INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

12 April 2021 (am)

Subject CM1A – Actuarial Mathematics Core Principles

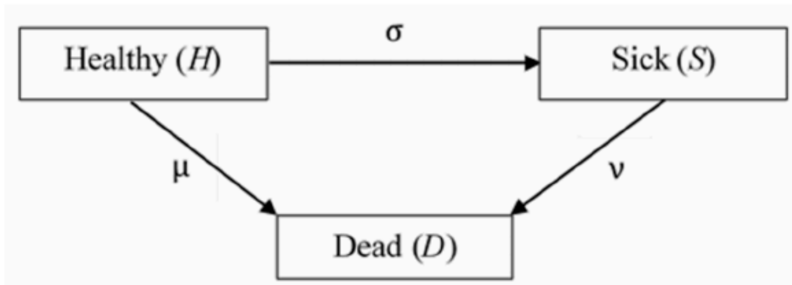
Time allowed: Three hours and fifteen minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 Calculate ${}_{3|5}q_{\overset{1}{45}:45}$ assuming AM92 mortality for both individuals and that the individuals are independent with regards to mortality. [4]

2 A life insurance company uses the following three-state model, with constant forces of transition, to price its stand-alone critical illness policies.



Under these policies, a lump sum benefit is payable when a life becomes critically ill during the policy term. No other benefits are payable.

A 30-year policy with sum assured $150,000 is issued to a healthy life aged 35 exact.

The expected present value of the benefit at outset is given by the following formula:

$$\boldsymbol{m} \times \int_{\boldsymbol{a}}^{\boldsymbol{b}} \boldsymbol{n} \times e^{\boldsymbol{z}t}\, dt.$$

(i) State the numerical values of ***a***, ***b***, ***m***, ***n*** and ***z***. [3]

(ii) Calculate the expected present value of the benefit for this policy based on your answer to part (i).

Basis: $\mu = 0.01$
$\sigma = 0.02$
$\nu = 0.04$

Interest: 3% p.a. effective

[3]
[Total 6]

3 A fixed interest security of nominal amount \$100,000 was issued on 1 March 2017 and was redeemed at par on 1 March 2020. Coupons were paid at the rate of 4% p.a. annually in arrears.

The value of the inflation index at various dates during the term of the security was as follows:

Date	*Inflation index*
1 March 2017	240.5
1 March 2018	256.0
1 March 2019	272.8
1 March 2020	286.6

(i) Demonstrate that the effective annual real rate of return achieved over the term of the security is approximately equal to –1.9% p.a. [5]

(ii) Comment on the result in part (i). [3]

[Total 8]

4 The force of interest, $\delta(t)$, is a function of time and at any time t, measured in years, is given by the formula:

$$\delta(t) = \begin{cases} 0.03 + 0.005t & 0 \le t \le 6 \\ 0.1 - 0.01t & t > 6 \end{cases}$$

$A(0,t)$, the accumulation at time t of a unit of money invested at time 0, can be written as:

$$A(0,t) = \begin{cases} e^{\boldsymbol{a} + \boldsymbol{b}t + \boldsymbol{c}t^2} & 0 \le t \le 6 \\ e^{\boldsymbol{f} + \boldsymbol{g}t + \boldsymbol{h}t^2} & t > 6 \end{cases}$$

(i) Calculate the values of ***a***, ***b***, ***c***, ***f***, ***g*** and ***h***. [5]

A sum of \$5,000 is invested at $t = 2$ for 5 years.

(ii) Calculate the annual nominal rate of return convertible monthly on the investment. [3]

[Total 8]

5 The force of mortality, μ_x^*, experienced by a particular population at all ages x (where x is not necessarily an integer) is 20% higher than that under the PMA92C20 table.

Calculate the following, based on μ_x^*, assuming a rate of interest of 7% p.a.

(i) $\ddot{a}_{70:\overline{3}|}$ [5]

(ii) $A_{70:\overline{3}|}$ [2]

[Total 7]

6 A special whole life assurance policy is issued to a life aged 45 exact. The policy provides a benefit of \$40,000 on death within 15 years of inception, and \$50,000 on death thereafter. Benefits are payable at the end of the year of death.

(i) Calculate the expected present value of the benefit payments. [3]

(ii) Calculate the variance of the present value of the benefit payments.

Basis:

Mortality:	AM92 Ultimate
Interest:	6% p.a.

[6]

[Total 9]

7 A life insurance company issues a 20-year with-profit endowment assurance policy to a life aged 45 exact for a sum assured of \$150,000. The sum assured, together with any attaching bonuses, is payable on survival to the end of the term or at the end of the year of death if earlier.

The company assumes that future reversionary bonuses will be declared on the policy annually at a rate of 1.92308% of the sum assured, compounded and vesting at the end of the policy year (i.e. the death benefit does not include any bonus relating to the policy year of death).

Calculate the level premium payable annually in advance throughout the policy term, and ceasing on earlier death.

Basis:

Mortality:	AM92 Select
Interest:	6% p.a.
Initial expenses:	\$200 plus 75% of the annual premium
Renewal expenses:	2.5% of each annual premium excluding the first
Claim expenses:	\$140 on death or maturity

[10]

8 On 1 January 2022, a student plans to take out a 10-year bank loan for \$15,000.

Under the repayment schedule, instalments will be paid monthly in arrears until the end of the term. The first instalment, at the end of January 2022, will be X, and the second instalment, at the end of February 2022, will be $2X$, and so on, until the instalment at the end of December 2026, which will be $60X$. The remaining instalments from the end of January 2027 will also be $60X$.

The bank charges a rate of interest of 12% p.a. effective.

(i) Write down an equation of value to calculate X. [2]

(ii) Calculate the value of X using the equation of value in part (i). [5]

(iii) Write down an equation to calculate the loan outstanding, after the instalment paid at the end of December 2026, using the retrospective method. [2]

(iv) Calculate the loan outstanding after the instalment at the end of December 2026 has been paid, using the equation in part (iii). [1]

(v) Comment on your answer to part (iv). [2]

(vi) Write down an equation to calculate the total interest paid during 2027. [2]

(vii) Calculate the total interest paid during 2027 using the equation in part (vi). [2]

The bank also offers the 10-year loan with the same interest rate but where the monthly instalments remain level throughout the term.

(viii) Comment on whether the total interest paid by the student under this revised offer would be greater or less than that paid under the original repayment schedule. You should not perform any further calculations. [2]

[Total 18]

9 The Green Investment Company has the opportunity to purchase a factory for \$400,000. The factory is to be leased and two different companies, A and B, are interested in the lease. The two companies have made the following proposals.

Company A
The Green Investment Company will need to spend another \$50,000 refurbishing the factory for Company A.

Company A will pay rent annually in advance for 20 years starting immediately. The rent will increase by 3% p.a. compound each year. At the end of 20 years, Company A will purchase the factory from the Green Investment Company for \$450,000.

Company B
Company B will pay rent at an initial level amount of \$44,600 p.a. payable monthly in advance starting immediately. The rent will increase by 50% at the end of the 10th year and remain at this level for the next 10 years. At the end of 20 years, ownership of the factory will pass to Company B at no further cost.

(i) Calculate the initial annual rent payable by Company A, to give the Green Investment Company an internal rate of return of 9% p.a. effective on the proposal. [3]

(ii) Demonstrate that the internal rate of return from Company B's proposal would be greater than 9% p.a. effective. [3]

The Green Investment Company does not have the capital available to purchase the factory but can take out a loan at an interest rate of 9.5% p.a. effective. The loan is to be repaid over 20 years in level instalments payable annually in arrears.

The Green Investment Company decides to accept the proposal from Company B, and takes out a loan in order to purchase the factory.

(iii) Calculate the accumulated profit of the investment after 20 years using an effective rate of interest of 9.5% p.a. [4]

[Total 10]

10 On 1 January 2011, a life insurance company planned to issue the following two policies to lives then aged 45 exact:

- a 15-year without-profit endowment assurance with a sum assured of S payable on maturity or immediately on earlier death, and with premiums of P payable annually in advance
- a 15-year temporary life annuity payable annually in advance purchased with a single premium of $50,000.

The annual annuity payments were calculated to be exactly sufficient to pay the premiums for the endowment assurance as they fell due.

(i) Calculate P and S. [7]

The policies were actually issued with S = $90,000 payable under each endowment assurance policy and an annual premium of P = $4,450.

On 31 December 2020, there were 550 policies still in force. During 2020, there were six deaths with no other decrements taking place.

(ii) Calculate the mortality profit for the calendar year 2020. [10]

(iii) Comment on your numerical result obtained in part (ii).

Basis:
Mortality: AM92 Ultimate
Interest: 4% p.a.
Expenses: Ignore

[3]
[Total 20]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2021

Subject CM1 - Actuarial Mathematics
Core Principles
Paper A

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision

Paul Nicholas
Chair of the Board of Examiners
July 2021

A. General comments on the *aims of this subject and how it is marked*

1. CM1 provides a grounding in the principles of modelling as applied to actuarial work - focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

2. Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations, but candidates are not penalised for this. However, candidates may not be awarded marks where excessive rounding has been used or where insufficient working is shown.

3. These solutions use full actuarial notation although candidates who used notation based on standard keystrokes were given full credit.

B. Comments on *candidate performance in this diet of the examination.*

1. The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well by most candidates. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those candidates.
2. There appeared to be a large number of ill-prepared candidates who had underestimated the quantity of study required for the subject.
3. The nature of the online exam format meant that there was little on the paper that could be answered via knowledge based alone.
4. Where candidates made numerical errors, examiners awarded marks for the correct method used and also for the parts of the calculation that were correct. However, many candidates often did not show enough of their working to fully benefit from this.
5. The examiners felt that the "open book" nature of the online exam led some candidates to rely on their notes much more than if the exam had been "closed book". The examiners strongly recommend that candidates prepare for online exams just as thoroughly as they would do if the exam were of the traditional "closed book" format. Candidates should treat it as a bonus that they can refer to their notes but they should not be relying on being able to do so.

C. Pass Mark

The Pass Mark was 58.
1,856 presented themselves and 941 passed.

Solutions for CM1A – April 2021

Q1 $\quad {}_{3|5}q^{\ 1}_{45:45} = \dfrac{l_{48}\times l_{48}}{l_{45}\times l_{45}}\times\dfrac{1}{2}\times {}_{5}q_{48:48} = \dfrac{l_{48}\times l_{48}}{l_{45}\times l_{45}}\times\dfrac{1}{2}\times\left(1-\dfrac{l_{53}\times l_{53}}{l_{48}\times l_{48}}\right)$ [2]

$=\dfrac{(9,753.4714)^2}{(9,801.3123)^2}\times\dfrac{1}{2}\times\left(1-\dfrac{(9,630.0522)^2}{(9,753.4714)^2}\right)$ [1]

$=0.990261683\times\dfrac{1}{2}\times(0.02514763)$ [1]

$=0.012451366$

This question was generally well-answered.

Q2

(i)

a=0,

b=30 [a and b together ½]

m= 150,000 [½]

n=0.02 [1]

$z = \ -ln\,(1.03) - 0.01 - 0.02 = -0.059559$ [1]

(ii)

$=-\dfrac{150000\times 0.02}{0.059559}\left[e^{-0.059559t}\right]_0^{30}$ [1½]

= 50,370.22 [1 – 0.167500] [1]

= 41, 933.21 (without rounding 41,933.28) [½]

[Total 6]

Most candidates scored some marks in part (i) and a significant number of candidates were then able to use the answers to part (i) to make an attempt at part (ii). However, a surprising number of candidates struggled to perform the straightforward integration in part (ii)

A common error in part (i) was to confuse the effective rate of interest, i, with the force of interest, δ .

Q3

(i)

Interest paid per year is $0.04\times 100,000 = 4,000$ [½]

Hence, by expressing cash flows in 1 March 2017 purchasing power, the effective annual real rate of return achieved, i, is found by solving:

$$100{,}000 = 4{,}000 \times \frac{240.5}{256.0} \times v_{i\%} + 4{,}000 \times \frac{240.5}{272.8} \times v_{i\%}^2 + \left(4{,}000 + 100{,}000\right) \times \frac{240.5}{286.6} \times v_{i\%}^3$$

$$\Rightarrow \quad 100{,}000 = 3{,}757.81 v_{i\%} + 3{,}526.39 v_{i\%}^2 + 87{,}271.46 v_{i\%}^3$$

[3]

Then, we have:

$$\left.\begin{array}{lll} i = -2\% & \Rightarrow & RHS = 100{,}230.69 \\ i = -1.5\% & \Rightarrow & RHS = 98{,}769.15 \end{array}\right\}$$

$$\Rightarrow i \approx -0.02 + \left[-0.015 - (-0.02)\right] \times \frac{100{,}230.69 - 100{,}000}{100{,}230.69 - 98{,}769.15} \approx -1.9\%$$

[1½]

(ii)
High actual inflation over the term of the loan has eroded the real return achieved. [1]
The nominal rate of return achieved is 4% per annum. However, as average inflation over the term of the loan (i.e. 6.02% pa) has exceeded 4% per annum, the real rate of return achieved by the lender is negative. [2]

[Total 8]

Part (i) was generally well answered. Common errors included: -
not discounting the individual cashflows back to time t=0,
ignoring the amounts of the cashflows and using an average inflation over the entire period,
getting the ratios of the inflation indices the wrong way round.

The question omitted to say that the bond was issued at par. In practice, nearly all the candidates assumed this to be the case; where an alternative assumption was made candidates were not penalised.

Where candidates made comments in part (ii), they were often of insufficient quality to demonstrate understanding. Candidates often failed to distinguish between nominal return and real return.

Q4
(i)
For $0 \le t \le 6$:

$$A(0,t) = \exp\left(\int_0^t \delta(s)\,ds\right) = \exp\left(\int_0^t 0.03 + 0.005s\,ds\right)$$

$$= \exp\left[0.03s + 0.0025s^2\right]_0^t = e^{0.03t + 0.0025t^2}$$

and $A(0,6) = e^{0.03\times 6 + 0.0025\times 36} = e^{0.27}$

For $t > 6$:

$$A(0,t) = \exp\left(\int_0^6 \delta(s)\,ds + \int_6^t \delta(s)\,ds\right) = A(0,6) \times \exp\left(\int_6^t 0.1 - 0.01s\,ds\right)$$

$$= e^{0.27}\exp\left[0.1s - 0.005s^2\right]_6^t = e^{0.27}\exp\left[0.1t - 0.005t^2 - (0.6 - 0.18)\right]$$
$$= e^{0.1t - 0.005t^2 - 0.15}$$

$a = 0$ [½]
$b = 0.03$ [½]
$c = 0.0025$ [1]
$f = -0.15$ [1]
$g = 0.1$ [1]
$h = -0.005$ [1]

(ii)

Nominal rate of return is $i^{(12)}$ where $\left(1 + \frac{i^{(12)}}{12}\right)^{60} = \frac{A(7)}{A(2)}$ [1]

$$= \frac{e^{0.1\times 7 - 0.005\times 49 - 0.15}}{e^{0.03\times 2 + 0.0025\times 4}} = \frac{e^{0.305}}{e^{0.07}} = e^{0.235}$$ [1]

Therefore $i^{(12)} = 12\left(e^{0.235/60} - 1\right) = 4.709\%$ [1]

[Total 8]

Part (i) was generally well answered. A common error was to evaluate A(0,6) incorrectly or omit it entirely in the derivation of f, g and h. It was not necessary to show the derivation of the numerical results in order to gain full marks. The derivation is included here to aid candidates' understanding.

In part (ii) some candidates did not appreciate that the accumulation factors derived in part (i) were only applicable if accumulated from time t=0. Many candidates attempted to derive new accumulation factors from time t=2, which is a perfectly valid approach but takes more time. Common errors included:
Integrating over an incorrect time period;
Using a formula for δ(t) over a time period for which it was not relevant.

Q5

(i)

$\mu_x^* = 1.2\mu_x$ where $t \le 1$

${}_tp_x^* = e^{-\int_0^t \mu_{x+s}^* ds}$ where $t \le 1$

$\Rightarrow {}_tp_x^* = e^{-\int_0^t 1.2\mu_{x+s} ds}$ where $t \le 1$ [1]

$\Rightarrow p_x^* = (p_x)^{1.2}$ [1]

$\ddot{a}^*_{70:\overline{3}|} = 1 + v \times p_{70}^* + v^2 \times {}_2p_{70}^*$ at 7% pa [1]

$$\Rightarrow \ddot{a}^*_{70:\overline{3}|} = 1 + v \times \left(\frac{9112.449}{9238.134}\right)^{1.2} + v^2 \times \left(\frac{8968.099}{9238.134}\right)^{1.2}$$ [1]

$= 2.762234$ [1]

(ii) $A_{70:\overline{3}|} = 1 - d\ddot{a}^{*}_{70:\overline{3}|} = 1 - \left(\dfrac{0.07}{1.07}\right) \times (2.762234)$ [1½]

$= 0.819293$ [½]

[Total 7]

This question was poorly answered. Many candidates did not appreciate that the values of μ_x shown in the tables are values at exact age x and do not apply over the whole period of x to x+1. It is therefore necessary to derive an average value of μ_x for age x from the tabulated value of p_x.

In part (i) many candidates forgot that the first payment in a life annuity in advance is 1 at time t=0, and that to receive the 3rd payment the life only needs to survive 2 years.
Another common error was to use only a one-year survival probability, rather than the survival probability from outset.

In part (ii) where candidates used the valid (but unnecessarily time consuming) approach of deriving the endowment assurance factor from first principles, a common error was to miss out the pure endowment benefit.

Q6

(i)

$$EPV = 40,000 \times A^{1}_{45:\overline{15}|} + 50,000 \times v^{15}_{6\%} \times \frac{l_{60}}{l_{45}} \times A_{60}$$ [1]

Where

$$A^{1}_{45:\overline{15}|} = A_{45:\overline{15}|} - v^{15}_{6\%} \times \frac{l_{60}}{l_{45}} = 0.42556 - 0.417265 \times \frac{9287.2164}{9801.3123}$$

[1]

$$= 0.030181218$$

$$EPV = 40,000 \times 0.030181218 + 50,000 \times 0.417265 \times \frac{9287.2164}{9801.3123} \times 0.32692$$

$= \$7,670.11$ [1]

(ii)

To get the variance, we calculate the 2nd moment by defining the benefit as a combination of a temporary assurance and a deferred whole life assurance.

Therefore:

Benefit from age 45 to 60:

$40,000^2 \times {}^2\mathrm{A}^{1}_{45:\overline{15}|}$ with i at 6% [1]

$$= 40,000^2 \times \left[{}^2A_{45} - v^{15}_{12.36\%} \times \frac{l_{60}}{l_{45}} \times {}^2A_{60} \right]$$

$$= 40{,}000^2 \times \left[0.04172 - 0.174110 \times \frac{9287.2164}{9801.3123} \times 0.14098 \right]$$ [1½]

=1,600,000,000 × 0.018461 = $29,537,600. [½]

Benefit from age 60:

$$50{,}000^2 \times v^{15}_{12.36\%} \times \frac{l_{60}}{l_{45}} \, {}^2A_{60} \text{ with } i \text{ at } 6\%$$ [1]

$$= 50{,}000^2 \times 0.174110 \times \frac{9287.2164}{9801.3123} \times 0.14098$$

$$= 50{,}000^2 \times 0.023259 = \$58{,}147{,}500$$ [½]

Then total 2nd moment = 29,537,600 + 58,147,500

= 87,685,100 (with no rounding 87,684,707) [½]

Variance = 87,685,100 – $(7{,}670.11)^2$ = $\$^2$28,854,513 = $(\$5{,}372)^2$ [1]

[Total 9]

Part (i) was generally well answered. Common errors included: -
Using an endowment assurance factor rather than a term assurance factor for the first benefit.
Missing out the survival probability to age 60 in the second benefit.

Part (ii) was poorly answered. The simplest approach is to treat the benefit payments as a term assurance and deferred whole life, which are independent and therefore the covariance between them is zero.

Candidates who treated the benefits as a whole life benefit plus a deferred whole life failed to appreciate that these are not independent and hence failed to address the covariance between them.

Other common errors when calculating the second moment in part (ii) included: -
Omitting the square of the sums assured,
Squaring the survival probability for the deferred whole life benefit.

Q7

EPV Premiums $P\ddot{a}^{6\%}_{[45]:\overline{20|}} = 11.888P$ [1]

The interest rate for valuing the benefits is 4% p.a. $\frac{1.0192308}{1.06} = \frac{1}{1.04}$ [1]

EPV Benefit

$$= \$150{,}000 \left(\frac{1}{1.0192308} A^{1}_{[45]:\overline{20|}} + A_{[45]:\overset{\;\;1}{\overline{20|}}} \right) \text{ at } 4\%$$ [2]

$$= \$150,000\left(\frac{1}{1.0192308}\left(A_{[45]:\overline{20}|} - v_{4\%}^{20}\frac{l_{65}}{l_{[45]}}\right) + v^{20}\frac{l_{65}}{l_{[45]}}\right)$$

$$= \$150,000\left(\begin{array}{l}\frac{1}{1.0192308}\left(0.46982 - (1.04)^{-20}\frac{8,821.2612}{9,798.0837}\right) \\ \\ +(1.04)^{-20}\frac{8,821.2612}{9,798.0837}\end{array}\right) \quad [1½]$$

$$= \$150,000(0.057821 + 0.41089) = \$70,306 \quad [½]$$

EPV Expenses

$$= \$200 + 0.75P + 0.025P\left(\ddot{a}^{6\%}_{[45]:\overline{20}|} - 1\right) + 140A^{6\%}_{[45]:\overline{20}|}$$

$$= \$200 + 0.75P + 0.025 \times 10.888P + 140 \times 0.32711 = \$245.80 + 1.0222P \quad [3]$$

Thus

$$10.866P = \$70,552 \Rightarrow P = \$6493.03 \quad [1]$$

[Total 10]

This question was generally well answered. Common errors included: -
Not splitting the death and survival benefits to correctly allow for bonuses vesting at the end of the policy year if a policyholder survives to the end of that policy year.
Using annuity and assurance factors at the adjusted interest rate to value cashflows that do not attract bonuses, in particular the claim expense

Q8

(i)

$$\$15,000 = X\,(Ia)_{\overline{60}|@j} + 60Xv_j^{60}a_{\overline{60}|@j} \quad [2]$$

where $j = \frac{i^{(12)}}{12}$

Alternative

$$\$15,000 = X\,(Ia)_{\overline{60}|@j} + 12 \times 60\,Xv_i^5 a^{(12)}_{\overline{5}|@i}$$

(ii)

$$(Ia)_{\overline{60}|@j} = \frac{\ddot{a}_{\overline{60}|j} - 60v_j^{60}}{j} = \frac{\frac{1 - 1.0094888^{-60}}{0.0094888/1.0094888} - 60 \times 1.0094888^{-60}}{0.0094888} = 1261.989$$

[2]

$$a^{(12)}_{\overline{5}|@i} = 3.6048 \times \frac{0.12}{0.11387} = 3.7990 \qquad \text{OR} \qquad a_{\overline{60}|@j} = 45.58779473 \quad [1]$$

where $i = 12\%$ and $j = \frac{i^{(12)}}{12} = 1.12^{1/12} - 1 = 0.0094888$ [1]

So $\$15{,}000 = 1261.989 \times X + 60 \times 45.58779473 \times (1.0094888)^{-60} \times X$

$$\Rightarrow X = \frac{15{,}000}{2{,}814.053} = \$5.33$$ [1]

(iii)
Loan outstanding at the end of December 2026

$$15{,}000(1.12)^5 - 5.33 \times (Ia)_{\overline{60}|@\,j}(1.12)^5$$ [2]

(iv)

$$\$26{,}435.13 - \$11{,}855.09 = \$14{,}580$$ [1]

(v)
As interest exceeds repayments at early durations the loan outstanding will increase [1]
Repayments will only start to reduce loan outstanding once the repayments increase beyond a certain amount [½]
Therefore by halfway through the term the loan outstanding has barely reduced - thus very little of the initial loan has been paid off. [½]

(vi)
Interest repaid during 2027 = Total repayments less capital repaid

$$12 \times 60X - (\text{loan o/s Dec 2026 } - \text{loan o/s Dec 2027})$$ [1]

$$= 12 \times 60X - \left(\left[15{,}000(1.12)^5 - 5.33 \times (Ia)_{\overline{60}|@\,j}(1.12)^5\right] - 60Xa_{\overline{48}|@\,j}\right)$$

$$= 12 \times 60X - \left(\$14{,}580 - 60Xa_{\overline{48}|@\,j}\right)$$ [1]

<u>Alternative</u>

$$= 12 \times 60X - \left(\left[15{,}000(1.12)^5 - 5.33 \times (Ia)_{\overline{60}|@\,j}(1.12)^5\right] - 12 \times 60Xa^{(12)}_{\overline{4}|@\,i}\right)$$

$$= 12 \times 60X - \left(\$14{,}580 - 12 \times 60Xa^{(12)}_{\overline{4}|@\,i}\right)$$ [2]

(vii)

$$60Xa_{\overline{48}|@\,j} = 60 \times 5.33 \times 38.411832159 = \$12{,}284$$ [1]

Total repayments during 2027 $= 12 \times 60X = \$3{,}837.88$ [½]
So interest repaid during 2027 = Total repayments less capital repaid

$$= \$3{,}837.88 - (\$14{,}580 - \$12{,}284) = \$1{,}542$$ [½]

<u>Alternative</u>

$$12 \times 60Xa^{(12)}_{\overline{4}|@\,i} = 720 \times 5.33 \times 3.0373 \times \frac{0.12}{0.11387} = \$12{,}285$$

So interest repaid during 2027 = Total repayments less capital repaid

$$= \$3{,}837.88 - (\$14{,}580 - \$12{,}285) = \$1{,}543$$

(viii)
Under the revised repayment schedule, more of the loan will be repaid earlier and so the loan outstanding at any one time will be less than under the original schedule [1]
and so less interest will be paid in total. [1]

[Total 18]

The early parts of this question were generally well answered.

In parts (i) and (ii) common errors included: -
Using $(Ia)^{(12)}_{\overline{5}|}$ to value the first 60 payments. This is incorrect as it allows for annual increases and not the monthly increases required. (This compound interest function is not currently covered by the CM1 syllabus.)
Where $a^{(12)}_{\overline{5}|}$ was used in the alternative solution many candidates missed that the payment needed to be multiplied by 12 to reflect the total annual payment was now 12 × 60X.
The commentary given by candidates for part (v) was often unclear.

For parts (vi) and (vii) many candidates attempted to calculate the interest by calculating (loan outstanding × interest rate) as you would do for a single payment.

The question asked for equations in parts (i), (iii) and (vi) to be used to calculate parts (ii), (iv) and (vii) respectively. Where this was not done, limited credit was given for (ii), (iv) and (vii).

Q9

(i)

$$\$450,000 = X\left[1 + (1.03)v^1_{9\%} + (1.03)^2 v^2_{9\%} + \ldots + (1.03)^{19} v^{19}_{9\%}\right] + \$450,000 v^{20}_{9\%}$$ [1]

With $\dfrac{1.03}{1.09} = \dfrac{1}{1+j} \Rightarrow j = 0.058252427$ [½]

$$\$450,000 = X\ddot{a}_{\overline{20}|j\%} + \$450,000 v^{20}_{9\%}$$ [½]

$\$450,000 = X \times 12.31216704 + \$80,293.9004 \Rightarrow X = \$30,027.70$ per annum [1]

(ii)

$i = 9\% \Rightarrow d^{(12)} = 8.58689942\%$

$\$44,600 \times \ddot{a}^{(12)}_{\overline{10}|9\%} + \$44,600 \times 1.5 \times \ddot{a}^{(12)}_{\overline{10}|9\%} \times v^{10}_{9\%}$ [1½]

$= \$44,600 \times 6.72639989 + \$44,600 \times 1.5 \times 6.72639989 \times 0.42241081$

$= \$299,997.4351 + \$190,083.2379 = \$490,080.67 > \$400,000$ *Purchase price* [1]

$\Rightarrow$ IRR > 9% per annum [½]

(iii)

Profit $=\left[\$44,600\times\ddot{a}^{(12)}_{\overline{10|}9.5\%}+\$44,600\times1.5\times\ddot{a}^{(12)}_{\overline{10|}9.5\%}\times(1.095)^{-10}-400,000\right]\times(1.095)^{20}$ [2½]

$\ddot{a}^{(12)}_{\overline{10|}9.5\%}=6.5974$

$=\left[472,342.60-400,000\right]\times(1.095)^{20}=72,342.60\times(1.095)^{20}=444,300.21$ [1½]

[Total 10]

Parts (i) and (ii) were generally well answered.

In part (iii) candidates who used a discounted payback period approach were given credit, but most candidates over-simplified the calculation and so did not score full marks.

Q10

(i)

$\$50,000=P\ddot{a}_{45:\overline{15|}}=P\left(11.386\right)\Rightarrow P=\$4,391.36$ [2]

$\$4,391.36\ddot{a}_{45:\overline{15|}}=S\bar{A}_{45:\overline{15|}}$ OR $\$50,000=S\bar{A}_{45:\overline{15|}}$ [1½]

$50,000=S\left[(1.04)^{0.5}\times A^{1}_{45:\overline{15|}}+A_{45:\overline{15|}}^{\ \ \ \ 1}\right]$ [1]

where

$(1.04)^{0.5}A^{1}_{45:\overline{15|}}+A_{45:\overline{15|}}^{\ \ \ \ 1}=(1.04)^{0.5}\times0.035920087+0.526139912=0.562771357$ [2]

$50,000=S\left[0.562771357\right]$

$S=\$88,846.03$ [½]

(ii)

Endowment:

${}_{10}V=\$90,000\bar{A}_{55:\overline{5|}}-\$4,450\ddot{a}_{55:\overline{5|}}=90,000\times0.824144965-4,450\times4.585$
$=\$53,769.80$ [1]

Where $(1.04)^{0.5}A^{1}_{55:\overline{5|}}+A_{55:\overline{5|}}^{\ \ \ \ 1}=(1.04)^{0.5}\times0.024993342+0.798656658=0.824144965$

[1]

DSAR = $\$90,000(1.04)^{0.5}-\$53,769.80=\$38,012.55$ [2]

E(deaths)= $q_{54}\times\left(550+6\right)=0.003976\times556=2.210656$ [1]

EDS $=2.210656\times\$38,012.55=\$84,032.67$ [½]

ADS = $6\times\$38,012.55=\$228,075.30$ [½]

EDS-ADS = $-\$144,042.63$ [½]

Annuity:

${}_{10}V=\$4,450\times\ddot{a}_{55:\overline{5|}}=\$20,403.25$ [1]

DSAR = $-\$20,403.25$ [½]

E(deaths)= $q_{54} \times (550+6) = 0.003976 \times 556 = 2.210656$

EDS $= 2.210656 \times -\$20,403.25 = -\$45,104.57$ [½]

ADS = $6 \times -\$20,403.25 = -\$122,419.5$ [½]

EDS-ADS = $\$77,314.93$ [½]

Total mortality profit: $-\$144,042.64 + \$77,314.93 = -\$66,727.70$ [½]

(iii)

The insurance company expected approximately 2.21 deaths, whereas 6 deaths actually occurred. So actual mortality was heavier than expected. [1]

With endowment assurances, earlier-than-expected deaths lead to an earlier payment of the benefit - the benefit is paid as a death benefit rather than as a maturity benefit. This implies earlier than expected deaths lead to a mortality loss. Here, as actual mortality was heavier than expected, there is a mortality loss on the endowments. [1]

With an annuity, early deaths imply no future benefits are paid. Thus earlier-than-expected deaths lead to a mortality profit. Here, as actual mortality was heavier than expected, there is a mortality profit on the annuities. [1]

The mortality loss on the endowments > the mortality profit on the annuities, thus overall there is a total mortality loss. [1]

[Marks available 4, maximum 3]

Part (i) was generally well answered. A common error was applying the claim acceleration adjustment to both the death benefit and the survival benefit when calculating $\bar{A}_{45:\overline{15|}}$. (This also applied in part (ii)).

In part (ii) many candidates only calculated the mortality profit arising from the endowment assurance and ignored the mortality profit of the annuity policy. Other common errors included: -
Making no adjustment to the sum assured to allow for the immediate payment on death when calculating the DSAR.
Using the number of policies in force at end of the year (550) rather than at the start of the year (550+6=556);
Using the mortality rate for the age at the end of year instead of the age at the start of year when calculating the expected number of deaths.

In part (iii) many candidates lost marks as they only commented on the results for the endowment assurance and not the annuity policy.

[Paper Total 100]

END OF EXAMINERS' REPORT