PABNA UNIVERSITY OF SCIENCE & TECHNOLOGY

ASSIGNMENT

on

Test of single mean and single variance

Department of

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1. Introduction

In statistics, hypothesis testing is a core component of data analysis. It helps us make decisions or inferences about population parameters based on sample data. Two common types of tests are the **Test of Single Mean** and the **Test of Single Variance**. The **Test of Single Mean** is used when we want to determine if a population mean is equal to a hypothesized value, while the **Test of Single Variance** assesses whether a population's variance equals a specified value. These tests are foundational in inferential statistics, allowing for evidence-based conclusions.

In both cases, we formulate hypotheses, calculate test statistics, and compare them to critical values to decide whether to reject or fail to reject the null hypothesis.

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2. Test of Single Mean

2.1 Definition

A **Test of Single Mean** is used to determine whether the sample mean ($\bar{x}$) differs significantly from a hypothesized population mean ($\mu 0$).

2.2 Types of Tests

There are two common types of tests for the single mean:

- **Z-test**: When the population standard deviation (σ) is known, we use the Z-test. This test is applicable for large sample sizes ($n > 30$).

- **t-test**: When the population standard deviation is unknown, we use the t-test, especially for smaller sample sizes ($n \leq 30$).

The choice between the Z-test and t-test depends on whether we have information about the population standard deviation and the sample size.

2.3 Hypotheses for the Test of Single Mean

In a Test of Single Mean, we define two hypotheses:

- **Null Hypothesis (H_0):** The population mean is equal to a specific value.

$$H^{0}: \mu = \mu 0$$

- **Alternative Hypothesis (H_1):** The population mean is not equal to the hypothesized value (two-tailed test), or it is greater than or less than the hypothesized value (one-tailed test).

$$H_{1}: \mu \neq \mu 0 (two - tailed)$$

or

$$H_{1}: \mu > \mu 0 (right - tailed)$$

or

$$H_{1}: \mu < \mu 0 (left - tailed)$$

2.4 Test Statistic

The test statistic for both Z-test and t-test is calculated as the difference between the sample mean ($\bar{x}$)and the population mean ($\mu 0$), divided by the standard error.

- **Z-test formula**:

$$Z = \frac{\bar{x} - \mu_0}{\frac{\sigma}{\sqrt{n}}}$$

where:

- x¯ = sample mean
- μ0 = hypothesized population mean
- σ = population standard deviation
- n = sample size

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

where:

- s = sample standard deviation (used when σ\sigmaσ is unknown)

2.5 Decision Rule

1. **Calculate the test statistic** (Z or t).
2. **Find the critical value** from the Z or t-distribution table based on the significance level (α) and degrees of freedom (for t-test).
3. **Compare the test statistic** to the critical value:
   - If the test statistic is **greater than** the critical value (or falls in the rejection region), **reject the null hypothesis**.
   - If the test statistic is **smaller than** the critical value, **fail to reject the null hypothesis**.

Alternatively, you can compute the **p-value** and:

- If the p-value is less than the significance level (α), reject the null hypothesis.
- If the p-value is greater than α, fail to reject the null hypothesis.

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3. Test of Single Variance

3.1 Definition

The **Test of Single Variance** is used to determine whether the variance of a population is equal to a hypothesized value. This test is important when examining the variability or consistency within a population.

3.2 Hypotheses for the Test of Single Variance

In the Test of Single Variance, the hypotheses are:

- **Null Hypothesis (H_0):** The population variance is equal to the hypothesized variance.

$$H_0: \sigma^2 = \sigma_0^2$$

- **Alternative Hypothesis (H_1):** The population variance is not equal to the hypothesized variance (two-tailed test), or it is greater than or less than the hypothesized variance (one-tailed test).

$$H_1: \sigma^2 \neq \sigma_0^2 (two - tailed)$$

or

$$H_1: \sigma^2 > \sigma_0^2 (right - tailed)$$

or

$$H_1: \sigma^2 < \sigma_0^2 (left - tailed)$$

3.3 Test Statistic

The test statistic for testing a single variance is the **Chi-square statistic**:

- **Chi-square formula**:

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

where:

- s^2 = sample variance
- σ_0^2 = hypothesized population variance
- n = sample size

3.4 Decision Rule

1. **Calculate the test statistic** $\chi 2$.
2. **Find the critical value** from the Chi-square distribution table based on the significance level (α) and degrees of freedom $(df = n - 1)$.
3. **Compare the test statistic** to the critical value:
   - If the test statistic is **greater than** the critical value or falls in the rejection region, **reject the null hypothesis**.
   - If the test statistic is **smaller than** the critical value, **fail to reject the null hypothesis**.

Alternatively, you can compute the **p-value** using the Chi-square distribution.

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4. Example 1: Test of Single Mean

Suppose you have a sample of 50 students' exam scores, and you want to test if the average exam score is 75. The sample mean is 78, the population standard deviation is 10, and the significance level is 0.05.

- **Null Hypothesis (H_0):** $\mu = 75$
- **Alternative Hypothesis (H_1):** $\mu \neq 75$

First, calculate the **Z-statistic**:

$$Z = \frac{78 - 75}{\frac{10}{\sqrt{50}}} = \frac{3}{\frac{10}{7.071}} = \frac{3}{1.414} = 2.12$$

At $\alpha = 0.05$, the critical Z-value for a two-tailed test is approximately 1.96. Since 2.12 > 1.96, we **reject the null hypothesis** and conclude that the average exam score is significantly different from 75.

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5. Example 2: Test of Single Variance

Suppose a factory claims that the variance in the diameter of machine parts is 0.04 cm². A sample of 30 parts is selected, and the sample variance is 0.06 cm². You want to test if the variance is significantly different from the claimed value, using a significance level of 0.05.

- **Null Hypothesis (H_0):** $\sigma^2 = 0.04$
- **Alternative Hypothesis (H_1):** $\sigma^2 \neq 0.04$

First, calculate the **Chi-square statistic**:

$$\chi^2 = \frac{(30 - 1) \times 0.06}{0.04} = \frac{29 \times 0.06}{0.04} = 43.5$$

With $df = 29$ and $\alpha = 0.05$, the critical values from the Chi-square distribution are approximately 16.95 and 41.57 (for a two-tailed test). Since 43.5 > 41.57, we **reject the null hypothesis**, suggesting that the variance is significantly different from 0.04 cm^2.

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6. Conclusion

The **Test of Single Mean** and **Test of Single Variance** are important tools in inferential statistics, allowing researchers to draw conclusions about population parameters based on sample data. By performing hypothesis tests, we can assess the validity of claims about population characteristics, such as the mean or variance. These tests help ensure that our decisions are based on statistical evidence, enabling data-driven conclusions.