Presentation on Estimation

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Introduction:

Estimation is the process of approximation an unknown population parameter using sample data. Since collecting data from an entire population is often impractical, estimation provides a way to make informed decisions using limited information. It helps in making predictions and decisions based on data analysis.

Estimation is important as used in real-world application like economics, medicine, machine learning and research. It helps in decision-making with incomplete data

The estimation can be divided into two types:

1. Point estimation
2. Interval estimation

Point Estimation: Point estimation is the process of estimating an unknown parameter by a single value, derived from sample data. Example (sample mean).

Interval Estimation: Provides a range within which the parameter lies(e.g., Confidence level)

Explain Basic concept of estimation with Example

What is estimator?

An estimator is a statistical function or rule used to estimate an unknown parameter based on sample data . It is a formula that helps approximate values like mean, variance or proportion when we cannot measure the entire population.

Example :

Sample Mean ($\bar{X}$) as an estimator of Population Mean (μ)

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} Xi$$

What is estimate?

A particular value of the estimator is called an estimate.

Example: The sample mean $\bar{X}$ = 7 is an estimate value of the estimator.

Difference between estimator and estimate

Estimator	Estimate
A formula or rule used to approximate a population parameter	A specific numerical value obtained using an estimator.
A function or statistical method	A single computed value from sample data
The sample mean formula($\bar{X} = \frac{1}{n}\sum_{i=1}^{n} Xi$) is an estimator of the population mean (μ)	If calculate $\bar{X}$ = 25 from a sample, then 25 is the estimate.
Used before collecting sample data	Obtained after applying the estimator to sample data

Criteria of a good estimators

1. Unbiasedness
2. Consistency
3. EfficiencyType equation here.
4. Sufficiency

Explain unbiasedness estimates with example

what is unbiasedness?

Any statistics whose mathematical expectation is equal to a parameter θ is called an unbiased estimator of the parameter θ . Otherwise the statistics is said to be biased.

Let Tn be a statistic calculated from a sample (x1,x2………,xn) of size n from density f(x|θ).
If for all n and $\theta, is\ E(Tn) = \theta$.

For example , if random sample (x1,x2,……..,xn) of size n is drawn from a normal distribution population with mean θ and variance ∂^2.

Then

$E(\overline{X}) = \frac{1}{n} E(x1+x2+x3+\ldots\ldots+xn)$

$= \frac{1}{n}[E(x1) + E(x2) + E(x3) + \ldots..+E(xn)]$

$= \frac{1}{n}.\ n\theta$

$E(\overline{X}) = \theta$

And

$E(s^2) = \frac{1}{n-1} E[\sum(xi - \bar{x})^2]$

$= \frac{\partial^2}{(n-1)} E[\frac{\sum(xi-\bar{x})^2}{\partial^2}]$

$= \frac{\partial^2}{(n-1)} E(x^2{}_{n-1})$

$= \partial^2(n-1)^{-1}.\ (n\text{-}1)$

$= \partial^2$

Thus $\bar{X}$ and s^2 are an unbiased estimator of θ and ∂^2 respectively .

Math Example:

The following observations constitute a random sample from an unknown population. Estimate the mean and standard deviation of the population. Also, find the estimate of standard error of sample mean.

14, 19, 17, 20, 25.

Solution:

The unbiased estimators of the population mean (μ) and the population variance (δ^2) are $\bar{x}= \sum xi$ / n and $s^2= \sum(xi - \bar{x})^2$/ (n-1) respectively. Here n = 5 and $\bar{x}=\sum xi$ / n = 95/5 = 19

$\sum(xi - \bar{x})^2$= (- 5) ^ 2 + 0^ 2 + (- 2) ^ 2 + 1 ^ 2 + 6 ^ 2 = 66

thus, s ^ 2= 66/4 = 16.5

s = sqrt(16.5) = 4.06

The estimates of μ and δ are 19 and 4.06 respectively.

The standard error of sample mean is S.E.($\bar{x}$) = $\delta/\sqrt{n}$. But as δ is not known, it is estimated by s,

Estimate of SE, ($\bar{x}$) = s/$\sqrt{n}$ = $\frac{\sqrt{66/4}}{\sqrt{5}} = \sqrt{\frac{66}{20}} = \sqrt{3.3}$ = 1.82

Explain consistent estimates with example

An estimator is consistent if it is converges to the true parameter value as the sample size increases. In others words, the probability of the estimator being close to the actual parameter increases with sample size.

Let Tn be a statistics calculated from a sample size from density f(x|θ) .
If p[Tn – θ <∈] = 1- δ, n→ ∞
where ∈ and δ are arbitrary small positive numbers then Tn is called a consistent estimator of θ

for example, if x1,x2,……xn is a random sample from a population with finite mean E(xi)= μ<∞

Now, we have $\bar{X} = n^{-1} \sum_{i=1}^{n} xi$

$$E(\bar{X}) = \frac{1}{n} E(x1+x2+x3+……+xn)$$
$$= \frac{1}{n} [E(x1) + E(x2) + E(x3) + …..+E(xn)]$$
$$= \frac{1}{n}.\ n\mu$$
$$E(\bar{X}) = \mu,,\ n \to \infty$$

Hence sample mean $\bar{X}$ is always a consistent estimator of the population mean μ

Comparison between Consistent and Unbiased Estimators

Consistent Estimator	Unbiased Estimator
Converges to true value as sample size increases	Expected value equals true value
Good for large datasets	Works for any sample size
May be biased for small samples	May not be consistent in small data
Example : Sample variance	Example: Sample Mean

Conclusion

Understanding the concepts of estimation, consistency, and unbiasedness is crucial for making informed decisions based on data. These principles are widely used in fields like economics, healthcare, engineering, and research, ensuring precise and dependable statistical analysis.


Thank you!