INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

14 April 2023 (am)

Subject CM1 – Actuarial Mathematics
Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 (i) Describe in words ${}_{a|b}q_{[x]+1}$. [2]

(ii) Calculate, showing all working, ${}_{5|3}q_{[38]+1}$.

Basis:
Mortality AM92 (select) [2]

[Total 4]

2 A general insurance company has set up a generalised cashflow model for the **claims** payments it will pay arising from its portfolio of car insurance policies.

Describe the cashflows with reference to the certainty and uncertainty of size and timings of payments from the insurance company's point of view. [3]

3 Calculate $a^{(4)}_{82\frac{1}{4}}$, showing all working.

Basis:
Mortality PMA92C20
Interest 4% per annum effective

[7]

4 A life insurance company sells 1-year pet insurance policies. If the pet dies during the year, the sum assured is payable at the end of the year. If the pet escapes, no benefit is payable and the policy is cancelled.

The company uses the following dependent decrement rates to price these policies:

$(aq)^{death}$	0.25
$(aq)^{escape}$	0.35

The company assumes that the forces of decrement are constant over the year, that the two decrements operate independently and that the independent and dependent forces of decrement are equal.

The company has proposed reducing the underlying **independent** mortality rate to 70% of the current value.

Calculate, showing all working, the proposed new dependent decrement rates. [7]

5 The gross future loss random variable at inception for a policy issued to a life, aged 50 exact, is given by:

$$1{,}500\,a_{\overline{\min(K_{50},\ 15)}|} + 35\left(\ddot{a}_{\overline{\min(K_{50}+1,\ 15)}|} - 1\right) + 200 - 0.97 \times P$$

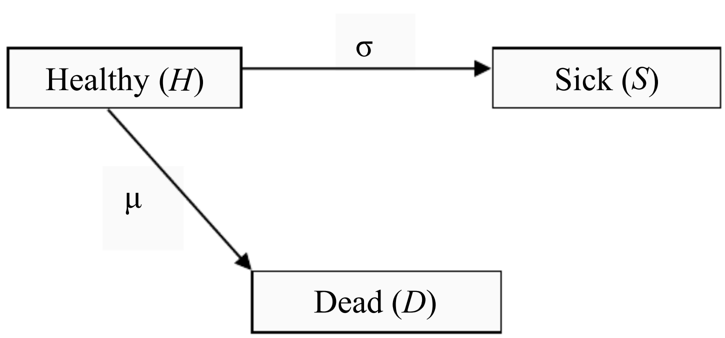
where:

P = premium paid.
K_x = curtate future lifetime of a person aged x exact.

Describe in words the timings and amounts of the benefits, expenses and premium of the policy implied by the gross future loss random variable. [5]

6 A life insurance company sells a special 15-year pure endowment assurance policy, where a sum assured of $10,000 is paid at the end of the term if the policyholder remains healthy throughout.

The following three-state transition model is used:



(i) Show that the expected present value of the benefit, in respect of a healthy life aged 50 exact, is approximately equal to $1,054. [4]

Premiums of P per annum are payable continuously throughout the policy term, ceasing on death or if the policyholder becomes sick.

(ii) Calculate P, showing all working. [5]

Basis:
$\mu = 0.04$
$\sigma = 0.08$
Force of interest: 3% per annum

[Total 9]

7 An organisation is required to pay $5 million in 18 years' time. It holds a cash fund equal to the present value of the liabilities and wishes to use the whole fund to purchase a combination of 9-year and 20-year zero-coupon bonds in order to immunise against small changes in the interest rate. The current interest rate is 8% per annum effective.

(i) Calculate, showing all working, the proportion of the cash fund that should be invested in each of the two zero-coupon bonds. [8]

The organisation invests in the two bonds in accordance with the proportions calculated in part (i).

(ii) Estimate, using the volatility, the percentage change in the present value of the assets, if the interest rate were to increase to 8.5% per annum effective. You should show your working. [4]

[Total 12]

8 A life insurance company issues a retirement policy to a male policyholder aged 60 exact. The policy provides the following benefits:

- An immediate whole life level annuity of £25,000 per annum payable monthly in advance.
- A reversionary annuity of £15,000 per annum payable monthly in advance to the policyholder's surviving female spouse, who is currently aged 62 exact.
- A lump sum of £200,000 payable immediately to the policyholder's family if the policyholder dies before age 65.

Calculate the expected present value of the policy benefits at outset. You should show all working.

Basis:

Mortality	Male PMA92C20
	Female PFA92C20
Interest	4% per annum effective

[13]

9 A government issues a fixed interest bond paying coupons at a rate of 9% per annum, payable half-yearly in arrears. The bond is to be redeemed at \$110 per \$100 nominal on any coupon payment date from 10 to 15 years after issue. The date of redemption is at the discretion of the government.

Investor A is subject to income tax at 25% and capital gains tax at 30%, and wishes to achieve a net redemption yield of at least 6% per annum effective.

(i) Calculate, showing all working, the maximum price per \$100 nominal that Investor A should offer for this bond on issue. [5]

Investor A purchases the bond at the price determined in part (i).

Three years after issue, immediately after a coupon payment has been made, Investor A decides to sell the bond to Investor B.

Investor B is subject to income tax at 10% and capital gains tax at 35%, and wishes to obtain a net redemption yield of at least 8% per annum effective.

(ii) Calculate, showing all working, the maximum price per \$100 nominal that Investor A can expect to receive from Investor B. [7]

The bond is sold to Investor B at the price determined in part (ii).

(iii) Calculate, using linear interpolation, the net effective annual redemption yield that will be obtained by Investor A. You must show all your working. [5]

[Total 17]

10 A life insurance company issued a 15-year special endowment assurance product on 1 January 2013, to a portfolio of policyholders aged 50 exact. The benefits were as follows:

- A death benefit, payable at the end of the year of death, of $100,000 during the first policy year, which reduces by $5,000 each subsequent year.
- Survival benefits of $20,000 payable on survival to each of the 10th and 15th policy anniversaries.

Level annual premiums are payable, annually in advance, ceasing after 10 years, or on earlier death.

(i) Show that the annual premium is approximately equal to $2,786. You must show all your working. [11]

On 1 January 2022, there were 225 policies in force, and 2 policyholders died during 2022.

(ii) Calculate, showing all working, the mortality profit for this portfolio for 2022. [10]

(iii) Explain your results in part (ii). [2]

Basis:

Mortality	AM92 Ultimate
Interest	6% per annum effective

[Total 23]

END OF PAPER

EXAMINERS' REPORT

CM1 - Actuarial Mathematics

Core Principles

Paper A

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

For some candidates, this may be their first attempt at answering an examination using open books and online. The Examiners expect all candidates to have a good level of knowledge and understanding of the topics and therefore candidates should not be overly dependent on open book materials. In our experience, candidates that spend too long researching answers in their materials will not be successful either because of time management issues or because they do not properly answer the questions.

Many candidates rely on past exam papers and examiner reports. Great caution must be exercised in doing so because each exam question is unique. As with all professional examinations, it is insufficient to repeat points of principle, formula or other text book works. The examinations are designed to test "higher order" thinking including candidates' ability to apply their knowledge to the facts presented in detail, synthesise and analyse their findings, and present conclusions or advice. Successful candidates concentrate on answering the questions asked rather than repeating their knowledge without application.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
July 2023

A. General comments on the *aims of this subject and how it is marked*

CM1 provides a grounding in the principles of modelling as applied to actuarial work - focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations, but candidates are not penalised for this. However, candidates may not be awarded full marks where excessive rounding has been used or where insufficient working is shown.

Although the solutions show full actuarial notation, candidates were generally expected to use standard keystrokes in their solutions.

Candidates should pay attention to any instructions included in questions. Failure to do so can lead to fewer marks being awarded. In particular, where the instruction, "*showing all working*" is included and the candidate shows little or no working, then the candidate will be awarded very few marks even if the final answer is correct.

Where a question specifies a method to use (e.g. *determine the present value of income using annuity functions*) then, if a candidate uses a different method, the candidate will not be awarded full marks, indeed, the candidate might even be awarded no marks.

Candidates are advised to familiarise themselves with the meaning of the command verbs (e.g. state, determine, calculate). These identify what needs to be included in answers in order to be awarded full marks.

B. Comments on *candidate performance in this diet of the examination.*

The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those candidates.

As in previous exam diets, there appeared to be a large number of insufficiently prepared candidates who had underestimated the quantity of study required for the subject.

The nature of the online exam format meant that there was little on the paper that could be answered via bookwork knowledge alone.

Where candidates made numerical errors, the examiners awarded marks for the correct method used and also for the parts of the calculation that were correct. However, many candidates often did not show enough of their working to fully benefit from this.

The Examiners felt that the "open book" nature of the online exam led some candidates to rely on their notes much more than if the exam had been "closed book". The Examiners

strongly recommend that candidates prepare for online exams just as thoroughly as they would do if the exam were of the traditional "closed book" format. Candidates are recommended to use their notes only as a tool to check or confirm answers where necessary, rather than as a source for looking up the answers.

C. Pass Mark

The Pass Mark for this exam was 62
1,903 presented themselves and 775 passed.

Solutions for Subject CM1 Paper A- April 2023

Q1
(i)

${}_{a|b}q_{[x]+1}$ is

The probability that a life aged *x+1 exact* who entered the select population at age *x exact* will survive *a* years and then die within the following *b* years, that is, they will die between exact ages *x+1+a* and *x+1+a+b*.

[2]

(ii)

$${}_{5|3}q_{[38]+1} = \frac{l_{[38]+1+5} - l_{[38]+1+3+5}}{l_{[38]+1}} \quad [1]$$

$$= \frac{l_{44} - l_{47}}{l_{[38]+1}} = \frac{9{,}814.3359 - 9{,}771.0789}{9{,}864.4047} = 0.004385161 \quad [1]$$

Alternative:

$${}_{5|3}q_{[38]+1} = {}_5p_{[38]+1} \times {}_3q_{44} = \frac{l_{44}}{l_{[38]+1}} \times (1 - {}_3p_{44})$$

$$= \frac{l_{44}}{l_{[38]+1}} \times \left(1 - \frac{l_{47}}{l_{44}}\right) = \frac{9{,}814.3359}{9{,}864.4047} \times \left(1 - \frac{9{,}771.0789}{9{,}814.3359}\right)$$

$l_{[38]+1} = 9{,}864.4047$

$l_{44} = 9{,}814.3359$

$l_{47} = 9{,}771.0789$

[Total 4]

This question was generally well answered.

Some candidates did not seem to appreciate that the individual entered the select population at age [x] and was currently aged [x]+1 (so the life had been in the select population for 1 year).

Q2
From the general insurance company's perspective:
The size of claims will be unknown [½]
Different type of claims give rise to different sizes of claims - range from small (for example for minor damage) through medium (for example for total loss), to high (for example for a large personal injury claim under liability cover) [½]
Some types of claims have an upper limit e.g. property claims limited to replacement value of assets [½]
The timings of cashflows will be unknown [½]

as the timing of future insured events occurring is unknown. [½]
The timing will also depend on the time between the insured event occurring and the time at which the claim is reported [½]
and on the length of time it takes to settle the claim [½]
It is normal for claims arising due to property claims to be settled more quickly than those arising from liability claims. [½]

[Marks available 4½, maximum 3]

[Total 3]

> *Candidates should read the question carefully and answer the question asked. Here the question referred to "claims" and so only cash flows relating to "claims" gained credit.*

Q3

We require payments to be made at ages 82.5, 82.75, 83, 83.25, 83.5
The present value of these payments is

$$\frac{0.25(l_{82.5}v^{0.25}+l_{82.75}v^{0.5})}{l_{82.25}}+\frac{l_{83}}{l_{82.25}}v^{0.75}\ddot{a}_{83}^{(4)}$$ [3]

We assume there is a uniform distribution of deaths between integer ages 82 and 83

$l_{82}=6{,}188.234$

$l_{83}=5{,}772.378$ [½]

Using interpolation to find $l_{82.25}$, $l_{82.5}$ and $l_{82.75}$

$l_{82.25}=-(6{,}188.234-5{,}772.378)*(82.25-83)+5{,}772.378=6{,}084.27$ [½]

$l_{82.5}=-(6{,}188.234-5{,}772.378)*(82.5-83)+5{,}772.378=5{,}980.306$ [½]

$l_{82.75}=-(6{,}188.234-5{,}772.378)*(82.75-83)+5{,}772.378=5{,}876.342$ [½]

$\ddot{a}_{83}^{(4)}\approx\ddot{a}_{83}-3/8=6.468-3/8=6.093$ [1]

PV of payments

$$=0.25\frac{(5{,}980.306v^{0.25}+5{,}876.342v^{0.5})}{6{,}084.27}+\frac{5{,}772.378}{6{,}084.27}v^{0.75}6.093$$

$=0.4801+5.6131=6.0932$ [1]

Alternative method:
We require payments to be made at ages 82.5, 82.75, 83, 83.25, 83.5
The present value of these payments is

$$\frac{0.25(l_{82.5}v^{0.25}+l_{82.75}v^{0.5})}{l_{82.25}}+\frac{l_{83}}{l_{82.25}}v^{0.75}\ddot{a}_{83}^{(4)}$$

We assume there is a constant force of mortality between integer ages 82 and 83.

$p_{82} = 1 - q_{82} = 1 - 0.067201 = 0.932799$

$\mu_{82} = -\ln(0.932799) = 0.069566$

We derive $l_{82.25}$, $l_{82.5}$ *and* $l_{82.75}$.

$l_{82.25} = l_{82} \exp^{-0.25\mu_{82}} = 6{,}188.234 \times \exp^{-0.25 \times 0.069566} = 6{,}081.543$

Similarly, $l_{82.5} = 5{,}976.691$ *and* $l_{82.75} = 5{,}873.646$ [½]

$\ddot{a}_{83}^{(4)} \approx \ddot{a}_{83} - 3/8 = 6.468 - 3/8 = 6.093$

PV of payments

$$= 0.25 \frac{\left(5{,}976.691 v^{0.25} + 5{,}873.646 v^{0.5}\right)}{6{,}081.543} + \frac{5{,}772.378}{6{,}081.543} v^{0.75}\, 6.093$$

$= 0.4801 + 5.6156 = 6.0957$

[Total 7]

There were several ways this question could be answered and all of them started by writing out the cash flow of the annuity in terms of the payments, the probability of payments being received and the discount factors.

There are then various ways of calculating the required probabilities depending on whether the candidate assumed a uniform distribution of deaths or a constant force of mortality. As the question did not state which method should be used then either was acceptable.

Q4

$(aq)_x^{death}$	0.25
$(aq)_x^{escape}$	0.35

$(ap)_x = 1 - 0.25 - 0.35 = 0.4$ [½]

Force of mortality = $\left(\frac{0.25}{0.25 + 0.35}\right) \times -\ln(0.4) = 0.381788$ [1½]

Force of Escape = $\left(\frac{0.35}{0.6}\right) \times -\ln(0.4) = 0.534503$ [½]

$q_x^{death} = 1 - e^{-0.381788} = 0.317360$ [1]

and 70% of this is 0.222152 [½]

Therefore, the adjusted force of mortality is $-\ln(1 - 0.222152) = 0.251224$ [½]

The adjusted ${}^{adj}(ap)_x = e^{-(0.251224 + 0.534503)} = 0.455788$ [½]

And ${}^{adj}(aq)_x^{death} = \left(\frac{0.251224}{0.251224 + 0.534503}\right) \times (1 - 0.455788) = 0.174003$

[1½]

$${}^{adj}(aq)_x^{escape} = \left(\frac{0.534503}{0.251224+0.534503}\right)\times(1-0.455788) = 0.370208$$ [½]

[Total 7]

This question was poorly answered. This question was about the application of the formulae in the multiple decrement model.

Most candidates were correctly able to derive forces of mortality and escape. But then made mistakes when adjusting the underlying mortality rate.

Many candidates directly adjusted the forces. Many candidates adjusted both the mortality and escape figures. These errors seem to point to poor reading of the question.

Q5

The benefit is a temporary life annuity:
of 1,500 per annum [½]
payable in arrears [½]
until the death of the policyholder [½]
up to a maximum term of 15 years. [½]
Initial expense (incurred at the outset of the policy i.e.at t=0) of 200 [½]
Initial expense (incurred at the outset of the policy i.e.at t=0) of 0.03P. [½]

Renewal expenses (or annuity) of 35 [½]
are incurred annually in advance from the beginning of the second year onwards
(until the death of the policyholder or for a maximum of 14 payments) [1]
iA single premium of P is payable at outset [½]

[Total 5]

Most candidates gained some of the available marks.

A common error was to exclude some of the necessary information from the description of the benefit and/or expenses.

Q6

(i)

The probability that the benefit will be paid is ${}_{15}p_{50}^{\overline{HH}} = e^{-\int_{50}^{65}(\mu_x+\sigma_x)\,dx}$ [1½]

Hence, EPV benefits is given by

$10,000\times v^{15}\times {}_{15}p_{50}^{\overline{HH}} = 10,000\times v^{15}\times {}_{15}p_{50}^{HH} = 10,000\times e^{-15\times 0.03}\times e^{-15\times(0.04+0.08)}$ [1½]

$= 10,000\times e^{-15\times(0.15)} = 10,000\times e^{-2.25} = 1,053.99 \approx \$1,054$ [1]

(ii)
Let annual premium be P then

EPV of premium $= P\int_0^{15} e^{-\delta t}\,.\,{}_tp_{50}^{\overline{HH}}dt = P\int_0^{15} e^{-\delta t}.e^{-(\mu+\sigma)t}\,dt$ [2]

$= P\int_0^{15} e^{-(\delta+\mu+\sigma)t}\,dt$

$= P\times\frac{-1}{0.15}\left[e^{-0.15t}\right]_0^{15}$

$= \frac{P}{0.15}\times\left(1-e^{-2.25}\right)$
= 5.96401 P [2]

Hence P = $\frac{1053.99}{5.96401} = \$176.73$ per annum [1]

[Total 9]

This question was poorly attempted.

In part (i), the benefit is valued at a specific point in time and so should be valued using a probability and discount factor only. A common error was to use an incorrect probability and to integrate the equation.

In part (ii) the premium is payable continuously and so the expected present value of the premium should be integrated over the premium payment period. A common error was to use an incorrect probability.

Q7
(i)
Let A = the nominal amount invested in the 9-year zero coupon bond
and B = the nominal amount invested in the 20-year zero coupon bond

PV of liabilities $= 5,000,000\times v_{8\%}^{18} = 1,251,245.1456$ [½]
PV of assets $= A\times v_{8\%}^{9} + B\times v_{8\%}^{20}$ [1]
PV of assets = PV of liabilities
$A\times v_{8\%}^{9} + B\times v_{8\%}^{20} = 1,251,245.1456\ldots\ldots \boxed{1}$ [½]
DMT of assets = $\frac{\left(9\times A\times v_{8\%}^{9} + 20\times B\times v_{8\%}^{20}\right)}{1,251,245.1456}$ [1]

DMT of liabilities= $= \dfrac{(18\times 1,251,245.14561)}{1,251,245.1456} = \dfrac{22\ 522\ 412.62044}{1,251,245.1456} = 18$ [½]

DMT of assets= DMT of liabilities

$\dfrac{\left(9\times A\times v_{8\%}^{9} + 20\times B\times v_{8\%}^{20}\right)}{1,251,245.1456} = 18$ [½]

$9\times A\times v_{8\%}^{9} + 20\times B\times v_{8\%}^{20} = 18\times 1,251,245.1456 = 22\ 522\ 412.62044\ldots\ldots \boxed{2}$

$\boxed{2} - 9\times\boxed{1} : (20-9)\times Bv_{8\%}^{20} = (18-9)\times 1,251,245.1456 = 11,261,206.31022$ [2]

$\Rightarrow Bv_{8\%}^{20} = 1,023,746.028202$ either/or $Av_{8\%}^{9} = 227,499.117378$ [1]

$\dfrac{1,023,746.028202}{1,251,245.1456}\times 100 = 81.8182\%$ [½]

Invest 81.8182% of cash into 20-year zero coupon bond
Invest (100 - 81.8182) = 18.1818% of cash into 9-year zero coupon bond [½]

(ii) DMT of assets = DMT liabilities = 18

Volatility (@ i=8%) = $\dfrac{18}{(1.08)} = 16.666667$ [2]

Percentage change in the present value of the assets: $-(0.085-0.08)\times 16.666667$ [1]
$= -8.33333\%$ [1]
a decrease in the value or negative answer

[Total 12]

In part (i) most candidates correctly set up the simultaneous equations to be solved, but then failed to correctly solve them. A common error was to calculate the nominal amounts (instead of the amounts to be invested). Many candidates missed marks by not giving the required proportions invested.

Part (ii) was not well answered. Many candidates did not know how to use the volatility to estimate the required percentage change in the present value of assets. A common error was to calculate the volatility at an interest rate of 8.5% per annum.

Q8

1st Benefit

$25,000\times \ddot{a}_{60}^{(12)}$ [1½]

$\ddot{a}_{60}^{(12)} = 15.632 - \dfrac{11}{24} = 15.173667$ [1]

$25,000\times 15.173667 = 379,341.67$ [½]

Present Value of 1st benefit is £379,342

2nd Benefit

$15,000 \times \ddot{a}^{(12)}_{60|62}$ [1½]

$\ddot{a}^{(12)}_{60|62} = \ddot{a}^{(12)}_{62} - \ddot{a}^{(12)}_{60:62} \approx \ddot{a}_{62} - \frac{11}{24} - \ddot{a}_{60:62} + \frac{11}{24} = 15.963 - 13.734 = 2.229$ [2]

$15,000 \times 2.229 = 33,435$ [½]

PV of 2nd benefit is £33,435

<u>3rd Benefit</u>

$200,000 \bar{A}^{1}_{60:\overline{5}|}$ [1½]

$= 200,000 \times 0.016251699 = 3,247.0075$ [½]

$\bar{A}^{1}_{60:\overline{5}|} = (1.04)^{0.5} A^{1}_{60:\overline{5}|}$ [1, claim acceleration]

$= (1.04)^{0.5} \left(A_{60} - v^5 \frac{l^m_{65}}{l^m_{60}} A_{65} \right)$ [1, term assurance]

$= (1.04)^{0.5} \left[\left(1 - d \times (\ddot{a}_{60})\right) - 0.80701 \times \left(1 - d \times (\ddot{a}_{65})\right) \right]$ [1, premium conversion]

$= (1.04)^{0.5} \left[\left(1 - d \times (15.632)\right) - 0.80701 \times \left(1 - d \times (13.666)\right) \right]$

$= 0.016251699$

$d = 0.038461538$

$\ddot{a}_{60} = 15.632$

$\ddot{a}_{65} = 13.666$

$l_{60} = 9,826.13$

$l_{65} = 9,647.797$ [1]

Total is 379,342 + 33,435 + 3,247 = 416,027.0075

Total present Value of Benefit is £416,027

[Total 13]

This question was well answered.

Most errors occurred with the lump sum benefit where some candidates did not know how to calculate the term assurance function from annuity values (i.e. using the premium conversion formula).

Q9

(i)

$i = 6\% \Rightarrow i^{(2)} = 5.9126028\%$ [½]

$\frac{0.09 \times (1 - 0.25)}{1.1} = 6.1363636\% > i^{(2)} = 5.9126028\%$ [1]

Thus a Capital loss is made, the worst case scenario for the investor is to redeem as early as possible

thus n= 10 [1½]

$P = (1-0.25)\times 9a^{(2)}_{\overline{10}|6\%} + 110v^{10}_{6\%} = \$111.838367013$ [2]

(ii)

$i = 8\% \Rightarrow i^{(2)} = 7.8460969\%$ [½]

$\dfrac{0.09\times(1-0.1)}{1.1} = 7.3636364\% < i^{(2)} = 7.8460969\%$ [1]

$\Rightarrow$ Capital gain is made, worst case scenario for the investor is to redeem as late as possible at t=15 that is at n= 12. [1½]

$P = (1-0.1)\times 9a^{(2)}_{\overline{12}|8\%} + 110v^{12}_{8\%} - 0.35\times v^{12}_{8\%}\times\left(110-P\right)$ [2]

$P\times(1-0.138989816) = 105.922103556 - 15.2888797$

$\Rightarrow P = \$105.2638232$ [2]

(iii)

$111.838367013 = (1-0.25)\times 9a^{(2)}_{\overline{3}|i\%} + 105.2638232\times v^{3}_{i\%}$ [2½]

Interest rate	Equation value
0.04	112.496502
i	111.838367
0.045	111.004293

[1]

Choosing appropriate interest rates - the two interest rates must straddle the final answer with a maximum gap of 1% if the final answer < 20% [1]

$i = 0.042 - \left(\dfrac{112.496502 - 111.838367}{112.496502 - 111.004293}\right)\times\left(0.04 - 0.045\right) = 4.2205\% = 4.2\%$ [½]

[Total 17]

This question was generally well answered.

In part (i) a common error was to include capital gains tax when calculating the price despite the capital gain (CG) test indicating a capital loss would be made.

In part (ii) a common error was not to repeat the CG test with the new information (but assume the same result as in (i)) and not to reduce the term by the 3 years that had elapsed.

In the CG test for both parts (i) and (ii) candidates needed to explicitly interpret the CG test and consider the worst case scenario for the investor in order to obtain full credit.

Q10

(i)

Death benefit

$105,000 \times A^{1}_{50:\overline{15}|} - 5,000\left(IA\right)^{1}_{50:\overline{15}|}$ [2]

$= 105,000 \times 0.052817362 - 5,000 \times 0.473339332 = 3,179.126357737$ [½]

$A^{1}_{50:\overline{15}|} = A_{50:\overline{15}|} - v^{15}_{6\%} \times \frac{l_{65}}{l_{50}} = 0.43181 - v^{15}_{6\%} \times \frac{8,821.2612}{9,712.0728} = 0.052817362$ [1½]

$A^{1}_{50:\overline{15}|} = A_{50} - v^{15}_{6\%} \times \frac{l_{65}}{l_{50}} \times A_{65} = \quad 0.20508 \; - v^{15}_{6\%} \times \frac{8,821.2612}{9,712.0728} \times 0.40177 = 0.05281213$

$\left(IA\right)^{1}_{50:\overline{15}|} = \left(IA\right)_{50} - v^{15}_{6\%} \times \frac{l_{65}}{l_{50}}\left[\left(IA\right)_{65} + 15A_{65}\right]$ [1]

$= 4.84555 - 0.417265061 \times 0.908277912\left[5.50985 + 15 \times 0.40177\right]$

$= 4.84555 - 0.378992638\left[5.50985 + 15 \times 0.40177\right]$ [1]

$= 0.4733399332$

Survival benefits

$20,000 \times v^{10}_{6\%} \times {}_{10}p_{50} + 20,000 \times v^{15}_{6\%} \times {}_{15}p_{50}$

$= 20,000 \times 0.533967695 + 20,000 \times 0.378992638$ [2]

$= 10,679.353906491 + 7,579.852758892 = 18,259.206665384$ [½]

${}_{10}p_{50} = \frac{l_{60}}{l_{50}} = \frac{9,287.2164}{9,712.0728} = 0.95625482$ [½]

${}_{15}p_{50} = \frac{l_{65}}{l_{50}} = \frac{8,821.2612}{9,712.0728} = 0.90827791$ [½]

The total value of benefits is $3,179.126 + 18,259.207 = 21,438.333$

$P\ddot{a}_{50:\overline{10}|6\%} = P \times 7.694$ [1]

$P = \$2,786.37$ per annum which is approximately equal to $2,786 p.a. [½]

(ii)

${}_{10}V = 55,000 A^{1}_{60:\overline{5}|} - 5,000\left(IA\right)^{1}_{60:\overline{5}|} + 20,000 \times v^{5}_{6\%} \times {}_{5}p_{60}$ [2½]

$= 2,296.41616 - 645.9750784 + 14,195.339578146 = 15,845.780659796$ [½]

$A^{1}_{60:\overline{5}|} = A_{60:\overline{5}|} - v^{5}_{6\%} \times \frac{l_{65}}{l_{60}} = 0.75152 - v^{5}_{6\%} \times \frac{8,821.2612}{9,287.2164} = 0.041753021$ [1]

$\left(IA\right)^{1}_{60:\overline{5}|} = \left(IA\right)_{60} - v^{5}_{6\%} \times \frac{l_{65}}{l_{60}}\left[\left(IA\right)_{65} + 5A_{65}\right]$

$$= 5.46572 - v_{6\%}^{5} \times \frac{l_{65}}{l_{60}} \times \left[5.50985 + 5 \times 0.40177\right] = 0.129195016$$ [2]

$q_{59} = 0.00714$ [½]

$DSAR = S - ({}_{10}V + R) = 55{,}000 - (15{,}845.78065 + 20{,}000) = 19{,}154.2193402039$ [1½]

$E(\#\ of\ \ deaths) = 225 \times q_{59} = 225 \times 0.00714 = 1.6065$ [½]

$EDS = 1.6065 \times DSAR = 30{,}771.25337$ [½]

$ADS = 2 \times DSAR = 38{,}308.43868$ [½]

$Mortality\ profit = EDS - ADS = \$ - 7{,}537.18531$ [½]

(iii)
The amount of money the company needs to pay for each life that dies is $55,000 [½]
The amount of money the company needs for each life that survives the year is $35,846 (= $20,000 survival benefit + a reserve of $15,846) [½]
Since $55,000 > $35,846 the death strain at risk is positive [½]

With a positive DSAR, excess deaths will lead to a mortality profit [½]

Here actual deaths (2) are greater than expected deaths (1.6), so experienced mortality was heavier than expected [½]
Thus, the company has experienced a mortality loss.

[Marks available 2½, maximum 2]
[Total 23]

This question was well answered.

In part (ii) common errors in the reserve calculation included using an incorrect sum assured for the death benefit and including premiums.

A common error in the DSAR calculation was not including the survival benefit.

[Paper Total 100]

END OF EXAMINERS' REPORT

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