INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

11 April 2022 (am)

Subject CM1 – Actuarial Mathematics Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team at T. 0044 (0) 1865 268 873.

1 State four data items relating to the holding of equity shares in a single company that could be used to model the future value of the equity shareholding. [2]

2 An investor is considering making an investment and is deciding between two possible alternatives:

- A 6-month investment which can be purchased at a simple rate of discount of 4.15% p.a.
- A bank deposit, for 6 months, offering an effective rate of interest of 4.35% p.a.

Determine which of these two alternatives offers the higher rate of return. [4]

3 An insurance company has liabilities of £4 million due in 7 years' time and £13 million due in 11 years' time. The company has assets consisting of two zero-coupon bonds, one paying £6.9617 million in 4 years' time and the other paying £11.4007 million in 18 years' time. The current interest rate is 6% p.a. effective.

Demonstrate that Redington's first two conditions for immunisation against small changes in the rate of interest are satisfied for this insurance company. [6]

4 (a) Write down the formula for the variance of the present value of an n-year temporary annuity of £1 p.a. payable at the end of each year, issued to a life aged x exact.

(b) Calculate, showing all working, the variance of the present value of a 20-year temporary annuity of £5,000 p.a., payable at the end of each year, issued to a life aged 44 exact.

Basis:

Mortality	AM92 Ultimate
Interest	4% p.a. effective

[7]

5 At a particular insurance company, actuarial students study for a maximum of 3 years.

Students are subject to the following decrements:

- Mortality
- Not progressing with their studies but staying with the company (withdrawal)
- Leaving the company to join another employer (transfer).

The forces of mortality, withdrawal and transfer are assumed to be independent and to be constant over individual years of study.

In addition, at the end of each year of study, a proportion of students will complete their studies and will be deemed to have qualified.

The following forces of decrement will apply for each year of study:

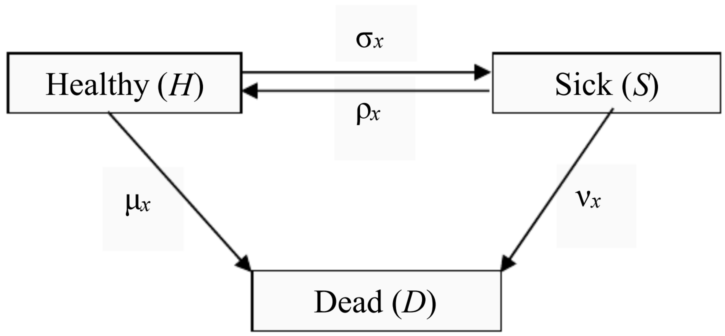
Year of study	*Force of mortality*	*Force of withdrawal*	*Force of transfer*
1	0	0.2	0.25
2	0.025	0.1	0.35
3	0.030	0	0.45

The proportion of students who qualify each year is as follows:

End of year of study	*Proportion of students who qualify (%)*
1	3
2	12
3	100

Calculate, showing all working, the probability that a student who starts the programme will qualify while still being at the company. [7]

6 An insurance company issues a 30-year combined death and sickness policy to a healthy life aged 25 exact. The company uses the following multiple state model in respect of the policy.



The expected present value of the benefits provided by the policy are set out in expressions (a), (b) and (c) below:

(a) $$50{,}000\int_0^{30} e^{-\delta t}\left({}_tp_{25}^{HH}\times\mu_{25+t}+{}_tp_{25}^{HS}\times\nu_{25+t}\right)dt$$

(b) $$20{,}000\int_0^{29.5} e^{-\delta t}\times{}_tp_{25}^{HH}\times\sigma_{25+t}\int_0^{29.5-t}{}_{0.5+s}p_{25+t}^{\overline{SS}}\times\nu_{25.5+t+s}\,ds\,dt$$

(c) $$5{,}000\int_0^{30} e^{-\delta t}\,{}_tp_{25}^{HS}\,dt$$

Describe, in words, the **benefits** provided by the policy. You may assume the time periods are measured in years. [6]

7 A company issues a fixed-interest security paying coupons at a rate of 6% p.a. payable half-yearly in arrears.

The security is to be redeemed at 103% on any coupon payment date from 10 to 15 years after issue, with the exact date of redemption at the discretion of the company.

An investor, liable to income tax at 20% and capital gains tax at 25%, purchases the security on the date of issue at a price which gives a minimum net yield to redemption of 5% p.a. effective.

Calculate, showing all working, the price per £100 nominal paid by the investor. [6]

8 (i) Show, using expressions in the form of integrals, that $\bar{A}^{\,2}_{xy:\overline{n}|}=\bar{A}^{\,1}_{x:\overline{n}|}-\bar{A}^{\,1}_{xy:\overline{n}|}$. [2]

(ii) Calculate 100,000 $\bar{A}^{\,2}_{50:50:\overline{1}|}$.

Basis:
Mortality AM92 Ultimate
Interest 7% p.a. effective

Assume that the lives are independent with respect to mortality. [5]
[Total 7]

9 The annual effective forward rate applicable over the period t to $t + r$ is defined as $f_{t,r}$ where t and r are measured in years.

You are given the following information:

$$f_{0,1} = 4.2\%, \ f_{1,1} = 4.8\%, \ f_{2,1} = 5.3\%, \ f_{2,2} = 5.9\% \ .$$

(i) Calculate $f_{3,1}$, showing all working and giving your answer as a percentage to four decimal places. [2]

(ii) Calculate, showing all working and giving your answers as a percentage to four decimal places, all possible spot rates of interest that the above information allows you to calculate. [4]

(iii) Calculate, using linear interpolation and showing your working, the annual effective gross redemption yield of a 4-year bond, redeemable at 105%, with a 2.5% coupon payable annually in arrears. [6]

[Total 12]

10 A life insurance company issues a 3-year without-profit endowment assurance. The same sum assured is payable on survival to the end of the policy term or at the end of year of death if earlier. Premiums are payable annually in advance throughout the term of the policy or until earlier death.

Surrenders are only allowed at the end of the policy year. The surrender value is calculated as the sum of the premiums paid to date and is payable at the time of surrender.

A colleague has started to complete the profit test for this policy. Unfortunately, they have mislaid the profit testing basis. The only details they can remember are:

- the annual effective interest rate earned on cashflows is constant over the 3-year term.
- renewal expenses payable at the beginning of the second and third policy years are the same.
- renewal commission is a fixed percentage of premium and is payable at the beginning of the second and third policy years.
- the risk discount rate is constant over the 3-year term.

The incomplete multiple decrement table is given by:

Age	q_x^{death}	q_x^{surr}	$(aq)_x^{death}$	$(aq)_x^{surr}$	$(ap)_x$	$_{t-1}(ap)_{62}$
62	0.003550	0.1	0.003550	0.099645	0.896805	1.000
63	0.004251	0.05	0.004251	(a)	(b)	(c)
64	0.005073	0	0.005073	0	0.994927	(d)

(i) Calculate, showing all working, the values of the missing entries (a), (b), (c) and (d) in the table. [2]

The incomplete calculation of the present value of profit is given by:

Policy year	*Premium*	*Initial expense*	*Renewal expense*	*Initial commission*	*Renewal commission*	*Interest*
1	5,000	200	0	1,250	0	(e)
2	5,000		20		125	242.75
3	5,000		(f)		(g)	(h)

Policy year	*Expected cost of death benefit*	*Expected cost of surrender payment*	*Expected cost of maturity payment*	*Profit vector*	*Profit signature*	*Discount factor*	*Present value of profit*
1	(i)	498.23		(l)	(n)	(p)	(s)
2	85.02	(j)		4,514.86	4,048.95	(q)	3,407.92
3	101.46	0	(k)	(m)	(o)	(r)	(t)

(ii) Calculate, showing all working, the values for the missing entries (e) to (t). [10]

[Total 12]

11 A life aged 30 exact purchases a 35-year term assurance policy. Level monthly premiums are payable in advance throughout the duration of the contract, ceasing on death, and the sum assured of $250,000 is payable immediately on death.

(i) Calculate, showing all working, the monthly premium.

Basis:

Mortality	AM92 select
Interest	4% p.a. effective
Expenses	Initial: $250 plus 60% of the first monthly premium
	Renewal: 3% of the second and subsequent monthly premiums

[7]

The insurance company actually charges a premium of $50 per month.

The company calculates gross premium **retrospective** reserves, assuming the same basis as in part (i) above but using a rate of interest of 6% p.a. effective.

(ii) Calculate, showing all working, the reserve held for the policyholder at age 55 exact, immediately before the premium then due. [6]

The insurance company is proposing to calculate reserves using the same basis as in part (ii) but adopting a gross premium **prospective** reserving approach.

(iii) Explain, without performing any further calculations, whether the proposal will result in a higher or lower reserve than that calculated in part (ii). [4]

[Total 17]

12 A life insurance company offers two alternative 10-year endowment assurance products, each with an initial sum assured of £50,000, to lives aged 55 exact. For both products, level premiums are payable annually in advance throughout the term of the policy or until earlier death.

The benefits are payable at the end of the year of death, or on survival to the end of the term.

Product A

A with-profit endowment assurance with an expected simple reversionary bonus of 2.1% p.a. added at the end of each year. Each increase is applied at the end of the year if the policyholder is still alive.

Product B

An endowment assurance where the initial benefit increases at a guaranteed compound rate of k% p.a. Each increase is applied at the end of the year if the policyholder is still alive.

The company sets k so that the maturity benefit will be the same under both products.

(i) Show that k is approximately equal to 1.924%. [2]

(ii) Calculate the annual premium for **each** product. Show all working. [12]

Basis:
Mortality AM92 Ultimate
Interest 6% p.a. effective

[Total 14]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2022

CM1 - Actuarial Mathematics
Core Principles
Paper A

Introduction
The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the Specialist Advanced (SA) and Specialist Principles (SP) subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
July 2022

A. General comments on the *aims of this subject and how it is marked*

CM1 provides a grounding in the principles of modelling as applied to actuarial work - focusing particularly on deterministic models which can be used to model and value known cashflows as well as those which are dependent on death, survival, or other uncertain risks.

Please note that different answers may be obtained to those shown in these solutions depending on whether figures obtained from tables or from calculators are used in the calculations, but candidates are not penalised for this. However, candidates may not be awarded full marks where excessive rounding has been used or where insufficient working is shown.

Although the solutions show full actuarial notation, candidates were generally expected to use standard keystrokes in their solutions.

Candidates should pay attention to any instructions included in questions. failure to do so can lead to fewer marks being awarded. In particular, where the instruction, "*showing all working*" is included and the candidate shows little or no working, then the candidate will be awarded very few marks even if the final answer is correct.

Where a question specifies a method to use (e.g. *determine the present value of income using annuity functions*) then where a candidate uses a different method the candidate will not be awarded full marks.

Candidates are advised to familiarise themselves with the meaning of the command verbs (e.g. state, determine, calculate). These identify what needs to be included in answers in order to be awarded full marks.

B. Comments on *candidate performance in this diet of the examination.*

The comments that follow the questions concentrate on areas where candidates could have improved their performance. Where no comment is made, the question was generally answered well. The examiners look most closely at the performance of the candidates close to the pass mark and the comments therefore often relate to those scripts.

Many candidates appeared to be inadequately prepared, in terms of not having sufficiently covered the entire breadth of the subject. We would advise candidates not to underestimate the quantity of study required for this subject.

Candidates should be aware that the questions cannot be answered using knowledge alone and well prepared candidates will demonstrate application of their knowledge to the questions asked.

Where candidates made numerical errors, the examiners awarded marks for the correct method used and for the parts of the calculation that were correct. However, many candidates often did not show enough of their working to fully benefit from this.

The Examiners felt that the "open book" nature of the online exam led some candidates to rely on their notes much more than if the exam had been "closed book". The Examiners strongly recommend that candidates prepare for online exams just as thoroughly as they would do if the exam were of the traditional "closed book" format. Candidates are recommended to use their notes only as a tool to check or confirm answers where necessary, rather than as a source for looking up the answers.

C. Pass Mark

The Pass Mark for this exam was 58.
1644 presented themselves and 520 passed.

Solutions for Subject CM1 Paper A- April 2022

Q1

Number of shares held [½]
Historical dividend amounts, current dividend (or current price of share and dividend yield) [½]
Historical Dividend dates [½]
Ex-dividend date of equity share [½]
Retained earnings [½]
Growth prospect of the company, sector or economy in general [½]
Other valid answers accepted [½]
[Marks available 3½, maximum 2]

This question was generally well answered. Full marks were not awarded where candidates included data items that did not relate to the holding of the equity share in a single company, and/or which were not relevant to modelling the future value of the equity shareholding.

Q2

Price (per £100 nominal) of the 6-month investment is given by:

$$P = 100 \times \left(1 - \frac{6}{12} \times 0.0415\right) = 97.925$$

[1½]

Thus, equivalent effective rate of interest per annum, i, is given by:

$97.925 \times (1+i)^{\frac{6}{12}} = 100 \Rightarrow (1+i)^{\frac{6}{12}} = 1.0212 \Rightarrow i = 0.04283$ or 4.283% per annum [1½]

Since 4.35% > 4.283% the Bank Deposit offers a better return. [1]
[Total 4]

A common error made by candidates, was to treat the simple discount rate as a compound discount factor.

Q3

The present value of the liabilities is

$$V_L = 4v^7 + 13v^{11} = 9.508466$$ [1]

and the present value of the assets is

$$V_A = 6.9617 * v^4 + 11.4007 * v^{18} = 9.508483$$ [1]

These are the same (to 4 decimal places) and so Redington's first condition is satisfied, present values equal [½]

The second condition is that the volatility of the assets and liabilities should be equal:

$$-V_L' = 28v^8 + 143v^{12} = 88.634165$$ [1]

$$\frac{-V_L'}{V_L} = \frac{88.6341}{9.5084} = 9.3216$$ [½]

$$-V_A' = 4 * 6.9617 * v^5 + 18 * 11.4007 * v^{19} = 88.634183$$ [1]

$$\frac{-V_A'}{V_A} = \frac{88.6341}{9.5084} = 9.3216$$ [½]

and since they are the same (to 4 decimal places) the second condition of volatility equal is also satisfied. [½]

[Total 6]

Acceptable alternatives for the second condition (having shown the first condition holds) was showing the equality of the Discounted Mean Terms of the assets and liabilities, the numerator of the DMT or the numerator of the Volatility, if accompanied by the reasoning that as the denominators are the same, only the numerators needed to be considered.

Some candidates did not calculate the required values for assets and liabilities but assumed the values were the same. Some candidates did not present their answers to sufficient decimal places, so full marks were not awarded when calculations were not shown. These answers were penalized as the candidate had not sufficiently demonstrated that the values were equal. The easiest way to show full workings is to include the values of the discount factors as shown in the solution above.

Q4

(a)

$$\text{Variance} = \frac{1}{d^2}\left[{}^2A_{x:\overline{n+1}|} - \left(A_{x:\overline{n+1}|}\right)^2\right] \text{ where } {}^2A_{x:\overline{n+1}|} \textit{ is } A_{x:\overline{n+1}|} \text{ calculated at } i' = (1+i)^2 - 1$$

[2½]

(b)

$$\text{Variance} = 5{,}000^2\,\frac{1}{d^2}\left[{}^2A_{44:\overline{21}|} - \left(A_{44:\overline{21}|}\right)^2\right]$$

$$= \frac{5{,}000^2}{d^2}\left[{}^2A_{44} - v^{2\times 21}\frac{l_{65}}{l_{44}}\,{}^2A_{65} + v^{2\times 21}\frac{l_{65}}{l_{44}} - \left(A_{44:\overline{21}|}\right)^2\right]$$

$$= \frac{5{,}000^2}{0.00147929}\left[\begin{array}{l} 0.08856 - 0.192575 \times \left(\dfrac{8{,}821.2612}{9{,}814.3359}\right) \times 0.30855 + 0.192575 \times \left(\dfrac{8{,}821.2612}{9{,}814.3359}\right) \\ \\ -\left(0.45258\right)^2 \end{array}\right]$$

$$= \frac{5{,}000^2}{0.00147929}\left[0.03515 + 0.17309 - 0.20483\right]$$

$= £^2\, 57.69m$ [4½]

[Total 7]

Common errors included: -
In (a) failing to write down the new interest rate, referring to 'n' rather than 'n+1'.
In (b) omitting the 5,000^2, being unable to calculate the $^2A_{44:\overline{21|}}$ factor.

Q5

Let μ_x^k be the independent force of decrement k in year x where $k = d, w, t$ for deaths, withdrawals, and transfers respectively.

The probability that a student is active at the end of year 1, before any student qualifies:

$= \exp\left[-\left(\mu_1^d + \mu_1^w + \mu_1^t\right)\right] = e^{-0.45} = 0.63763$ [1]

The probability of a student at the company qualifying at the end of year 1:

$= 0.63763 \times 0.03 = 0.0191289$ [1]

The probability that a student is active at the end of year 2, given they were active at the start of year 2, and before any student qualifies:

$= \exp\left[-\left(\mu_2^d + \mu_2^w + \mu_2^t\right)\right] = e^{-0.475} = 0.62189$ [½]

The probability that a student is still at the company and not yet qualified at the end of year 2:

$0.63763 \times 0.62189 \times 0.97 = 0.384635477$ [1]

The probability of a student at the company qualifying at the end of year 2 is:

$0.384635477 \times 0.12 = 0.04616$ [½]

The probability that a student is active at the end of year 3, given they were active at the start of year 3, and before any student qualifies:

$= \exp\left[-\left(\mu_3^d + \mu_3^w + \mu_3^t\right)\right] = e^{-0.48} = 0.61878$ [½]

The probability that a student is still at the company and not yet qualified at the end of year 3:

$0.384635477 \times 0.88 \times 0.61878 = \ 0.20945$ [1]

The probability of a student at the company qualifying at the end of year 3 is:

$0.20945 \times 1 = 0.20945$ [½]

Thus, the total probability of qualifying whilst still with the company:

$= 0.01913 + 0.04616 + 0.20945 = 0.27474$ [1]

[Total 7]

A common error made by candidates when calculating the probability that a student is still at the company and not yet qualified at the end of years 2 and 3 was ignoring the students needing to survive the previous years' death, withdrawal, transfer and qualifying decrements.

Q6

(a) A lump sum of £50,000 is payable immediately on death whether from the healthy state or from the sick state. The period of cover is to age 55 (for 30 years).

(b) A lump sum payable immediately on entering the sick state (for the first and any subsequent bouts of sickness). Where the amount payable is £20,000 multiplied by the probability that the policyholder will be continuously sick for at least six months and then die from the sick state before age 55. The cover is to age 54.5 (for 29.5 years).

(c) £5,000 per annum payable continuously whilst a policyholder is sick (for the first and any subsequent bouts of sickness). The period of cover is to age 55 (for 30 years).

[Total 6]

Although the benefit described in part (b) could exist and could be described fully, it was not the benefit the examiners had intended to present and neither was it one which candidates were used to seeing. Adding an extra term $e^{-\delta(s+0.5)}$ to the second integral the solution becomes: -

A lump sum of £20,000 is payable immediately on death from the sick state having been continuously sick for at least six months at the time of death. The cover is from age 25.5 to 55 (for 29.5 years).

The question as it appears in the paper is harder than the examiners had intended. We recognised this and to mitigate the increased difficulty, marks were awarded either for the correct solution as it appeared in the paper or for the solution to the intended question.

Q7

We have $i^{(2)} = 0.049390$ and $\frac{D}{R}\times(1-t_1) = \frac{6}{103}\times(1-0.20) = 0.046602$

$0.049390 > 0.046602$

Since $i^{(2)} > \frac{D}{R}(1-t_1)$ there is a capital gain on redemption and the worst-case scenario for the investor is if the stock is redeemed as late as possible (i.e., at time 15).

The price (per £100 nominal) is:

$$
\begin{aligned}
& P && = 6\times(1-0.20)\times a_{\overline{15|}}^{(2)} + 103v^{15} - 0.25\times(103-P)v^{15} \\
\Rightarrow\ & (1-0.25\times 0.48102)\times P && = 4.8\times 1.012348\times 10.3797 + 0.75\times 103\times 0.48102 \\
\Rightarrow\ & P && = £99.57
\end{aligned}
$$

[Total 6]

This question was generally well answered.

Q8

(i)

$$\bar{A}^{2}_{xy:\overline{n}|} = \int_0^n e^{-\delta t}\,{}_tp_x\left(1 - {}_tp_y\right)\mu_{x+t}\,dt$$ [1]

$$= \int_0^n e^{-\delta t}\,{}_tp_x\,\mu_{x+t}\,dt - \int_0^n e^{-\delta t}\,{}_tp_{xy}\,\mu_{x+t}\,dt = \bar{A}^{1}_{x:\overline{n}|} - \bar{A}^{1}_{xy:\overline{n}|}$$ [1]

(ii)
Using this result with x = y = 50 and n = 1

$$\bar{A}^{1}_{50:\overline{1}|} - \bar{A}^{1}_{50:50:\overline{1}|} = \bar{A}^{1}_{50:\overline{1}|} - \tfrac{1}{2}.\bar{A}_{\overbrace{50:50}^{1}:\overline{1}|}$$

[1]

$$\bar{A}^{1}_{50:\overline{1}|} = (1.07)^{\frac{1}{2}}.q_{50}v$$

$$= 0.002508 \times (1.07)^{-\frac{1}{2}} = 0.00242458$$ [1½]

$$\bar{A}_{\overbrace{50:50}^{1}:\overline{1}|} = (1.07)^{\frac{1}{2}}.v.\left(1 - p_{50}^2\right)$$ [1½]

$$= (1.07)^{-\frac{1}{2}}\left(1 - (1 - 0.002508)^2\right) = 0.00484307$$

$$100{,}000\,\bar{A}^{2}_{xy:\overline{n}|} = 100{,}000 \times (0.00242458 - \tfrac{1}{2} \times 0.00484307)$$

$$= 0.3040$$

[1]
[Total 7]

A common error made by candidates in part (i) was to just write out the benefits in integral form, but not show how the equality was derived.

In part (ii) candidates who correctly answered the question using an integral approach or otherwise were given credit.

Q9

(i) $f_{3,1}$ is such that:

$$1.053 \times \left(1 + f_{3,1}\right) = 1.059^2$$ [1]

$$\Rightarrow f_{3,1} = 6.5034\%\,pa$$ [1]

(ii) Let $i_y = y -$ year spot rate p.a.

$$i_1 = f_{0,1} = 4.2000\%$$

[1]

$$\left(1 + i_2\right)^2 = 1.042 \times 1.048$$

$$\Rightarrow i_2 = 4.4996\%$$

[1]

$$\left(1 + i_3\right)^3 = 1.042 \times 1.048 \times 1.053$$

$$\Rightarrow i_3 = 4.7657\%$$

[1]

$$(1+i_4)^4 = 1.042 \times 1.048 \times 1.053 \times 1.065034$$

$$\Rightarrow i_4 = 5.1974\%$$

[1]

(iii)

PV of payments of bond is:

$$P = 2.5\left(1.042^{-1} + 1.044996^{-2} + 1.047657^{-3} + 1.051974^{-4}\right) + 105 \times 1.051974^{-4}$$

$$\Rightarrow P = 94.6412$$

[3]

The equation of value to find the gross redemption yield is:

$94.6412 = 2.5\, a_{\overline{4}|} + 105\, v^4$ [1]

Try 5%, RHS = $2.5 \times 3.5460 + 105 \times 0.82270$

=95.2485 [½]

Try 6%, RHS $= 2.5 \times 3.4651 + 105 \times 0.79209$

= 91.8322 [½]

$\Rightarrow i = 0.05 + \dfrac{95.2485 - 94.6412}{95.2485 - 91.8322} \times 0.01$ [½]

= 0.05178 (i.e. 5.178% pa) (accurate answer 5.1746%) [½]

[Total 12]

Parts (i) and (ii) were generally well answered. Candidates who did not present their answer as a percentage to four decimal places did not receive full marks.

In part (iii) many candidates missed out the derivation of the price of the bond using the term structure of interest rates given in the question. Candidates also often incorrectly used coupon payments of 0.025 or 0.25 instead of 2.5.

Where the full workings for the linear interpolation were not shown, candidates did not receive full marks.

Q10

(i)

$$(a) = (1 - 0.004251) \times 0.05 = 0.049787$$

$$(b) = 1 - .004251 - 0.049787 = 0.945962$$

$$(c) = 0.896805 \times 1 = 0.896805$$

$$(d) = 0.896805 \times 0.945962 = 0.848343$$

[2]

(ii)

Interest earned on cashflows is given by

$$\left(\frac{242.75}{5{,}000-20-125}\right)=5\%$$ [1]

Interest in year 1,
$(e)=0.05\times\left(5{,}000-200-1{,}250\right)=177.50$ [½]

Renewal expense in year 3 is the same as in year 2, therefore (f) = 20.

Renewal commission in year 3 is the same as in year 2, therefore (g) = 125.

Interest in year 3 is the same as in year 2, therefore (h) = 242.75. [1½]

Sum Assured is given by

$$\frac{85.02}{0.004251}=20{,}000$$ [½]

Expected cost of death payment in year 1
$(i)=20{,}000\times 0.00355=71$ [½]

Expected cost of surrender payment in year 2
$(j)=\left(2\times 5{,}000\right)\times 0.049787=497.87$ [½]

Expected cost of maturity payment in year 3
$(k)=20{,}000\times 0.994927=19{,}898.54$ [½]

Alternative method $20{,}000-101.46=19{,}898.54$

Profit vector for year 1
$(l)=5{,}000-200-1{,}250+177.5-71-498.23=3{,}158.27$ [½]

Profit vector for year 3
$(m)=5{,}000-20-125+242.75-101.46-19{,}898.54=-14{,}902.25$ [½]

Profit Signature for year 1
$(n)=3{,}158.27\times 1=3{,}158.27$ [½]

Profit Signature for year 3
$(o)=-14{,}902.25\times 0.848343=-12{,}642.22$ [½]

Risk discount rate is given by

$$\sqrt{\left(\frac{4{,}048.95}{3{,}407.92}\right)}-1=9\%$$ [½]

Discount factor for year 1
$(p)=\left(1.09\right)^{-1}=0.917431$

Discount factor for year 2
$(q) = (1.09)^{-2} = 0.841680$

Discount factor for year 3
$(r) = (1.09)^{-3} = 0.772183$ [1½]

PV profit for year 1
$(s) = 3{,}158.27 \times 0.917431 = 2{,}897.49$

PV profit for year 3
$(t) = -12{,}642.23 \times 0.772183 = -9{,}762.12$

[1]

[Total 12]

This was a new type of profit testing question which required some logical thinking. Candidates did not appear to be intimidated by this new presentation and generally performed well.

A common error was not showing all workings.

Q11

Let P denote the monthly premium.

Then, equation of value is:

$$12P \times \ddot{a}^{(12)}_{[30]:\overline{35}|} = 250{,}000 \times \bar{A}^{\;1}_{[30]:\overline{35}|} + 0.57P + 0.03 \times 12P \times \ddot{a}^{(12)}_{[30]:\overline{35}|} + 250$$ [3½]

From tables, we have:

$$\ddot{a}^{(12)}_{[30]:\overline{35}|} = \ddot{a}_{[30]:\overline{35}|} - \frac{11}{24} \times \left(1 - v^{35}\frac{l_{65}}{l_{[30]}}\right) = 19.072 - \frac{11}{24} \times \left(1 - 1.04^{-35}\frac{8821.2612}{9923.7497}\right) = 18.7169$$

$$\bar{A}^{\;1}_{[30]:\overline{35}|} = 1.04^{0.5} \times \left(A_{[30]} - v^{35}\frac{l_{65}}{l_{[30]}}A_{65}\right) = 1.04^{0.5} \times \left(0.16011 - 1.04^{-35}\frac{8821.2612}{9923.7497}0.52786\right) = 0.04202$$

[3]

Thus, we have:

$$\begin{aligned} 12P \times 18.7169 &= 250{,}000 \times 0.04202 + 0.57P + 0.36P \times 18.7169 + 250 \\ \Rightarrow 217.2947 \times P &= 10{,}755 \\ \Rightarrow P &= \$49.49 \end{aligned}$$

[½]

(ii)
Retrospective gross premium reserve at time 25 is given by:

$$_{25}V = v^{-25}\frac{l_{[30]}}{l_{55}}\times\left(0.97\times 12\times 50\times \ddot{a}^{(12)}_{[30]:\overline{25}|} - 0.57\times 50 - 250{,}000\times \bar{A}^{\ 1}_{[30]:\overline{25}|} - 250\right) \quad \text{at 6\% p.a.} \qquad [2½]$$

Thus, at 6% p.a. interest, we have:

$$v^{25}_{6\%}\times\frac{l_{55}}{l_{[30]}} = 0.23300\times\frac{9{,}557.8179}{9{,}923.7497} = 0.224408$$

$$\ddot{a}^{(12)}_{[30]:\overline{25}|} = \ddot{a}^{(12)}_{[30]} - v^{25}_{6\%}\times\frac{l_{55}}{l_{[30]}}\times\ddot{a}^{(12)}_{55} = \left(16.374-\frac{11}{24}\right)-0.224408\times\left(13.057-\frac{11}{24}\right) = 13.0884$$

$$\bar{A}^{\ 1}_{[30]:\overline{25}|} = 1.06^{0.5}\times\left(A_{[30]} - v^{25}_{6\%}\times\frac{l_{55}}{l_{[30]}}\times A_{55}\right) = 1.06^{0.5}\times\left(0.07316-0.224408\times 0.26092\right) = 0.015039$$

[3]

Thus, the reserve at time 25 is given by:

$$_{25}V = \frac{1}{0.224408}\times\left(0.97\times 12\times 50\times 13.0884 - 0.57\times 50 - 250{,}000\times 0.015039 - 250\right) = \$15{,}950$$

[½]

(iii)
Given that the premium of \$50 is near to the \$49.49 calculated in part (i) we can reasonably assume that the basis too may be near to that given in part (i).
Using the premium basis with a rate of interest of 4% p.a., we have
$GPV^{pro}_{4\%} = GPV^{ret}_{4\%}$. [1]
Then, using a rate of interest of 6% p.a. rather than 4% p.a., the retrospective provision at time 25 will increase (as PV of past income will be accumulated at a higher rate of interest).
Thus, we have $GPV^{ret}_{6\%} > GPV^{ret}_{4\%}$. [1]
However, using a rate of interest of 6% p.a. rather than 4% p.a., the prospective provision at time 25 will decrease (as PV of future outgo will be discounted at a higher rate of interest).
Thus, we have $GPV^{pro}_{6\%} < GPV^{pro}_{4\%}$. [1]
Hence, we have $GPV^{ret}_{6\%} > GPV^{ret}_{4\%} = GPV^{pro}_{4\%} > GPV^{pro}_{6\%}$. [½]
So the answer to (ii) would be lower if we calculated gross premium prospective reserves using the same basis. [½]
[Total 17]

Part (i) Common errors included: -

Using an incorrect method for adjusting an annual annuity to monthly;
Allowing for the renewal expense incorrectly;
Not allowing for the death benefit to be payable immediately on death;
Using ultimate mortality instead of select mortality as specified in the basis.

In part (ii) candidates seem to be unfamiliar with the concept of retrospective reserves. Many candidates did not attempt this question part or calculated a prospective reserve, which gained very little credit.

A common error was not including the initial expenses when calculating the retrospective reserve.

In Part (iii) candidates performed very well. General comments about reserves gained very little credit.

Q12

(i)

$50,000 + 50,000 \times 0.021 \times 10 = £60,500$ [1]

$50,000 \times (1+k)^{10} = 60,500$

$(1+k)^{10} = 1.21 \Rightarrow k = 1.9244876\%$ [1]

(ii)

Policy A

$P\ddot{a}_{55:\overline{10}|} = 48,950 A^{1}_{55:\overline{10}|} + 1,050 (IA)^{1}_{55:\overline{10}|} + 60,500 \frac{l_{65}}{l_{55}} v^{10}_{6\%}$ [3]

$\frac{l_{65}}{l_{55}} v^{10}_{6\%} = \frac{8,821.2612}{9,557.8179} \times 0.558395 = 0.51536305$ [½]

$(IA)^{1}_{55:\overline{10}|} = (IA)_{55} - \frac{l_{65} \times v^{10}}{l_{55}} \left[(IA)_{65} + 10 A_{65} \right]$ [1]

$= 5.22868 - 0.51536305 \times [5.50985 + 10 \times 0.40177] = 0.318532773$ [1]

$A^{1}_{55:\overline{10}|} = A_{55} - \frac{l_{65} \times v^{10}}{l_{55}} A_{65} = 0.26092 - 0.51536305 \times 0.40177 = 0.053862587$ [1]

$\ddot{a}_{55:\overline{10}|} = 7.610$ [½]

$48,950 \times 0.053862587 + 1,050 \times 0.318532773 + 60,500 \times 0.51536305$

$= £34,150.50$

$P \times 7.61 = 34,150.50 \Rightarrow P = £4,487.58$ [½]

Policy B

$$P\ddot{a}_{55:\overline{10}|} = £50,000 \begin{pmatrix} q_{55} \times v^{1} + (1.01924) \times {}_{1|}q_{55} \times v^{2} + (1.01924)^{2} \times {}_{2|}q_{55} \times v^{3} \\ + \ldots + (1.01924)^{9} \times {}_{9|}q_{55} \times v^{10} \end{pmatrix}$$

$$+ £60,500 \frac{l_{65}}{l_{55}} v^{10}_{6\%}$$

The new interest rate is given by $\left(\frac{1.01924}{1.06} \right)^{-1} - 1 = 4\%$ [½]

$P\ddot{a}_{55:\overline{10}|i=6\%} = \frac{50,000}{1.01924} A^{1}_{55:\overline{10}|i=4\%} + 60,500 \frac{l_{65}}{l_{55}} v^{10}_{6\%}$ [2½]

$$A^{1}_{55:\overline{10}|i=4\%} = A_{55} - \frac{l_{65} \times v^{10}_{4\%}}{l_{55}} A_{65} = 0.3895 - 0.623502985 \times 0.52786 = 0.060377714$$

[1]

$$P \times 7.61 = \frac{50,000}{1.0192} \times 0.060377714 + 60,500 \times 0.51536305 = 34,141.47951$$

[½]

$$\Rightarrow P = £4,486.40$$

[Total 14]

This question focused on the difference between simple and compound increases in benefits. Part (i) was generally well answered. In Part (ii) common errors for product A included: -
Treating benefit increases as compound increases rather than simple;
Not splitting the benefit into component parts (term assurances and a pure endowment);
Incorrectly calculating the benefit amounts for each of component parts of the benefit (level term assurance, increasing term assurance and pure endowment);
Using an incorrect formula for evaluating the increasing term assurance benefit.

In Part (ii) a common error for product B was incorrectly calculating the benefit amounts for each of the component of the benefits (term assurance and pure endowment).

[Paper Total 100]

END OF EXAMINERS' REPORT