Pabna University Of Science And Technology

Department of Information And Communication Engineering

Assignment

Course Name: Engineering Statistics
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Name of the Topic: Comparison of two sample Means.

The **comparison of two sample means** is a statistical technique used to determine whether there is a significant difference between the average values (means) of two independent groups or samples. This is a core concept in inferential statistics, commonly used in experiments, surveys, and observational studies.

Sample Mean:

The **sample mean** is the average value of a sample — a subset of a population. It's a point estimate of the population mean and is calculated as:

$$\bar{x} = \frac{\sum x_i}{n}$$

Where:

- $\bar{x}$ = sample mean
- x_i = each value in the sample
- n = number of observations in the sample

Types of Two-Sample Mean Comparisons

There are two major cases:

1. Independent Samples

When the two samples are unrelated (e.g., group A and group B are made up of different individuals).

Test Used: Two-Sample (Independent) t-Test

This tests the null hypothesis:

$$H_0 : u_1 = u_2$$

Versus an alternative hypothesis (one-sided or two-sided):

$H_1 : u_1 \neq u_2$ (two-tailed)

$H_1 : u_1 > u_2$ or $u_1 < u_2$ (one-tailed)

Assumptions:

- Data are normally distributed (especially for small samples)
- Samples are independent
- Variances can be equal (pooled t-test) or unequal (Welch's t-test)
- Test Statistic:

If variances are equal:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Where s_p is the pooled standard deviation:

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

If variances are unequal (Welch's t-test):

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Degrees of freedom (approximate) are calculated using the **Welch–Satterthwaite equation**.

2. Paired Samples

When the same individuals are measured twice (e.g., before and after treatment) or matched pairs.

Test Used: Paired t-Test

It compares the **mean of the differences** between paired observations.

$$H_0: \mu_d = 0 \qquad \text{(no difference)}$$

$$t = \frac{\bar{d}}{s_d/\sqrt{n}}$$

Where:

- $\bar{d}$ = mean of the differences
- S_d = standard deviation of the differences
- n = number of pairs

Interpreting Results

- If the **p-value** < significance level (e.g., 0.05), reject the null hypothesis
- This suggests that the difference in means is **statistically significant**.
- A **confidence interval** (typically 95%) can also show the range of possible differences in means.

Example (Independent Samples)

A researcher wants to know if there's a difference in test scores between students in two different schools.

Sample A (n = 30): Mean = 75, SD = 10
Sample B (n = 30): Mean = 80, SD = 12

Using a two-sample t-test, the researcher can determine if the difference in means (80 - 75 = 5) is statistically significant or due to random variation.