Department of ***Information & Communication Engineering***

Course Code: STAT-2201
Course Title: Engineering Statistics

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Session: 2021-22

2nd Year 2nd Semester

Dept. of Information & Communication Engineering

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Date of Submission: 19.04.2025

Three-way contingency tables, Large Sample Test of Significance.

Three-Way Contingency Tables:

- A **three-way contingency table** is a type of table used in statistics to summarize the relationship between **three categorical variables**. It extends the idea of a two-way table (which examines the relationship between two variables) by adding a third dimension.
- When we perform a **Large Sample Test of Significance** for a three-way contingency table, we're usually testing hypotheses about **independence or interaction** among the variables. The most common method for this is the **Chi-Square Test of Independence** extended to three dimensions.

Basic Structure:

Let's say we have three variables:

1. **A: Machine Type** → M1, M2
2. **B: Operator** → O1, O2
3. **C: Defect Type** → D1, D2

Or,

- **A** with I levels (e.g., Gender: Male, Female)
- **B** with J levels (e.g., Education: High School, College, Graduate)
- **C** with K levels (e.g., Income: Low, Medium, High)
- The data is arranged in a 3-dimensional array: Oijk = observed frequency for level i of A, level j of B, and level k of C.

Hypotheses We Can Test

Here are some typical hypotheses:

1. Complete Independence

All three variables are mutually independent:

$H_0: P(A=i, B=j, C=k) = P(A=i) \cdot P(B=j) \cdot P(C=k)$

2. Conditional Independence

Two variables are independent **given** the third:

$H_0: A \perp B \mid C$ (A and B are conditionally independent given C)

This is especially important in **log-linear modeling** and **causal inference**.

3. No Three-Way Interaction

- Tests whether the interaction among the three variables is only due to two-way interactions.

Objective of Large Sample Test of Significance

The goal is to test **independence** or **interaction** among variables using a **Chi-Square Test** when you have a **large sample size**.

Common Hypotheses:

1. **Complete Independence**:
    1. All three variables are independent:
    2. Ho:$P(A,B,C)=P(A)\cdot P(B)\cdot P(C)$
2. **Partial/Conditional Independence**:
    1. For example:
    2. Ho:$A \perp B \mid C \rightarrow$ A and B are independent **given** C.
3. **No Three-Way Interaction**:
    1. Only pairwise relationships exist; no higher-order interaction.

Large Sample Test: Chi-Square Approach

To conduct the test:

1. Calculate expected frequencies Ei under the null hypothesis (independence model).

2. Use the Chi-Square test statistic:

$$\chi^2 = \sum_{i=1}^{i} \frac{(Oi - Ei)^2}{Ei}$$

3. Degrees of Freedom:

For testing A⊥B|C

df=(I−1)(J−1)K

Where:

- I: levels of A
- J: levels of B
- K: levels of C

5. Decision:

- Compare the test statistic with the **Chi-Square distribution** at your chosen alpha level (e.g., 0.05). If p-value < α, **reject H_0**.

Example Use Cases

- Analyzing if gender and education level are associated differently across income groups.
- Understanding relationships between treatment, outcome, and hospital site in clinical studies.