INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

22 September 2022 (am)

Subject CM2 – Financial Engineering and Loss Reserving Core Principles

Paper A

Time allowed: Three hours and twenty minutes

In addition to this paper you should have available the 2002 edition of the Formulae and Tables and your own electronic calculator.

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1 An individual has the following utility function:

$$U(w) = \frac{(w^{\gamma} - 1)}{\gamma}, \ (w > 0),$$

where w is wealth in \$000s. Their current wealth is \$8,000 and their current utility is 2.1012.

(i) Show that $\gamma = 0.01$ to two decimal places. [1]

(ii) Show that $U(w)$ exhibits declining absolute risk aversion and constant relative risk aversion. [3]

The individual has been offered a ticket to enter a lottery with a 1 in 10,000 chance to win \$1m.

(iii) Calculate, to the nearest \$, the maximum price, P, that the individual would pay for the ticket. [3]

(iv) Discuss why this form of utility function with $\gamma > 1$ would be inconsistent with common utility theory. [2]

[Total 9]

2 (i) Describe the differences between a structural credit risk model and a reduced form credit risk model. [4]

A firm issues a 15-year zero-coupon bond with a maturity value of \$100m. The current value of the firm's assets is \$150m and the estimated volatility of the firm's assets is 33%. The risk-free rate of interest is 1% p.a. continuously compounded.

(ii) Calculate the credit spread on the debt, using the Merton model. [6]

[Total 10]

3 The table below gives the following values for a market as at time 0:

Time t	*f(t – 1, t)* (%)	*P(0, t)* ($)	*R(0, t)* (%)	*B(t)* ($)
0	–	–	–	100.00
1	0.9	99.10	0.9	(a)
2	(b)	96.46	1.8	103.67
3	3.3	(c)	2.3	107.14
4	3.1	90.48	(d)	110.52

- $f(t, T)$ is the continuously compounded forward rate p.a. applying between time t and T.
- $P(t, T)$ is the price at time t for a zero-coupon bond maturing at time T, with a nominal value of $100.
- $R(t, T)$ is the continuously compounded spot rate of interest p.a. at time t for the period t to T.
- $B(t)$ is the value of a cash account at time t.

(i) Calculate the values of (a), (b), (c) and (d) in the table above. [4]

At time 0, an investor purchased $500 nominal of zero-coupon bonds that mature at time 3 and $1,000 nominal of zero-coupon bonds that mature at time 4. At time 2, interest rate expectations have changed as set out below.

Time t	*f(t – 1, t)* (%)
2	–
3	2.5
4	2.0

(ii) Calculate the profit or loss the investor would make if they sold all of their bonds at time 2. [4]

(iii) (a) Explain the meaning of an inverted yield curve.

(b) Explain why an inverted yield curve is unusual.

(c) Suggest possible reasons why a yield curve may be inverted.

[5]

[Total 13]

4 A financial derivative is held for a 2-year period. An analyst assumes that the change in the value of the derivative per year, i, is the same for each year. They assume that i follows a Normal distribution with parameters $\mu = -1$ and $\sigma = 1$. Let X_2 denote the accumulated value of this amount, i.e.

$$X_2 = (1+i)^2.$$

(i) Show that the probability that $X_2 \leq k$ for any non-negative k is given by

$$\frac{2}{\sqrt{2\pi}} \int_0^{\sqrt{k}} e^{-\frac{1}{2}x^2} dx. \qquad [3]$$

(ii) Show, by using an appropriate substitution or otherwise, that the answer to part (i) is the same as the probability that $G \leq k$, where G is a gamma distributed variable with parameters $\alpha = \frac{1}{2}$, $\lambda = \frac{1}{2}$. You may use without proof the fact that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$. [4]

(iii) Calculate the mean and variance of X_2. [2]

(iv) Discuss the appropriateness of the analyst's modelling assumptions. [4]
[Total 13]

5 A non-dividend paying stock has a price at time $t = 0$ of \$8. In any unit of time $(t, t+1)$, the price of the stock either increases by 25% or decreases by 20%, and \$1 held in cash at time t receives interest to become \$1.04 at time $t + 1$. The stock price after t time units is denoted by S_t.

A derivative contract is written on the stock with expiry date $t = 2$, which pays \$10 if and only if S_2 is not \$8 (and otherwise pays \$0).

(i) Explain what is meant by a risk-neutral probability measure. [2]

(ii) Calculate the up-step and down-step probabilities under the risk-neutral probability measure for this model. [1]

(iii) Calculate the price (at $t = 0$) of the derivative contract. [4]
[Total 7]

6 Consider a European call option, C, and a European put option, P, both written on a non-dividend paying stock, S, with the same strike price and maturity.

(i) Determine, for C and P:

(a) the put–call parity relationship by constructing and comparing two portfolios.

(b) a relationship between the deltas.

(c) a relationship between the gammas.

[6]

Consider now a portfolio of cash: n units of P and 1 million units of S. The delta of P is –0.212, and the gamma of P is 0.377.

(ii) Calculate the value of n that would give a portfolio a delta of zero. [2]

Two derivatives are now added to the portfolio: the call option C and a new derivative, D, which has a delta of 0.222 and a gamma of 0.111.

(iii) Calculate the number of derivatives C and D that would need to be added to the portfolio so that both the delta and gamma of the expanded portfolio are zero. [5]

[Total 13]

7 The run-off triangle below shows the cumulative claims incurred on a portfolio of general insurance policies.

	Development year			
Accident year	*0*	*1*	*2*	*3*
2018	1355	1876	2140	2288
2019	1456	2007	2232	
2020	1412	1986		
2021	1347			

The claims inflation over the 12 months up to the middle of the given year is as follows:

Year	*Rate (%)*
2019	2.00
2020	1.80
2021	2.40

It is estimated that corresponding claims inflation rates for future years will be as follows:

Year	*Rate (%)*
2022	2.80
2023	2.60
2024	1.90

(i) Calculate the outstanding claims, using the inflation-adjusted chain ladder method. [10]

(ii) Explain how you could validate whether the method in part (i) is appropriate for modelling this portfolio. [2]

Following a review, the insurer has decided to reduce the number of staff working on claim settlement.

(iii) Discuss, without performing further calculations, how you may adapt your calculations in part (i) to reflect this change. [2]

The law requires the insurer to hold a reserve higher than the expected future claims to allow for possible adverse experience. The required reserve is 1.75 × the present value of expected claims. Claims are assumed to be paid halfway through each year.

(iv) Calculate the required reserve using the following discount rates:

(a) 3% p.a.

(b) 4% p.a.

[2]

(v) Calculate the implied duration of the insurer's reserve value. [1]

(vi) Suggest criteria that the insurer may use to determine an appropriate asset in which to invest the reserve. [4]

[Total 21]

8 Consider the following assets in a world where the capital asset pricing model holds. These are the only risky assets in the market.

Asset	*Expected return (% p.a.)*	*Total value of assets in market ($m)*	*Beta*
Risky asset A	3.5	20	1.5
Risky asset B	2.2	30	0.2
Risky asset C	4.4	10	2.4

(i) Calculate:

(a) the risk-free rate of interest.

(b) the expected return on the market portfolio.

[4]

The standard deviation of the return on the market portfolio is 10%.

(ii) Calculate the market price of risk. [1]

The risk-free rate of interest now increases to 3% p.a.

(iii) Explain why one or more of the figures in the table must change. [2]
[Total 7]

9 A pension fund has been offered two investment opportunities.

Asset A gives an annual return of 3*B*%, where *B* is a binomial random variable with parameters $n = 4$ and $p = 0.4$.

Asset B gives an annual return of 4*P*%, where *P* is a Poisson random variable with parameter $\mu = 2$.

Calculate the following three measures of investment risk for each asset:

(a) Variance [1]

(b) Semi-variance [4]

(c) Shortfall probability versus a benchmark return of 4%. [2]
[Total 7]

END OF PAPER

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EXAMINERS' REPORT

CM2 - Financial Engineering and Loss Reserving

Core Principles

Paper A

September 2022

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Sarah Hutchinson
Chair of the Board of Examiners
December 2022

A. General comments on the *aims of this subject and how it is marked*

The aim of Subject CM2 is to develop the necessary skills to construct asset liability models, value financial derivatives and calculate reserves for insurance or guarantees. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.

The marking approach for CM2 is flexible in the sense that different answers to those shown in the solution can earn marks if they are relevant and appropriate. Marks for the methodology are also awarded including marks for using the right method even if an error in an earlier part of the question prevents the final answer from being correct. The marking focusses on rewarding candidates' understanding of the concepts, including their ability to articulate algebra and arguments clearly.

B. Comments on *candidate performance in this diet of the examination*

This exam was sat online and most questions focussed on applied calculations and analysis of the results. Some of the questions required students to apply concepts from the Core Reading to scenarios they might not have seen before and the stronger students scored highly here. Average marks were slightly lower than the historic norm for the subject but the pass mark was also set slightly lower.

As in previous sessions, there was evidence that some students found algebra tricky when answering questions in Word. Students should note that rearranging and solving algebra on screen can sometimes be hard if you are used to using pen and paper, so this is a worthwhile skill to practice before the exams. It's also worth saying that using the equation editor in Word to set out formulae is not necessary, your workings just need to be clear enough for the examiner to follow them.

Q4 proved to be the most challenging. The algebra required here was fairly brief but the content was quite technical.

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C. **Pass Mark**

The Pass Mark for this exam was 58
1301 presented themselves and 528 passed.

Solutions for CM2A - September 2022

Q1
(i)
U(w) = (8^0.01 - 1) / 0.01 = 2.1012 [1]

(ii)
U'(w) = w^(-0.99) [½]
U''(w) = -0.99w^(-1.99) [½]
A(w) = 0.99/w [½]
A'(w) = -0.99/w^2 <0 [½]
Therefore it exhibits declining absolute risk aversion
R(w) = 0.99 [½]
R'(w) = 0 [½]
Therefore it exhibits constant relative risk version

(iii)
Maximum P solves the following equation, where X represents the win from the lottery and w is current wealth (w=8):
E(U(w-P+X)) = U(w) [1]
1/10,000*U(8-P+1,000) + 9,999/10,000*U(8-P) = 2.1012 [1]
P = 0.004 [½]
Therefore the maximum price he will pay is $4. [½]

(iv)
If γ>1 then U''(w)>0 [½]
The function would not satisfy the principle of diminishing marginal utility of wealth. [½]
This would suggest the individual was risk seeking (i.e. not risk averse). [½]
Common utility theory assumes that individuals are risk averse. [½]
If γ>1 then the utility function would exhibit increasing absolute risk aversion. [½]
This is not plausible. [½]

[Marks available 3, maximum 2]

[Total 9]

Parts (i) and (ii) of this question were generally answered well. Fewer candidates identified the equation they needed to solve in part (iii), with common mistakes including using the wrong units for wealth or solving for utility of expected wealth rather than expected utility of wealth.

Part (iv) was answered well, although some candidates suggested that U'(w) would become negative which will not happen for any value of γ.

Q2
(i)
Structural, or firm-value, models are used to represent a firm's assets and liabilities (or capital and debt) and define a mechanism for default. [½]
These models deliver an explicit link between a firm's default and the economic

conditions and provide a sound basis for estimating default correlations amongst different firms. [½]
The Merton model is an example of a structural model. [½]
The disadvantage is identifying the correct model and estimating its parameters. [½]
Reduced form models do not attempt to deliver a representation of a firm, like structural models do. [1]
Rather they are statistical models that use observed data, both macro and micro, and so can usually be 'fitted' to data. [½]
The market statistics most commonly used are the credit ratings. [½]
Default is no longer tied to the firm value falling below a threshold-level, as in structural models. [1]
Rather, default occurs according to some exogenous hazard rate process. [½]
[Marks available 5½, maximum 4]

(ii)
The value of a firm (V) is the sum of its debt (D) and equity (E), therefore the value of the debt is calculated as D = V - E = 150 - E [½]
The value of a firm's equity is a call option on the value of the firm with a strike of the face value of the debt, and can be calculated using Black-Scholes: [½]
$d_1 = 1.0736$ [½]
$d_2 = -0.2044$ [½]
$N(d_1) = 0.8585$ [½]
$N(d_2) = 0.4190$ [½]
E = 92.712 [½]
D = 150 - E = 57.29 [½]
Bond yield = -ln(P(t,T))/(T-t) [½]
-ln(57.29/100)/15 = 3.71% [½]
Spread = yield - risk-free rate [½]
= 2.71% [½]
[Total 10]

Many candidates performed well in this question, making good points in part (i), some only described the models rather than providing the required comparison to score strongly.

Part (ii) was answered well, except from some errors towards the end when calculating the bond yield from its value.

Q3
(i)(a) = \$100 * exp(0.009) = \$100.90 [1]
(i)(b) = ln(exp(0.018*2)/exp(.009)) = 2.7% [1]
(i)(c) = \$100 * exp(-3*.023) = \$93.33 [1]
(i)(d) = ln(exp(3*.023)*exp(0.031))/4 = 2.5% [1]

(ii)
P(2,3) = 100*exp(-0.025) = \$97.53 [½]
P(2,4) = 100*exp(-0.025)*exp(-0.02) = \$95.60 [½]
At time zero investor buys 5 bonds with maturity 3 and 10 bonds with maturity 4

Cost = 5*93.33 + 10*90.48 = $1,371.50 [1]
At time 2 they are worth 5*97.53 + 10*95.60 = $1,443.65 [1]
Therefore investor makes a profit of $72.15 [1]

(iii)(a)
An inverted yield curve is when rates are higher at the shorter term [½]
i.e. the curve is downward sloping [½]

(iii)(b)
Usually the yield curve increases with maturity [½]
Reflecting uncertainty about future rates [½]
And as compensation for investing money for a longer time [½]
Liquidity preference theory also notes that investors might prefer shorter-dated assets [½]
[Marks available 2, maximum 1]

(iii)(c)
Market segmentation theory argues that different agents in the market have different objectives [½]
And rates are determined by supply and demand [½]
Therefore high demand for longer term bonds might drive rates down [1]
An inverted yield curve is an indicator of an impending recession [1]
An inverted yield curve suggests that investors believe short-term interest rates are going to fall sharply at some point in the future [1]
[Marks available 4, maximum 3]
[Total 13]

Some candidates did not perform well in this question, with fewer candidates correctly calculating the values in part (i) and fewer still correctly calculating the profit in part (ii).

Many candidates made some good points in part (iii), but to score highly, it was important to look beyond the supply/demand imbalance at different durations and think about the reasons for this.

Q4
(i)
$P\left(X_2 \le k\right) = P\left((1+i)^2 \le k\right)$ [½]

$= P\left(Z^2 \le k\right), Z \sim N(0,1)$ [½]

$= P(-\sqrt{k} \le Z \le \sqrt{k})$ [½]

$= 2P(0 \le Z \le \sqrt{k})$ by symmetry of the Normal distribution [½]

The probability of a continuous random variable being observed over a range is the integral of its pdf over the range:

$= 2\int_0^{\sqrt{k}} f_Z(x)\,dx$ [½]

$$=2\int_0^{\sqrt{k}} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}\,dx$$

[½]

(ii)

Using the substitution $t = x^2$ [½]

We note that $dt = 2xdx = 2\sqrt{t}dx$ [½]

So then the integral becomes:

$$\frac{2}{\sqrt{2\pi}}\int_0^k e^{-\frac{1}{2}t}\left(\frac{dt}{2\sqrt{t}}\right)$$

[½]

$$=\frac{1}{\sqrt{2\pi}}\int_0^k t^{-\frac{1}{2}} e^{-\frac{1}{2}t}\,dt = \int_0^k \frac{\alpha^\lambda}{\Gamma(\alpha)} t^{-\alpha} e^{-\lambda t} dt$$

, where $\alpha = \lambda = \frac{1}{2}$ [1]

This is the integral of a Gamma distribution with α and λ parameters set out above [½]

Because the Gamma distribution is bounded below by 0 [½]

this integral is exactly $P(G \leq k)$,), where G is as set out in the question [½]

(iii)

From the tables:

$$Mean = \frac{\alpha}{\lambda} = 1$$

[1]

$$Variance = \frac{\alpha}{\lambda^2} = 2$$

[1]

(iv)

Pros:

The model is simple to understand [½]

It uses a tractable distribution, which means the owner may be able to use it more easily [½]

The model produces non-negative accumulated values, which is a useful 'inbuilt feature' [½]

Cons:

The model is only valid over two-year periods [½]

which is quite a long time for a single period… [½]

and may not be useful for e.g. looking at the results on a common basis like annual [½]

The model also assumes that the same rate applies across each year in the two-year period [½]

But in practice, investment rates are sometimes correlated with rates in the past. [½]

So that this assumption may not be reasonable [½]

The Normal distribution might not be appropriate for modelling the annual return [½]

for example because it understates the likelihood of extreme events [½]

The analyst is assuming a symmetrical distribution of returns [½]

which might not apply in practice [½]

[Marks available 6½, maximum 4]

[Total 13]

This proved to be the most challenging question on the paper, perhaps because of the technical content or perhaps because of the algebra. The CM2 exam will always need to include some algebra and the examiners will mark it leniently as long as they can follow your workings.

Parts (iii) and (iv) were answered better, although in part (iv), some candidates overfocused on the disadvantages of using a Normal distribution, when many of the marks on offer are for broader concepts as would be expected for a "discuss" question.

Q5

(i)

Under Q the expected return on the risky stock is the same as that on a risk-free investment in cash [1]

In other words, under the probability measure Q investors are neutral with regard to risk: they require no additional return for taking on more risk [1]

Under the risk-neutral probability measure the discounted stock price is a martingale [1]

[Marks available 3, maximum 2]

(ii)

We use the standard formula for q but with *(1+i)* instead of e^i:

q = (1.04 - 0.8)/(1.25 - 0.8) = 8/15

So the risk neutral probability measure has:

An up jump probability of q = 8/15 [½]

A down jump probability of (1-q) = 7/15. [½]

(iii)

There are three possible stock prices at t=2

1. S_2= \$8 x 1.25 x 1.25 = \$12.5 [½]
2. S_2= \$8 x 1.25 x 0.8 = \$8 [½]
3. S_2= \$8 x 0.8 x 0.8 = \$5.12 [½]

The corresponding payoffs are:

1. \$10
2. \$0
3. \$10 [½]

So the option price is:

$$V_0 = (1.04)^\wedge(-2)[10\,x\,q\,x\,q + 0\,x\,2\,x\,q\,x\,(1-q) + 10\,x\,(1-q)\,x\,(1-q)]$$ [½]

$$= 10\,x\,(1.04)^{-2}\left[\left(\frac{8}{15}\right)\left(\frac{8}{15}\right) + \left(\frac{7}{15}\right)\left(\frac{7}{15}\right)\right]$$ [½]

$$= \$4.64$$ [1]

[Total 7]

This question was mainly answered well. The most common errors were treating the interest rate as continuously compounded in part (ii) instead of annually compounded. Although it was still possible to score full marks in part (iii) using the value of q from part (ii).

Question 3 saw some similar issues so it is always worth checking the type of interest rate in each question.

Q6

(i)(a)

Consider two portfolios at time t as follows:

1. Consisting of a European Call Option with price c_t, strike price K, strike price K and strike date T,
on a non-dividend-paying stock with price S_t, plus a cash lump sum equal to the discounted present value of the strike price K [½]
2. Consisting of one share of the same non-dividend paying stock with price S_t, plus a European put option with price p_t, strike price K and strike date T, on the same stock [½]

If the stock price at the strike date exceeds the strike price, ie $S_T > K$, then:
The call option will be exercised by handing over the cash, which will have accumulated in value to K, giving a portfolio of one share worth S_T [½]
The put option will expire worthless and the second portfolio will consist of one share worth S_T [½]

If $S_T \leq K$ then:
The call option will expire worthless leaving a portfolio of cash worth K [½]
The put option will be exercised and the stock sold for K, again giving a portfolio of cash worth K [½]

So irrespective of S_T, the portfolios will have the same value at time T. Thus, if the investment market is arbitrage free and there are no transaction costs or taxes, the two portfolios must also have equal value now:

$$c_t + Ke^{-r(T-t)} = p_t + S_t$$ [1]

This is the put-call parity relationship.

(b)

Partially differentiating this with respect to S_t gives:

$$\Delta_{call} = \Delta_{put} + 1$$ [1]

(c)

Partially differentiating this again with respect to S_t gives:

$$\Gamma_{call} = \Gamma_{put}$$ [1]

(ii)

For a portfolio with n put options like P and one million shares to have a delta of zero we need:

$\Delta_{portolio} = -0.212n + 1{,}000{,}000 = 0$ [1]

so $n = \dfrac{1{,}000{,}000}{0.212} = 4{,}716{,}981$ [1]

(iii)
$\Delta_{call} = \Delta_{put} + 1 = \ -0.212 + 1 = 0.788$ [½]
$\Gamma_{call} = \Gamma_{put} = 0.377$ [½]
As the original portfolio has a delta of zero we need:
$\Delta_{extended\ portolio} = 0.788C + 0.222D = 0$ [1]
In addition, given that cash and the stock both have a gamma of zero, we require that:

$\Gamma_{extended\ portfolio} = 0.377n + 0.377C + 0.111D = 0$ [1]

From above n = 4,716,981
Solving simultaneously:
C = 104,605,990 [1]
D = -371,304,147 [1]
[Total 13]

In this question, most candidates correctly identified the portfolios in part (i), although not all were able to differentiate the put-call parity formula to derive the relationships for Delta and Gamma.

Candidates' performance was lower in parts (ii) and (iii), but the stronger candidates were able to identify the portfolios needed.

The question contained a minor typo before part (ii) where the colon should have been a comma, but candidate scripts suggest that the meaning was still clear.

Q7
(i)
Incremental Claim Amounts: [1]

	0	1	2	3
2018	1355	521	264	148
2019	1456	551	225	
2020	1412	574		
2021	1347			

Incremental Claim Amounts in mid-2021 prices: [1]

	0	1	2	3
2018	1440.7	543.1	270.3	148.0
2019	1517.8	564.2	225.0	
2020	1445.9	574.0		
2021	1347.0			

Cumulative Claim Amounts in mid-2021 prices: [1]

	0	1	2	3
2018	1440.7	1983.9	2254.2	2402.2
2019	1517.8	2082.0	2307.0	
2020	1445.9	2019.9		
2021	1347.0			

Development Factors:
DF(0,1) = (1983.9 + 2082 + 2019.9) / (1440.7 + 1517.8+ 1445.9) = 1.3817 [1]
DF(1,2) = (2254.2 + 2307) / (1983.9 + 2082) = 1.1218 [1]
DF(2,3) = 2402.2 / 2254.2 = 1.0657 [1]

Completed Cumulative Claims in mid-2021 prices: [1]

	0	1	2	3
2018				
2019				2458.5
2020			2266.0	2414.7
2021		1861.2	2087.9	2225.0

Incremental Claims in mid-2021 prices: [1]

	0	1	2	3
2018				
2019				151.5
2020			246.1	148.8
2021		514.2	226.7	137.1

Incremental Claims adjusted for inflation: [1]

	0	1	2	3
2018				
2019				155.7
2020			253.0	156.9
2021		528.6	239.2	147.3

Outstanding Claims = 1,480.68 [1]

(ii)
To check how well the inflation adjusted chain ladder technique performs:
The cumulative claim payments in years for which data is already available can be estimated using the development factors calculated in (i) [1]
These estimates can be compared to the actual claim values and the differences can be reviewed to assess if errors are large enough to suggest that the model is inaccurate [1]
Similarly, the actual claims experience for each accident year in 2022 could be compared to the estimates produced by the model [1]
The model assumes claims arise uniformly throughout the year. Depending on the type of general insurance this may not be valid and therefore the inflation adjustments are inaccurate [1]

[Marks available 4, maximum 2]

(iii)
The reduction in staff might lead to increased claim settlement times. [½]
It would be sensible to reflect this in the estimate of future claim payments [½]
The insurer may need to adjust the calculated development factors in the light of this information [1]
An increased number of development years may be required [½]
Claims might increase in absolute terms as claims underwriting might be less stringent [½]
[Marks available 3, maximum 2]

(iv)(a)
1.75 * ((528.6 + 253.0 + 155.7) * 1.03^-0.5 + (239.2+156.9) * 1.03^-1.5 + 147.3 * 1.03^-2.5 [½]
= 2,518.75 [½]

(iv)(b)
1.75 * ((528.6 + 253.0 + 155.7) * 1.04^-0.5 + (239.2+156.9) * 1.04^-1.5 + 147.3 * 1.04^-2.5 [½]
= 2,495.69 [½]

(v)
(2518.74 - 2495.69) / 2518.74 = 0.01*D [½]
So D = 0.92 years [½]

(vi)
The asset should deliver at least the rate of return used to calculate the reserve [1]
The asset should have an interest rate duration of around 0.92 years [1]
The asset might also need to have inflation sensitivity to minimise the insurer's exposure to this [1]
The asset should have minimal probability of default [1]
The asset should be in the same currency as the reserve and expected claims [1]
The asset should be liquid enough to pay claims when needed [1]
[Marks available 6, maximum 4]
[Total 21]

Candidates performed well on part (i) of this question, with some mistakes creeping in but only being penalised in the specific step where they appeared.

In parts (ii) and (iii) often needed candidates to make more distinct points to score full marks. The later question parts saw generally lower marks awarded, but most candidates made some good points in part (vi) and those that scored highest were those who were able to think broadly around the topic.

Q8
(i)
We know that $E_i = r + \beta_i * (E_M - r)$ [½]
So $3.5 = r + 1.5 * (E_M - r)$ [½]

And $2.2 = r + 0.2 * (E_M - r)$ [½]

Subtracting the second equation from the first:
$3.5 - 2.2 = 1.3 * (E_M - r)$ [½]
So $E_M - r = 1$ [½]

Also subtracting 7.5 times the second equation from the first:
$3.5 - 7.5 * 2.2 = -6.5r$ [½]
So r = 2% [½]
And E_M = 3% [½]

(ii)
MPR = $(E_M - r) / \sigma_M$ = (3% - 2%) / 10% = 0.1 [1]

(iii)
Every asset must have an expected return at least as high as the risk-free rate [½]
So the return on asset B must increase to at least 3% [½]
Intuitively if the risk-free rate increases, then investors will require a higher return on risky assets as well [½]
If the market price of risk remains unchanged, then the returns on all assets might just increase by 1% [½]

[Total 7]

Candidates performed well in this question as it was a straightforward question and saw some of the highest average marks in this paper. Most candidates correctly solved the risk-free rate and market return, and most correctly identified the likely impacts of the risk-free rate increasing.

Q9
(a)
$Var_A(3B\%) = 8.64\%\%$ [½]
$Var_B(4P\%) = 32\%\%$ [½]

(b)
$\mu_A = 3x4x0.4 = 4.8\%$ [1]
Semi-variance$_A$ = $(4.8 - 0)^2$ x 81/625 + $(4.8 - 3)^2$ x 216/625 = 4.106%% [1]
$\mu_B = 4x2 = 8\%$ [1]
Semi-variance$_B$ (4P%) = $(8 - 0)^2$ x e^{-2} + $(8 - 4)^2$ x e^{-2} x 2^1 / 1! = 12.992%% [1]

(c)
$Prob_A$ (3B < 4) = $Prob_A$ (B < 1.333) = $Prob_A$ (B = 0) + $Prob_A$ (B = 1) = 0.4752 [1]
$Prob_B$ (4P < 4) = $Prob_B$ (P < 1) = $Prob_B$ (P = 0) = 0.13534
[1]

[Total 7]

Most candidates answered part (a) of this question correctly, but parts (b) and (c) proved more of a challenge. Some candidates used their answer from part (a) to answer part (b) and this was also a valid approach.

[Paper Total 100]

END OF EXAMINERS' REPORT

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