INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

10 April 2025

Subject CS2 – Risk Modelling and Survival Analysis Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

1 A pension scheme pays an annual pension on members' birthdays beginning on their 65th birthday. The scheme guarantees that at least five pension payments will be made whether the member survives or not. The scheme actuary uses an exponential model for mortality up to age 70 and then uses Gompertz' Law from age 70 onwards. The force of mortality below age 70 is assumed to be equal to that given by Gompertz' Law at age 70.

(i) Write down an expression for the force of mortality from age 70 in terms of age, x and two constants B and c. [1]

(ii) Calculate the values of B and c if $\mu_{75} = 0.043$ and $\mu_{80} = 0.062$. [3]

(iii) Calculate the probability that a member aged 50 dies between the ages of 66 and 68. [2]

(iv) Calculate the expected number of additional pension payments the scheme will make per 1,000 members as a result of the five payments guarantee. [4]

[Total 10]

2 Claims on a health insurance policy follow the lognormal distribution with mean 1,000 and standard deviation 300.

(i) Calculate the parameters of the distribution of claims. [4]

The insurer effects an individual excess of loss reinsurance treaty with a retention limit of 400.

(ii) Calculate the probability that the claim amount paid by the reinsurer is more than 600. [4]

(iii) Comment on your answer to part (ii). [1]

[Total 9]

3 An economic advisor to the government of a country has stated that income inequality has increased in the last decade when compared to the decade before that (the 'previous decade'). The government's economic department determines income inequality by looking at the ratio between the minimum threshold of the top 5% of earners and the average annual income. The ratio was 2.5 in the previous decade. Average annual income in the last decade has been 0.785 (working in units of \$100,000).

Income distribution is assumed to follow a Burr distribution with parameters α, λ and γ (as given in the IFoA Formulae and Tables). During the last decade, it is believed that $\alpha = 2$ and $\gamma = 2$. The values of α, λ and γ are derived by working in units of \$100,000.

(i) Determine λ, rounded to the nearest integer given $\Gamma(1.5) = 0.886$. [4]

(ii) Calculate, showing all workings, the minimum income threshold of the top 5% of earners and the income inequality ratio. [5]

(iii) Comment on the economic advisor's statement given your answers in part (ii). [1]

[Total 10]

4 A medical statistician wishes to compare the treatment for a virus in the two hospitals in a certain city. These hospitals are called North and South. The researcher collects data on the number of days to recovery for patients admitted to hospital with this virus and fits Cox's Proportional Hazard model with three covariates:

- z_1, which is 0 for patients in North and 1 for patients in South
- z_2, which measures the number of chronic illnesses patients have
- z_3, which measures the number of previous times the patient has had the same virus in the last 10 years.

(i) State the lives who are represented by the baseline hazard. [1]

When the model is fitted, the regression coefficients and their associated *p*-values are as follows:

Covariate z_i	*Coefficient β_i*	*p-value*
1. Hospital	0.06	0.337
2. Chronic illnesses	−0.21	0.004
3. Previous times	−0.09	0.023

(ii) Comment on the model findings on the effect of the covariates on time to recovery using the model results in the table above. [5]

A second, separate study finds that the probability of a patient who has no chronic illnesses and who has never had this virus before, is still unwell 5 days after admission to the South hospital, is 0.92.

(iii) Calculate the probability that a patient with two chronic illnesses who had this virus once before will still be unwell 5 days after admission to the North hospital, assuming that the second study result can be used to determine the baseline hazard. [5]

[Total 11]

5 Consider the following time series model:

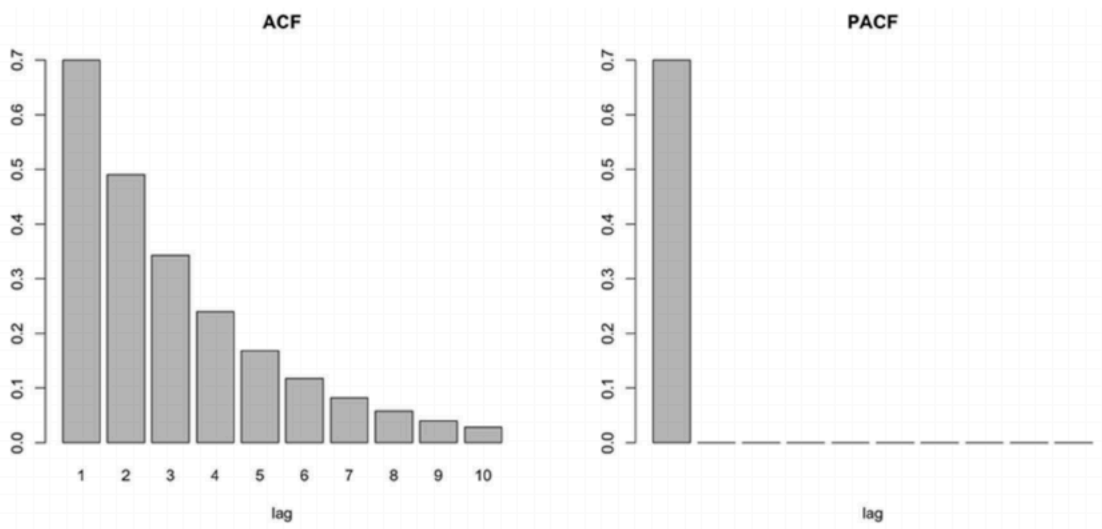
$$y_t = \alpha y_{t-1} + \beta e_{t-1} + e_t$$

where $e_t \sim N(0,\sigma^2)$ and the e_t are assumed to be independent for any t.

(i) Identify the conditions that parameters α and β should satisfy for the process to be stationary. [3]

(ii) Derive the expressions for the Autocorrelation Function (ACF) of this process assuming it is stationary. [8]

For a particular choice of α and β the ACF and Partial Autocorrelation Function (PACF) plots of the model above are shown in the following figure:



(iii) State the values of α and β chosen, giving a reason for your answer for each one. [4]

[Total 15]

6 The insurance regulator in a certain country rates all insurance companies each year based on a set of financial statements they submit to the regulator on 1 January each year. Each insurance company is rated on its ability to meet claims in the coming year with the ratings being 'No Concern', 'On Watch' and 'Insolvent'.

The regulator estimates that 3% of companies that were No Concern last year will move to On Watch and that 0.2% of companies that were No Concern will be Insolvent in the coming year. Of the companies On Watch last year, 35% return to No Concern and 1% will be Insolvent in the coming year. Once a company is Insolvent its business operations are assumed to cease.

Changes in rating are modelled using a Markov chain.

(i) Write down the 1-year transition matrix. [1]

The regulator monitors 250 companies all of which start as being of No Concern.

(ii) Calculate the expected number of Insolvent insurance companies after 2 years. [2]

(iii) Calculate the probability that an insurance company will never be On Watch. [4]

The regulator would like to improve their monitoring by considering real-time data on claims events.

(iv) Explain how the Markov chain model would need to be adapted to achieve this. [4]

(v) Describe two methods for simulating the new model. [4]

[Total 15]

7 An actuary is required to model the consumer price index Q_t, based on some time series observations: $Q_1, Q_2, \ldots, Q_{101}$.

The actuary considers a stationary time series model for the corresponding log ratios $r_1, r_2, \ldots, r_{100}$ where $r_i = \log\left(\frac{Q_{i+1}}{Q_i}\right)$.

The numerical summaries for the ACF and PACF values for the log ratios r_i are:

	Lag 1	*Lag 2*	*Lag 3*
ACF	0.522	0.156	0.046
PACF	0.522	−0.160	0.050

The sample mean and sample variance for r_i are 0.8 and 0.2, respectively.

You are given the model, $r_t = a_0 + a_1 r_{t-1} + \varepsilon_t$ where ε_t is a white noise process with mean zero and variance σ^2.

(i) Fit the model for r_t based on the information above. [4]

The last three observed values of Q_t are $Q_{99} = 6.87$, $Q_{100} = 7.20$ and $Q_{101} = 7.45$.

(ii) Derive the forecast value for r_{101} together with the corresponding 95% confidence interval. [6]

(iii) Derive, using the result from part (ii), the forecast value for the next observation, Q_{102}, together with the corresponding 95% confidence interval. [4]

[Total 14]

8 A sports club operates a membership scheme for its supporters consisting of three membership tiers, basic, medium and advanced, with each tier providing different benefits.

Each supporter's membership tier is determined at the beginning of the month, based on how many game tickets they purchased in the previous month. Supporters who purchase two or more game tickets in a month will be promoted to the next higher tier for the following month (or remain in the advanced tier). Supporters purchasing exactly one ticket in a month will remain in their existing tier. Supporters who do not purchase any tickets in a month will be demoted to the next lower tier (or remain in the basic tier).

Let α, β and δ be the proportion of supporters buying zero, one or at least two tickets respectively, where $\alpha, \beta, \delta > 0$.

(i) Explain why the membership scheme can be modelled as a Markov chain. [1]

(ii) Write down the transition matrix for the process. [2]

(iii) Explain if the chain is:

(a) irreducible. [1]

(b) aperiodic. [1]

Past data shows that in any given month 50% of supporters bought no tickets, 40% bought exactly one ticket and 10% bought at least two tickets.

(iv) Calculate the probability that a supporter who is currently in the medium tier will be in the advanced tier 3 months later. [2]

(v) Determine the stationary distribution of this process, showing all of your workings. [5]

The cost of running the scheme per member per month is £0 for basic members, £10 for medium members and £20 for advanced members. The club earns a profit of S per member for every game ticket sold before deducting the costs associated with the membership scheme.

(vi) Determine the range of values of S for which the club will be able to make an overall profit from the members of the scheme. You should assume that members in the advanced tier buy three tickets on average per month. [4]

[Total 16]

END OF PAPER

Institute and Faculty of Actuaries

EXAMINERS' REPORT

CS2 Risk Modelling and Survival Analysis

Paper A

Core Principles

April 2025

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

For some candidates, this may be their first attempt at a professional qualification examination. The Examiners expect all candidates to have a good level of knowledge and understanding of the topics and therefore candidates should revise thoroughly to prepare for closed-book and in-person examinations. In our experience, candidates that are insufficiently prepared are not successful because of lack of knowledge, time management issues and/or because they do not properly answer the questions.

Many candidates rely on past exam papers and examiner reports in preparing for exams. Great caution must be exercised in doing so because each exam question is unique. As with all professional examinations, it is insufficient to repeat points of principle, formula or other textbook works. The examinations are designed to test "higher order" thinking including candidates' ability to apply their knowledge to the facts presented in detail, synthesise and analyse their findings, and present conclusions or advice. Successful candidates concentrate on answering the questions asked rather than repeating their knowledge without application.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
June 2025

A. General comments on the *aims of this subject and how it is marked*

The aim of the Risk Modelling and Survival Analysis Core Principles subject is to provide a grounding in mathematical and statistical modelling techniques that are of particular relevance to actuarial work, including stochastic processes and survival models.

Some of the questions in this paper admit alternative solutions from those presented in this report, or different ways in which the provided answer can be determined. All mathematically correct and valid alternative solutions received credit as appropriate.

In cases where an error was carried forward to later parts of the answer, candidates were not penalised a second time for the same error if those later parts were otherwise answered correctly.

In higher-order skills questions, where comments were required, well-reasoned comments that differed from those provided in the solutions also received credit as appropriate.

Candidates are advised to take careful note of all instructions that are provided with the exam in order to maximise their performance in future CS2A examinations.

B. Comments on *candidate performance in this diet of the examination*

Overall performance in CS2 examinations continues to improve compared to previous sessions with both the mean mark and the pass rate increasing. The examiners are encouraged to see evidence that candidates are better prepared for the extensive CS2 syllabus. This session the examination moved to a "closed book" format and hence the nature of some of the questions is a little different to reflect this. However we would not want to overstate this for CS2 where in recent Examiners' Reports we encouraged candidates in the "open book" era to prepare for the examination as if it were closed book.

Most questions on the CS2A paper begin by asking for some explanation or more simple calculations related to a particular syllabus area and then progress to later question parts that seek to apply that syllabus area to a particular scenario or to comment on the earlier results. Successful candidates are therefore able to evidence both understanding of the module content and application of the statistical and modelling techniques to a new scenario.

The questions where candidates scored the highest proportion of the available marks were Q2 on risk distributions and Q8 on stochastic processes. Historically it has been the survival models questions that candidates have found the easiest, so it is good to see a wider representation of the syllabus here. The question where candidates scored the lowest proportion of available marks was Q7 on time series applications, an area where we often see lower marks.

C. Pass Mark

The Pass Mark for this exam was 57
948 presented themselves and 456 passed.

Solutions for Subject CS2A – April 2025

Q1

(i)

$\mu_x = Bc^x$ [½]

(ii)

$\mu_{75} = Bc^{75} = 0.043$ (A)

$\mu_{80} = Bc^{80} = 0.062$ (B) [½]

(B) / (A) gives $c^5 = \frac{62}{43}$ [½]

So $\log(c) = \frac{1}{5}\log\left(\frac{62}{43}\right) = 0.073187$ [½]

And c = 1.075932 [½]

Substitute back into (A) $B = {}^{0.043}/_{1.075932^{75}}$ [½]

Gives B = 0.000178 [½]

(iii)

First calculate the exponential force of mortality

$\mu = 0.000178.1.075932^{70} = 0.029823$ [½]

We need the probability of survival for 16 years and then death in next 2 years

Prob = exp(-16 * 0.029823).[1 – exp(-2 * 0.029823)] [1]

= 0.03593 [½]

(iv)

We need the curtate expectation of future life at age 65 for 4 years (because the payments are annual in advance)

Expected pension payments in first 5 years before guarantee is $\sum_{k=0}^{4} {}_kp_{65}$ [1]

t	t_p_65
0	1
1	0.970618
2	0.942099
3	0.914418
4	0.88755
sum	4.714684

[1]

Expected number of pension payments per 1000 members = 4714.684

Therefore number of additional pension payments = 5000 – 44714.684 [1]

=285.3157 (accept 285) [1]

[Total 10]

Commentary:
The first three parts of this question were generally well answered. In part (iii) a number of candidates had not read carefully at which ages Gompertz and exponential mortality applied. Part (iv) was surprisingly not so well answered. The question is a simple application of the expectation of future life but could also have been answered from first principles.

Q2

(i)

$\exp(\mu+1/2\sigma^2) = 1000$ (1) [½]

$\exp(2\mu+\sigma^2)*(\exp(\sigma^2)-1) = 9{,}00{,}00$ (2) [½]

Dividing equation (2) by the square of (1) we get [½]

$\exp(\sigma^2)-1) = 9{,}00{,}00/ 1000000 = .09$ [½]

$\sigma^2 = \ln(1.09)= .086178$ [½]

$\sigma = 0.29356$ [½]

Substituting in (1) we get:

$\mu = \ln(1000)- ½* .086178$ [½]

= 6.86467 [½]

[4 marks]

(ii)

Let X denote the individual claim amount variable.

Then $X \sim \log N(6.86467, 0.086178)$. [½]

The claim amount paid by the reinsurer exceeds 600 if $X> 1000$

$P(X>1000) = \int_{1000}^{\infty} f(x)dx$, where f(x) is p.d.f of log N(6.86467,0.086178) [½]

$= \Phi(U_0) - \Phi(L_0)$ [½]

Where $U_0 = \infty$ and

$L_0 = (\ln(1000)- 6.86467)/0.29356$ [½]

$= (\ln(1000)- 6.86467) /.29356$

= 0.146666 [½]

So, $P(X>1000) = \Phi(\infty) - \Phi(L_0)$

$= 1- \Phi(L_0)$

$=1- \Phi (0.146666)$ [½]

$= 1- 0.558302$ [½]

$= 0.44169$ or 44.1698% [½]

[4 marks]

(iii)

P(amount paid > 600) = P(amount of claim > 1000)

= P(amount of claim > mean) [½]

Which in this case < 50% given the shape of lognormal distribution [½]

Note: no marks given for comments on the commercial implications of reinsurance or speculation about the reinsurance premium. Marks awarded are for comments about the probability distribution.

[1 mark]

[Total 9]

Commentary:
Parts (i) and (ii) were well answered and is a relatively straightforward application of the log normal distribution to a reinsurance scenario. Although only one mark, part (iii) was not well answered. Many candidates speculated about the commercial implications of reinsurance rather than commenting on the probability distribution.

Q3

(i)

Mean =lambda ^ (1/gamma) * Γ(alpha)Γ(1+1/gamma)·Γ(alpha −1/gamma)

0.785 = lambda ^ 0.5 * Γ2 Γ(1+1/2)·Γ(2 −1/2) [1½]

= lambda ^ 0.5 * 1* Γ1.5 Γ1.5 [1]

= lambda ^ 0.5 * 1* 0.886 * 0.886 [½]

= lambda ^ 0.5 * 1* 0.785 [½]

Giving, lambda = 1 [½]

[4 marks]

(ii)

F(x >X)=(lambda / (lambda + x ^ gamma)) ^ alpha [2]

$0.05 = (1 / (1 + x \wedge 2)) \wedge 2$ [½]

$0.2236 = (1 / (1 + x \wedge 2))$ [½]

$(1 + x \wedge 2) = 1 / 0.2236$ [½]

Giving; x = 1.86 [1]

Ratio = 2.37 [½]

[5 marks]

(iii)

Ratio is lower than 2.5. Hence, inequality has not increased as per the criteria [1]

[1 mark]
[Total 10]

Commentary:
This question covers a risk distribution in a novel scenario but with the aid of the distribution function given in the Formulae and Tables book this question was well answered.

Q4
(i)

A patient in North hospital with no chronic illnesses who has not had the virus before [1]

[1 mark]

(ii)

We minimise recovery times by maximising the hazard [1]

Hazard is slightly higher (recovery shorter) for South [½]

But the difference between hospitals does not appear be large (beta close to 0) [½]

And the coefficient is not statistically significant even at 10% [½]

Chronic illnesses reduce the hazard (longer recovery) [½]

This covariate is statistically significant [½]

prior infections of the virus reduce the hazard (longer recovery) [½]

This covariate is statistically significant [½]

Suggesting overall medical history is more important than place of treatment [1]

[5½ marks available, maximum 5]

(iii)

For the person in the second study z1=1, z2=0, z3=0 [½]

So their hazard function is h_0 * exp(0.06) where h_0 is baseline hazard [1]

And their survival function is S(5) = $\exp\left(-\int_0^5 h_0 e^{0.06}\, dt\right) = 0.92$ [1]

Therefore $\exp\left(-\int_0^5 h_0\, dt\right) = 0.92^{\exp(-0.06)} = 0.924478$ [½]

For the person in the question z1=0, z2=2, z3=1 [½]

So their hazard function is h_0 * exp(-0.51) [½]

Their survival function is S(5) = $\exp\left(-\int_0^5 h_0 e^{-0.51}\, dt\right) = 0.92\,4478^{\exp(-0.51)}$ [½]

$= 0.95394$ [½]

[5 marks]

[Total 11]

Commentary:
The question is an application of Cox's Proportional Hazard model which was somewhat well answered although there was much variability in the quality of answers to part (ii). Key to this part is understanding which way around the hazard function operates in the scenario. The hazard is for recovery not mortality therefore we seek to maximise not minimise the hazard. Candidates who knew the formula for the survival function in terms of the baseline hazard in a proportional hazards model did well in part (iii) but a disappointing number of candidates did not apply the baseline hazard correctly.

Q5

(i)

$|\alpha| < 1$ [2]

and no additional condition for β. [1]

[3 marks]

(ii)

$cov(e_t, Y_t) = \alpha\, cov(e_t, Y_{t-1}) + cov(e_t, e_t) + \beta cov(e_t, e_{t-1}) = \sigma^2$

due to the zero correlation between e_t and e_{t-1} [1]

Similarly

$cov(e_{t-1}, Y_t) = \alpha\, cov(e_{t-1}, Y_{t-1}) + cov(e_{t-1}, e_t) + \beta cov(e_{t-1}, e_{t-1})$

$= (\alpha + \beta)\sigma^2$ [2]

Implying that for k=0,1

$$cov(Y_{t-k}, Y_t) = \alpha\, cov(Y_{t-k}, Y_{t-1}) + \beta cov(Y_{t-k}, e_{t-2}) + cov(Y_{t-k}, e_t)$$

$\gamma_0 = \alpha\gamma_1 + (1 + \alpha\beta + \beta^2)\sigma^2$ [1]

$\gamma_1 = \alpha\gamma_0 + \beta\sigma^2$ [1]

And for k>1

$cov(Y_{t-2}, Y_t) = \alpha cov(Y_{t-1}, Y_t) + \beta cov(Y_{t-2}, Y_t) + cov(e_{t-2}, Y_t) = \alpha\gamma_{k-1}$ [1]

And therefore

$$\rho_1 = \frac{(1+\alpha\beta)(\alpha+\beta)}{(1+\beta^2+2\alpha\beta)}$$

[1]

$\rho_k = \alpha^{k-1}\rho_{1,}$ k > 1

[1]

[8 marks]

(iii)

As there is only one lag at the PACF or because the ACF shows exponential decay [1]

Then $\beta = 0$ [1]

and as the first lag entries of both ACF and PACF are equal to 0.7 [1]

Then $\alpha = 0.7$ [1]

[4 marks]

[Total 15]

Commentary:
Of the two time series analysis questions on the paper this was the one that generated the better answers. In part (ii) there are a number of ways to develop the recursive relationship in rho_k and all were given full credit as long as a number of steps were shown developing the gamma and then the rho functions step by step. Part (iii) is a reminder that candidates are well advised to familiarise themselves with the output from common time series models as well as the equations that characterise these models.

Q6
(i)

$$P = \begin{bmatrix} 0.968 & 0.03 & 0.002 \\ 0.35 & 0.64 & 0.01 \\ 0 & 0 & 1 \end{bmatrix}$$

[1]

[1 mark]

(ii)

By matrix multiplication the two-year transition matrix is $P^2 =$

$$\begin{bmatrix} 0.947524 & 0.04824 & 0.004236 \\ 0.5628 & 0.4201 & 0.0171 \\ 0 & 0 & 1 \end{bmatrix}$$ [½]

And reading from the matrix the required probability = 0.004236 [½]

So the expected number is 250 * 0.004236 [½]

= 1.059 [½]

[2 marks]

(iii)

For a company to never go on watch it must stay of No Concern and then go straight to Insolvent [1]

Pr(NC to I after 1 year) = 0.002

Pr(NC to I after 2 years) = 0.968 * 0.002 [½]

...

Pr(NC to I after k years) = 0.968^(k-1) * 0.002 [1]

Sum of all these probabilities to k=infinity = 0.002/ (1 – 0.968) [1]

= 0.0625 [½]

[4 marks]

(iv)
We move from a Markov Chain to a Markov jump process [1]
With varying claims conditions this must be time in-homogeneous [1]
We would need to estimate transition rates or intensities over time [1]
For this we need waiting times in the different states or residual holding times between jumps and the number of transitions [1]
[4 marks]

(v)
We simulate the Markov jump process as a Markov chain [1]
Can do this approximately by considering very small time periods [½]
and consider the transition intensity to be proportionate to the time period [½]
then model the jump process as a series of discrete chains [½]
Or an exact method needs some other way of estimating the transition probabilities [½]
Then the holding times are a series of exponential random variables [½]
With a rate parameter derived from the transition probabilities [½]
[4 marks]
[Total 15]

Commentary:
The first three parts of this question were somewhat well answered. They are a straightforward application of Markov chains. Parts (iv) and (v) were not well answered which was disappointing because, although this part has not been examined recently, the move from a Markov chain to a jump process and then estimation of a time in-homogeneous process are both set out in the CS2 Core Reading. Whilst the majority of questions in this CS2 A paper will continue to focus on calculations based on the models and distributions covered in the syllabus, candidates should also be prepared to discuss modelling and estimation processes.

Q7
(i)

Simple moments' estimation conforms that

$\hat{a}_1 = 0.522$ [1]

And $\hat{a}_0 = \bar{r}(1 - \hat{a}_1) = 0.8 * (1 - 0.522) = 0.382$ [1½]

Additionally

$\widehat{\sigma^2} = \gamma_0 - a_1\gamma_1 = \gamma_0 * (1 - a_1^2) = 0.2 * (1 - 0.522\text{^}2) = 0.146$ [1½]

[4 marks]

(ii)

From the definition, the observed values for $r_{100} = \log \frac{Q_{101}}{Q_{100}}$

$r_{100} = \log \dfrac{7.45}{7.2} = 0.0341$ [1]

The forecast function is simply obtained

$$\hat{r}_{101} = a_0 + a_1 r_{100}$$

As r_{100} is observed and is 0.04691692 then $\hat{r}_{101} = 0.382 + 0.522 * 0.0341$ [1]

= 0.400 [1]

As the error for this forecast is up to the value of the white noise at time 101, ε_{101}.

The 95%CI for the true value of r_{101} is

$\hat{r}_{101} \pm 1.96\,\sigma$ [1]

$0.400 \pm 1.96 \; * sqrt(0.146) =$ [1]

(-0.347,1.148) [1]

[6 marks]

(iii)

Since $Q_{102} = \exp(r_{101}) * Q_{101}$ the corresponding values are [1]

Therefore $\widehat{Q_{102}} = \exp(0.400) * 7.45 = 11.117$ [1]

And the interval is then

$(\exp(-0.3474223), \exp(1.147857)) * 7.45 =$ [1]

= (5.263, 23.478) [1]

[4 marks]

[Total 14]

Commentary:
This question was not well answered. The application of time series here involved a method of moments estimation to parameterise the series followed by forecasting using that series. The

underlying time series calculations are quite straightforward once what is being modelled (the log ratio function of an economic series) is clarified. This question illustrates well the two key attributes of success in the later questions in a CS2 A examination: detailed understanding of the core modelling material and careful application of that modelling to a novel scenario.

Q8

(i)

The probability that a supporter has a particular membership status next month depends only on their membership status in the current month (i.e. the status in previous month is not relevant) [½]

Therefore the process is Markov. [½]

[1 mark]

(ii)

P=

$\alpha+\beta$	δ	0
α	β	δ
0	α	$\beta+\delta$

[2]

[2 marks]

(iii)

(a) The chain is irreducible, [½]

as every state can be reached from every other. [½]

(b) It is aperiodic, [½]

as the chain may remain at its current state for all three states. [½]

[2 marks]

(iv)

The transition matrix becomes

P =

0.9	0.1	0
0.5	0.4	0.1
0	0.5	0.5

[½]

P^3 =

0.839	0.143	0.018
0.715	0.214	0.071
0.450	0.355	0.195

[1]

The required probability is 0.071 [½]

[2 marks]

(v)

Let the probability that a supporter is in class i according to the stationary distribution

be π_i (i = 1-Basic, 2-Medium, 3-Advanced). [½]

The π_i are given by the general formula $\pi = \pi P$.

That is

(a) $\pi 1 = 0.9\pi 1 + 0.5\pi 2$

(b) $\pi_2 = 0.1\pi_1 + 0.4\pi_2 + 0.5\pi_3$

(c) $\pi_3 = 0.1\pi_2 + 0.5\pi_3$ [1]

Also, $\pi_1 + \pi_2 + \pi_3 = 1$ [½]

From (a), $\pi_1 = 5\pi_2$. [½]

From (c), $\pi_3 = 0.2\pi_2$ [½]

Hence, $\pi_2 = 1/(5 + 1 + 0.2) = 0.1612903$ [½]

$\pi_1 = 5\pi_2 = 0.8064515$ and $\pi_3 = 0.2\pi_2 = 0.03225806$ [1]

Hence, the stationary distribution is (0.8064515, 0.1612903, 0.03225806) [½]

[5 marks]

(vi)
Expected cost of the scheme per member per month is

$£0\times 0.80645 + £10\times 0.16130 + £20\times 0.03226 = £2.2582$ [1½]

For the scheme to be worth running, therefore, the average profit per member per month must exceed £2.2582. [½]

The average monthly profit per ticket is

$£S\times 50\%\times 0 + £S\times 40\%\times 1 + £S\times 10\%\times 3 = £0.7S$ [1½]

Hence, the required range value for S is

$S > 2.2582/0.7$

i.e. $S > 3.226$ or $S > £3.23$ [½]

[4 marks]
[Total 16]

Commentary:
This question showed a continuation of a recent trend in this examination of improvement in the quality of answers to stochastic processes questions. The first four parts were generally well answered. The last part required application of the stationary distribution. Where candidates calculated prices using profit per member rather than profit per ticket, substantial credit was still given.

[Paper Total 100]

END OF EXAMINERS' REPORT

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