INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

8 April 2024 (am)

Subject CM1 – Actuarial Mathematics for Modelling
Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 Calculate ${}_{4.25}q_{87.25}$ assuming PMA92C20 mortality and that there is a uniform distribution of deaths between integer ages. You should show your working. [3]

2 An insurance company is required to pay an annuity annually in advance for 4 years. The first payment is $1.5 million and each subsequent payment is 3% greater than the previous one.

(i) Calculate, showing all working and using an effective rate of interest of 8% p.a., the convexity of the annuity. [3]

The company is considering investing in a single zero-coupon bond of suitable term to cover the present value of the annuity.

(ii) Explain if it is possible to find a zero-coupon bond such that the fund would be immunised against small changes in the rate of interest. [2]

[Total 5]

3 Three bonds, A, B and C, each pay coupons at 4% annually in arrears and are redeemed at 103%. The outstanding terms of the bonds are exactly 1, 2 and 3 years, respectively. The prices of the bonds per £100 nominal are £104, £105 and £106, respectively.

(i) Using the information above and showing all working:

(a) Determine all possible discrete spot rates.

(b) Determine all possible discrete future rates.

[8]

A bank offers a 3-year regular savings plan and adds interest in line with the rates calculated in part (i).

An individual invests £12,000 at the beginning of each of the 3 years.

(ii) Calculate, showing all working, the total accumulated value of the investments at the end of year three. [4]

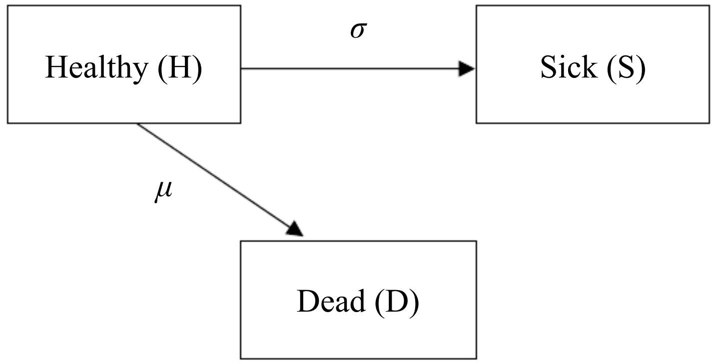
[Total 12]

4 An individual pays £4,000 p.a. into a savings account for 10 years. During the first 4 years, the payments are made quarterly in advance. For the remaining years, the payments are made continuously.

The investor achieves a yield of 6% p.a. convertible quarterly on the investment.

Calculate, showing all working, the accumulated amount in the savings account at the end of 10 years. [7]

5 A life insurance company uses the following three-state continuous-time Markov model, with constant forces of transition σ and μ, to price a sickness policy.



The company issues a 25-year policy to a healthy life aged 35 exact.

The benefits provided are:

- £150,000 payable immediately on death from the healthy state.
- £300,000 payable immediately on becoming sick.
- £500 payable at the end of the term of the policy if no claim has been made on the policy.

(i) Calculate, showing all working, the total expected present value of the benefits. [7]

Basis:

μ:	0.025 for all ages
σ:	0.001 for all ages
Force of interest:	5% p.a.

Premiums are payable continuously for the term of the policy while the policyholder is in the healthy state.

(ii) Write down an integral formula for the expected present value of the annual premium. You are not required to do any calculations. [2]

The company is investigating introducing a new death benefit that is payable on death from the sick state.

To value this new benefit the transition probability ${}_tp_{35}^{HS}$ would be required.

(iii) Write down an integral formula for ${}_tp_{35}^{HS}$ in terms of other transition probabilities and forces of transition. You are not required to do any calculations. [2]

[Total 11]

6 A government issues a fixed-interest security paying coupons at a rate of 8% p.a. quarterly in arrears.

The security is to be redeemed at 103% on any coupon payment date from 15 to 18 years after issue, with the exact date of redemption at the discretion of the government.

Investor A, who is liable to pay income tax at 20% and capital gains tax at 25%, purchases the fixed-interest security on the date of issue at a price that gives a net yield to redemption of 6.2% p.a. effective.

(i) Calculate, showing all working, the price per £100 nominal paid by Investor A. [6]

Investor A sells the fixed-interest security 5 years after purchase to Investor B for £106.50 per £100 nominal.

(ii) Calculate, using linear interpolation and showing your working, the net effective yield Investor A earns on the total transaction. Give your answer as a percentage rounded to three decimal places. [5]

[Total 11]

7 (i) Determine the missing entries, (a) and (b), for the following multiple-decrement table.

Age x	$(al)_x$	$(ad)_x^{\text{death}}$	$(ad)_x^{\text{withdrawal}}$
50	100,000	123	3,100
51	96,777	125	3,050
52	(a)	128	2,950
53	90,524	132	2,800
54	87,592	(b)	2,550
55	84,906	141	2,100
56	82,665	148	1,400
57	81,117	155	950
58	80,012	162	700
59	79,150	173	250
60	78,727	188	0

[2]

(ii) Calculate the following probabilities, showing all working:

(a) A life aged 50 exact will not be in the population at the end of the 5th year.

(b) A life aged 54 exact will remain in the population for at least 2 years.

(c) A life aged 55 exact will die during the next 3 years.

(d) A life aged 50 exact will withdraw between the ages of 55 and 59.

[5]
[Total 7]

8 A pension scheme provides retirement benefits to a group of 200 pensioners currently aged 65 exact. The scheme pays an annual income of £19,000 annually in arrears if the pensioner is alive. A lump sum of £50,000 is paid at the end of the year of death.

During the first year 3 members died.

(i) Calculate, showing all working, the mortality profit or loss for the year. [7]

(ii) Explain why the mortality profit or loss has arisen. [3]

Basis:
Mortality: AM92 Ultimate
Interest: 6% p.a. effective

[Total 10]

9 A life insurance company issued a last survivor whole-life assurance policy on 1 January 2020 to a male life aged 60 exact and a female life aged 57 exact. The sum assured of £100,000 is paid at the end of the year of the second death.

Annual premiums of £1,495 are payable annually in advance while at least one of the lives is alive.

Calculate, showing all working, the prospective reserve in respect of this policy at 31 December 2022 if:

(a) both lives are alive

(b) only the female life is alive.

Basis:

Mortality:	PMA92C20 for the male
	PFA92C20 for the female
Interest:	4% p.a. effective

[8]

10 The force of interest is a function of time and at any time t (measured in years) is given by the formula:

$$\delta(t) = \begin{cases} a + bt & 0 \le t < 5 \\ c & 5 \le t \end{cases}$$

where a, b and c are constants.

You are given $v(t)$, the present value of a unit sum of money due at time t:

$$v(t) = \begin{cases} e^{-\left[0.04t + 0.01t^2\right]} & 0 \le t < 5 \\ e^{-[0.05t + 0.20]} & 5 \le t \end{cases}$$

(i) Calculate, showing all working, the values of a, b and c. [5]

£1,000 is invested at $t = 2$ for a period of 7 years.

(ii) Calculate, showing all working, the accumulated value of this investment at $t = 9$. [3]

(iii) Calculate, showing all working, the rate of interest earned on the investment in part (ii). Express your answer as a percentage rounded to three decimal places as a nominal rate of interest per annum convertible quarterly. [2]

A continuous payment stream is received at a rate of $5e^{0.03t}$ units per annum between $t = 8$ and $t = 13$.

(iv) Calculate, showing all working, the present value of the payment stream at $t = 0$. [4]

[Total 14]

11 A life aged 35 exact purchases a 30-year term assurance with the sum assured of £200,000 payable immediately on death. Level monthly premiums are payable in advance throughout the term, ceasing on death.

(i) Calculate, showing all working, the monthly premium. [7]

(ii) Write down an expression for the retrospective reserve after 25 years. You are not required to do any calculations. [3]

The retrospective reserve expressed as a percentage of the sum assured is 3.3%.

(iii) Explain why this percentage is relatively small. [2]

Basis:

Mortality:	AM92 Select
Interest:	4% p.a. effective
Initial expenses:	£120 at issue plus 30% of the first monthly premium
Renewal expenses:	2% of the second and subsequent monthly premiums

[Total 12]

END OF PAPER

Institute and Faculty of Actuaries

EXAMINERS' REPORT

CM1 - Actuarial Mathematics Core Principles

Paper A

April 2024

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

For some candidates, this may be their first attempt at answering an examination using open books and online. The Examiners expect all candidates to have a good level of knowledge and understanding of the topics and therefore candidates should not be overly dependent on open book materials. In our experience, candidates that spend too long researching answers in their materials will not be successful either because of time management issues or because they do not properly answer the questions.

Many candidates rely on past exam papers and examiner reports. Great caution must be exercised in doing so because each exam question is unique. As with all professional examinations, it is insufficient to repeat points of principle, formula or other text book works. The examinations are designed to test "higher order" thinking including candidates' ability to apply their knowledge to the facts presented in detail, synthesise and analyse their findings, and present conclusions or advice. Successful candidates concentrate on answering the questions asked rather than repeating their knowledge without application.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
June 2024

A. General comments on the *aims of this subject and how it is marked*

CM1 provides a grounding in the principles of modelling as applied to actuarial work focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations, but candidates are not penalised for this. However, candidates may not be awarded full marks where excessive rounding has been used or where insufficient working is shown.

Although the solutions show full actuarial notation, candidates were generally expected to use standard keystrokes in their solutions.

Candidates should pay attention to any instructions included in questions. Failure to do so can lead to fewer marks being awarded. In particular, where the instruction, "showing all working" is included and the candidate shows little or no working, then the candidate will be awarded very few marks even if the final answer is correct.

Where a question specifies a method to use (e.g. determine the present value of income using annuity functions) then, if a candidate uses a different method, the candidate will not be awarded full marks, indeed, the candidate might even be awarded no marks.

Candidates are advised to familiarise themselves with the meaning of the command verbs (e.g. state, determine, calculate). These identify what needs to be included in answers in order to be awarded full marks.

B. Comments on *candidate performance in this diet of the examination.*

The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those candidates.

As in previous exam diets, there appeared to be a large number of insufficiently prepared candidates who had underestimated the quantity of study required for the subject.

The nature of the online exam format meant that there was little on the paper that could be answered via bookwork knowledge alone.

Where candidates made numerical errors, the examiners awarded marks for the correct method used and also for the parts of the calculation that were correct. However, many candidates often did not show enough of their working to fully benefit from this.

The Examiners felt that the "open book" nature of the online exam led some candidates to rely on their notes much more than if the exam had been "closed book". The Examiners strongly recommend that candidates prepare for online exams just as thoroughly as they would do if the exam were of the traditional "closed book" format. Candidates are recommended to use their notes only as a tool to check or confirm answers where necessary, rather than as a source for looking up the answers.

Pass Mark

The Pass Mark for this exam was 63.
1981 presented themselves and 872 passed.

When setting the papers for the April 2024 diet, the examiners reduced the level of repetitive calculations from the A paper and reduced the number of higher order skills questions from the B paper. Rather than replacing these removed items with further questions, the available marks were reallocated to the remaining questions. In this way the examiners hoped to ease the time pressure that many candidates have in the past experienced.

To assess the Pass Mark, the examiners considered how a candidate with the minimum required level of competence would perform on the papers. This estimate was then compared to actual performance of a sample of candidates who achieved marks in the range of 60 to 65. The result of this analysis was that the Pass Mark was set at 63.

The script review criteria were extended such that those candidates where the awarded mark was between 60 and 63 were review marked to ensure that no candidate at the minimum required level of competence was disadvantaged by the setting of the Pass Mark at a level higher than 60%.

Solutions for Subject CM1A - April 2024

Q1

$${}_{4.25}q_{87.25} = 1 - {}_{4.25}p_{87.25}$$

$${}_{4.25}p_{87.25} = {}_{0.75}p_{87.25} \times {}_{3}p_{88} \times {}_{0.5}p_{91}$$

[1]

Using UDD ${}_{t-s}p_{x+s} = 1 - \dfrac{(t-s)q_x}{1-sq_x}$ for $0 \le s < t < 1$ and ${}_{t}q_x = tq_x$ for $0 \le t < 1$

$${}_{0.75}p_{87.25} = 1 - \frac{0.75q_{87}}{1-0.25q_{87}} = 1 - \frac{0.75(0.112969)}{1-0.25(0.112969)} = 0.912811$$

Or $\dfrac{1-q_{87}}{1-0.25q_{87}}$

[½]

$${}_{3}p_{88} = \frac{l_{91}}{l_{88}} = \frac{2,278.869}{3,534.054} = 0.644831$$

[½]

$${}_{0.5}p_{91} = 1 - 0.5(0.161088) = 0.919456$$

[½]

$${}_{4.25}q_{87.25} = 1 - 0.912811 \times 0.644831 \times 0.919456 = 1 - 0.541200$$

$${}_{4.25}q_{87.25} = 0.458800$$

[½]

Total [3]

Commentary:
Generally well answered. *The most serious error was applying UDD to the q values. This showed a clear lack of understanding.*

Q2

(i)

Working in millions. The convexity is given by:

$$\frac{1.5 \times \left(1 \times 2 \times 1.03v^3 + 2 \times 3 \times (1.03)^2 v^4 + 3 \times 4 \times (1.03)^3 v^5\right)}{1.5 \times \left(1 + 1.03v + (1.03)^2 v^2 + (1.03)^3 v^3\right)}$$

[1½]

$$= \frac{1.5 \times (1.635294 + 4.678759 + 8.924300)}{1.5(1 + 0.953704 + 0.909551 + 0.867442)}$$

[1]

$$= \frac{22.85753}{5.5960446}$$

=4.0846 [½]

(ii)

To achieve immunisation the spread of assets must be greater than the spread of liabilities. Here the spread of assets is zero and the spread of liabilities is 3 years. [1]

Therefore, the 3rd Redington condition cannot be satisfied, and so immunisation cannot be achieved. [1]

Total [5]

Commentary:
(i) was poorly answered as many candidates did not know the formula for convexity. *Many candidates struggled to derive the 2nd derivative,* *Another common problem was that candidates struggled to correctly deal with the growth factor in the 2nd derivative (as the derivative is relative to i, the growth factor remains unchanged from that which is found in the PV calculations).* *(ii) was very poorly answered. The question required candidates to understand the concept of convexity and apply it to the specific scenario in the question.*

Q3

(i)(a)

Define y_t to be the t-year spot rate.

$$1.04 = 1.07v_{y_1}$$

$$y_1 = \left(\frac{1.07}{1.04}\right) - 1 = 2.8846\%$$ [1]

$$1.05 = 0.04v_{y_1} + 1.07v_{y_2}^2$$

$$y_2 = \left(\frac{1.07}{1.05 - \frac{0.04}{1+y_1}}\right)^{\frac{1}{2}} - 1 = 2.8703\%$$ [1½]

$$1.06 = 0.04v_{y_1} + 0.04v_{y_2}^2 + 1.07v_{y_3}^3$$

$$y_3 = \left(\frac{1.07}{1.06 - \frac{0.04}{(1+y_1)} - \frac{0.04}{(1+y_2)^2}}\right)^{\frac{1}{3}} - 1 = 2.8559\%$$ [1½]

(i)(b)

The forward rate $f_{t,r}$ is the annual interest rate agreed at time 0 for an investment made at time t for a period of r years.

$f_{0,1} = 2.8846\%$; $f_{0,2} = 2.8703\%$; $f_{0,3} = 2.8559\%$ [½]

$$(1+y_1)\times(1+f_{1,1}) = (1+y_2)^2 \Rightarrow f_{1,1} = \left(\frac{(1+y_2)^2}{(1+y_1)}\right) - 1 = 2.856\%$$ [1]

$$(1+y_1)^1\times(1+f_{1,2})^2 = (1+y_3)^3 \Rightarrow f_{1,2} = \left(\frac{(1+y_3)^3}{(1+y_1)}\right)^{\frac{1}{2}} - 1 = 2.842\%$$ [1]

$$(1+y_2)^2\times(1+f_{2,1}) = (1+y_3)^3 = f_{2,1} = \left(\frac{(1+y_3)^3}{(1+y_2)^2}\right) - 1 = 2.827\%$$ [1½]

(ii)

£12,000 is deposited at time 0 and at the end of year 1 is worth

$$12,000\times(1+y_1) = 12,346.15$$. [1]

£12,000 is deposited at time 1 and so the total sum at the end of year 2 is:

$$(12,346.15+12,000)\times(1+f_{1,1})=25,041.50$$ [1½]

A further £12,000 is deposited at time 2 and the total sum at the end of year 3 is:

$$(25,041.50+12,000)\times(1+f_{2,1})=38,088.68$$ [1½]

Alternatively: $12,000\left[(1+y_3)^3+(1+f_{1,2})^2+(1+f_{2,1})\right]=38,088.68$

Total [12]

Commentary:
(i) was well answered. One concern is that the notation for the forward rates was often not correct, so although the calculations may have been correct, it was unclear which rate candidates were calculating. Some candidates only gave their answers to 2 or fewer decimal points. Given how close the answers can be to a correct answer even when an incorrect method is used, more decimal places should have been given. *(ii) was poorly answered. The answers showed that many candidates do not understand the meaning of spot rates and forward rates and the periods over which they apply. A common error was to only allow for the deposit at time zero.*

Q4

Accumulation after 10 years = $4,000\left[\ddot{s}_{\overline{4}|}^{(4)i\%}(1+i_1)^6+\bar{s}_{\overline{6}|}\right]$ [3]

Where $1+i_1=(1.015)^4$ and so $i_1=6.13636\%\,p.a.$

[½]

$\ddot{s}_{\overline{4}|}^{(4)}=\dfrac{(1.0613636)^4-1}{d^{(4)}}$ where $(1-\dfrac{d^{(4)}}{4})^4=\dfrac{1}{1.0613636}$ and so $d^{(4)}=0.0591133$

And $\ddot{s}_{\overline{4}|}^{(4)}=4.55034$ [1½]

$$\bar{s}_{\overline{6}|}=\frac{(1.0613636)^6-1}{ln(1.0613636)}=7.21194$$ [1½]

Accumulation after 10 years

$=4,000\left[4.55034(1.0613636)^6+7.21194\right]$

=£54,866.66 [½]

Total [7]

Commentary:
Reasonably well answered. *Common errors included using an incorrect interest rate for the quarterly in advance annuity and confusing the time periods for which the terms have to be accumulated or discounted.*

Q5

(i) Death and sickness benefit

$${}_tp_{35}^{HH} = \exp\left[-\int_0^t (\mu+\sigma)ds\right] = \exp\left[-\int_0^t (0.025+0.001)ds\right] = \exp(-0.026t) = {}_tp_{35}^{\overline{HH}}$$ [1]

Expected Present Value of Benefits is given by

$$150,000\int_0^{25} {}_tp_{35}^{HH}\times e^{-0.05t}\times\mu dt = 150,000\int_0^{25} e^{-0.026t}\times e^{-0.05t}\times 0.025dt$$ [1]

$$+\quad 150,000\int_0^{25} {}_tp_{35}^{HH}\times e^{-0.05t}\times 2\sigma dt = 150,000\int_0^{25} e^{-0.026t}\times e^{-0.05t}\times 2\times 0.001$$ [1]

$$PV = 150,000\times 0.027\int_0^{25} e^{-0.076t}dt = 4,050\times\left[\frac{e^{-0.076t}}{-0.076}\right]_0^{25}$$ [1]

$$= 4,050\times 11.18988659 = 45,319.04069 \quad (41,962.075+3,356.9659)$$ [½]

No claim benefit

$$500\times {}_{25}p_{35}^{\overline{HH}}\times e^{-0.05\times 25}$$ [1½]

$$= 500\times e^{-0.65}\times e^{-1.25}$$

$$= 500\times e^{-1.9}$$

$$= 74.78430961$$ [½]

Total benefit

45,319.04069 + 74.78430961 = 45,393.825 [½]

(ii)

Let P be the annual premium payable continuously.

$$P\int_0^{25} {}_tp_{35}^{\overline{HH}}\times e^{-0.05t}dt = P\int_0^{25} {}_tp_{35}^{HH}\times v^t dt = P\int_0^{25} e^{-0.026t}\times e^{-0.05t}dt = P\int_0^{25} e^{-0.076t}dt$$ [2]

(iii)

$${}_tp_{35}^{HS} = \int_0^t {}_sp_{35}^{\overline{HH}} \times \sigma_{35+s} \times {}_{t-s}p_{35+s}^{\overline{SS}} ds \quad \text{or} \quad {}_tp_{35}^{HS} = \int_0^t {}_sp_{35}^{HH} \times \sigma_{35+s} \times {}_{t-s}p_{35+s}^{SS} ds$$ [2]

Total [11]

Commentary:
(i) was poorly answered. In questions involving integration such as this one, sufficient steps must be shown to gain full marks. *Common errors included using an incorrect probability for ${}_tp_{35}^{HH}$ and integrating the no claims benefit which is paid only at one point in time.* *(ii) was generally well answered.* *(iii) was very poorly answered.*

Q6

(i)

We have $i^{(4)} = 0.060608511 < g \times (1 - t_1) = \dfrac{0.08}{1.03} \times (1 - 0.20) = 0.062135922$

Thus, there is a capital loss on redemption (i.e. no capital gain), so the worst-case scenario is to redeem as soon as possible i.e. n= 15. [3]

Thus, the price (per £100 nominal) is:

$$P = 8 \times (1 - 0.20) \times a_{\overline{15}|6.2\%}^{(4)} + 103v_{6.2\%}^{15} = £104.542819594$$

$$a_{\overline{15}|6.2\%}^{(4)} = 9.806674961 \quad v_{6.2\%}^{15} = 0.405632037$$ [3]

(ii)

Capital gain: 106.5 - 104.542819594 = 1.957180406

CGT: 1.957180406 x 0.25 = 0.489295101 [1]

$$104.542819594 = 8 \times (1 - 0.20) \times a_{\overline{5}|i\%}^{(4)} + (106.5 - 0.489295101)v_{i\%}^5$$

$$104.542819594 = 8 \times (1 - 0.20) \times a_{\overline{5}|i\%}^{(4)} + 106.0107049 \times v_{i\%}^5$$ [1½]

Try 6.5%, RHS $= 104.611311615$ [½]

Try 7%, RHS $= 102.504555533$ [½]

$$\Rightarrow i = 0.065 + \frac{104.611311615 - 104.5428196}{104.611311615 - 102.504555533} \times 0.005 = 0.065162553$$ [1]

=6.516% p.a [½]

[Total 11]

Commentary:
(i) was well answered. Some candidates struggled to convert the interest rate into a quarterly rate. A common error was allowing for capital gains tax despite the capital gains test showing there was no gain. *(ii) a common error was not allowing for capital gains tax when calculating the yield. The capital gains tax was easily determined by comparing the purchase and sale prices.*

Q7

(i)(a)

$(al)_{52} =$ 96,777 - 125 - 3,050 = 93,602 [1]

(i)(b)

$(ad)^{d}_{54} =$ 87,592 - 2,550 - 84,906 = 136 [1]

(ii)(a)

$$\frac{(100,000 - 84,906)}{100,000} = 0.15094$$ [1]

(ii)(b)

$$\frac{82,665}{87,592} = 0.94375$$ [1]

(ii)(c)

$$\frac{(141+148+155)}{84,906} = \frac{444}{84,906} = 0.0052293$$ [1½]

(ii)(d)

$$\frac{(2,100+1,400+950+700)}{100,000} = \frac{5,150}{100,000} = 0.0515$$ [1½]

Total [7]

Commentary:
A surprisingly large number of candidates did not know how to calculate these probabilities. This showed a clear lack of knowledge on the basic theory underlying multiple decrement tables.

Q8

(i)

$$V = 50{,}000 \times A_{66} + 19{,}000 \times a_{66}$$

$$= 50{,}000 \times 0.41758 + 19{,}000 \times (10.289 - 1)$$

=197,370 [2½]

DSAR = (50,000 - 19,000) - 197,370 = -166,370 [2]

$$EDS = 200 \times q_{65} \times (-166{,}370) = 200 \times 0.014243 \times (-166{,}370) = 2.8486 \times (-166{,}370)$$

= -473,921.582 [1½]

ADS = 3 x -166,370 = -499,110 [½]

Mortality profit = -473,921.582 + 499,110 = £25,188.42 (profit) [½]

(ii)

More people died than expected 3 vs 2.8486. [½]

For the lump sum benefit this created a mortality loss as the benefits paid out were more than the reserve held. [1]

For the annual income benefit, a death will lead to a release of reserve as no future benefit will be necessary. [1]

With more deaths than expected the release of reserve was larger than expected which led to a mortality profit on the annual income benefit. [1]

In total the mortality profit of the annual income benefit outweighs the mortality loss of the lump sum benefit and an overall mortality profit was made. [½]

[Marks available 4, maximum 3]

Total [10]

Commentary:
(i) was poorly answered. Common errors included • *Not allowing for the pension (annuity) in the DSAR.* • *Trying to calculate a premium for the reserve, which was unnecessary as this was a pension fund.* • *Using incorrect ages for the reserve calculation.* • *Using an incorrect age (relative to the age used in the reserve calculation) for the mortality rate in the EDS.*

Q9

(i)(a)

Reserve as at 31 December 2022 assuming both lives are alive:

$$V_3 = 100,000 \times A_{\overline{63:60}} - 1,495 \times \ddot{a}_{\overline{63:60}}$$ [1½]

$$\ddot{a}_{\overline{63:60}} = \ddot{a}_{63} + \ddot{a}_{60} - \ddot{a}_{63:60}$$

= 14.475 + 16.652 - 13.287

= 17.840 [2]

$$A_{\overline{63:60}} = 1 - d \times \ddot{a}_{\overline{63:60}}$$

=0.3138461538 [1]

= $V_3 = £4,713.8154$ [½]

(i)(b)

Reserve as at 31 December 2022 assuming the male life has died:

Single life (female) reserve

$$V_{3(female)} = 100,000 \times A_{60} - P \times \ddot{a}_{60}$$ [1½]

$$= 100,000\left(1 - \frac{0.04}{1.04} \times 16.652\right) - 1,495 \times 16.652$$ [½]

$$= 100,000 \times 0.35953845 - 1,495 \times 16.652$$ [½]

= £11,059.1062 [½]

[Total 8]

Commentary:
Generally well answered. *Common errors included using incorrect ages and not using a last survivor annuity to value the premium.*

Q10

(i)

For $0 \le t < 5$

$$v(t) = e^{-\int_0^t (a+bs)ds} = e^{-\left[as+0.5bs^2\right]_0^t} = e^{-[at+0.5bt^2]}$$ [1]

Comparing this with the value for *v(t)* given in the question we see that

$$e^{-\left[0.04t+0.01t^2\right]} = e^{-\left[at+0.5bt^2\right]}$$

Hence a=0.04 and b=0.02 [1]

For $5 \le t$

$$v(t) = v(5)\,e^{-\int_5^t c\,ds} = e^{-0.45}.\ e^{-c(t-5)} = e^{-0.45-ct+5c}$$ [2]

Hence,

-0.45- ct + 5c needs to equal -0.05t - 0.20

which suggests that c = 0.05 in order that the "t" term is correct.

Then, putting c=0.05, the left-hand side is -0.45-0.05t+5x0.05 = -0.05t-0.20 which is the required term. [1]

(ii)

Accumulation at t=9 is $1000 \times \dfrac{v(2)}{v(9)}$ [1]

$$v(2) = e^{-\left[0.04(2)+0.01(2)^2\right]} = e^{-0.12}$$

$$v(9) = e^{-\left[0.05(9)+0.20\right]} = e^{-0.65}$$

$$= 1{,}000 \times \frac{e^{-0.12}}{e^{-0.65}} = 1{,}000 \times e^{0.53} = £1{,}698.93$$ [2]

(iii)

$$1{,}000 \times \left(1+\frac{i^{(4)}}{4}\right)^{4\times7} = 1698.93$$

$$\Rightarrow i^{(4)} = 7.644\%\,pa$$ [2]

(iv)

$$PV = \int_8^{13} e^{-[0.05t+0.20]}\, 5e^{0.03t}\,dt$$ [2]

$$= 5e^{-0.20} \times \int_8^{13} e^{-0.02t}\,dt$$ [1]

$= 5e^{-0.20} \times \left[\dfrac{e^{-0.02t}}{0.02} \right]_8^{13}$

$= 4.09365 \times 4.05461$ [½]

= 16.598 [½]

[Total 14]

Commentary:
This question was reasonably well answered. *The biggest misunderstanding arose from the fact that v(t) has already been integrated from the δ(t) function, so when using the provided v(t) function, integration was not required.* *A common error in (ii) was not understanding that v(t) runs from 0 to t.* *Common errors in (iv) included trying to integrate the v(t) in the equation for the continuous cashflow and not knowing that when using v(t), cashflow values are already discounted to time t=0.*

Q11

(i)

Let P = monthly premium. Then equation of value is:

$$12P\ddot{a}^{(12)}_{[35]:\overline{30|}} = 200,000\bar{A}^{1}_{[35]:\overline{30|}} + 120 + 0.02 \times 12P\left(\ddot{a}^{(12)}_{[35]:\overline{30|}} - \frac{1}{12}\right) + 0.30P$$

[3½]

$$\Rightarrow 0.98 \times 12P\ddot{a}^{(12)}_{[35]:\overline{30|}} - 0.28P = 200,000\bar{A}^{1}_{[35]:\overline{30|}} + 120$$

$$\bar{A}^{1}_{[35]:\overline{30|}} = (1.04)^{0.5} \ .\left(A_{[35]:\overline{30|}} - v^{30}\ {}_{30}p_{[35]}\right)$$

$$\bar{A}^{1}_{[35]:\overline{30|}} = (1.04)^{0.5}\left(0.32187 - (1.04)^{-30}\frac{8,821.2612}{9,892.9151}\right) = 0.047880$$

[1½]

$$\ddot{a}^{(12)}_{[35]:\overline{30|}} = \ddot{a}_{[35]:\overline{30|}} - \frac{11}{24}\left(1 - v^{30}\ {}_{30}p_{[35]}\right)$$

$$\ddot{a}^{(12)}_{[35]:\overline{30|}} = 17.631 - \frac{11}{24}\left(1 - (1.04)^{-30}\frac{8,821.2612}{9,892.9151}\right) = 17.299$$

[1½]

$$\Rightarrow (0.98 \times 12 \times 17.299 - 0.28)P = (200,000 \times 0.047880) + 120$$

P = £47.73 per month [½]

(ii)

$$_{25}V = \frac{(1.04)^{25}}{_{25}p_{[35]}}\left[12P\ddot{a}^{(12)}_{[35]:\overline{25|}} - 200,000\bar{A}^{1}_{[35]:\overline{25|}} - 120 - 0.02\times 12P\left(\ddot{a}^{(12)}_{[35]:\overline{25|}} - \frac{1}{12}\right) - 0.30P\right]$$

$$= \frac{(1.04)^{25}}{_{25}p_{[35]}}\left[0.98\times 12P\ddot{a}^{(12)}_{[35]:\overline{25|}} - 0.28P - 200,000\bar{A}^{1}_{[35]:\overline{25|}} - 120\right]$$

[3]

$$\frac{(1.04)^{25}}{_{25}p_{[35]}}\left[12P\ddot{a}^{(12)}_{[35]:\overline{25|}} - 200,000\bar{A}^{1}_{[35]:\overline{25|}} - 120 - 0.02\times 12P\left(\ddot{a}^{(12)}_{[35]:\overline{25|}} - \frac{1}{12}\right) - 0.30P\right]$$

(iii)

The reserve is a very small percentage of the sum assured. This is because the benefit is only payable on death, and there is only a small probability of death occurring each year during the term. [2]

[Total 12]

Commentary:

(i) was well answered. Common errors included:

- *Errors with expenses, in particular not dealing with the initial expense correctly, and not deducting the appropriate value from the annuity used.*
- *Confusion with whether P is the annual premium or the monthly premium. Where an annual premium was used the initial expenses were often wrong.*

(ii) was reasonably answered. Candidates who gave the formula for a prospective reserve were not awarded any marks. Common errors included:

- *Using an incorrect term.*
- *Excluding the initial expenses.*

(iii) was poorly answered.

[Paper Total 100]

END OF EXAMINERS' REPORT

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