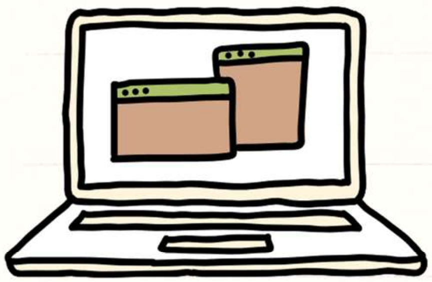
Exact Test for 2×2 and r×c Tables

Presented By



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Introduction to Exact Tests

Definition:

- Exact tests are statistical tests used to evaluate the association or independence between categorical variables.

Why are they important??

- They are especially useful when sample sizes are small, and other tests (like Chi-square) may not hold.
- They don't rely on large sample assumptions or expected frequency assumptions that Chi-square does

Why Use Exact Tests??

- Small sample sizes: Ideal for cases when expected cell frequencies are less than 5.
- Violation of assumptions: Useful when the assumptions for other tests (like Chi-square) are violated (e.g., the assumption of large expected counts).

2×2 Tables Overview

What is a 2×2 Table??

- A 2×2 table is a contingency table where both rows and columns represent two categories each, forming four possible combinations.
- Example: Testing the relationship between gender (Male/Female) and smoking status (Yes/No).

	Group 1 (e.g., Smokes)	Group 2 (e.g., Does not smoke)	Total
Category 1 (e.g., Male)	a	b	a+b
Category 2 (e.g., Female)	c	d	c+d
Total	a+c	b+d	N

Fisher's Exact Test (2×2 Table)

Definition:

- Fisher's Exact Test is used to assess whether there are nonrandom associations between the two categorical variables in a 2×2 table. It is called "exact" because it computes the exact probability of obtaining the observed table, based on the marginal totals.

When to Use:

- Used when sample sizes are small (typically less than 20).
- When expected cell frequencies are less than 5, which violates the assumption required for the Chi-square test.

Formula:

- p-value calculation is based on the hypergeometric distribution.

$$p = \frac{(a+b)!(c+d)!(a+c)!(b+d)!}{(a!b!c!d!)N!}$$

Example of Fisher's Exact Test (2×2)

Example Data:

- Testing if smoking is independent of gender. Suppose the data is:

	Male	Female	Total
Smokes	3	1	4
Does not	6	10	16
Total	9	11	20

Step by Step Process:

- Compute the p-value using Fisher's Exact Test formula.
- Interpretation: If the p-value is less than 0.05, we reject the null hypothesis and conclude that smoking is not independent of gender.

Test for r×c Tables (Generalization)

What are r×c Tables ??

- r×c tables are contingency tables with more than two categories in both rows and columns. For example, a table with 3 rows and 3 columns would have 9 cells
- Example: Testing if there's an association between educational level (High School, Bachelor, Master's) and employment status (Employed, Unemployed).

Why Use Exact Tests in r×c Tables ??

- Generalization: Exact tests (e.g., Fisher's Exact Test) can be extended to larger r×c tables, but computational complexity increases.
- Computational Challenges: Larger tables make p-value calculations more expensive, requiring tools like R or Python.
- When to Use: Exact tests work best for small r×c tables; for large datasets, Chi-square is more efficient

Fisher's Exact Test for Larger Tables

- Definition: Fisher's Exact Test can be extended to r×c tables, but as the table size increases, the computational complexity increases.
- Limitations:
  - For large r×c tables, computing the exact p-value becomes computationally impractical.
  - Fisher's Exact Test is rarely used for larger tables due to its computational burden.

When to Use:

- Fisher's Exact Test is best suited for small r×c tables where other tests like Chi-square fail due to small sample sizes or sparse data.

- It is also useful when Chi-square assumptions (such as large expected frequencies) are violated in larger tables

Comparison: Exact Tests vs. Chi-Square Test

- Chi-Square Test:
  - When to Use: Best for large sample sizes where expected frequencies are at least 5.
  - Assumptions: Assumes large sample sizes and expected frequencies > 5.
  - Limitations: Not suitable for small sample sizes or sparse data.
- Exact Tests:
  - When to Use: Ideal for small sample sizes or sparse data.
  - Advantages: No assumptions about sample size or expected frequencies.
  - Limitations: Computationally intensive for larger tables.
- Choosing the Right Test: If sample size is small or Chi-square assumptions are violated, use exact tests. For larger samples, Chi-square is more efficient.

Advantages of Exact Tests

- Accuracy: Provides exact p-values, unlike approximations used in Chi-square.
- No Sample Size Assumptions: Works well even with small datasets.
- Versatility: Can be applied to both 2×2 and r×c tables.
- Real-World Example: Used in clinical trials where small sample sizes make approximations less reliable.
- Precise Control Over Type I and Type II Errors

Applications of Exact Tests

- Medicine: Clinical trials with small sample sizes.
    - Example: Testing if a new drug affects patients differently based on gender or age.
- Education: Comparing student performance with different teaching methods in small cohorts.
    - Example: Testing if the teaching method impacts the scores in a small class.
- Social Sciences: Analyzing voting patterns or preferences in small survey samples.
    - Example: Testing if voter preferences are independent of geographical regions.

Assumptions and Limitations of Exact Tests

Assumptions:

- Independence: Each observation should be independent.
- Small sample sizes: Ideal for datasets with low expected frequencies (e.g., fewer than 5 counts).

Limitations:

- Computational Complexity: As table size increases, exact tests become difficult to compute manually and computationally intensive.
- Efficiency: For large datasets, approximate methods like Chi-square are more efficient.

Conclusion

- Summary:
  - Exact tests (like Fisher's Exact Test) are crucial for small datasets or when Chi-square assumptions don't hold.
  - Fisher's Exact Test is the go-to method for 2×2 tables, but it can also be extended to r×c tables with limitations in computation.
  - Choose Exact Tests when sample sizes are small or when other methods fail.

- Final Thought: Exact tests provide a robust solution for analyzing categorical data when traditional methods are not feasible due to small sample sizes or other issues.

Any Question???


Thanks
to
Everyone