INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

15 April 2024 (am)

Subject CM2 – Economic Modelling
Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of
the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on
T. 0044 (0) 1865 268 873.

1   **(i)**   Explain whether an investor who believes that semi-strong form Efficient Markets Hypothesis (EMH) applies would invest in an actively managed fund.

                                                           [3]

    (ii)   Describe, in your own words, the behavioural heuristic known as 'overconfidence' and how this may arise.   [3]

    (iii)   Describe an example of a situation that illustrates overconfidence in the context of fund management.   [2]

                                                               [Total 8]

2   An insurer writes insurance policies that pay out a fixed benefit of $250 on a claim. The number of claims is assumed to follow a Poisson process with parameter λ.

    Premiums are received continuously and a premium loading of 10% is applied.

    The insurer is considering entering a reinsurance agreement and is looking at two different types of contract:

- Proportional reinsurance
- Excess of loss reinsurance.

    (i)   Describe the differences between these two forms of reinsurance.   [2]

    The insurer decides to use proportional reinsurance and will retain a proportion α of each risk. The reinsurer uses a premium loading of 12%.

    (ii)   Show that with this reinsurance, the adjustment coefficient R for the insurer is defined by the following equation:

$$e^{250\alpha R} = (280\alpha - 5)R + 1$$

                                                              [4]

    (iii)   Find the value of α that maximises the adjustment coefficient, by differentiating the formula from (ii) with respect to α.

             [**Note**: You can assume that when you find a turning point it is a maximum without checking the second derivative.]   [7]

                                                              [Total 13]

3   Consider two individuals with the following utility functions:

- Individual A: $U(w) = w + 0.01w^2$, $w > 0$
- Individual B: $U(w) = ln(w)$, $w > 0$.

Each individual has a current wealth of $300.

(i)   Calculate the current utility of wealth for each individual.   [1]

(ii)   Show that individual A has increasing absolute risk aversion and individual B has decreasing absolute risk aversion.   [3]

Each individual is offered the chance to gamble for free. The outcomes of the gamble are distributed as follows: $100 gain (i.e. increase in wealth) with probability 20%, no change in wealth with probability 70%, and $200 loss with probability 10%.

(iii)   Calculate the expected change in wealth from the gamble.   [1]

(iv)   Determine, for each individual, whether they should accept the gamble.   [3]

(v)   Discuss the relationship between your answers to parts (iii) and (iv).   [3]

The person organising the gamble now wants to charge participants an entry fee.

(vi)   Show that the maximum entry fee that individual A should be willing to pay for the gamble is $8.68.   [3]

[Total 14]

4 An investment analyst assumes the return on a fund follows a discrete distribution as set out in the table below:

Return (% p.a.)	Probability (%)
0	10
2	20
4	40
6	25
10	5

(i) Calculate the expected return and the variance of the return. [3]

(ii) Calculate the following risk measures:

   (a) The semi-variance of return

   (b) The shortfall probability relative to a benchmark return of 5%

   (c) The expected shortfall relative to a benchmark return of 5%. [3]

A second analyst believes the true distribution of the return on the fund is as set out in the table below:

Return (% p.a.)	Probability (%)
−10	10
2	20
4	40
6	25
30	5

(iii) Comment on how using this distribution would affect your answers to parts (i) and (ii), without performing any further calculations. [3]

[Total 9]

5 An individual has a liability of $1,000 payable in exactly 3 years' time. To determine the present value of the liability, they assume an annual return that follows a log-normal distribution with parameters μ and σ^2. The return in each year is independent of the return in any other year.

(i) Derive the formula for:

   (a) the expected present value of the liability.

   (b) the variance of the present value of the liability.

[4]

The investor has calculated the expected present value as $862 and the variance as $2,232.48.

(ii) Determine the values of μ and σ^2 used by the investor. [4]

The investor chooses to invest $900 and uses the distribution in part (ii) to model this investment.

(iii) Calculate the probability that this investment will **not** be sufficient to meet the liability of $1,000 in 3 years' time. [2]

[Total 10]

6 An analyst working at a bank wishes to model the return on two securities. They have recommended the following single period multifactor model of security returns:

$$R_i = \alpha + \beta_{i1} M_1 + \beta_{i2} M_2 + \xi_i$$

where R_i = return on security i, α = constant, β_{ij} for i, j and $i \neq j$ are fixed parameters specific to each security, M_k for $k = 1,2$ are correlated rates of change, and ξ_i is the independent random component of return that is also independent of M_k for $k = 1,2$.

(i) Derive an expression for the covariance between the returns of the two securities. [5]

(ii) State how the expression in part (i) would change if M_k for $k = 1,2$ were independent rates of change. [1]

Now suppose that the multifactor model takes the form:

$$R_i = (1 - X)^2 M_1 + M_2$$

where X is a fixed parameter.

(iii) (a) Derive an expression for the single period multifactor model of security returns using principal components.

You may assume that $M_2 = (1 - X)^2 M^*_1$, where M^*_1 is the first principal component.

(b) Discuss a conclusion that can be drawn from the result in part (iii)(a).
[6]
[Total 12]

7 The table below shows the cost of claims settled per calendar year for a portfolio of car insurance policies in $000s:

	Development year		
Accident year	*0*	*1*	*2*
2020	1,729	199	57
2021	2,274	318	
2022	2,511		

The number of settled claims in each period is as follows:

	Development year		
Accident year	*0*	*1*	*2*
2020	247	43	19
2021	291	50	
2022	317		

(i) Calculate the outstanding claims reserve for this portfolio, using the average cost per claim method with grossing up factors. [7]

(ii) Explain how an insurer may adjust this calculation to allow for inflation. [3]
[Total 10]

8 Consider a process, $\{N(t)\}_{t>0}$, denoting the number of claims an insurer experiences over time. Suppose that $N(1)$ follows a Poisson distribution with parameter λ.

(i) Derive, from first principles, the mean of $N(1)$. [3]

One of the requirements for the process $N(t)$ to be Poisson is to assume that, when $s < t$, the number of claims in the time interval $(s, t]$ is independent of the number of claims up to time s.

(ii) Explain, in your own words, what this assumption implies about the number and/or rate of claims that the insurer experiences. [1]

(iii) Discuss whether it is reasonable for the insurer to make this assumption. [2]
[Total 6]

9 An insurance company sells inflation-linked policies to customers. The policy includes a guarantee that the rate of return will have a minimum of 1% p.a. and a maximum of 2.5% p.a., applied at maturity. The insurance company would like to delta hedge the position.

The current rate of inflation is 1% p.a.

You may assume that:

- continuously compounded risk-free interest rate, $r = 2\%$ p.a.
- implied volatility, $\sigma = 15\%$.
- term of policy, $t = 8$ years.
- the assumptions underlying the Black–Scholes formula apply.

    (i)   Explain how to construct a portfolio using European options that replicates the required payoff profile.   **[2]**

    (ii)   Calculate the policy's delta.   **[5]**

    (iii)   Explain how to construct a delta neutral portfolio.   **[1]**

Due to unforeseen events, the insurance company has been unable to rebalance its portfolio for delta hedging purposes. The policy will expire at the end of the day.

Current inflation stands at 10% p.a.

    (iv)   State, with reasons, the value for this policy of:

        (a)   delta.

        (b)   gamma.

        (c)   vega.
   [2]
   [Total 10]

10 Consider a forward contract on a share over a period where no dividends are payable. S_0 is the initial price of the share, r is the continuously compounded risk-free rate of interest and t is the time to delivery of the contract in months.

    (i)   Show, by constructing two portfolios and assuming no arbitrage, that the price of this forward contract is $S_0 e^{rt}$.   **[3]**

We are now told that $S_0 = \$10$, $t = 20$ months and $r = 7\%$ p.a. The share will pay a dividend of 3% of the share price every 6 months and the next dividend is due in 1 month's time. You may assume that dividends are immediately reinvested.

    (ii)   Determine the fair price for this forward contract.   **[5]**
   [Total 8]

END OF PAPER

EXAMINERS' REPORT

CM2 - Financial Engineering and Loss Reserving

Core Principles

Paper A

April 2024

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

For some candidates, this may be their first attempt at answering an examination using open books and online. The Examiners expect all candidates to have a good level of knowledge and understanding of the topics and therefore candidates should not be overly dependent on open book materials. In our experience, candidates that spend too long researching answers in their materials will not be successful either because of time management issues or because they do not properly answer the questions.

Many candidates rely on past exam papers and examiner reports. Great caution must be exercised in doing so because each exam question is unique. As with all professional examinations, it is insufficient to repeat points of principle, formula or other text book works. The examinations are designed to test "higher order" thinking including candidates' ability to apply their knowledge to the facts presented in detail, synthesise and analyse their findings, and present conclusions or advice. Successful candidates concentrate on answering the questions asked rather than repeating their knowledge without application.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
June 2024

A. General comments on the *aims of this subject and how it is marked*

The aim of subject CM2 is to develop the necessary skills to construct asset liability models, value financial derivatives and calculate reserves for insurance or guarantees. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.

The marking approach for CM2 is flexible in the sense that different answers to those shown in the solution can earn marks if they are relevant and appropriate. Marks for the methodology are also awarded including marks for using the right method even if an error in an earlier part of the question prevents the final answer from being correct. The marking focusses on rewarding candidates' understanding of the concepts, including their ability to articulate arguments clearly.

B. Comments on *candidate performance in this diet of the examination.*

In this exam most questions focussed on applied calculations and analysis of the results. Some of the questions required application of concepts in the Syllabus to scenarios that candidates might not have seen before and the stronger candidates scored highly here.

As in previous sessions, there was evidence that some candidates found algebra complex when answering questions in Word. Future candidates should note that rearranging and solving algebra on screen can sometimes be hard if you are used to using pen and paper, so this is a worthwhile skill to practise before the exams. It's also worth saying that using the equation editor in Word to set out formulae is not necessary, your workings just need to be clear enough for the examiner to follow them.

Q6 and Q9 proved to be the most challenging questions on the paper. Q6 required candidates to derive and work with a multifactor model of security prices, and algebraic slip-ups were common here. Q9 required candidates to derive and work with a portfolio of options to give a defined payoff - few produced the right portfolio but most still worked through the later question parts and picked up marks.

C. Pass Mark
Average marks were roughly in line with the historic norm for the subject but the pass mark was set slightly lower than in the last few sessions due to the demand of several questions as outlined in the commentary in this and the B paper report.
The Pass Mark for this exam was 56
1307 presented themselves and 583 passed.

Solutions for Subject CM2A - April 2024

Q1

(i)

Active fund managers attempt to detect exploitable mis-pricings, since they believe that markets are not universally efficient. [½]

Fund managers have an advantage in terms of being able to form an opinion on the competence of the management team and the strategy of the company. [½]

Fund managers are also increasingly utilising 'alternative data' (e.g. satellite images, web searches, social media etc) to generate excess performance. [½]

Semi-strong form EMH assumes that all publicly available information is included in stock prices [½]

If an investor believes semi-strong EMH applies, then they are likely to think actively managed funds can outperform the market. [½]

However, there is a cost to obtaining information quickly and accurately. [½]

Active fund managers trade more frequently than passive fund managers… [½]

…incurring transaction costs. [½]

The active fund manager is likely to charge a higher fee for these extra costs [½]

The investor will need to consider whether he believes the impact on expected fund return is greater than the increased costs. [½]

Or, if we assume that active managers have access only to public information (and not insider information)… [½]

…then active managers should not be able to outperform the market… [½]

…because semi-strong form EMH says that market prices already include this information [½]

[Marks available 6½, maximum 3]

(ii)

The concepts in this question part have been removed from the syllabus. [3]

(iii)

The concepts in this question part have been removed from the syllabus. [2]

[Total 8]

Commentary:

Part (i) was answered well by most candidates. To score full marks it was important to relate the concepts behind semi-strong market efficiency to an actively managed fund and draw conclusions from the points of view of an investor, so some answers were too generic to score full marks.

The concepts tested in parts (ii) and (iii) were removed from the 2024 syllabus so all candidates were awarded full marks for these question parts.

Q2

(i)

Under proportional reinsurance, the reinsurer covers an agreed proportion of each risk… [½]

…and the reinsurance premium is in proportion to this risk ceded. [½]

Under excess of loss reinsurance, the cost to an insurer of a large claim, a cluster of claims arising from a single event (such as an explosion) or claims over a given period (such as a catastrophe) is capped… [½]

…with the liability above a certain level being passed to a reinsurer. [½]

In most cases the reinsurer's maximum liability is limited. [½]

[Marks available 2½, maximum 2]

(ii)

R must satisfy

$$\lambda M_X(R) = cR + \lambda = (1.1 \times 250 \times \lambda - 1.12 \times 250 \times (1-\alpha) \times \lambda)R + \lambda$$ [1]

$$= \lambda\big(275 - 280 \times (1-\alpha)\big)R + \lambda$$ [1]

$$M_X(R) = \exp(250 \times \alpha \times R)$$ [1]

$$\exp(250 \times \alpha \times R) = (280 \times \alpha - 5)R + 1$$ [1]

[Marks available 4, maximum 4]

(iii)

Differentiating with respect to α gives:

$$\frac{d}{d\alpha}\left[\exp(250 \times \alpha \times R)\right] = (280 \times \alpha - 5)\frac{dR}{d\alpha} + 280 \times R$$ [1]

$$\exp(250 \times \alpha \times R) \times \frac{d}{d\alpha}[250 \times \alpha \times R] = (280 \times \alpha - 5) \times \frac{dR}{d\alpha} + 280 \times R$$ [1]

$$\exp(250 \times \alpha \times R) \times \left(250 \times R + 250 \times \alpha \times \frac{dR}{d\alpha}\right) = (280 \times \alpha - 5) \times \frac{dR}{d\alpha} + 280 \times R$$ [1]

The value of α which maximises the adjustment coefficient occurs when $\frac{dR}{d\alpha} = 0$ [1]

$$\exp(250 \times \alpha \times R) \times 250 \times R = 280 \times R$$

$$R = \frac{1}{250 \times \alpha}\ln\left(\frac{28}{25}\right)$$ [1]

Substituting into the equation from (ii) gives

$$\frac{28}{25} = \frac{(280 \times \alpha - 5)}{250 \times \alpha}\ln\left(\frac{28}{25}\right) + 1$$

$$\alpha \times \left(\frac{3}{25} \times 250 - 280 \times \ln\left(\frac{28}{25}\right)\right) = -5 \times \ln\left(\frac{28}{25}\right)$$ [1]

$$\alpha = \left(\frac{3}{25} \times 250 - 280 \times \ln\left(\frac{28}{25}\right)\right) = \frac{5 \times \ln\left(\frac{28}{25}\right)}{280 \times \ln\left(\frac{28}{25}\right) - 30} = 0.3272$$ [1]

[Marks available 7, maximum 7]

[Total 13]

Commentary:

Part (i) and (ii) of this question were answered well by most candidates.

In part (i) some candidates did not give enough detail for a two-mark question - it is always worth bearing in mind how many marks are on offer as a guide to the level of detail required in an answer.

Part (iii) proved challenging, with many candidates completing the first half of the solution correctly but struggling to solve the equation to find a value for α.

Q3

(i)

Individual A:

U(300) = 300 + 0.01*300^2 = 1200 [½]

Individual B:

U(300) = ln(300) = 5.704 [½]

[Marks available 1, maximum 1]

(ii)

Individual A:

U'(w) = 1 + 0.02w

U''(w) = 0.02

A(w) = -U''(w)/U'(w) = -0.02/(1+0.02w) [½]

A'(w) = 0.0004/(1+0.02w)^2 [½]

A'(w) > 0 therefore individual A has increasing absolute risk aversion. [½]

Individual B:

U'(w) = 1/w

U''(w) = -1/w^2

A(w) = 1/w [½]

A'(w) = -1/w^2 [½]

A'(w)<0 therefore individual B has decreasing absolute risk aversion. [½]

[Marks available 3, maximum 3]

(iii)

0.2*100 + 0.7*0 + 0.1*(-200) = 0 [1]

[Marks available 1, maximum 1]

(iv)

The individual will accept the gamble if the expected utility after taking the gamble
is greater than their current utility.

Individual A:

E(U(gamble)) = 0.2*U(300+100) + 0.7*U(300) + 0.1*U(300-200)

= 0.2*2000 + 0.7*1200 + 0.1*200

= 1260   [1]

This is greater than individual A's current utility so they will accept the gamble.   [½]

Individual B:

E(U(gamble)) = 0.2*U(300+100) + 0.7*U(300) + 0.1*U(300-200)

= 0.2*5.991 + 0.7*5.704 + 0.1*4.605

= 5.651   [1]

This is less than individual B's current utility so they will not accept the gamble.   [½]

[Marks available 3, maximum 3]

(v)

The expected outcome of this gamble is zero, i.e. it is a fair gamble.   [½]

A risk averse individual will reject a fair gamble.   [½]

A risk seeking individual will accept a fair gamble.   [½]

Individual A is risk seeking as U''(w) = 0.02 > 0.   [½]

Individual B is risk averse as U''(w) = -1/w^2 <0   [½]

Therefore, individual A will accept the gamble, whereas individual B will reject it.   [½]

[Marks available 3, maximum 3]

(vi)

Let w be current wealth, X be the outcome of the gamble and P be the maximum price the individual is willing to pay.

P will satisfy the equation:

E(U(w+X-P)) = U(w)   [1]

We know U(w) = 1200

E(U(w+X-P)) = 0.2*U(300+100-8.68) + 0.7*U(300-8.68) + 0.1*U(300-200-8.68)

= 0.2*1922.63 + 0.7*1139.99 + 0.1*174.71

= 1200   [1]

This is equal to the current utility therefore P = 8.68 satisfies the equation.   [1]

[Marks available 3, maximum 3]

[Total 14]

Commentary:

This question was mostly answered well, with candidates showing a good understanding of utility functions and how they relate to decision-making.

It was also possible to answer part (iv) by considering whether each individual is risk-seeking or risk-averse and this approach was given credit.

Part (vi) of this question caused some difficulty, with candidates calculating utility of expected wealth rather than expected utility of wealth.

Q4

(i)

E(X) = 0.1*0% + 0.2*2% + 0.4*4% + 0.25*6% + 0.05*10%   [½]

= 4%   [1]

Var(X) = 0.1*(4%-0%)² + 0.2*(4%-2%)² + 0.4*(4%-4%)² + 0.25*(4%-6%)² +

0.05*(4%-10%)²   [½]

=0.052%   [1]

[Marks available 3, maximum 3]

(ii)(a)

semi-var(X) = 0.1*(4%-0%)² + 0.2*(4%-2%)² + 0.4*(4%-4%)²   [½]

=0.024%   [½]

(ii)(b)

shortfall prob = 10% + 20% + 40% = 70%   [1]

(ii)(c)

expected shortfall = 0.1*(5%-0%) + 0.2*(5%-2%) + 0.4*(5%-4%)   [½]

=1.5%   [½]

[Marks available 3, maximum 3]

(iii)

The expected return is likely to be about the same	[½]
The distribution now has much more extreme upsides and downsides	[½]
The distribution has fatter tails/increased kurtosis	[½]
The variance has increased significantly	[½]
The semi-variance has also increased significantly	[½]
There is no change in the shortfall probability	[½]
This is one of the limitations of this measure - it doesn't capture how bad the shortfall could be	[½]
The expected shortfall has increased	[½]
The semi-variance, shortfall probability and expected shortfall are all measures of downside risk and take no account of the upside risk	[½]
The maximum return is now significantly higher	[½]

[Marks available 5, maximum 3]

[Total 9]

Commentary:

Most candidates completed the calculations in this question well. Most candidates were also able to explain how the changes to the distribution in part (iii) would impact the risk metrics. Calculations were not required in part (iii) but were given credit where candidates chose this approach.

Q5

(i)

Annual return year t = i_t

EPV of 1 in 3 years will be

$1/(1+i_1) \times 1/(1+i_2) \times 1/(1+i_3)$

[1]

Variable $1+i$ follows a Lognormal distribution with parameters μ and $\sigma2$, so $\log(1+i)$ follows a Normal distribution with parameters (and hence mean and variance) μ and $\sigma2$. So, considering logs, log EPV will be:

$- \log(1+i_1) - \log(1+i_2) - \log(1+i_3)$

[½]

The mean of a series of any iid random variables is the sum of the individual means, and the variance of a series of any iid random variables is the sum of the individual variances

[½]

Therefore the mean of Log EPV = -3μ, and the variance of Log EPV is $3\sigma^2$

Therefore Log EPV follows a Normal distribution with parameters -3μ and $3\sigma^2$

[½]

Hence EPV follows a Lognormal distribution with parameters -3μ and $3\sigma^2$

[½]

Therefore the mean is $1000 \exp(-3\mu + 3/2 \ \sigma^2)$ from the *Formulae and Tables*, and

[½]

The variance is $1000^2 \exp(-6\mu + 3\sigma^2) \times (\exp(3\sigma^2)-1)$ from the *Formulae and Tables*

[½]

[Marks available 4, maximum 4]

(ii)

Using the formulae from (i):

Mean is $1000 \exp(-3\mu + 3/2 \ \sigma^2)$, and Variance is $1000^2 \exp(-6\mu + 3\sigma^2) \times (\exp(3\sigma^2)-1)$

from the *Formulae and Tables* from part (i)

Therefore $1000 \exp(-3\mu + 3/2 \ \sigma^2) = \$862$ (a)

[1]

And $1000^2 \exp(-6\mu + 3\sigma^2) \times (\exp(3\sigma^2)-1) = \$2,232.48$ (b)

[1]

Dividing (b) by (a)^2 we get:

$(\exp(3\sigma^2)-1) = 2,232.48 / 862^2$

Solve for $\sigma^2 = (1/3) \log [(2,232.48 / 862^2) + 1] = 0.001$

[1]

Using (a) we solve for $\mu = [0.00150 - \log (0.862)]/3 = 0.05$

[1]

[Marks available 4, maximum 4]

(iii)

The shortfall probability is the probability that $900 invested at time t=0, is less

than $1000 at time t=3

[½]

From (i) and (ii), EPV ~ Normal (-3 x 0.05, 3 x 0.001)

The probability is therefore P(EPV > 0.9) = 1 - P(EPV < 0.9)

[½]

Which is 1 - Φ [(log(0.9) - (-3 x 0.005))/sqrt(3x0.001)]

[½]

Which is 1 - Φ [0.815002] = 0.207536

[½]

[Marks available 2, maximum 2]

[Total 10]

Commentary:
This question assesses competence in, and application of, Syllabus item 2.1, where different statistical metrics are used as a measure of investment risk. It required candidates to derive formulae in (i) using results from the Formulae and Tables, then use these formulae in parts (ii) and (iii).
Credit was given in later question parts for candidates who derived incorrect results in (i) if they followed these through correctly in their workings. For example, the question stated the assumption that the annual return followed a lognormal distribution and gave the parameters. Where candidates assumed that i, rather than (1+i) followed this distribution, they were given appropriate credit for their working to ensure they were not disadvantaged.

Q6

(i)

$$\text{Cov}(R_i, R_j) = \text{Cov}(\alpha + \beta_{i1} M_1 + \beta_{i2} M_2 + \xi_i; \alpha + \beta_{j1} M_1 + \beta_{j2} M_2 + \xi_j) \qquad [1]$$

$$= \text{Cov}(\beta_{i1} M_1, \beta_{j1} M_1) + \text{Cov}(\beta_{i1} M_1, \beta_{j2} M_2) + \text{Cov}(\beta_2 M_2, \beta_{j1} M_1) + \text{Cov}(\beta_{i2} M_2, \beta_{j2} M_2) \qquad [1]$$

$$= \beta_{i1} \times \beta_{j1} \times \text{Cov}(M_1, M_1) + \beta_{i1} \times \beta_{j2} \times \text{Cov}(M_1, M_2) + \beta_{i2} \times \beta_{j1} \times \text{Cov}(M_2, M_1) + \beta_{i2} \times \beta_{j2} \times \qquad [1\frac{1}{2}]$$

$$\text{Cov}(M_2, M_2)$$

$$= \beta_{i1} \times \beta_{j1} \times \sigma^2_{M1} + \text{Cov}(M_1, M_2) \times (\beta_{i1} \times \beta_{j2} + \beta_{i2} \times \beta_{j1}) + \beta_{i2} \times \beta_{j2} \times \sigma^2_{M2} \qquad [1\frac{1}{2}]$$

[Marks available 5, maximum 5]

(ii)

$$\text{Cov}(R_i, R_j) = \beta_{i1} \times \beta_{j1} \times \sigma^2_{M1} + \beta_{i2} \times \beta_{j2} \times \sigma^2_{M2} \qquad [1]$$

[Marks available 1, maximum 1]

(iii) (a)

Let $M_1 = M^*_1$ (the first principal component) [1]

Then $M^*_2 = M_2 - (1 - X)^2 M^*_1$ (second principal component) [1]

$\Rightarrow R_i = 1 + (1 - X)^2 M^*_1 + M^*_2$ [1]

$R_i = 1 + M_2$ [1]

(iii) b)

R_i can be written as a function of M_2 (only). [1]

This could be interpreted to mean that M_1 does not provide information over and above M_2 [1]

[Marks available 6, maximum 6]
[Total 12]

Commentary:

This question was not well answered although marks were awarded where possible for partially correct algebra.

One common mistake was neglecting the correlation of the variables, hence ending up with the answer to part (ii) without properly working through the steps in part (i).

Part (iii) is not a topic that has been examined often in the past, and a significant number of candidates did not attempt part (iii) at all. It is a reminder that any part of the syllabus can be assessed.

Q7
(i)
Cumulative amounts:

	0	1	2
2020	1729	1928	1985
2021	2274	2592	
2022	2511		

	0	1	2
2020	247	290	309
2021	291	341	
2022	317		

[2]

Average cost per claim (cumulative):

	0	1	2
2020	7	6.6483	6.4239
2021	7.8144	7.6012	
2022	7.9211		

[1]

Cost per claim ratios (bold are projections):

	0	1	2	Ultimate
2020	7 1.0897	6.6483 1.0349	6.4239 1	6.4239
2021	7.8144 1.0640	7.6012 **1.0349**		7.3447
2022	7.9211 **1.0768**			7.3561

[1]

Claim number ratios:

	0	1	2	Ultimate
2020	247 0.7994	290 0.9385	309 1	309.0
2021	291 0.8009	341 **0.9385**		363.3
2022	317 **0.8001**			396.2

[1]

Finally, the total claims projected are

1985 + 363.3 * 7.3447 + 396.2 * 7.3561 = 7568.0 [1]

Allowing for claims already paid, the outstanding claims reserves is

7568.0 - (1985 + 2592 + 2511) = 480.0, or $480,000 [1]

[Marks available 7, maximum 7]

(ii)
The insurer would need to adjust historic claims to bring them into 'today's money'…   [1]
using actual inflation figures   [1]
They would then apply assumed future inflation to the projected claims   [1]
[Marks available 3, maximum 3]
[Total 10]

Commentary:
This question tested a claims run-off method that is not often examined, but most candidates understood what was needed and worked through the steps well. Most candidates also set out enough detail to pick up marks even if they had made some mistakes in their method, which was pleasing to see.   *Part (ii) was answered fairly well, though not all candidates described separately the adjustments needed for past and expected future inflation.*

Q8
(i)
The expectation is given by:

$$\sum_{k \geq 0} k \frac{e^{-\lambda} \lambda^k}{k!}$$
[½]

$$= \sum_{k \geq 1} k \frac{e^{-\lambda} \lambda^k}{k!}$$
[½]

$$= \sum_{k \geq 1} \frac{e^{-\lambda} \lambda^k}{(k-1)!}$$
[½]

$$= \lambda \sum_{k \geq 1} \frac{e^{-\lambda} \lambda^{k-1}}{(k-1)!}$$
[½]

$$= \lambda \sum_{k \geq 0} \frac{e^{-\lambda} \lambda^k}{k!}$$
[½]

$$= \lambda$$ [½]
[Marks available 3, maximum 3]

(ii)
This property means that, in a given interval, the number of claims is unaffected by
the number of claims in the past   [1]
This is also known as the process being 'memoryless'   [½]
[Marks available 1½, maximum 1]

(iii)

On the one hand, using a Poisson process leads to tractable mathematics in the field of ruin theory [½]

And on the surface it appears intuitive e.g. we would not expect the number of claims e.g. ten years ago to affect the number of claims now [½]

However, it is easy to imagine scenarios in which the assumption of independence breaks down. [½]

e.g. during a mass claims event, it is highly likely that there will be more claims next week given there were a large number of claims this week [½]

This means the assumption may in practice be questionable [½]

[Marks available 2½, maximum 2]

[Total 6]

Commentary:

Part (i) required candidates to 'Derive, from first principles' so simply stating the result did not score marks.

Part (ii) was answered well.

Candidates should bear in mind the number of marks available as some answers to part (iii) were too brief for a two-mark question.

Q9

(i)

The requisite payoff profile can be created using an option spread [½]

By buying a forward or exposure to the inflation index [½]

And buying a put option at the 1% pa strike [½]

And selling a call option at the 2.5% pa strike [½]

Both options with an 8 year term [½]

[Marks available 2½, maximum 2]

(ii)

Delta at 1% strike:

$d_1 = (\log(1/\exp(0.01*8)) + (0.02 + 0.5 \times 0.15^2) \times 8)/(0.15 \times 8^{0.5}) = 0.4021$ [1]

Delta $= N(0.4007) - 1 = 0.6557 - 1 = -0.344$ [1]

Delta at 2.5% strike:

$d_1 = (\log(1/\exp(1.025*8)) + (0.02 + 0.5 \times 0.15^2) \times 8)/(0.15 \times 8^{0.5}) = 0.1179$ [1]

Delta $= N(0.1179) = 0.547$ [1]

Net delta $= 1 - 0.344 - 0.547$

$= 0.109$ per unit [1]

[Marks available 5, maximum 5]

(iii)

The insurance company would need to sell delta units of the inflation-linked index	[½]
To achieve a portfolio delta of zero	[½]
e.g. by selling 0.109 units.	[½]

[Marks available 1½, maximum 1]

(iv)

Delta = Gamma = Vega = 0	[1]
because the policy is about to expire	[1]

[Marks available 2, maximum 2]

[Total 10]

Commentary:

This was one of the harder questions on the paper. Few candidates constructed a correct portfolio in part (i), but many identified the need for two options with different strike prices and were given credit for this. The portfolio shown above is not the only possible portfolio that produces the required payoff and credit was given for valid alternatives.

In part (ii), marks were awarded for correct calculations based on the portfolio that had been proposed in part (i).

Parts (iii) and (iv) were not attempted by some. They were largely independent of parts (i) and (ii) so candidates should bear in mind that it's always worth attempting to answer later question parts even if they have not been able to answer earlier parts.

Q10

(i)

Let K be the forward price. Now compare the setting up of the following portfolios

at time 0:

Portfolio A: one long forward contract [½]

Portfolio B: borrow Ke^{-rT} cash and buy one share at S_0. [½]

If we hold both portfolios at time T then both have the value of $S_T - K$ at time T. [½]

By the principle of no arbitrage these portfolios must always have the same value

before T. [½]

In particular at time 0, portfolio B has value $S_0 - Ke^{-rT}$, which must equal the

value of the forward contract. [½]

This can only be zero (the value of the forward contract at t=0) if $K = S_0e^{rT}$. [½]

[Marks available 3, maximum 3]

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(ii)
The dividends will be paid at the end of months 1, 7, 13 and 19. So there will be 4 payments during the life of the forward. Since the dividends are reinvested in the share at the prevailing price, the size of the shareholding will increase by a factor of 1.03 when each dividend is paid.

Let the forward price be F*.

Consider to portfolios:

A*: 1 forward contract + cash of F*e^{-rt}

[½]

B*: 1.03^{-4} shares

[½]

Portfolio A* will contain 1 share at time t.

[½]

With reinvestment of the dividends, Portfolio B* will contain 1 full share at time t.

[½]

So again, using the principle of no arbitrage, the two portfolios must have the same value at time 0.

[½]

$0 + F^* e^{-rt} = 1.03^{-4} S_0$.

[½]

So:

$S_0 = 10, r = 0.07, t = 20/12$

[1]

F*= 1.03^{-4} x 10 x $e^{0.07 \ x \ 20/12}$ = \$9.98

[1]

[Marks available 5, maximum 5]

[Total 8]

[Paper Total 100]

Commentary:
Part (i) of this question was standard and is examined fairly often. Most candidates scored well and the portfolios above are not the only possible portfolios that can be used to prove this, though some candidates proposed portfolios that did not work.
Part (ii) was more challenging, but many candidates attempted to adjust the standard forward pricing formula to allow for the amounts and timing of dividends and were given partial credit for this.

END OF EXAMINERS' REPORT