INSTITUTE AND FACULTY OF ACTUARIES

EXAMINATION

22 April 2021 (am)

Subject CM2A - Financial Engineering and Loss Reserving Core Principles

Time allowed: Three hours and fifteen minutes

If you encounter any issues during the examination please contact the Assessment Team on T. 0044 (0) 1865 268 873.

1   An investor has $800 to invest, for a period of 1 year, and has identified two investment opportunities in which to invest. The first is a direct investment in a stock index for a period of 1 year. The annual return, X, on the index follows a Normal distribution with mean $\mu = 7\%$ p.a. and standard deviation $\sigma = 5.5\%$ p.a.

(i)   Calculate the following in respect of the investment at the end of 1 year:

(a)   The shortfall probability below a value of $720

(b)   The 99.5% value at risk.

[5]

The second opportunity is a derivative that offers the following payoff in 1 year's time based on the performance of the index during the year.

Payoff ($)	Scenario
730	when $X \leq -7.1\%$
750	when $-7.1\% < X \leq 7\%$
962	when $X > 7\%$

(ii)   Calculate the expected payoff from the derivative at the end of the year.   [4]

(iii)   Calculate the following in respect of the payoff from the derivative:

(a)   The shortfall probability below a value of $720

(b)   The 99.5% value at risk.

[2]

(iv)   Comment on how the investor may choose between the two investments.   [3]

[Total 14]

2   A dividend-paying share is currently priced at $120. Every 3 months, the share price will either increase by 25% or decrease by 20%.

An insurer is considering purchasing a 6-month European call option on the share with a strike price of $120. Each share will pay a dividend of $4 just before the option expires.

The risk-free force of interest is 0% per annum.

(i)   State the difference between American and European options.   [1]

(ii)   Calculate the risk neutral price of the option, using a binomial tree.   [5]

(iii)   Calculate the volatility of the share implied by the option price.   [2]

(iv)   State the benefits of using a risk-neutral derivative pricing approach.   [2]

(v)   Explain why it may be difficult to price this option using the Black–Scholes option pricing model.   [2]

Now consider an American option on the same underlying share with all other parameters unchanged.

(vi)   Explain why you may expect the price of the American option to be different to your answer in part (ii).   [2]

(vii)   Calculate the price of the American put option.   [2]

[Total 16]

3   Two assets are available for investment. Asset A returns 2X%, where X is a Binomial random variable with parameters $n = 6$ and $p = 0.4$. Asset B returns 1.5Y%, where Y is a Normal random variable with parameters $\mu = 3.2$ and $\sigma = 2$.

   (i)   Calculate the following separately for each of assets A and B:

   (a)   The variance of the return

   (b)   The shortfall probability below a return of 3%.

   [4]

   An investor with a quadratic utility function wishes to invest in either Asset A, or Asset B.

   (ii)   Explain which asset the investor should choose.   [3]

   The investor is now considering whether to split their investment between both assets.

   (iii)   Explain whether the investor may decide to split the investment in each of the following circumstances:

   (a)   If the assets are independent

   (b)   If the assets exhibit some positive correlation.

   [4]
   [Total 11]

4   (i)   Explain the similarities and differences between the basic chain ladder method and the inflation-adjusted chain ladder method for calculating run-off triangles.   [3]

   (ii)   Discuss when it may be more appropriate to use the inflation-adjusted chain ladder method than the basic chain ladder method.   [2]

   (iii)   Discuss possible reasons why neither method may be appropriate for calculating run-off triangles.   [2]
   [Total 7]

5   A risk-averse, non-satiated investor is trying to determine their utility of wealth function, $U(w)$. They have decided to use the utility function $U(w) = w + dw^2$, where $d < 0$ is a constant that the investor has chosen.

(i)   Derive an upper bound in terms of d for the range of values of w over which $U(w)$ can be used.   [2]

(ii)   Explain why $d < 0$ is a necessary condition for $U(w)$ to be a valid utility function.   [2]

The investor lives on a tropical island. On this island, root vegetables can be bought once a week for $10 per box. The investor knows that they will be able to sell any vegetables they buy for $30, $12, $10 or $0.5 per box with equal probability. All boxes of vegetables sold at a given time will be sold for the same price.

The investor's current wealth is $100. If the investor were to buy seven boxes of vegetables, their expected utility of wealth after selling them would be 50.

(iii)   Calculate the value of d.   [5]

(iv)   Calculate the expected utility of wealth of the investor, if they do not buy any vegetables.   [1]

The investor has decided they want to buy seven boxes of vegetables.

(v)   Discuss whether $U(w)$ is appropriate for the investor.   [2]
[Total 12]

6 An insurer assumes that the number of claims it incurs follows a Poisson process with parameter μ. The adjustment coefficient, R, for this insurer is a risk measurement parameter associated with its surplus process. It is the unique positive root of the following equation:

$$\mu M_X(R) = \mu + cR$$

where $M_X(R)$ is the moment generating function of the individual claim amount distribution, X.

(i) State what quantity is represented by c. [1]

The insurer assumes that claim amounts follow the gamma distribution with parameters $\alpha = 2$ and $\lambda = 1$.

(ii) Write down the moment generating function of this gamma distribution. [1]

(iii) Show that R satisfies the following equation:

$$cR^2 + R(\mu - 2c) + (c - 2\mu) = 0.$$ [4]

The insurer sets $\mu = 3.5$ and $c = 1$.

(iv) Solve the equation in part (iii) to determine the adjustment coefficient. [2]

[Total 8]

7 Explain, for each of the following scenarios, which behavioural biases are exhibited.

(i) A portfolio manager says that 50% of their portfolio performed well because of their careful stock selection process, while 50% underperformed because of a global economic crash. [1]

(ii) An individual investor claims that their stock-picking skills are very strong because 75% of their portfolio produced a 3% return. The other 25% of their portfolio produced a 50% loss. [2]

(iii) A risk modelling team at an insurance company decides to continue to use a modelling technique for a risk because it is used by modelling teams at other similar companies. The team makes this decision despite academic research suggesting that a new approach is substantially better. [2]

(iv) An actuary, with no knowledge of equity risk modelling, strongly opposes a new equity risk model that has suggested that the 1-in-50 value at risk of an equity is a 25% loss. The actuary's rationale for their opposition is that the previous model suggested a 10% loss. [3]

[Total 8]

8 A general insurer specialising in catastrophes is trying to understand its future probability of ruin. It models future claims payments in millions of dollars over each yearly interval as Exponentially distributed with parameter $\lambda = 1$. The insurer assumes that the claims payments are independent each year.

The insurer holds an initial surplus of $3m and assumes no future premium income.

(i) Calculate the probability that the insurer is solvent at the following times measured in years:

(a) $t = 0$

(b) $t = 1$.

[2]

(ii) State, without performing any further calculations, how your answers to part (i) would change if the insurer changed its net claims distribution to be Exponentially distributed with parameter $\lambda = 2$. [1]

(iii) Discuss the appropriateness of the insurer's approach to assessing its future probability of ruin. [4]

[Total 7]

9 In a particular market, there are two assets, A and B, for which the annualised rates of return have the following characteristics:

Asset	Expected value (%)	Standard deviation (%)
A	10	20
B	5	0

An investor is considering investing a proportion of their wealth x_A in asset A and x_B in asset B.

(i) State the formula for the market price of risk in this market. [1]

(ii) Show that the efficient frontier for this investor is a straight line passing through the points (0, 0.05) and (0.1, 0.075) in (Standard Deviation, Expected Return) space. [5]

A third asset, C, becomes available in the market. It has an annualised expected return of 6% and an annualised standard deviation of 10%. It is uncorrelated with assets A and B.

(iii) Show that the new efficient frontier using A, B and C, passes through the point (0.1, 0.0769). [6]

[**Hint:** the market price of risk for a portfolio involving only assets A and C is maximised when $x_A = {}^5/_9$ and $x_C = {}^4/_9$.]

[Total 12]

10 Consider two investment portfolios, A and B, with cumulative probability distribution functions of returns F_A and F_B respectively.

Assume that portfolio A exhibits first-order stochastic dominance over portfolio B.

(i) Explain in words:

    (a) what this tells us about the returns on portfolio A and portfolio B.

    (b) why a non-satiated investor will prefer portfolio A to portfolio B.

                                                                       [2]

Assume now that portfolio A exhibits second-order stochastic dominance over portfolio B.

(ii) State the requirements on an investor for them to prefer portfolio A over portfolio B. [1]

Assume now that portfolio B provides a return of either –5%, or +10%, with equal probability.

(iii) Set out how you may determine a distribution that the return on portfolio A could take, such that A is second-order stochastic dominant over B. [2]

                                                          [Total 5]

END OF PAPER

INSTITUTE AND FACULTY OF ACTUARIES

EXAMINERS' REPORT

April 2021

Subject CM2 – Financial Mathematics and Loss Reserving Core Principles Paper A

Introduction

The Examiners' Report is written by the Chief Examiner with the aim of helping candidates, both those who are sitting the examination for the first time and using past papers as a revision aid and also those who have previously failed the subject.

The Examiners are charged by Council with examining the published syllabus. The Examiners have access to the Core Reading, which is designed to interpret the syllabus, and will generally base questions around it but are not required to examine the content of Core Reading specifically or exclusively.

For numerical questions the Examiners' preferred approach to the solution is reproduced in this report; other valid approaches are given appropriate credit. For essay-style questions, particularly the open-ended questions in the later subjects, the report may contain more points than the Examiners will expect from a solution that scores full marks.

The report is written based on the legislative and regulatory context pertaining to the date that the examination was set. Candidates should take into account the possibility that circumstances may have changed if using these reports for revision.

Paul Nicholas
Chair of the Board of Examiners
July 2021

A. General comments on the *aims of this subject and how it is marked*

1. The aim of Subject CM2 is to develop the necessary skills to construct asset liability models, value financial derivatives and calculate reserves for insurance or guarantees. These skills are also required to communicate with other financial professionals and to critically evaluate modern financial theories.

2. The marking approach for CM2 is flexible in the sense that different answers to those shown in the solution can earn marks if they are relevant and appropriate. Marks for the methodology are also awarded including marks for using the right method even if an error in an earlier part of the question prevents the final answer from being correct. The marking focusses on rewarding candidates' understanding of the concepts, including their ability to articulate algebra and arguments clearly.

B. Comments on *candidates' performance in this diet of the examination.*

1. Some of the questions required candidates to apply concepts from the Core Reading to scenarios they might not have seen before and the stronger candidates scored highly here. Average marks were a little lower than the last couple of sittings but within the historic norm for the subject.

2. Because of the format of the exam paper there was less algebra, but even so there was evidence that some candidates found the algebra tricky when answering questions in Word. The examiners were lenient with notation when marking these questions. Candidates should note that rearranging and solving algebra on screen can sometimes be hard if you are used to using pen and paper, so this is a worthwhile skill to practice before the exams.

C. Pass Mark

The Pass Mark for this exam was 60.
1,309 candidates presented themselves and 445 passed.

Solutions for CM2A - April 2021

Q1
(i)(a)
X = 720/800 – 1 = -10%   [1]
P(X<-10%) = P(Z<(-10% - 7%)/5.5%) = P(Z<-3.09) = 0.1%   [1]

(b)
P(Z<(t – 7%)/5.5%) = 0.005   [½]
(t – 7%)/5.5% = -2.5758   [½]
t = -7.1669%   [1]
800 * (1-7.1669%) = $742.66   [1]

(ii)
P(X≤-7.1%) = P(Z≤-2.56)
=0.00518   [1]
P(X>7%) = 0.5 (as 7% is the mean)   [1]
P(-7.1%<X≤7%) = 1-0.5-0.00518 = 0.49482   [1]
Expected pay out = 730*0.00518 + 750*0.49482 + 962*0.5 = $855.90   [1]

(iii)(a)
0%   [1]

(b)
Probability pay out is ≤ 730 is 0.52% therefore the 99.5% VaR is $730   [1]

(iv)
The expected return from investing in the index is 800*1.07 = $856   [½]
So the expected returns are very similar for each investment   [½]
Based on the expected shortfall below $720 the derivative is less risky as there is no
possibility of this   [½]
If the investor has a utility function with a discontinuity at the minimum required return then
he may base his decision on this measure   [½]
The 99.5% VaR is higher (i.e. a greater loss) for the derivative, so based on this measure the
investor may prefer to invest in the stock index   [1]
The pay off on the derivative is significantly higher than the index when the return is slightly
above the mean, so the investor may prefer this   [1]
[Marks available 4, maximum 3]
[Total 14]

> *This question was answered well on the whole, with many candidates scoring full
> marks in parts (i) to (iii). Part (iv) was not answered quite so well, with some
> candidates not making enough distinct points for the number of marks on offer.*

Q2
(i)
A European option is an option that can only be exercised at expiry   [½]
An American option is one that can be exercised on any date before its expiry   [½]

(ii)
Risk neutral 'up' move probability $q = (e^r - d) / (u - d) = 0.2 / 0.45 = 0.44$ [1]

Time 0		Time 1		Time 2	
Share price	Option value	Share price	Option value	Share price (net of dividend)	Option value
				$S(0)u(1)u(2) - D = 183.5$	Payoff = 63.5
		$S(0)u(1) = 150$	Call = 27.94		
				$S(0)u(1)d(2) - D = 116$	Payoff = 0
$S(0) = 120$	Call = 12.29			$S(0)d(1)u(2) - D = 116$	Payoff = 0
		$S(0)d(1) = 96$	Call = 0		
				$S(0)d(1)d(2) - D = 72.8$	Payoff = 0

for all pre-dividend share prices in tree [1]
for correct allowance for dividends [1]
for all option values in tree [1]

Final option value = 12.54 [1]

(iii)
Using step 1 in the tree: First we check that ud = 1 [½]
$u = 1.25 = e^{\sigma * sqrt(3/12)}$
Re-arranging and solving for σ = 44.6%. [2]
[Note: The question asked for the volatility of the share price implied by the option price and students were also awarded marks for calculating an implied volatility using the Black-Scholes model.]

[Marks available 2½, maximum 2]

(iv)
The risk neutral approaches offers a convenient computational tool [1]
Investors are assumed to be indifferent to risk [½]
All assets are assumed to earn a risk-free return [½]
No assumptions need to be made about an asset's expected return [1]
The price produced is the no-arbitrage price [½]
A wide range of derivatives can be priced in this way [½]

[Marks available 4, maximum 2]

(v)
The Black-Scholes model assumes that the share price follows a lognormal distribution whereas the price here is binomial [1]
The Black-Scholes model assumes that dividends are continuous whereas here there is a one-off assumed dividend at a set time [1]
The Black-Scholes constant volatility assumption is flawed here as well because of the dividend [1]

[Marks available 3, maximum 2]

(vi)

It could be optimal to exercise the American call option before maturity in order to capture the dividend [1]

This is because you have to own the stock on the ex-dividend date in order to receive the dividend [½]

The ability to exercise represents an additional benefit of holding the American-style call option compared to the European-style call option [½]

Therefore, the value of an American-style call option will be higher than the equivalent European-style call option [½]

[Marks available 2½, maximum 2]

(vii)

Exercising the put option early will never be optimal. No interest will be earned on the proceeds for doing so [1]

Therefore, we can apply put-call parity:

C – P = S – K + r – D => P = 12.54 – 120 + 120 + 4 = 16.54 [1]

[Total 16]

This question was answered fairly well by most candidates. Many did not manage to allow for the dividend payment correctly but it was still possible to earn most of the marks on offer. Some of the answers provided to parts (v) and (vi) were rather generic and needed to be tailored more to the scenario in the question to score full marks.

Q3

(i)(a)

Asset A

Variance = 2^2 * npq = 5.76%% [1]

Asset B

Variance = 1.5^2 * σ^2 = 9%% [1]

(b)

Asset A

P(2X<3) = P(X<1.5) = P(X≤1) = 0.2333 [1]

Asset B

P(1.5Y<3) = P(Y<2) = P(Z<-0.6) = 0.27425 [1]

(ii)

Mean returns: Asset A = 2*6*0.4 = 4.8%, Asset B = 1.5*3.2 = 4.8%. [1]

Both assets have same mean therefore investor will choose the asset with the lowest variance. [1]

The investor will choose Asset A. [1]

The quadratic utility function also implies variance as the risk measure [1]

[Marks available 4, maximum 3]

(iii)(a)
If the assets are independent then a combination of both assets will give a lower variance than either asset on its own [1]
But with the same expected return [½]
So the investor would prefer to invest partially in each asset [½]

(b)
If the assets exhibit correlation and short selling is not allowed then the variance of a combined portfolio will be higher than if they were uncorrelated [1]
But it will still be lower than investing in a single asset [½]
So the investor would still prefer to invest partially in each asset [½]
Or, if short selling is allowed, the investor could short one asset in order to achieve a lower portfolio variance [2]

[Marks available 4, maximum 2 for part (b)]
[Total 11]

Parts (i) and (ii) of this question were answered well, with many candidates scoring full marks. Part (iii) caused more difficulty with many candidates failing to identify the opportunity to reduce risk through diversification, with some risk reduction still possible if the assets are correlated as long as the correlation is not perfect.

Q4
(i)
Similarities
Both methods are based on the same assumption that payments from each accident year will develop in the same way. [½]
Or in effect, the same development factors are used to project outstanding claims for each accident year [1]
Changes in the rate at which claims emerge can only be incorporated by adjusting those development factors [½]
Both methods assume that the first year is fully run off [½]

Differences
The basic chain ladder method assumes that weighted average past inflation will be repeated in the future [½]
The inflation-adjusted approach does not do this and instead modifies historic claims to be consistent (in real terms) with one another [½]
It does this with the use of historic inflation rates, which are a key input to the model [½]
However, the method does not automatically allow for future inflation [½]
And this is normally given as an input for inflation-adjusted chain ladder [½]

[Marks available 5, maximum 3]

(ii)
The advantages of inflation-adjusted chain ladder are its ability to make all history consistent in real terms [½]
This will be most useful when inflation has historically been large [1]
so that the adjustment will be bigger [½]

For example if inflation has historically been 0.5% for several years and is expected to stay at that rate for the foreseeable future, there may be very little reason to use inflation-adjusted chain ladder over basic chain ladder [1]

[Marks available 3, maximum 2]

(iii)
Both methods assume that all years behave and develop identically [1]
So they should not be used in situations where this is less likely to be true [½]
For example if the business entity has changed from start-up to major market player… [½]
… or one year experienced particularly severe claims due to a catastrophe [½]

[Marks available 2½, maximum 2]

[Total 7]

> *This question saw a variety of responses, with many candidates making some valid points but not enough for all of the marks on offer. Most candidates gave some credible examples for when the inflation-adjusted method might be the best method to use, which was encouraging.*

Q5
(i)
For a utility function to be valid, it must respect the fact that the investor is non-satiated [½]
This is equivalent to requiring that $U'(w)>0$ [½]
Substituting the given form of (w) we find the following:
$U'(w) = 1+2dw > 0$ [½]
$w < -1/2d$ [½]

(ii)
For a utility function to be valid, it must also respect the investor being risk-averse [½]
This is equivalent to requiring that $U''(w)<0$ [½]
Substituting the given form of (w) we find the following:
$U''(w) = 2d < 0$, which requires that d is negative [1]

(iii)
If the investor buys 7 boxes of vegetables, they have £30 remaining in cash [½]
For each of the payoffs, their final wealth will be:

Payoff	Final Wealth
30	210 + 30 = 240
12	84 + 30 = 114
10	70 + 30 = 100
0.5	3.5 + 30 = 33.50

[½ per correct value]

So their expected utility is:
$(w) = 0.25(240 + 114 + 100 + 33.5) + 0.25(240^2 + 114^2 + 100^2 + 33.5^2)$ [1]
$(w) = 121.875 + 20429.5625d = 50$ [½]
$d = (50−121.875) / 20429.5625 = −0.003518$ [1]

(iv)
$(100) = 100 − 0.003518*100^2 = 64.82$ [1]

(v)
The investor bought the vegetables despite this strategy having lower expected utility according to the utility function they chose [½]
The investor is risk-averse, so they should make the decision with higher expected utility [½]
So the utility function may not be appropriate for the investor [½]
because it is not consistent with the decisions they are making [½]
or because for this decision the investor is not as risk-averse i.e. they may be risk-seeking when it comes to buying these vegetable [½]
Also, the maximum wealth this utility function can be used with is £142 [½]
The investor can easily exceed this wealth in the highest payoff scenario [½]
[Marks available 3½, maximum 2]
[Total 12]

> *Parts (i) and (ii) of this question were answered well with many candidates scoring full marks. Part (iii) caused more difficulty and many candidates were not able to correctly calculate the wealth in each scenario, which then led to finding the wrong value for d. Parts (iv) and (v) were answered well on the whole and it was possible to score full marks here even if part (iii) had been answered incorrectly.*

Q6
(i)
c denotes the rate of premium income of the insurer. [1]

(ii)
From the tables, $(x)=(1−x)^{−2}$ [1]

(iii)
Given the information in the question:
$(1−R)^2=\mu+cR$ [½]
$\mu=(\mu+cR)(1−R)^2=(\mu+cR)(1−2R+R^2)$ [½]
$\mu=\mu+(c−2\mu)+R^2(\mu−2c)+cR^3$ [½]
$0=(cR^2+R(\mu−2c)+(c−2\mu))$ [1]
So $R=0$ or $cR^2+(\mu−2c)+(c−2\mu)=0$ [½]
As $R>0$, we can reject the former so $cR^2+(\mu−2c)+(c−2\mu)= 0$ [1]

(iv)
$R^2+1.5R−6=0$ [1]
The roots are R = -3.312 or R = 1.812 [1]
[Total 8]

This question was mostly answered well, with many candidates scoring full marks or close to full marks. The most common areas where marks were lost were not applying the MGF to the specific scenario in (ii) and not noting that we can divide by R because it's greater than zero in (iii).

Q7
(i)

This is an example of self-serving bias	[½]
Where people credit favourable events to their own credit or skills	[½]
while attributing negative events to external forces	[½]

[Marks available 1½, maximum 1]

(ii)

This is an example of confirmation bias	[½]
Where people look for evidence that confirms their point of view	[½]
or dismiss evidence that does not	[½]
It is likely the investor is also being overconfident	[½]
they are overestimating their own skills and abilities to pick stocks	[½]
because they are claiming they have 'very strong' skills	[½]
but have lost half the value of a quarter of their portfolio	[½]
In fact, confirmation bias is a specific manifestation of overconfidence	[½]

[Marks available 4, maximum 2]

(iii)

There is status quo bias here	[½]
the team is sticking with the current situation	[½]
even in the face of a favourable alternative	[½]
The team are also showing herd behaviour	[½]
They are following/mimicking the actions of others	[½]
in order to manage an uncertain situation	[½]

[Marks available 3, maximum 2]

(iv)

The actuary is anchoring and adjusting	[½]
They have started with an idea of an answer	[½]
(in this case the previous model output)	[½]
and are then adjusting away from this to arrive at their final judgement	[½]
The actuary is also overconfident	[½]
because they are offering strong critique of an area in which they have no expertise	[½]
The actuary is arguably also following status quo bias	[½]
again because they are seeking to stay with the current model output	[½]
They may also be engaging in familiarity bias	[½]
where the 'known' model is preferred to the 'new' one	[½]

[Marks available 5, maximum 3]
[Total 8]

This question saw plenty of good answers from candidates, but few made enough distinct points to score full marks. Some candidates gave generic definitions of each of the biases without applying them to the specific scenarios in the question, and this was not enough to score full marks.

Q8
(i)(a)
No time has elapsed and we know that the insurer is currently solvent [½]
So the probability of solvency is 1 [½]

(b)
$(Solvency)=(Exp(1)<3)$ [½]
Using the tables for the CDF of the Exponential distribution, this is $1-e^{-1*3} = 0.95021$ [½]

(ii)
The answer to part (a) would be unchanged [½]
The answer to part (b) would increase [½]

(iii)
There are many issues with the modelling approach the insurer is proposing:
The insurer specialises in catastrophe insurance [½]
so the Exponential distribution may not be appropriate for this business [½]
for example because it does not have the high kurtosis/'fat tails' commonly seen in catastrophe events [1]
for example because catastrophe events tend to be 'low frequency, high impact', so modelling them with a continuous distribution may not even be appropriate in the first place [1]
The insurer is also assuming that net claims are independent in each annual period [½]
In practice this is unlikely to be true [½]
because the number/size of claims are likely to be highly correlated to the previous year's [1]
The insurer has also assumed its initial surplus is constant [½]
in practice it could invest these funds to generate return at e.g. the risk-free rate [½]
Solvency is only assessed on an annual basis, but ruin can occur in reality at any time. [½]
[Marks available 6½, maximum 4]
[Total 7]

Parts (i) and (ii) of this question were answered well by many candidates. Part (iii) generated lots of good ideas but few candidates scored full marks. To score highly in part (iii) needed plenty of ideas and they needed to be specific to the scenario in the question.

Q9
(i)

$$MPR = \frac{expected\ return - 5\%}{standard\ deviation}$$

[1]

(ii)
The expected return on the portfolio is:

$$E = x_A E_A + x_B E_B$$

[½]

Substituting $xA=1-xB$ into this gives:

$$E = 0.1(1 - x_B) + 0.05x_B = 0.1 - 0.05x_B$$

[½]

Rearranging this gives:

$$x_B = \frac{0.1 - E}{0.05} \ and \ x_A = 1 - x_B = \frac{E - 0.05}{0.05}$$

[½]

We also have:

$$\sigma^2 = x_A^2 V_A + x_B^2 V_B + 2x_A x_B C_{AB}$$

[½]

$$= 0.2^2 x_A^2$$

[½]

i.e. $\sigma = 0.2x_A$

[½]

Substituting our formula for xA into this gives the equation of the efficient frontier as:

$$\sigma = 0.2 \left(\frac{E - 0.05}{0.05}\right) = 4(E - 0.05)$$

[½]

This is a straight line in $E-\sigma$ space [½]
Furthermore:
when $E=0.05, \sigma=0$ and [½]
when $E=0.075, \sigma=0.1$ as required [½]

(iii)
Asset B is risk-free, so it has zero standard deviation, and any combination of Asset B and a portfolio of assets involving A and C only will lie somewhere along the straight line in the $E-\sigma$ space joining Asset B and the other portfolio. [1]

The efficient frontier involving all three assets will be the one that maximises the market price of the risk (MPR). This must be the straight-line through Asset B that is tangential to the efficient frontier involving Assets A and C only. [1]

At the point of tangency:
- we have no holding of Asset B, $x_B = 0$

[½]

- $x_A = \frac{5}{9}$ and $x_C = \frac{4}{9}$ [½]

Here:

$$E = 0.1 \times \frac{5}{9} + 0.06 \times \frac{4}{9} = \frac{37}{450}$$ [½]

$$\sigma = \sqrt{0.04\left(\frac{5}{9}\right)^2 + 0.01\left(\frac{4}{9}\right)^2} = \frac{\sqrt{116}}{90}$$ [½]

So, the equation of the efficient frontier involving all three assets is:

$$E = 0.05 + \left(\frac{\frac{37}{450} - \frac{1}{20}}{\frac{\sqrt{116}}{90}}\right)\sigma = 0.05(1 + \sigma\sqrt{29})$$ [1]

When $\sigma = 0.1$:

$$E = 0.05(1 + 0.1 \times \sqrt{29}) = \frac{1}{20} + \frac{\sqrt{29}}{200} = 0.07693$$ [½]

So the efficient frontier involving all three assets passes through the point (0.1, 0.0769) as required. [½]

[Total 12]

> *This was probably the trickiest question in the paper and few candidates scored really strongly. A number of candidates worked through the algebra in parts (ii) and (iii) with some success but didn't explain their steps clearly or skipped some key steps which meant they scored well but did not receive full marks.*

Q10
(i)(a)
For all possible returns on the portfolio (x) [½]
B is at least as likely as A to produce a return smaller than that x [½]
For some return x, B is more likely to produce a return smaller than that x [½]

(b)
A non-satiated investor will prefer portfolio A because they can be sure that it improves) their expected return and expected utility [½]

(ii)
For second order stochastic dominance to matter, the investor must be risk-averse [½]
and prefer more to less (be non-satiated) [½]

(iii)
The requirement for second order stochastic dominance is that:

$$\int_a^x F_A(y)\,dy \leq \int_a^x F_B(y)\,dy$$ [½]

For all values of x, with strict inequality holding for at least one x. a is the lowest value either portfolio can take. [½]

Given the form of B, many distributions meet this condition – a simple example is A offering a return of 11% guaranteed (or any value above 10%). This needs to be checked only at -5% (the integral of A is 0, of B it is 0.5), 10% (at A it is still 0, at B is it 1) and 11% (1 for both A and B) [1]

[Total 5]

> *Parts (i) and (ii) of this question were answered fairly well, though part (i) required an explanation in words so candidates who answered it with only algebra did not score highly. Part (iii) required an example distribution for full marks, and there were lots of possible options here but only the stronger students score full marks.*

[Paper Total 100]

END OF EXAMINERS' REPORT