INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

13 September 2023 (am)

Subject CM2 – Financial Engineering and Loss Reserving Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 (i) State, with reasons, whether each of the following scenarios are examples of evidence for or against the Efficient Markets Hypothesis (EMH):

(a) A stock price is still moving in response to a public earnings announcement that was made 3 months ago.

(b) The performance of an actively managed index fund is the same as a passive fund tracking the same index.

(c) The volatility of a share is greater than that implied by a discounted cashflow model.

(d) Two equity analysts at different firms have determined target prices for the stock of a specific company, and their target prices are different. Both have access to the same information.

[4]

(ii) Give an example of a scenario that demonstrates semi-strong form EMH. Your example should be different to those in part (i). [1]

(iii) Explain how your scenario given in part (ii) demonstrates semi-strong form EMH. [2]

[Total 7]

2 Consider four assets that deliver a return of r% over a fixed period, T, with defined probabilities as set out in the table below:

	$P(r = -2\%)$ (%)	$P(r = -1\%)$ (%)	$P(r = 0\%)$ (%)	$P(r = 1\%)$ (%)	$P(r = 2\%)$ (%)
Asset A	10	30	20	30	10
Asset B	30	10	20	10	30
Asset C	25	25	0	25	25
Asset D	40	10	10	20	20

Determine which pairs of assets demonstrate:

(a) first-order stochastic dominance.

(b) second-order stochastic dominance.

[9]

3 The price at time t of a portfolio of stocks, S_t, follows a lognormal distribution with parameters $\mu = 0.09t$ and $\sigma^2 = 0.04t$, where t is the time from now measured in years and $S_0 = 1$. An investor needs to repay a loan of \$150,000 exactly 5 years from now.

(i) Determine the amount the investor would need to invest in the portfolio at time $t = 0$ to give an 80% probability of having at least enough to repay the loan at time $t = 5$. [4]

The investor has \$100,000 available, which they invest in the portfolio today.

(ii) Calculate the following risk measures applied to the difference at time $t = 5$ between the value of the investor's portfolio and the amount required to repay the loan:

(a) 95% value at risk

(b) Expected shortfall or surplus.

[5]

(iii) Discuss what conclusions and actions the investor may draw from your answers to parts (i) and (ii). [2]

[Total 11]

4 An insurance company sells car insurance policies that each pay out a fixed amount of \$25,000 if a car is damaged. Let X be a random variable representing the number of insured cars damaged. X is assumed to follow a binomial distribution with parameters n = number of policies and p = 1%. Let the random variable L_P represent the loss per policy, i.e. $L_P = \$25{,}000X/n$. Each policy is for one car.

(i) Calculate the mean and standard deviation of L_P for the following scenarios:

(a) $n = 1$

(b) $n = 20$

(c) $n = 200$.

[4]

(ii) Explain which insurance concept is demonstrated by your results from part (i). [2]

The government of Country Z, where this insurance is sold, has set the premium for this type of policy at \$260.

(iii) Discuss, with reference to your answer to part (i), whether individuals are likely to buy this policy and whether insurers are likely to offer it. [3]

(iv) Comment on how adverse selection and moral hazard could impact this policy. [3]

[Total 12]

5 An analyst at a bank wants to model interest rates and is considering using either the Vasicek or Cox–Ingersoll–Ross models.

(i) State two similarities and two differences between these models. [2]

The analyst is looking to calibrate their model to the yield curves of a single country. In the period modelled, the short-term interest rates in this country were consistently negative over a period of multiple years.

(ii) Explain why the Cox–Ingersoll–Ross model will not be appropriate for the analyst. [2]

(iii) Explain how the analyst could adjust the Cox–Ingersoll–Ross model to reflect the points you have identified in part (ii). [2]

[Total 6]

6 (i) Define, in your own words, the market price of risk in modern portfolio theory. [1]

A panel of investment experts has provided annual expected investment returns for three asset classes that vary depending on the state of the economy (recession, normal or bubble). These are shown in the table below. You may assume the risk-free annual rate of return is 2%.

	Asset class			*Probability*
	Property	*Stock*	*Bonds*	
Recession	–1%	–2%	6%	0.1
Normal	4%	6%	2%	0.6
Bubble	8%	12%	5%	0.3
Market capitalisation ($ billion)	25	50	50	

(ii) Calculate the market price of risk. [5]

An analyst is looking at a portfolio (P_A) that offers a mean return, $\mu_A = 6\%$, and standard deviation of return, $\sigma_A = 3.15\%$. They are unsure whether a more efficient portfolio (P_B) can exist that offers either of:

- $\mu_B \geq \mu_A$ and $\sigma_A = \sigma_B$
- $\sigma_B \leq \sigma_A$ and $\mu_A = \mu_B$.

(iii) Determine whether a more efficient portfolio, P_B, can be found. [3]

[Total 9]

7 Consider the process A_t defined by the following integral:

$$\int_0^t W_s ds$$

where W_s is a standard Brownian motion.

(i) Explain why this process is not a martingale. [2]

(ii) Show that A_t is Normally distributed with mean 0 and variance $\frac{t^3}{3}$.

[**Hint**: Note that $tW_t = \int_0^t t dW_s$.] [6]

[Total 8]

8 Under certain assumptions, the adjustment coefficient r for an insurer's surplus process is a parameter that is the unique positive root of the following equations:

$$\lambda M_X(r) - \lambda - cr = 0$$

$$c = (1 + \theta)\lambda m_1$$

$M_X(r)$ is the moment generating function of the individual claim amount distribution.

(i) Write down the key assumption that is required to make these equations valid. [1]

(ii) Define the following terms from the equations above:

(a) λ

(b) c

(c) θ

(d) m_1

[2]

The insurer is looking at a product where claims take a value 1 with probability p, and 0 otherwise.

(iii) Show that the approximate relationship $r \approx 2\theta$ holds. You may assume r is small enough that terms of order r^3 and above can be discarded. [4]

[Total 7]

9 An insurance company is considering offering a 10-year investment product. The product will provide returns linked to a stock index. The total return at the end of the term is guaranteed to be no less than 10% and no more than 50%.

You may assume:

- continuously compounded risk-free rate, $r = 2\%$ p.a.
- implied stock price volatility, $\sigma = 10\%$ p.a.

(i) Calculate, using the Black–Scholes model, the price at time $t = 0$ of the investment product. [7]

(ii) Explain why the treatment of σ in the Black–Scholes model in part (i) may be a simplification of the real-world situation. [2]

[Total 9]

10 A stock, S, is currently priced at $S_0 = \$9$.

A derivative contract is written on S. The contract will pay an amount, Y, in exactly 1 year's time, where $Y = S_1^{\,2}$.

An analyst has estimated the price volatility of S at 20% p.a. The risk-free rate of interest is 12% p.a.

(i) Calculate the fair price of the derivative contract. You may assume that the assumptions underlying the Black–Scholes model hold. [5]

The analyst wishes to set up a delta hedged portfolio for the derivative contract, consisting of S and a holding in cash.

(ii) Determine the analyst's hedging portfolio at $t = 0$. [4]

[Total 9]

11 A surplus process for an insurer is denoted as $U(t)$ with initial surplus U.

(i) Define the surplus process for this insurer in terms of the initial surplus, premium income and aggregate claims process. All notations used should be defined. [1]

An insurer writes a 1-year pet insurance policy. The policy covers 100 pets and will pay out \$10 on each death of an insured pet during the policy term.

The pet mortality rate is assumed to be 2.5% per month with claims being independent and assumed to follow a Poisson process. Premiums are paid continuously at \$29 per month.

The insurer has an initial surplus of \$50.

(ii) Calculate the probability that the time to the first claim is longer than 1 month. [2]

(iii) Calculate the expected value of $U(t)$ at:

(a) the end of month 1.

(b) the end of the year.

[4]

(iv) Calculate the probability that the initial surplus and the insurer's premium income will be insufficient to cover the total claims in the first year. [3]

The insurer increases the estimate of pet mortality to 3.25% per month.

(v) Comment on the effect that this increase in the mortality estimate will have on the:

(a) probability of ruin in the next year.

(b) ultimate probability of ruin.

[3]
[Total 13]

END OF PAPER

EXAMINERS' REPORT

CM2 - Financial Engineering and Loss Reserving

Core Principles

Paper A

September 2023

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

For some candidates, this may be their first attempt at answering an examination using open books and online. The Examiners expect all candidates to have a good level of knowledge and understanding of the topics and therefore candidates should not be overly dependent on open book materials. In our experience, candidates that spend too long researching answers in their materials will not be successful either because of time management issues or because they do not properly answer the questions.

Many candidates rely on past exam papers and examiner reports. Great caution must be exercised in doing so because each exam question is unique. As with all professional examinations, it is insufficient to repeat points of principle, formula or other text book works. The examinations are designed to test "higher order" thinking including candidates' ability to apply their knowledge to the facts presented in detail, synthesise and analyse their findings, and present conclusions or advice. Successful candidates concentrate on answering the questions asked rather than repeating their knowledge without application.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
November 2023

A. General comments on the *aims of this subject and how it is marked.*

The aim of Subject CM2 is to develop the necessary skills to construct asset liability models, value financial derivatives and calculate reserves for insurance or guarantees. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.

The marking approach for CM2 is flexible in the sense that different answers to those shown in the solution can earn marks if they are relevant and appropriate. Marks for the methodology are also awarded, including marks for using the right method even if an error in an earlier part of the question prevents the final answer from being correct. The marking focusses on rewarding candidates' understanding of the concepts, including their ability to articulate algebra and arguments clearly.

B. Comments on *candidate performance in this diet of the examination.*

Most questions focussed on applied calculations and analysis of the results. Some of the questions required candidates to apply concepts presented in the Core Reading, to scenarios they might not have seen before and some candidates scored highly here.

.Candidates should note that rearranging and solving algebra on screen can sometimes be hard if you are used to using pen and paper, so this is a worthwhile skill to practise before the exams. It's also worth saying that using the equation editor in Word to set out formulae is not necessary, your workings just need to be clear enough for the examiner to follow them.

Question 7 and Question 9 proved to be the most challenging questions on the paper. Question 7 required candidates to work with Standard Brownian Motion, and in many answers the algebra was not explained clearly enough to score strongly. Question 9 required candidates to price an unusual option from first principles, and while few candidates scored full marks, many started down the right route and earned some credit for this.

C. Pass Mark

The Pass Mark for this exam was 57
1261 presented themselves and 531 passed.

Solutions for CM2A - September 2023

Q1
(i)(a)
Against, because market prices should immediately incorporate all available information. [1]

(b)
For, because active managers using the same information as other market participants should achieve the same returns. [1]

(c)
Against, because if the cashflow model incorporates all market information then it should match prices and therefore have the same volatility (e.g. as explained by Shiller). [1]

(d)
Against, because analysts using the same information should reach the same target price. [1]
(Could also be for since it depends on the models each analyst uses.)

(ii)
Example answer:
The CEO of a company buys stock in their own company as they know a merger is going to be announced publicly soon. [½]
The stock rises significantly in value following the announcement, whilst the overall market performance is moderate. [½]
(Any example showing semi-strong or strong form EMH is acceptable.)

(iii)
Example answer:
Semi-strong form EMH means that the market price of an investment incorporates all publicly available information [½]
but it does not incorporate information only available to insiders. [½]
Those with inside information can exploit this to produce excess performance. [½]
In the example given the CEO uses their insider knowledge of the upcoming merger to buy stock before the public announcement and resulting rise in stock price. [½]
(Any other valid points can score marks depending on the example given in (ii))

[Total 7]

This question was answered well by most candidates.

In part (i) only well prepared c stated the result and explained why.

In part (iii), only well prepared candidates that gave enough explanation were awarded full marks.

Q2

CDFs:

	F(-2%)	F(-1%)	F(0%)	F(1%)	F(2%)
Asset A	10%	40%	60%	90%	100%
Asset B	30%	40%	60%	70%	100%
Asset C	25%	50%	50%	75%	100%
Asset D	40%	50%	60%	80%	100%

[2]

Integrated CDFs:

	ICDF(-2%)	ICDF(-1%)	ICDF(0%)	ICDF(1%)	ICDF(2%)
Asset A	10%	50%	110%	200%	300%
Asset B	30%	70%	130%	200%	300%
Asset C	25%	75%	125%	200%	300%
Asset D	40%	90%	150%	230%	330%

[2]

A exhibits first order stochastic dominance over B if CDF(A) <= CDF(B) for all outcomes and CDF(A) < CDF(B) for some outcome.

A exhibits second order stochastic dominance over B if $\int \mathrm{CDF(A)}$ <= $\int \mathrm{CDF(B)}$ for all outcomes and $\int \mathrm{CDF(A)}$ < $\int \mathrm{CDF(B)}$ for some outcome.

(a)
Asset B displays first order stochastic dominance over asset D [1]
Asset C displays first order stochastic dominance over asset D [1]

(b)
Asset A displays second order stochastic dominance over assets B, C and D [1]
Asset B displays second order stochastic dominance over asset D [1]
Asset C displays second order stochastic dominance over asset D [1]

[Total 9]

This question was generally answered well. Most candidates were able to perform the correct calculations, though not all candidates identified all of the dominance pairs.

Q3

(i)
S_5 is lognormal with parameters 0.09*5=0.45 and 0.04*5=0.2 [½]
P(S_5<X) = 20% [½]
P(Z<(lnX - 0.45)/sqrt(0.2)) = 20% [½]
(lnX - 0.45)/sqrt(0.2) = -0.8416 [½]
X = 1.0764 [½]
150000/1.0764 = 139353 [½]

The investor needs to invest $139,353 in the portfolio to give an 80% probability of having at least enough to repay the loan. [1]

(ii)(a)
P(S5<X) = 5% [½]
P(Z<(lnX - 0.45)/sqrt(0.2)) = 5% [½]
(lnX - 0.45)/sqrt(0.2) = -1.6449 [½]
X = 0.751547 [½]
150000-100000X = 74845 [½]
The 95% VaR is $74,845 [½]

(b)
E(S5) = exp(0.45 + 0.2/2) = 1.733253 [1]
The expected value of the portfolio is 100000*1.733253 = $173,325 [½]
Therefore there is an expected surplus of 173325-150000 = $23,325 [½]

(iii)
If the investor had $139,353 then they would have at least an 80% chance of repaying the loan. [½]
However they only have $100,000, so they have a less than 80% chance. [½]
The investor has an expected surplus of $23,325, so they do expect to be able to repay the loan. [½]
However there is a chance of a big shortfall. [½]
The VaR shows that there is a 5% chance of a shortfall of at least $74,845. [½]
The investor might want to choose less risky investments to increase the chance of being able to repay the loan. [½]

[Marks available 3, maximum 2]

[Total 11]

Most candidates understood what was needed here, though mistakes in the calculations using the Normal distribution were fairly common.

In part (iii) most candidates scored marks, but note that we needed conclusions and suggested actions so simply stating what parts (i) and (ii) showed did not score marks.

Q4
(i)
E(X) = np = 0.01n [½]
sd(X) = sqrt(npq) = sqrt(0.0099n) [½]
E(LP)=25000E(X)/n=250 [½]
sd(LP)=25000sd(X)/n=25000*sqrt(0.0099n)/n [½]
E(LP) does not depend on n, therefore is equal to $250 for all scenarios [½]
sd(LP) is equal to $2,487 for n=1 [½]
$556 for n=20 [½]
and $176 for n=200 [½]

(ii)

The results from part (i) demonstrate the benefits of pooling resources. [1]
Pooling resources reduces the variability of losses due to adverse outcomes. [½]
As shown in part (i) increasing the number of policies does not affect the expected loss per policy [½]
But it does reduce the standard deviation of the loss per policy. [½]
Lower variability is desirable for insurance companies. [½]
[Marks available 3, maximum 2]

(iii)
The premium is 260/250-1 = 4% greater than the expected loss per policyholder. [½]
A policyholder may be willing to pay more than the expected loss if they are risk averse. [½]
A policyholder may be willing to pay more than the expected loss if they cannot withstand the value of a loss (i.e. $25,000). [½]
If the above apply then they may be likely to buy the policy. [½]
If the insurer is risk averse they will want the insurance premium to be greater than the expected loss. [½]
The insurer will also need a margin over the expected loss to cover costs, e.g. overheads, labour costs, advertising costs. [½]
They may need to charge more than the expected loss for regulatory reasons or to cover capital requirements. [½]
If the 4% margin is sufficient to cover the above then they may be willing to offer the policy. [½]
The insurance contract will only be feasible if the minimum premium that the insurer is prepared to charge is less than the maximum the potential policyholder is prepared to pay. [½]
The insurer might be more likely to offer the product if they can issue a large number of policies as this will reduce the variance of outcomes for them. [½]
The premium is about 10% of the payout, so customers are likely to be happy paying premiums for 10 years before feeling like they've overpaid. [½]
[Marks available 5½, maximum 3]

(iv)
Adverse selection: *(maximum 1½ marks)*
Individuals who think or know they are more likely to experience a loss than the assumed 1% are more likely to take out the insurance. [½]
This risk is reduced if this type of insurance is compulsory in Country Z. [½]
The payout is a fixed amount of $25,000. Owners of low value cars might take out the insurance looking for the payout. [½]
The insurance company can reduce this risk by underwriting [½]
and by putting policyholders in homogeneous pools [½]
and by monitoring their experience and adjusting assumptions if required. [½]

Moral hazard: *(maximum 1½ marks)*
Policyholders may be less careful because they have insurance, making damage to their car more likely. [½]
Policyholders might be especially careless if their car is worth much less than the $25,000 insurance payout. [½]
The insurance company can reduce this risk by monitoring their experience and adjusting assumptions if required [½]

or introducing exceptions in the policies. [½]
Moral hazard makes insurance more expensive and could push the price of insurance above the maximum premium that a person is prepared to pay. [½]

[Marks available 5½, maximum 3]

[Total 12]

Most candidates scored well in parts (i) and (ii) of this question, calculating the correct results and explaining how this is a key concept in insurance.

Parts (iii) and (iv) were not always answered so well, and (iv) needed more than just an explanation of what adverse selection and moral hazard are to score full marks.

Q5

(i)

Similarities:
They are both short rate models. [½]
They are both one-factor models. [½]
They both have mean-reversion. [½]

Differences:
Vasicek permits non-positive interest rates, whereas CIR does not. [½]
The distribution of Vasicek is Normal, with CIR it is a non-central chi-squared distribution. [½]

[Marks available 2½, maximum 2]

(ii)

The data the analyst has suggests that the short rate of this nation is in fact negative. [½]
As this is not a one-off in the data and there are no errors. [½]
However, CIR does not permit negative short rates. [½]
Because when the short rate of a CIR model hits 0, it immediately rebounds. [½]
Due to the square root term. [½]
This means it will not be possible to accurately calibrate CIR to the curve. [½]

[Marks available 3, maximum 2]

(iii)

One option could be to use CIR to instead model $r_t' = r_t + \alpha$ where α is some positive value. [1]
Then the restrictions of CIR become $r_t' \geq 0$ or $r_t \geq -\alpha$ so that negative short rates are now permitted. [1]
A suitable choice of α would make sure that all negative rates seen in the data are possible values for r_t, [1]
(Credit given for reasonable answers that address a key issue found in part (ii), provided they are clearly argued)

[Marks available 3, maximum 2]

[Total 6]

Most candidates scored well in this question.

Part (i) needed a comparison of the two models, so candidates who simply described the characteristics of each model did not score marks for this.

Part (iii) was the trickiest part, and needed suggestions for how the Cox-Ingersoll-Ross model could be adapted rather than suggestions to use a different model.

Q6

(i)

The market price of risk is a measure of the return in excess of the risk-free rate that investors demand to bear one unit of risk. [1]

(ii)

Market return in a recession = (25 x -1% + 50 x -2% + 50 x 6%) / 125 = 1.4% [½]
Market return normally = (25 x 4% + 50 x 6% + 50 x 2%) / 125 = 4.0% [½]
Market return in a bubble = (25 x 8% + 50 x 12% + 50 x 5%) / 125 = 8.4% [½]
E_M = 1.4% x 0.1 + 4.0% x 0.6 + 8.4% x 0.3 [½]
= 5.06% [½]
$\sigma_M^2 = 0.014^2 \times 0.1 + 0.04^2 \times 0.6 + 0.084^2 \times 0.3 - 0.0506^2$
[½]
= 0.000536 [½]
$\sigma_M = 0.000536^{0.5}$ = 2.32% [½]
Market price of risk = (5.06% - 2%) / 2.32% = 1.319 per unit of risk [1]

(iii)

The capital market line can be used to determine portfolios that sit on the efficient frontier [½]
$(E_P - r) / \sigma_P = (E_M - r) / \sigma_M$ [½]
(6% - 2%) / 3.15% = 1.269 per unit of risk [½]
This is lower than the market price of risk calculated in part (ii). [½]
Therefore, portfolio (A) is not the most efficient portfolio for the stated μ_A and σ_A. [1]
(Or, for full marks, candidate can demonstrate a portfolio with higher expected return for the same risk or lower risk for the same expected return.)

[Total 9]

This question caused more difficulty than expected, though many candidates still scored well.

Part (i) needed an explanation of the market price of risk, so stating the formula (either in algebra or in words) did not score marks.

Part (ii) was straightforward, and the most common slip-up was incorrectly calculating the volatility of the market portfolio.

Part (iii) was only answered correctly by the strongest candidates, and the key here was to identify that portfolio A does not sit on the efficient frontier.

Q7

(i)

$\frac{dA_t}{dt} = W_t$, so

We can write this as $dA_t = W_t dt + 0 dW_t$ [½]

This process has a non-zero drift [½]

So it cannot be a martingale [1]

(ii)

Using the results from part (i) we can write:

$dA_t = W_t dt = d(tW_t) - t dW_t$ [1]

Or equivalently, using the hint:

$A_t = tW_t - \int_0^t s dW_s$ [1]

$A_t = \int_0^t (t-s) dW_s$ [1]

The integrand $(t-s)$ is deterministic, so A_t's distribution has the following properties:

A_t is Normally distributed [1]

The expectation of A_t is 0 [1]

The variance of A_t is $\int_0^t (t-s)^2 ds$ [1]

Evaluating the integral, we find that $A_t \sim N\left(0, \frac{t^3}{3}\right)$

[Total 8]

This question was one of the harder ones in the paper.

In part (i) most candidates understood what was needed, but not all were able to demonstrate clearly why A_t is not a martingale.

In part (ii) many candidates scored some marks, but there were often unexplained leaps in the algebra that meant it could only earn partial marks.

Q8
(i)
The key assumption is that the number of claims follows a Poisson process (with parameter λ) [1]

(ii)
λ is the parameter of the Poisson process in part (i) [½]
c is the insurer's premium income (per unit time) [½]
θ is the premium loading factor [½]
m_1 is the mean (first moment) of the claim amount [½]

(iii)
The moment generating function of this claim is (either from Tables or by definition):
$M_X(t) = E(e^{tX}) = 1 - p + pe^t$ [½]
and $m_1 = p$ [½]

So then the equation becomes:
$\lambda(1 - p + pe^r) - \lambda - (1 + \theta)\lambda pr = 0$ [½]
$(e^r - 1) = (1 + \theta)r$ [½]

Using the information in the question:
$\left(1 + r + \frac{r^2}{2} - 1\right) \approx (1 + \theta)r$ [½]
$\frac{r^2}{2} = r\theta$ [½]
and given $r > 0$, we may divide by r [½]
to get $r \approx 2\theta$ [½]
[Total 7]

Parts (i) and (ii) of this question were answered well by most candidates.

Part (iii) caused more difficulty, and some candidates had algebra that appeared to reach the right conclusion but without clear steps along the way. The examiners are often lenient with marking algebra given that it has to be typed in Word, but candidates need to show clear reasoning to evidence understanding.

Q9
(i)
The price can be considered as a long call spread with a lower and upper strike of 10% and 50% respectively.
Lower strike call option
$d_1 = (\log(1/1.1) + (0.02 + 0.5 \text{ x } 0.1^2) \text{ x } 10)/(0.1 \text{ x } 10^{0.5}) = 0.489$ [½]
$d_2 = 0.489 - 1 \text{ x } 10^{0.5} = 0.173$ [½]
$N(d_1) = 0.6876$ [½]
$N(d_2) = 0.5678$ [½]
=> $c_{10\%} = 1 \text{ x } N(0.489) - 1.1 \text{ x } e^{-0.2} \text{ x } N(0.173)$

=> $c_{10\%} = 1 \text{ x } 0.6876 - 1.1 \text{ x } e^{-0.2} \text{ x } 0.5687 = 0.176$ [1]
Higher strike call option
$d_1 = -0.4916$ [½]
$d_2 = -0.8079$ [½]
$N(-0.4916) = 0.3115$ [½]
$N(-0.8079) = 0.2096$ [½]
=> $c_{50\%} = 0.054$ [1]
Price = $0.176 - 0.054 + 0.1 \text{ x } e^{-0.02 \text{ x } 10} = 0.204$ per unit [1]

(ii)
Black Scholes assumes that the value of σ does not change with the strike. [1]
This is not consistent with the σ levels seen in practice, which typically varies by strike. [1]
The model also assumes that σ is constant over time. [1]

[Marks available 3, maximum 2]

[Total 9]

This was a challenging question and few candidates scored highly. Most identified that the option in the question needed to be priced as two separate options with strike prices of 110 and 150, but these both needed to be call options and they needed to be combined with a final adjustment to produce the required payoff profile.

Part (ii) was often answered better, but many candidates strayed into a general discussion of the Black-Scholes model when we needed points related specifically to σ.

Q10
(i)
The fair price for the new derivative security at time 0 is given by
$V_0 = e^{-r} E_Q[D|F_0] = e^{-r} E_Q[{S_1}^2|F_0]$ [1]

From Black-Scholes theory, we know under Q:
$\ln(S_t) \sim N\left[\ln(S_0) + \left(r - \frac{\sigma^2}{2}\right) t, \sigma^2 t\right]$ [1]
$\Rightarrow \ln(S_1) \sim N\left[\ln(S_0) + \left(r - \frac{\sigma^2}{2}\right), \sigma^2\right]$ [½]

From the Tables (moments of lognormal):
$V_0 = e^{-r}\left[\exp\left(2\left\{\ln(S_0) + \left(r - \frac{\sigma^2}{2}\right)\right\} + \frac{(2\sigma)^2}{2}\right)\right]$ [½]
$= (S_0)^2 e^{r+\sigma^2}$ [1]
$= (9)^2 e^{0.12+0.2^2} = \$95.05$ [1]

(ii)
Delta for the new derivative is given by:
$\Delta = \frac{\partial V_t}{\partial S_t} = \frac{\partial}{\partial S_t}\left((S_t)^2 e^{r+\sigma^2}\right)$ [1]
$= 2S_t e^{r+\sigma^2}$ [½]

When t=0

$\Delta = 2 \times 9 \times e^{0.12+0.2^2} = 21.123$ [1]

The delta of the stock is one so 21.123 units of stock are needed to hedge. [1]
The cost of this is 21.123 x 9 = $190.11, so a short holding in cash equal to $95.05 is needed to make the total cost equal to the cost of the derivative, which is also $95.05. [½]

[Total 9]

This question required an option to be priced from first principles, and only the strongest candidates produced correct answers. Partial marks were awarded for starting down the right route with risk-neutral pricing .

Q11

(i)

$U(t) = U + ct - S(t)$ [½]

Where:
U is the initial surplus.
c is the rate of premium income per unit time
S(t) is the aggregate claims process in the time interval [0,t] [½]

(ii)

Let N(t) represent the number of claims to time t and T(n) represent the time until claims n:
P(T(1)>t) = P(N(t)=0) = exp(-λt) [1]
P(T(1)>1) = P(N(1)=0) = exp(-2.5) = 8.2% [1]

(iii)(a)

E[S(t)] = 10E[N(t)] = 10 x 2.5 x t = 25t [1]
E[U(t)] = E[U+ct-S(t)] = U + ct - E[S(t)] [1]
E[U(1)] = E[U+c-S(1)] = U + c - E[S(1)] = 50 + 29 - 25 x 1 = 54 [1]

(b)

E[U(12)] = E[U+12 x c-S(12)]=U+ 12 x c -E[S(12)] = 50 + 12 x 29 - 25 x 12 = 98 [1]

(iv)

P(U(12)<0)=P(U+12 x c-S(12)<0)
= P(50 +12 x 29 < S(12)) [1]
= P(398 < S(12)) = P(39.8 < N(12)) [1]
= 1 - P(N(12) < 39.8)
= 4.6% [1]

(v)(a)

Increasing the mortality rate will increase the Poisson parameter λ. [1]
This increases the expected number of claims and hence the probability that the insurer's income will be insufficient to cover the claims over the year will increase. This increases the probability of ruin in the next year. [1]

(b)
Increasing the mortality rate will increase the ultimate probability of ruin, because the policy has finite length and U = U(12). [1]

[Total 13]

The last question on the paper was answered well by most candidates. There were plenty of correct answers, and the most common mistake was treating premiums as being per pet rather than per policy.

Part (iii) also needed to include reasons for the impact on the probability of ruin and some candidates did not write enough here for the number of marks on offer.

The model solution assumes a constant number of pets covered by the policy. The question could also be read as if the number of pets decreases over time and any candidates who assumed this were given appropriate credit.

[Paper Total 100]

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