INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

19 September 2024 (am)

Subject CS2 – Risk Modelling and Survival Analysis Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 An insurance company has a portfolio of policies where the loss amounts follow an exponential distribution (with parameter λ). The company has an individual excess of loss reinsurance arrangement with a retention level \$12,000.

The following twelve claim amounts (all in \$ and net of reinsurance) are observed:

8,760	10,510	7,800	6,900	11,000	9,000
9,500	11,570	10,400	12,000	12,000	12,000

(i) Calculate the maximum likelihood estimate of parameter λ. [4]

The insurance company wishes to replace the current reinsurance treaty with a proportional reinsurance arrangement such that the population mean claim amount after retention arising from the new treaty is the same as that of the current treaty.

(ii) Derive the retained percentage for the proportional reinsurance arrangement. [7]

[Total 11]

2 The mortality of the population of a particular country has been studied. A recent investigation suggests that for individuals aged 10 exact and over, the force of mortality is constant, μ. Further, the study suggests that 25% of those born survived to exact age 20 and 20% of those born survived to exact age 25 years.

(i) Calculate:

- the constant force of mortality, μ.
- the survival probability, ${}_{10}p_0$.

[3]

A new study suggests that mortality at older ages follows Makeham's Law and that:

$\mu_{70} = 0.026742$
$\mu_{75} = 0.149824$
$\mu_{80} = 0.358927.$

(ii) Determine the probability that a life age 72 exact will survive 15 years. [6]

(iii) Comment on the validity of the assumptions made about forces of mortality in parts (i) and (ii) above. [2]

[Total 11]

3 Consider the following time series process:

$$y_t = a_7 y_{t-7} + \varepsilon_t$$

where ε_t represents a white noise process with zero mean.

(i) Identify the value of a_7 for which y_t is stationary. [2]

(ii) Derive the autocovariance function (ACF) for y_t. [4]

(iii) Comment, with reference to this ACF, on the suitability of the process for modelling seasonal data. [2]

(iv) Specify the value of a_7 for which seasonal differencing is applicable. [2]

[Total 10]

4 An insurance company has a portfolio of 350 policies and assumes that the policies exist in three possible states of claim costs: low, moderate and high. For each of these states the overall claim sizes are assumed to be £200, £300 and £500, respectively. The policies change states according to a Markov jump process with the generator matrix:

$$A = \begin{pmatrix} * & 0.1 & 0.5 \\ 0.3 & * & 0.4 \\ 0.1 & * & -0.2 \end{pmatrix}$$

(i) State the missing entries A_{11}, A_{22} and A_{32} marked * in the generator matrix A above. [2]

(ii) Derive the transition matrix, P, for the corresponding Markov jump process. [2]

(iii) Derive the stationary distribution related to the matrix P. [4]

(iv) Calculate the expected value and the standard deviation of the total claims for this portfolio based on the distribution in part (iii). [7]

[Total 15]

5 The generating function for the Clayton copula is given by the following expression:

$$\frac{1}{\alpha}(t^{-\alpha} - 1)$$

(i) Derive the Clayton copula function $C(u,v)$ using the above generating function and showing each step clearly. [3]

The annual returns from two investment portfolios, X and Y, are assumed to follow the following Normal distributions:

$$X \sim N(0.05, 0.05^2) \text{ and } Y \sim N(0.05, 0.15^2)$$

Their dependant behaviour can be modelled using a Gumbel(2) copula.

(ii) Calculate the probability that both funds fall in value over the next year. [3]

(iii) Calculate the probability that both funds return more than 10% over the next year. [3]

(iv) Explain why the answers to parts (ii) and (iii) are different. [2]

A junior analyst wants to build a new model that does not incorporate any allowance for dependent behaviour between portfolios X and Y. They will then use both models to estimate the total return from a fund consisting of equal amounts of portfolios X and Y.

(v) Identify the simplest adjustment the analyst could make to the original Gumbel(2) model to obtain the new model. [1]

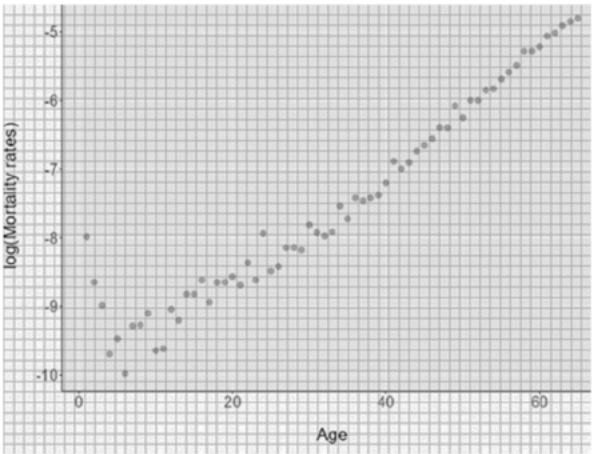
(vi) Discuss, with reference to the difference between the two model approaches, the consequences of using an inappropriate model in this scenario. [4]

[Total 16]

6 The number of customers arriving at a bank branch after time $t = 0$ is modelled as a Poisson process at the rate of 120 per hour. It is known that exactly one customer arrives between $t = 1$ and $t = 2$ minutes.

Determine, explaining your reasoning, the joint probability that exactly two customers arrive between $t = 0$ and $t = 2$ minutes and three customers arrive between $t = 1$ minute and $t = 4$ minutes. [9]

7 The graph below shows the crude mortality rates from age 1 to 65 on a logarithmic scale in a population.



You have recently joined an insurance company as a risk analyst and your team is interested in using this data to project mortality rates into ages above 65.

One of your colleagues proposes a model of the form:

$$y_x = \exp(\alpha + \beta x + \varepsilon_x) \text{ with } \varepsilon_x \sim N(0, \sigma^2) \qquad \text{(Model 1)}$$

where x represents age, y_x represents the mortality rate at age x, and α and β are two unknown parameters that can be estimated.

(i) Comment on the suitability of Model 1 for projecting mortality rates into ages above 65 years. [3]

After fitting Model 1, your line manager suggests exploring alternative models in the following form:

$$y_x = \exp\left(\sum_k f_k(x) a_k + \varepsilon_x\right) \qquad \text{(Model 2)}$$

where $f_k()$ are known functions of your choice and the a_k are unknown coefficients that can be estimated.

(ii) Discuss two variants of Model 2 that can be used to tackle the shortcomings of Model 1 stating the advantages and limitations of each. [6]

Your company has recently acquired new mortality data, separate from the graph above. These data include aggregated death counts and exposed-to-risk by age x and calendar year t. Your company aims to use these data for forecasting future mortality rates.

Your manager recently participated in a mortality modelling seminar where the presenter successfully used the following simplified Lee–Carter model to forecast mortality rates:

$$y_{x,t} = \exp(a_x + k_t + \varepsilon_{x,t}) \text{ with } \varepsilon_{x,t} \sim N(0, \sigma^2) \qquad \text{(Model 3)}$$

where a_x and k_t are unknown parameters that can be estimated.

Based on that experience, your line manager suggests that the team should consider Model 3.

(iii) Demonstrate that Model 3 is not identifiable. [1]

(iv) Describe how you would achieve identifiability when fitting Model 3. [2]

(v) Comment on the suitability of Model 3. [3]

[Total 15]

8 Three friends, X, Y and Z, are throwing a ball to each other. They usually throw it only in one circular direction, from X to Y to Z to X to Y and so on, with the following exceptions:

- Occasionally Y throws the ball to X.
- All three are allowed to throw the ball to themselves (with no limit).
- Every so often Y mis-throws the ball into a nearby river (R), after which the ball is lost.

There are no other ball transitions except the ones described above.

A teacher wishes to model this game as a Markov chain and observes the friends playing for an hour. The teacher estimates the following two-step probabilities:

- $P^2_{ZZ} = 0.5625$
- $P^2_{XX} = 0.4$
- $P^2_{ZY} = 0.125$
- $P^2_{YR} = 0.1$.

The teacher also estimates that the one-step probability that Y throws the ball to themself is the same as the one-step probability of mis-throwing it into the river.

(i) Derive the one-step transition matrix from the two-step probabilities above. [8]

(ii) Calculate the probability that, if Z currently has the ball, Z will have it again after exactly four subsequent throws. [3]

(iii) State, with a reason, if this Markov chain is irreducible. [1]

(iv) State the stationary probability distribution of this Markov chain. [1]

[Total 13]

END OF PAPER

EXAMINERS' REPORT

CS2 Risk Modelling and Survival Analysis

Paper A

Core Principles

September 2024

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

For some candidates, this may be their first attempt at answering an examination using open books and online. The Examiners expect all candidates to have a good level of knowledge and understanding of the topics and therefore candidates should not be overly dependent on open book materials. In our experience, candidates that spend too long researching answers in their materials will not be successful either because of time management issues or because they do not properly answer the questions.

Many candidates rely on past exam papers and examiner reports. Great caution must be exercised in doing so because each exam question is unique. As with all professional examinations, it is insufficient to repeat points of principle, formula or other text book works. The examinations are designed to test "higher order" thinking including candidates' ability to apply their knowledge to the facts presented in detail, synthesise and analyse their findings, and present conclusions or advice. Successful candidates concentrate on answering the questions asked rather than repeating their knowledge without application.

Candidates should note that from the April 2025 exam session, all examinations will continue to be delivered virtually and will have online proctoring. Exams will be closed book and closed web. The ability to refer to past examiner reports and past papers during the exam is not permitted. Candidates attempting to do so will be in breach of the Assessment Regulations and subject to inappropriate conduct investigations. Further details of the new exams can be found on the IFOA website.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
December 2024

A. General comments on the *aims of this subject and how it is marked*

The aim of the Risk Modelling and Survival Analysis Core Principles subject is to provide a grounding in mathematical and statistical modelling techniques that are of particular relevance to actuarial work, including stochastic processes and survival models.

Some of the questions in this paper admit alternative solutions from those presented in this report, or different ways in which the provided answer can be determined. All mathematically correct and valid alternative solutions received credit as appropriate.

In cases where an error was carried forward to later parts of the answer, candidates were not penalised a second time for the same error if those later parts were otherwise answered correctly.

In higher order skills questions, where comments were required, well-reasoned comments that differed from those provided in the solutions also received credit as appropriate.

Candidates are advised to take careful note of all instructions that are provided with the exam in order to maximise their performance in future CS2A examinations.

B. Comments on *candidate performance in this diet of the examination*

Overall performance in this subject continues to improve session-by-session with the average mark increasing in each of the last four sittings and the proportion of candidates passing CS2 is higher than at any time since the first sitting under the new syllabus in April 2019.

Questions in the A paper cover a wide range of syllabus areas and generally set a problem which requires the application of one or two syllabus areas to a particular scenario. Questions often finish with a higher order element that requires interpretation of the results or analysis of the limitations of the approach undertaken. Throughout, candidates are rewarded for well-structured answers that relate to the problem scenario. The majority of marks are awarded for evidence of understanding of the statistical or modelling principles and their accurate application to the scenario given. A correct, final numerical answer often accounts for only ½ or 1 of the marks available.

The question where candidates averaged the highest proportion of the available marks was Q4 on Markov jump processes. This was welcome as it has nearly always been the survival analysis questions that gave the best performance in recent years. Linked to that, the question where candidates averaged the lowest proportion of the available marks was Q7 on mortality projections. This topic is not regularly examined in paper A and many candidates struggled to evidence understanding of the extrapolation approach to projection outlined in the Core Reading. The question with the greatest range of marks relative to those available was Q6 on Poisson processes. This question is an excellent example of where structured thinking and evidence of understanding could score the majority of marks even where the final numerical result was incorrect.

C. Pass Mark

The Pass Mark for this exam was 55.
1167 candidates presented themselves and 549 passed.

The pass mark reflects the level attained by a minimally competent candidate in this examination setting. The pass mark may therefore vary between different examination sessions. The minimally competent candidate is first assessed through a "bottom-up" summation of marks appropriate to each part question in each examination paper. This is then confirmed during marking with both quantitative and qualitative analysis of individual question performance.

Solutions for Subject CS2A – September 2024

Q1

(i)

MLE = lambda hat= n1 / (n2 * M + sum(values < M])) [1]

Where n1 = 9, n2 =3 , M =12000, sum(values < M] = 85440 [1]

lambda hat = 0.0000741 [2]

[Marks available 4, maximum 4]

(ii)

E (X) = ∫(from 0 to M) *x * lambda * exp(-lambda x) dx* + *M*P(X > M)* [1]

= [-*x* exp(-lambda x)](o to M)* +∫(from 0 to M) *exp(-lambda x) dx+12000* exp(-lambda *12000)*

{integration by parts} [1]

= ∫(from 0 to M) *exp(-lambda x) dx* *{ first and last term cancel out}* [1]

= [1 - *exp(-lambda M)] /lambda* [½]

= 7948.494 [1]

Proportional insurance alpha * Mean claim = 7948.494 [1]

Mean claim for the population is calculated as = 1/ lambda = 13493.333 [1]

Proportional insurance alpha = 58.91% [½]

[Marks available 7, maximum 7]
[Total 11]

Commentary:
This is a relatively straightforward question on the exponential as a claim distribution in the presence of reinsurance. Part (i) was well answered but part (ii) was less so with the average candidate scoring 3 out of 7. A common error here was to calculate the mean claim direct from the sample data rather than through the exponential pdf and the lambda result from (i).

Q2

(i)

<u>μ and ${}_{10}P_0$</u>

Given ${}_{20}P_0 = 0.25$ and ${}_{25}P_0 = 0.20$

ie. ${}_{10}P_0 * {}_{10}P_{10} = 0.25$ and ${}_{10}P_0 * {}_{15}P_{10} = 0.20$ [½]

Since force of mortality above age 10 years is constant, the above equations can be written as

. ${}_{10}P_0 * \exp(-10\,\mu) = 0.25$ (A) [½]

and ${}_{10}P_0 * \exp(-15\,\mu) = 0.20$ (B) [½]

Dividing equation (A) by the (B):

$\exp(5\,\mu) = 0.25/0.20 = 1.25$ [½]

$\mu = 1/5 \ln(1.25) = 1/5 * 0.223144 = 0.044629$ [½]

${}_{10}P_0 = 0.25 * \exp(10\,\mu) = 0.25 * 1.5625 = 0.390625$ [½]

[Marks available 3, maximum 3]

(ii)

<u>Makeham's law</u>

If the table follows Makeham's law, $\mu_x = A + Bc^x$

$\mu_{70} = A + Bc^{70}$, $\mu_{75} = A + Bc^{75}$, $\mu_{80} = A + Bc^{80}$ [½]

$\mu_{75} - \mu_{70} = B * \{ c^{75} - c^{70} \} = Bc^{70}(c^5 - 1)$ (A) [½]

$\mu_{80} - \mu_{75} = B * \{ c^{80} - c^{75} \} = Bc^{70}(c^{10} - c^5)$ (B) [½]

Dividing equation (B) by (A):

$(\mu_{80} - \mu_{75}) / (\mu_{75} - \mu_{70}) = (c^{10} - c^5) / (c^5 - 1)$ [½]

$(0.358927-0.149824)/ (0.149824- 0.026742) = c^5$ [½]

$1.698892 = c^5$

$c = 1.111817$ [½]

Substituting in (A),

(0.149824- 0.026742) = B* (1.111817^75 -1.111817^70) [½]

0.123082 =B*1166.121

B= 0.000106 [½]

Substituting in the value for μ_{70},

0.026742= A +0.000106* 1.111817^70

A= 0.26742- 0.176105 = -0.14936 [½]

Probability that a life aged 72 will survive 15 years

Using the formula tpx = s^t* g^(c^x(c^t-1))

${}_{15}p_{72}$ =s^15* g^(c^72(c^15-1))

S=exp(-A) =exp(-0.26742)= 1.161094 [½]

g= exp(-B/logc) =0.999005 [½]

Probability that a life aged 72 will survive 15 years =
1.161094^15* 0.999005^ (1.111817^72*(1.111817^15-1)) =0.003100 [½]

[Marks available 6, maximum 6]

(iii)
In Part (i)
The advantage of this method is that is makes calculations quick and easy [½]
The assumption that the force mortality is constant over ten years needs to be investigated. [1]
Concept of a constant force of mortality is a simplified assumption, but it may not always reflect the complexities of real-world mortality patterns [½]
More sophisticated models are used in practice when greater accuracy is required [½]

[Marks available 2½, maximum 1]

In Part (ii)
Makeham's law is a well-known mortality model used in actuarial science and demography to describe the force of mortality as a function of age. [1]
Makeham's law assumes that the force of mortality consists of two components:
Age-Independent Component (A), Age-Dependent Component (B) [1]
[Other sensible comments on advantages of Makeham, 1 mark each.]

[Marks available 2, maximum 1]

[Total 11]

Commentary:
This question was well answered as is typically the case with survival analysis questions in paper A.

Q3
(i)

In this case the back-shift polynomial is B(L) = 1-a7L^7 [1]
and stationarity holds for |a7| < 1. [1]

[Marks available 2, maximum 2]

(ii)

Using the Yule-Walker equations we can see that

$\gamma_s = a_7\gamma_{s-7}$ for $s \neq 0$ [1]

So for s = 1 the equation above implies that

$\gamma_1 = a_7\gamma_{1-7} = a_7\gamma_6$ and [½]

similarly for $s = 6$

$\gamma_6 = a_7\gamma_{7-6} = a_7\gamma_1$ [½]

Both of these equations imply that, $\gamma_6 = \gamma_1 = 0$. [½]

Similar arguments can be used for showing that

$\gamma_2 = \gamma_5 = 0$ $\gamma_3 = \gamma_4 = 0$ [½]

The only non-zero solutions are

$\gamma_7 = a_7\gamma_0$, $\gamma_{14} = a_7\gamma_7$ and so on [1]

[Marks available 6, maximum 4]

(iii)

As the autocovariance (and autocorrelation) is non-zero only for observations which are 7 time units apart, this can be used to model dependence with time period 7 [1]

Most common examples will be weekly series (e.g. grocery shopping) [1]

[Marks available 2, maximum 2]

(iv)

If $a_7 = 1$ [2]

[Marks available 2, maximum 2]

[Total 10]

Commentary:
The answers to this question were better than for many time series questions in recent years. Part (i) was very well answered. Part (ii) is a straightforward application of the Yule Walker equations. Where candidates scored less well in the later parts it was often because they were preconditioned to think about annual or monthly seasonality which is not really the point of this time series.

Q4

(i)

$$A = \begin{pmatrix} \mathbf{-0.6} & 0.1 & 0.5 \\ 0.3 & \mathbf{-0.7} & 0.4 \\ 0.1 & \mathbf{0.1} & -0.2 \end{pmatrix}$$

[2]

[Marks available 2, maximum 2]

(ii)

From the definition of a transition matrix

$$P = \begin{pmatrix} 0 & \frac{0.1}{0.6} & \frac{0.5}{0.6} \\ \frac{0.3}{0.7} & 0 & \frac{0.4}{0.7} \\ \frac{0.1}{0.2} & \frac{0.1}{0.2} & 0 \end{pmatrix}$$

[2]

[Marks available 2, maximum 2]

(iii)

From the transition matrix above we get

$$\pi_1 * 0 + \pi_2 \frac{3}{7} + \pi_3 \frac{1}{2} = \pi_1$$

[1]

$$\pi_1 * \frac{1}{6} + \pi_2 * 0 + \pi_3 \frac{1}{2} = \pi_2$$

[1]

$$\pi_1 * \frac{5}{6} + \pi_2 \frac{4}{7} + \pi_3 * 0 = \pi_3$$

[1]

Solving these gives

$$\pi = (0.3208556, 0.2620321, 0.4171123)$$

[1]

[Marks available 4, maximum 4]

(iv)

The expected claim costs per policy is

(200*0.3208556+300*0.2620321+500* 0.4171123)

[1]

= 351.3369 [½]

And so the expected overall cost is =350*351.3369 [1]

=122967.9 [½]

The standard deviation per policy claim is

$\sqrt{}(200\text{^}2 * 0.3208556 + 300\text{^}2 * 0.2620321 + 500\text{^}2 * 0.4171123 - 351.3369\text{^}2\ )$ [2]

= 131.3681 [½]

And for the overall portfolio is 350*131.3681 [1]

=45978.83 [½]

[Marks available 7, maximum 7]

[Total 15]

Commentary:
This question was the best performing one across the whole paper in terms of proportion of available marks scored by candidates. More than 96% of candidates gained full marks in part (i). Parts (ii) and (iii) were well answered too. In these parts, both fraction and decimal forms were acceptable. In part (iv) because the question did not say anything about the independence of the 350 policies, full marks were given for portfolios standard deviations of both 350 and sqrt(350) times per policy deviation.

Q5

(i)

From the generating function we have:

$1/\alpha\ (u^{-\alpha} + v^{-\alpha} - 1\text{-}1)$ [1]

$\alpha\ /\alpha\ (u^{-\alpha} + v^{-\alpha} - 2)$ [½]

$(u^{-\alpha} + v^{-\alpha} - 1)$ [½]

$(u^{-\alpha} + v^{-\alpha} - 1)^{-(1/\alpha)}$

this is the Clayton copula function, C(u,v) [1]

[Marks available 3, maximum 3]

(ii)

P(X<0) = 0.159

P(Y<0) = 0.371 [½]

P(X<0) and P(Y<0) = $\exp\{-\ [\ (-\ln 0.159)^2\ +\ (-\ln 0.371)^2\ ]^{0.5}\ \}\ = \exp\text{-}(4.365^{\ 0.5})$ [1½]

= exp(-2.09) = 0.124 [1]

[Marks available 3, maximum 3]

(iii)

Required probability = 1-0.841-0.629+C(.841,.629) [1½]

Now C(.841,.629) = $\exp\{-\ [\ (-\ln 0.841)^2\ +\ (-\ln 0.629)^2\ ]^{0.5}\ \}\ = 0.61\ )$ [1]

Therefore probability [½]

= 1 - 0.841 - 0.629 + 0.61 = 0.139

[Marks available 3, maximum 3]

(iv)
Gumbel copula is not symmetrical [1]
Gumbel exhibits higher tail correlation approaching (1,1) compared to approaching (0,0) [1]

[Marks available 2, maximum 2]

(v)
Simply change the Gumbel parameter value to 1 [1]

[Marks available 1, maximum 1]

(vi)
The variance of total returns would be greater under the Gumbel copula due to the non-zero correlation. [1]
Hence probability of extreme total returns would be higher under the Gumbel model [1]
If the Gumbel model is a better reflection of the data it is likely that adoption of the independent model would underestimate the risks of investing in both X and Y. [1]
If the independent model is a better reflection of the data it is likely that adoption of the Gumbel model would over-estimate the risks of investing in both X and Y. [1]

[Marks available 4, maximum 4]
[Total 16]

Commentary:
The first two parts of this question were well answered. A wide range of interpretative comments were eligible for marks in parts (iv) and (vi) where the important element was evidencing understanding of tail dependence and how it varies both between the two tails within a copula and also between different copulas.

Q6
(i)
Working in minutes , $\lambda = 2$ per minute [½]

Two intervals (0,2) and (1,4) are not disjoint sets [½]

Let X, Y, and Z be the numbers of arrivals in (0,1], (1,2] and (2,4) respectively. [1]

Then X, Y, and Z are independent as the variables are Poisson increments [½]

$X \sim Poisson(\lambda), Y \sim Poisson(\lambda), Z \sim Poisson(2\lambda)$ [1]

Let *A* be the event that there are two customers arriving between 0 and 2 minutes and three two customers arriving between 1 & 4 minutes given that one customer has arrived between 1 and 2. [½]

$P(A)$ = $P(X+Y=2$ and $Y+Z=3$ / Y = 1) [1]

$=P(X=1$ and $Z=2$ and Y = 1) / $P(Y=1)$

{given that one customer has arrived between (1,2), one customer must have arrived between 0 and 1 , 2 must have arrived between 2 and 4} [1½]

$= \lambda\, e^{-\lambda} \; * e^{-2\lambda} \; (2\lambda)^2 / 2$ {pdf of poisson} [1½]

= 3.966% or 0.04 [1]

[Total 9]

Commentary:
This was probably the most challenging question on the paper requiring clear thinking about the structure for an answer whilst using a relatively straightforward statistical framework (the Poisson pdf). The most common error was not noting that (0,2) and (1,4) overlap and therefore a simple chaining of two probabilities is insufficient. The average mark scored was 3½ out of 9 for this question.

Q7

(i)

Model (1) assumes that log mortality rates are linear in age overall; [1]
however, the overall pattern of the data is not linear. [½]
We are using data for ages up to 65 to project mortality at ages over 65 [1]
There are particular issues doing this given the nature of the under 65 data at certain ages [1]
In particular, the graph shows that the mortality rate decreases in the first few years after birth and then rises. [½]
Model (1) will not be able to capture that pattern appropriately [½]
Fitting Model (1) to the data will result is under fitting of infant mortality rates [½]
A more flexible model/approach will tend to fit the data better [½]
However, excluding infant period, the rest of the data pattern is not far from linear [½]
A simple approach could be to fit model 1 to the data excluding the infant period, check goodness of fit and if appropriate, use the fitted line to project mortality rates into higher age. [1]

[Marks available 7, maximum 3]

(ii)

Variant 1: P-splines [½]
i.e. f_k are b-splines basis [½]
and a penalty is used to achieve smoothing and forecasting [½]
Advantages:
Allow to capture non-linear and complex patterns from the data [½]
Smoothing parameters can be adjusted to avoid overfitting [½]
Limitations: [½]

Not straightforward to implement
Risk of overfitting if smoothing parameter too low [½]
Can lead to unreasonable or unstable forecast [½]
Variant 2: polynomials [½]
f_k(x) = x^k [½]
Advantages:
Help to capture non-linear patterns [½]
Easy to implement [½]
Limitations:
Risk of overfitting if too many terms are included. [½]
High order polynomials can yield unreasonable forecasts [½]
[Marks available 7, maximum 6 *(3 marks for each variant)*]

(iii)
The model is not identifiable because the parameter specifications (alpha_x, kappa_t) and specifications (alpha_x + a, kappa_t – a) would yield identical value of the fitted mortality rates. [1]
[Marks available 1, maximum 1]

(iv)
Identifiability can be achieved by imposing constraints on the parameters [2]
[Marks available 2, maximum 2]

(v)
Model (3) is an age-period model [½]
It is one of the simplest models for mortality projection [½]
Hence it's not guaranteed that it will fit this data well [½]
In particular, Model (3) will not be appropriate if cohort effects are present in the data [½]
A simple way to know if it's appropriate for this particular data is to fit it to the data and then assess the residuals (or perform goodness of fit test) [½]
If it fits the data well, then kappa_t can be projected for example using ARIMA model [½]
and the projected kappa_t can then be used to derive projected mortality rates [½]
[Marks available 3½, maximum 3]
[Total 15]

Commentary:
This question scored the lowest proportion of marks available across the two CS2 papers in this sitting. That might be because candidates have not taken the time to familiarise themselves with the mortality projection material at the end of the survival analysis sections of the Core Reading. Those candidates who were familiar with extrapolation methods and the Lee-Carter models scored well. In part (i) many candidates who scored poorly simply missed the major element of the scenario that mortality at ages up to 65 is being used to project mortality above age 65. In part (ii) full marks were available for candidates who contrasted two polynomial approaches or used Gompertz / Makeham as one of their polynomials instead of writing about splines.

Q8
(i)
pzz2=.5625, so pzz=sqrt .5625 = 0.75 [1]

and now pzx = 1-.75=0.25 as zy and zr = 0 [1]

pzy2=.125, so zx xy = .125 (only way) pxy = .125/.25 = .5 [1]

pxx = 1-.5=.5 as xz and xr = 0 [1]

pxx2 = .4 = xx xx + xy yx, so yx = (.4-.5^2)/.5 = .3 [1]

pyr2 = .1 = yy yr + yr rr = yr(yy+1) = .1, and yy^2+yy=.1=0 as yr=yy (given) [1]

solving using quad form gives yy = .0916 = yr [1]

```
     X       Y        Z        R
X 0.50 0.500000 0.000000 0.000000
Y 0.30 0.091608 0.516784 0.091608
Z 0.25 0.000000 0.750000 0.000000
R 0.00 0.000000 0.000000 1.000000
```
[1]

[Marks available 8, maximum 8]

(ii)
we need to consider each of the possible 4 step routes from Z to Z

zzzz

zxyz

xxyz

xyyz

xyzz [1]

Probability = .5625^2 +.25*.5*.5*.5168 +.25*.5*.0916*.5168

+2*(.25*.5*.5168*.75) [1]

= 0.4515 [1]

[Marks available 3, maximum 3]

(iii)
No [½]
Because can't move out of R. [½]
[Marks available 1, maximum 1]

(iv)
[0,0,0,1] [1]
[Marks available 1, maximum 1]
[Total 13]

Commentary:
This stochastic processes question was reasonably well answered although many candidates did poorly in part (ii) by not considering the different routes.

[Paper Total 100]

END OF EXAMINERS' REPORT

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