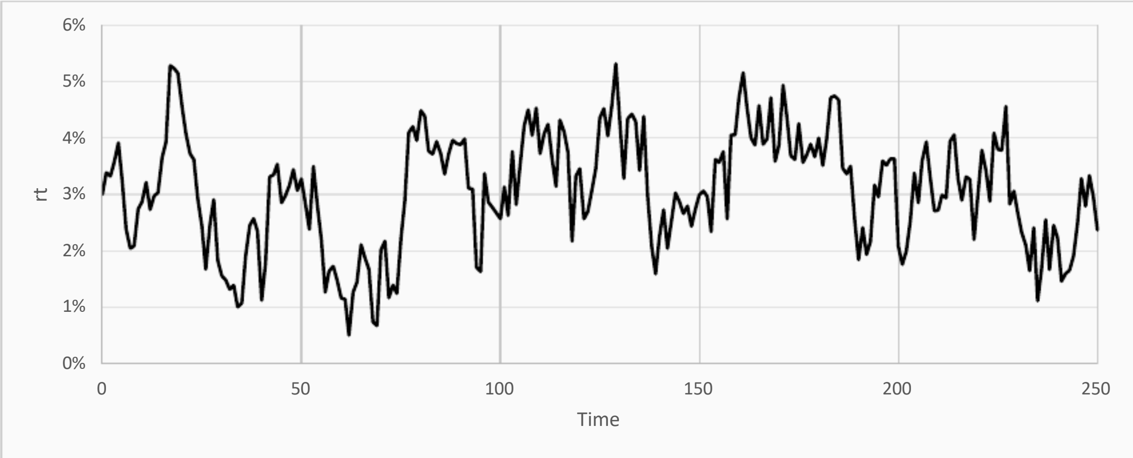
The charts below show sample paths for three different stochastic processes.



(ii) For each chart explain if, and why, the path is likely to be a Wiener process. [3]

The chart below shows a single sample path of a Vasicek model represented by the following stochastic differential equation:

$$dr_t = \kappa(\theta - r_t)dt + \sigma d\widehat{W}_t$$



(iii) Describe the likely impact on the chart of the following parameter changes:

(a) An increase in σ

(b) An increase in θ

(c) A reduction in κ.

[4]
[Total 10]

3 A company specialises in long-term insurance contracts. It holds reserves in respect of these contracts and it wants to build a model of its future investment returns.

(i) Identify the similarities and differences between a deterministic and stochastic model of investment returns. [3]

(ii) Explain why the company may prefer to use a stochastic model. [3]

The company has decided to build a stochastic model. It assumes that the rate of return in each calendar year is Normally distributed and independent of the rates of return in previous years. The company plans to review this model once every 2 years.

(iii) Set out **two** possible disadvantages of using this stochastic model. [2]

(iv) Suggest how the company may overcome each disadvantage identified in part (iii). [2]
[Total 10]

4 A biologist is modelling the population of a group of tortoises on an island over the next 5 years. There are currently 100 tortoises and the biologist has decided it is appropriate to model the population after n years as:

$$S_n = 100(1 + i_1)(1 + i_2)\ldots.(1 + i_n)$$

where:

- i_k is the growth rate of the tortoise population in year k.
- i_k are independent and identically distributed.
- $\log(1 + i_k) \sim N(\mu, \sigma^2)$ for each k.

(i) Write down the distribution of $\log(S_5)$. [1]

The biologist has calculated that:

- there is a 99% chance that $S_5 \geq 75$.
- there is a 99% chance that $S_5 \leq 140$.

(ii) Determine the values of μ and σ. [5]

A university research team has asked the biologist to determine how likely it is that there will be at least 100 tortoises after 10 years. The biologist assumes that the model can be extended to S_{10} with $i_6, i_7, \ldots i_{10}$ having the same properties as $i_1, i_2, \ldots i_5$.

(iii) Calculate the probability that $S_{10} \geq 100$. [3]

(iv) Identify the risks of the biologist using this model to produce the answer in part (iii). [2]

[Total 11]

5 In a Black–Scholes market, the stock price is given by:

$$S_t = S_0 \exp(0.2B_t + 0.2t)$$

where B_t is a standard Brownian motion under the real-world probability measure.

A derivative security written on the stock in the same market at time t has price:

$$D_t = 2\exp\left(0.6\left(\widetilde{B}_t - ct\right) + 0.39t\right)$$

where $\widetilde{B}_t$ is a standard Brownian motion under the equivalent martingale measure.

(i) Determine the value of c such that $B_t + ct$ is a standard Brownian motion under the equivalent martingale measure. [7]

(ii) Calculate the risk-free rate of interest. [1]

[Total 8]

6 An insurer has been offered two possible investments.

Investment A: a derivative that will produce a payoff $\$X_A$ as set out below:

X_A	*Probability*
3.0	$^1/_3$
2.6	$^1/_3$
1.0	$^1/_3$

Investment B: a diversified portfolio that will produce a payoff of $\$X_B$, where

$$X_B = 0.2 + N$$

and N is a Normal random variable with $\mu = 2$ and $\sigma = 2$.

The insurer uses a quadratic utility function.

(i) Calculate, for each investment:

(a) the expected payoff.

(b) the variance of the payoff.

[3]

(ii) State, with reasons, which investment the insurer would choose. [2]

(iii) Calculate, for each investment, the shortfall probability of the payoff falling below:

(a) \$0.5

(b) \$2.0.

[3]

(iv) Explain whether either investment would be more attractive for this investor in all circumstances. [2]

[Total 10]

7 An investor is considering investing in an asset with a payoff of X, where X has a probability density function f, given by:

$$f(s) = 0 \text{ for } s < 1$$

$$f(s) = 4/s^5 \text{ for } s \geq 1$$

(i) Calculate the semi-variance of return. [5]

(ii) Calculate the value at risk in the:

(a) worst 5% of outcomes.

(b) worst 10% of outcomes.

[4]

(iii) Discuss the differences between your answers to part (ii). [3]

[Total 12]

8 The run-off triangle below shows the incremental claims incurred on a portfolio of car insurance policies.

	0	*1*	*2*	*3*
2018	362	272	506	350
2019	444	116	165	
2020	487	195		
2021	518			

(i) Calculate the outstanding claims using the basic chain ladder method. [6]

(ii) Discuss whether the chain ladder method is likely to be suitable for this pattern of claims. [4]

(iii) Discuss, with reference to the pattern of claims, whether any other methods would be better than the basic chain ladder method for this portfolio. [2]

[Total 12]

9 Consider a market that satisfies the assumptions of the capital asset pricing model.

The market contains only three assets, with the following attributes:

	Market capitalisation (\$m)	*Expected return (p.a.) (%)*
Asset 1	6	5.0
Asset 2	4	7.0
Asset 3	10	8.5

The variance/covariance matrix is as follows:

	Asset 1	*Asset 2*	*Asset 3*
Asset 1	0.0005	0.0006	0.0005
Asset 2	0.0006	0.0015	0.0026
Asset 3	0.0005	0.0026	0.0042

(i) Show, using the figures provided, that the market portfolio has a beta of 1. [4]

A portfolio, P, on the efficient frontier is invested 30% in Asset 1 and 70% in Asset 3.

(ii) Calculate the expected return and standard deviation of portfolio P. [2]

(iii) Determine the market price of risk and the risk-free rate. [3]

(iv) Suggest reasons why it may not be appropriate to estimate parameters for an asset pricing model using past data. [3]

[Total 12]

END OF PAPER

Institute
and Faculty
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EXAMINERS’ REPORT

CM2 - Financial Engineering and Loss Reserving

Core Principles

Paper A

April 2023

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

For some candidates, this may be their first attempt at answering an examination using open books and online. The Examiners expect all candidates to have a good level of knowledge and understanding of the topics and therefore candidates should not be overly dependent on open book materials. In our experience, candidates that spend too long researching answers in their materials will not be successful either because of time management issues or because they do not properly answer the questions.

Many candidates rely on past exam papers and examiner reports. Great caution must be exercised in doing so because each exam question is unique. As with all professional examinations, it is insufficient to repeat points of principle, formula or other text book works. The examinations are designed to test "higher order" thinking including candidates' ability to apply their knowledge to the facts presented in detail, synthesise and analyse their findings, and present conclusions or advice. Successful candidates concentrate on answering the questions asked rather than repeating their knowledge without application.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
July 2023

A. General comments on the *aims of this subject and how it is marked*

The aim of Subject CM2 is to develop the necessary skills to construct asset liability models, value financial derivatives and calculate reserves for insurance or guarantees. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.

The marking approach for CM2 is flexible in the sense that different answers to those shown in the solution can earn marks if they are relevant and appropriate. Marks for the methodology are also awarded including marks for using the right method even if an error in an earlier part of the question prevents the final answer from being correct. The marking focusses on rewarding candidates' understanding of the concepts, including their ability to articulate algebra and arguments clearly.

B. Comments on *candidate performance in this diet of the examination.*

This exam was sat online and most questions focussed on applied calculations and analysis of the results. Some of the questions required candidates to apply concepts from the Core Reading to scenarios they might not have seen before and the stronger candidates scored highly here. Average marks were roughly in line with the historic norm for the subject but the pass mark was set slightly lower than normal.

As in previous sessions, there was evidence that some candidates found algebra tricky when answering questions in Word. Candidates should note that rearranging and solving algebra on screen can sometimes be hard if you are used to using pen and paper, so this is a worthwhile skill to practice before the exams. It's also worth saying that using the equation editor in Word to set out formulae is not necessary, your workings just need to be clear enough for the examiner to follow them.

Question 4 and Question 5 proved to be the most challenging questions on the paper. Question 4 required candidates to derive and work with a lognormal distribution, and algebraic slip-ups were common here. Question 5 required candidates to apply the Cameron-Martin-Girsanov theorem, which is not often examined, though most candidates who started down the right route managed to complete the question well.

C. Pass Mark

The Pass Mark for this exam was 57
1297 presented themselves and 558 passed.

Solutions for Subject CM2A – April 2023

Q1

(i)

U'(w) = 1/w [½]

U''(w) = -1/w^2 [½]

R(w) = w*-U''(w)/U'(w) = 1 [½]

R'(w) = 0 [½]

Therefore this utility function exhibits constant relative risk aversion [½]

so it is iso-elastic [½]

(ii)

Expected loss = 5% * 200,000 + 0.1% * 1,490,000 [½]

= $11,490 [½]

(iii)

The maximum premium, P, is given by the following equation:

E(U(w-X)) = U(w-P) [1]

Where w is current wealth and X is the loss

94.9%*U(1,500,000) + 5%*U(1,300,000) + 0.1%*U(10,000) = U(w-P) [1]

14.20881 = ln(1,500,000-P) [½]

P = $18,138 [½]

(iv)

The premium is higher than the expected loss [½]

Which implies that the singer is risk averse [½]

Also implied by the singer's iso-elastic utility function [½]

Initially it appears the singer would be better off not purchasing the insurance [½]

However, people buy insurance to reduce risk and increase certainty [½]

And buying the insurance at any premium lower than this would give a higher expected utility [½]

So they might prefer to buy the insurance [½]

[Marks available 3½, maximum 3]

(v)

Insurers manage their exposure by pooling risks [1]

This is a low likelihood but high severity risk [½]

And it might be difficult for an insurer to pool with other risks due to the unusual risk being covered [½]

So they might not be keen to take on such a risk [½]

Insurers also need to be able to estimate the probability of a risk event [1]

In particular, the insurer's view on the likelihood of each risk might differ from the singer's view [1]

which would mean that the insurance price differs from the value the singer places on the insurance [½]

There is also potential for adverse selection [1]

if the singer wants the insurance because they believe they are particularly likely to damage their vocal cords [1]

which will make them a higher risk for the insurer [½]

and moral hazard [1]

If the singer might take less care of their health	[1]
because they have insurance	[½]
Reinsurers might not want to accept this risk	[1]
This risk might be hard to estimate based on past data	[1]
The market for this risk might be very small	[1]

[Marks available 13, maximum 5]

[Total 15]

This question was answered well apart from part (iii), where many candidates were unable to find the certainty equivalent required.

There were lots of possible ideas in part (v) for why an insurer might not be willing to cover this sort of risk and most candidates managed to suggest some good points.

Q2

(i)

To estimate the return on an asset/portfolio	[½]
because a stochastic model gives the distribution of the return or because the return profile might not use a tractable distribution.	[½]
To estimate the volatility/risk on an asset/portfolio	[½]
because we need to know the range of possible outcomes	[½]
To estimate VaR/shortfall probability on an asset/portfolio	[½]
because we need to know the range of possible outcomes and the return profile might not use a tractable distribution	[½]
To evaluate guarantees on products	[½]
because we will need to know how likely a guarantee is to bite and the value in each case	[½]
To price options	[½]
because the distribution of the underlying assets might be complex or the options might be path-dependent	[½]
To calculate capital requirements	[½]
because the underlying movements might be complex and the values in the tails might be important	[½]

[Marks available 6, maximum 3]

(ii)

Path 1:

A Wiener process has independent, normally distributed increments	[½]
It is not simple to judge this from the chart	[½]
Path 1 appears to be a Wiener process	[½]

Path 2:

For a Wiener process $W_0 = 0$	[½]
Therefore Path 2 is not a Wiener process	[½]

Path 3:

A Wiener process has continuous sample paths	[½]
Path 3 appears to have a discontinuity at time 100	[½]

Therefore Path 3 is unlikely to be a Wiener process [½]

(iii)

a. σ controls the volatility of the process [½]
   So an increase in σ will result in an increase of the overall volatility [½]
   The maximum and minimum points on the chart would likely be higher [½]

b. θ represents the mean level of the short rate [½]
   An increase will result in an increase in the average level of the path (or shift the path upwards) [½]

c. κ determines the speed of reversion [½]
   A reduction means a lower speed of reversion [½]
   So the path is likely to drift more before reverting to θ [½]

[Total 10]

This question was generally answered well, though part (ii) caused some difficulties. The key here was to identify that path 2 does not start at zero and path 3 has a large jump at time 100.

Q3

(i)

Similarities:

Both models produce outputs given a specified set of inputs [½]
e.g. data and assumptions [½]
Both models will make assumptions about the future behaviour of financial variables [½]
e.g. asset returns, inflation etc [½]

Differences:

A deterministic model is based on one set of parameters [½]
and it can in practice be very hard to pick the 'correct' set [½]
In stochastic models, no single value is used [½]
and variations are allowed for by the application of probability theory [½]
So a stochastic model produces a different output every time it is run [½]
which will allow us to see the distribution of the results [½]

[Marks available 5, maximum 3]

(ii)

The company might prefer a stochastic model because:

For a deterministic model, deciding which set of input variables to use may be a challenge [½]
And it will be hard for the company to show it has picked the 'right' investment assumptions [1]
A stochastic model avoids this issue by assuming a range of possible outcomes could apply [½]
This is particularly useful for the insurer because they hold long-term contracts [½]
which are likely to be uncertain in the nature/term/currency [½]
and the variation built into a stochastic model is much more likely to capture this properly [½]

Running the stochastic model lots of times will produce a range of results with associated probabilities [½]
which gives us more information than a single figure from a deterministic model [½]
[Marks available 4½, maximum 3]

(iii)
There are many reasons that could reasonably be argued. Some are:
The company is assuming that investment returns are independent year-on-year.
In practice this is not appropriate [½]
As the returns in the past year sometimes inform the returns in the current year [½]
i.e. the two are correlated [½]
The company is assuming that returns are Normally distributed each year
The Normal distribution assigns a non-zero probability to returns smaller than -100% [½]
And in general it is not appropriate to assume a greater loss value than the initial investment outlay is possible [½]
[Marks available 2½, maximum 2]

(iv)
The company could correct for lack of dependence by assuming the returns of each year are a function of the previous year's returns [1]
Or it could consider changing its approach to a different type of model [1]
The company could change to a distribution that does not permit returns below -100% [½]
e.g. Like the LogNormal distribution [½]
[Marks available 3, maximum 2]
[Total 10]

Part (i) of this question asked for similarities and differences between deterministic and stochastic models. Most candidates identified the key features of each model but to score full marks candidates needed to provide a clear comparison of the two modelling approaches.

The later parts were answered well, with many candidates scoring highly in parts (iii) and (iv).

Q4
(i)
$\log(S_5) \sim N(\log(100) + 5\mu, 5\sigma^2)$ [1]

(ii)
$P(\log(S_5) \leq \log(75)) = 0.01,$ [½]
$P(\log(S_5) \geq \log(140) = 0.01$ [½]
$\log(S_5)$ is Normally distributed, so by symmetry
$\log(100) + \ 5\mu = \frac{\log(75)+\log(140)}{2}$ [1]
$\mu = 0.004879 \ (4sf)$ [½]
Then by definition of the Normal distribution:
$\log(75) = \log(100) + 5\mu + \ \sqrt{5}\sigma(-2.32635)$ [1]

$\sigma = \frac{\log(100) - \log(75) + 5\mu}{\sqrt{5} * 2.32635}$ [1]
$\sigma = 0.06000\ (4sf)$ [½]

(iii)
$P(S_{10} \geq 100) = P(\log(S_{10}) \geq \log(100))$ [½]
$= P(N(\log(100) + 10\mu, 10\sigma^2) \geq \log\ (100))$ [½]
$= P\left(N(0,1) \geq\ -\frac{10\mu}{\sqrt{10}\sigma}\right)$ [½]
Using μ and σ from part (iii):
$= P(Z \geq\ -0.25718)$ [½]
$= \Phi(0.25718) = 0.6015$ [1]

(iv)
The biologist has extended the model beyond the original time frame they were using it [½]
This could be a dangerous if the assumptions used to build them only apply over the next 5 years and not over the full 10 [½]
The biologist may also have only built this model a one-off exercise and not intended to share its output or probabilities it produces with third parties such as the university [1]
The model may have other limitations e.g. in how the biologist set μ and σ [1]
The biologist should be careful to disclose these issues to the university [1]
[Marks available 4, maximum 2]

This unexpectedly proved to be one of the hardest questions on the paper, with many candidates failing to identify the correct distribution in part (i). Credit was given in parts (ii) and (iii) for using whichever distribution was proposed in part (i), but candidates often still did not solve the algebra correctly.

Part (iv) tended to be answered better, with many candidates suggesting some good reasons why extending the model for a longer time period might not be a good idea.

Q5
(i)
The risk neutral and real world probability measures are connected by the market price of risk, which equals c.
The stock price and derivative price are given by:
$S_t = S_0 \exp\ (0.2B_t + 0.2t)$
$D_t = 2\exp\ (0.6B_t + 0.39t)$
If we convert these using the CMG theorem to a standard Brownian motion under the risk-neutral measure and assume that $\tilde{B}_t = B_t + ct$, they become: [1]
$S_t = S_0 \exp\big(0.2(\tilde{B}_t - ct) + 0.2t\big) = S_0 \exp\ (0.2(1-c)t + 0.2\tilde{B}_t)$ [1]
$D_t = 2\exp\big(0.6(\tilde{B}_t - ct) + 0.39t\big) = 2\exp\ ((0.39 - 0.6c)t + 0.6\tilde{B}_t)$ [1]
These processes are both geometric Brownian motions and the corresponding SDEs are:
$dS_t = S_t\left(\left(0.2(1-c) + \frac{0.2^2}{2}\right)dt + 0.2d\tilde{B}_t\right) = S_t\left(\big((0.22 - 0.2c)\big)dt + 0.2d\tilde{B}_t\right)$ [1]

$dD_t = D_t\left(\left(0.39 - 0.6c + \frac{0.6^2}{2}\right)dt + 0.6d\tilde{B}_t\right) = D_t\left((0.57 - 0.6c)dt + 0.6d\tilde{B}_t\right)$ [1]

Under the risk-neutral measure both assets have the same rate of drift (equal to the risk free rate). [½]

This means:

$0.22 - 0.2\text{c} = 0.57 - 0.6\text{c}$ [½]

$\Rightarrow 0.4\text{c} = 0.35 \;\Rightarrow \text{c} = 0.875$ [1]

(ii)

$r = 0.22 - 0.2c = 0.22 - 0.2x0.875 = 0.045$ [1]

[Total 8]

This question required candidates to apply the Cameron-Martin-Girsanov theorem in part (i), and those who identified this often went on to score full marks. The Examiners were fairly lenient with algebra here since this can be tricky to type out in Word.

Q6

(i)

Investment A:

E(A) = 1/3 x 3 + 1/3 x 2.6 + 1/3 x 1 = $2.2 [1]

Var (A) = 1/3 x (9 + 6.76 + 1) – 2.2^2 = $^{2}0.747 [1]

Investment B:

E(B) = $2.2 [½]

Var(B) = $^{2}4 [½]

(ii)

They would choose Investment A [½]

An insurer with a quadratic utility function will maximise their expected utility based on the first two moments of the distribution of return… [½]

alternatively, they will select the investment that has a higher return per unit variance… [½]

or the lowest variance per unit of return [½]

The expected returns from the two investments are identical [½]

However, Investment A has a lower variance of return than Investment B [½]

[Marks available 3, maximum 2]

(iii)

(a)

$P(X_A < 0.5) = 0$ [½]

$P(X_B < 0.5) = P(Z < (0.3 – 2) / 2) = P(Z < -0.85) = 0.19766$ [1]

(b)

P(XA < 2) = 1/3 [½]

P(XB < 2) = P(Z < (1.8 – 2)/2) = P(Z< -0.1) = 0.46017 [1]

(iv)
Investment A gives the same mean, a lower variance and lower shortfall probabilities at the two levels measured in part (iv) [½]
But Investment B can deliver a return above 3 in some scenarios [½]
So Investment A is not better in all scenarios [½]
For example Investment A does not show absolute or first order stochastic dominance over Investment B [½]
So neither investment is more attractive in every circumstance [½]
[Marks available 2½, maximum 2]
[Total 10]

Many candidates scored well on the mathematical parts of this question, but average marks tended to be lower in the 'wordy' parts, especially part (iv).

In part (iv) the key point was that the risk measures in the question all suggest that Investment A is better, but these risk measures do not tell the whole story and in some circumstances Investment B might be better.

Q7
(i)
The mean $\mu = \int_1^{\infty} sf(s)ds$ [½]
$= \int_1^{\infty} 4s^{-4}ds$ [½]
$= -\frac{4}{3}s^{-3}$ [½]
$= \frac{4}{3}$ [½]
Semi variance $= \int_1^{\mu}(\mu - s)^2 f(s)ds$ [½]
$= 4\int_1^{\frac{4}{3}}\left(\frac{4}{3} - s\right)^2 s^{-5}ds$ [½]
$= 4\int_1^{\frac{4}{3}}\frac{16}{9}s^{-5} - \frac{8}{3}s^{-4} + s^{-3}ds$ [½]
$= 4\left[-\frac{4}{9}s^{-4} + \frac{8}{9}s^{-3} - \frac{1}{2}s^{-2}\right]_1^{\frac{4}{3}}$ [½]
$= 4\left(\left(-\frac{9}{64} + \frac{3}{8} - \frac{9}{32}\right) - \left(-\frac{4}{9} + \frac{8}{9} - \frac{1}{2}\right)\right)$ [½]
$= \frac{5}{144} = 0.0347222$ [½]

(ii) (a)
$P(X < t) = 0.05$ [½]
$\Rightarrow 1 - P(X \geq t) = 0.05$ [½]
$\Rightarrow 1 - 4\int_t^{\infty} s^{-5}ds = 0.05$ [½]
$\Rightarrow 1 + [s^{-4}]_t^{\infty} = 0.05$ [½]
$\Rightarrow 1 - t^{-4} = 0.05$ [½]
$\Rightarrow t = 1.0129$ [½]

(b)
$1 - t^{-4} = 0.1$ [½]

$\Rightarrow t = 1.0267$ [½]

(iii)
The 10% VaR is higher than the 5% VaR [½]
which we would expect [½]
It is more than double the 5% VaR [½]
because of the curved shape of the PDF [½]
Even the 5% VaR is still higher than 1 [½]
so this asset has a very small chance of losing the investor money [½]
[Total 12]

In this question many candidates were not able to identify the correct integrals to work with in parts (i) and (ii), and those that did often slipped up when solving them. However, most candidates showed that they understood the risk measures and scored some marks for this.

Q8
(i)
Cumulative claim amounts:

	0	1	2	3
2018	362	634	1140	1490
2019	444	560	725	
2020	487	682		
2021	518			

[1]

Development Factors:
DF(0,1) = (634 + 560 + 682) / (362 + 444+ 487) = 1.4509 [1]
DF(1,2) = (1140 + 725) / (634 + 560) = 1.5620 [½]
DF(2,3) = 1490 / 1140 = 1.3070 [½]

Completed Cumulative Claims:

	0	1	2	3
2018				
2019				947.6
2020			1065.3	1392.3
2021		751.6	1173.9	1534.3

[1]
Outstanding Claims = (1534.3 – 518) + (1392.3 – 682) + (947.6 – 725) [1]
= 1949.2 [1]

(ii)
The basic chain ladder method might not be suitable [½]
Claims in year 0 appear to be increasing quite fast [½]
This could mean that the insurer's business is growing [½]
Or it could be a sign of high inflation which might invalidate the basic chain ladder method [½]
Claim development for policies issued in 2018 appears quite different to later years [½]

with claims roughly doubling in year 1 then doubling again in year 2 [½]
This could mean the development factors are not appropriate unless this pattern of claims is expected to be repeated [½]
If the claim pattern from 2018 is normal, then there are a lot of claims from later years yet to develop [½]
and they might not all be developed by year 3 [½]

[Marks available 4½, maximum 4]

(iii)
The insurer could use the inflation-adjusted chain ladder method… [½]
because this would correct for possible high inflation over the last few years [½]
The insurer could use the Bornhuetter-Ferguson method [½]
because this would allow a consistent assumption about how total claims relate to premiums [½]
The insurer could use the Average Cost Per Claim method… [½]
which would allow it to split out patterns in claim numbers and claim amounts [½]
The insurer could remove 2018 from the calculation if experience in that year was abnormal [½]
or perhaps adjust to correct it [½]

[Marks available 4, maximum 2]

[Total 12]

Part (i) of this question was answered well by most candidates, though some did not correctly calculate the final figure for outstanding claims which meant they missed out on scoring full marks.

Parts (ii) and (iii) were not answered so well. The key here was to identify that the claims show a strange pattern and the development factors do not decrease by year as they usually would, which means this reserving method might not be appropriate.

Q9
(i)
Cov(1,M) = 6/20*var(1) + 4/20*cov(1,2) + 10/20*cov(1,3) = 0.00052 [½]
Cov(2,M) = 6/20*cov(1,2) + 4/20*var(2) + 10/20*cov(2,3) = 0.00178 [½]
Cov(3,M) = 6/20*cov(1,3) + 4/20*cov(2,3) + 10/20*var(3) = 0.00277 [½]
Var(M) = (6/20)^2*var(1) + (4/20)^2*var(2) + (10/20)^2*var(3) + 2*(6/20)*(4/20)*0.0006 + 2*(6/20)*(10/20)*0.0005 + 2*(4/20)*(10/20)*0.0026= 0.001897 [½]
Beta(1) = cov(1,M)/var(M) = 0.274 [½]
Beta(2) = cov(2,M)/var(M) = 0.938 [½]
Beta(3) = cov(3,M)/var(M) = 1.460 [½]
Beta(M) = 0.3*Beta(1) + 0.2*Beta(2) + 0.5*Beta(3) = 1 [½]

(ii)
E(P) = 0.3*5% + 0.7*8.5% = 7.45% [1]
Var(P) = 0.3^2*var(1) + 0.7^2*var(3) + 2*0.3*0.7*cov(1,3) = 0.002313 [½]
Sd(P) = 4.8% [½]

(iii)
Using the statement that P is on the efficient frontier:
E(P) – r = (E(M) – r)*sd(P)/sd(M)
E(M) = 7.15% [½]
Sd(M) = sqrt(var(M)) = 4.3555% [½]
7.45% - r = (7.15% - r) * 1.104216
r = 4.27% [1]
Market price of risk = (E(M) – r)/sd(M) [½]
= 0.661 [½]

Or solving using figures for asset 1 gives r = 4.72%, MPR = 0.5579
Or solving using figures for asset 2 gives r = 4.72%, MPR = 0.5579
Or solving using figures for asset 3 gives r = 4.22%, MPR = 0.6727

(iv)
The past is not necessarily a guide to the future [½]
The data may be incomplete/inaccurate [½]
There may not be sufficient historical data… [½]
resulting in substantial statistical error [½]
The 'true' parameters of the model may change over time [½]
The assumptions underlying the asset pricing model may not be accurate [½]
[e.g. any of the CAPM assumptions] [½]
Any other sensible suggestions [½]

[Marks available 4, maximum 3]

[Total 12]

Part (i) of this question asked candidates to show that the market beta is 1 'using the figures provided'. Many candidates earned some credit for an algebraic proof, but a mathematical proof using the figures in the question was required to score highly.

For part (iii) the figures in the question unintentionally produce slightly different answers depending on which portfolio or asset is used, so the Examiners gave credit for any valid derivation.

[Paper Total 100]

END OF EXAMINERS' REPORT

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