Signal

Signal: A signal is defined as any physical quantity that varies with time, space, or any other independent variable or variables. Mathematically, we describe a signal as a function of one or more independent variables. Signal serves as carriers of information between communication devices. They can convey different types of information depending on the application required. For example, the functions

$$S_1(t) = 5t$$

$$S_2(t) = 20t^2$$

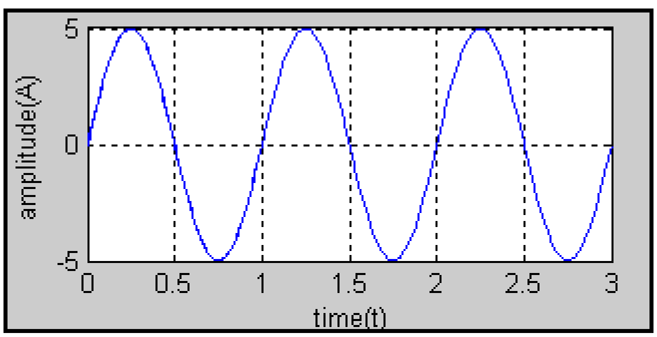
$$S_3(t) = sin(t) + sin(2t) + sin(3t)$$

where t is independent variable and $S_1(t)$, $S_2(t)$, $S_3(t)$ are dependent variable.

Another example, consider the function

$$s(x, y) = 3x + 2xy + 10y^2$$

The function describes a signal of two independent variables x and y that could represent the two spatial coordinates in a plan.



Examples of Signals

- Human voice and sound waves.
- Voltage in electrical circuits
- Room temperature controlled by a thermostat system
- Position, speed, and acceleration of an aircraft
- Force measured with force sensors in robotic systems
- Electromagnetic waves used to transmit information in wireless computer networks
- Digital photographs
- Digital Music Recording.

Types of Signals

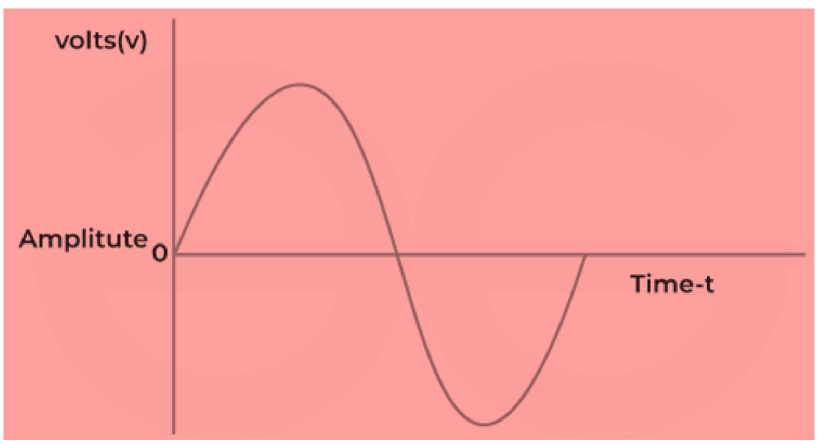
There are different types of signals which are given below:

- Analog Signals
- Digital Signals
- Real and Complex Signals
- Deterministic and Random Signals
- Periodic and Non-periodic Signals

1. Analog Signals

These signals are continuing (e.g., a real variable) and infinitely varying with time parameter or can take any value within a given range. These signals are represented by the sine wave. Examples of analog signals are audio signals, temperature readings, sound waves or television waves.

Example of analog signals are voice, speed, pressure, temperature, etc.



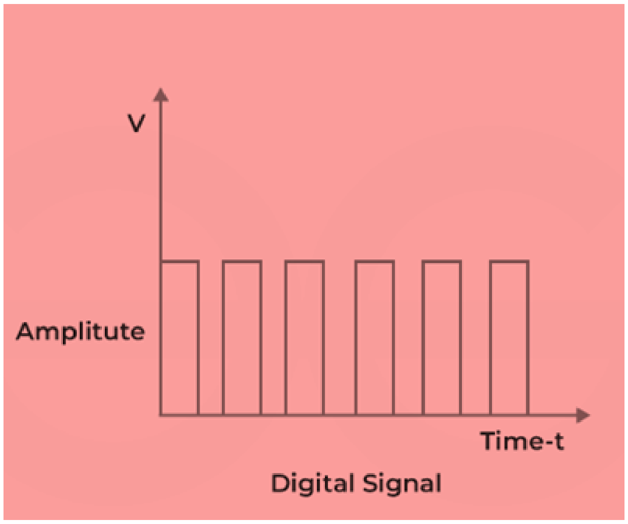
Properties of Analog Signal

The following are main properties of analog signals −

- Analog signals are continuous signals in both amplitude and time.
- Analog signals have a certain value or magnitude at any given instant of time.
- Analog signals have infinite resolution.
- Analog signals are best suited for representing the real-world phenomena.
- Analog signals are represented by the continuously varying smooth waveforms.

2. Digital Signals

A signal that is a function of discrete variables (e.g., an integer variable) is said to be discrete time and this are represent in binary form (0s and 1s). More robust against noise. Commonly used in computer systems and telecommunications.



Properties of Digital Signal

The following are some key characteristics of digital signals −

- Digital signals have discrete or discontinuous values in terms of both amplitude and time.
- Digital signals do not have values defined between any two distinct instants of time.
- Digital signals are represented using binary system by sampling the values of the signals at specific time instants.
- Digital signals represent information in the form of a sequence of binary 0s and 1s.
- Digital signals have a finite resolution.
- Digital signals are capable to perform logical operations.
- Digital signals are more efficient and reliable when it comes to storage and transmission.

Difference between Analog and Digital Signals

Let us now discuss the important differences between analog and digital signals −

Key	Analog Signals	Digital Signals
Representation	Analog signals are represented as continuous functions or waveforms of time.	Digital signals are represented as discrete functions of time.
Nature	Analog signals are continuous as they have infinite values within a specified range.	Digital signals are discontinuous as they have distinct values sampled at specific time instants.
Resolution	Analog signals have infinite resolution.	Digital signals have a finite resolution.
Accuracy	Analog signals are more accurate.	Digital signals are relatively less accurate.
Storage	Analog signals are difficult to store.	Digital signals are efficient to store.
Noise immunity	Analog signals are less immune to noise.	Digital signals have high immunity against noise.
Examples	Voice signals, temperature, speed, etc.	Data transmitted over internet, computer generated signals, etc.

Applications of Signals

Some of the applications of signal are listed below:

- These are mainly used in telecommunication industry for transmitting data and information through signals.
- Signals are used in satellite communication and radar systems to detect or communicate to various objects like planes and satellites.
- Signals also finds their application in medical field for various purposes like ECG tests, ultrasound and CT scan.
- Signals are also used in weather forecasting and also in other monitoring techniques like checking pollution levels, estimating temperature and humidity of the day, etc.
- Media channels broadcasting through televisions and radios takes place by via signals.

3. Real and Complex Signals

If the value of the signal x(t) is a real number, the signal x(t) is also a real signal; If the value of the signal x(t) is a complex number, the signal x(t) is complex signal. In general, the complex signal x(t) is a function of the form

x(t)= x1(t) + jx2(t)

where x1(t) and x2(t) are real symbols, j is complex.
The above equation represents a constant variable or the difference between the two.

4. Deterministic and Random Signals

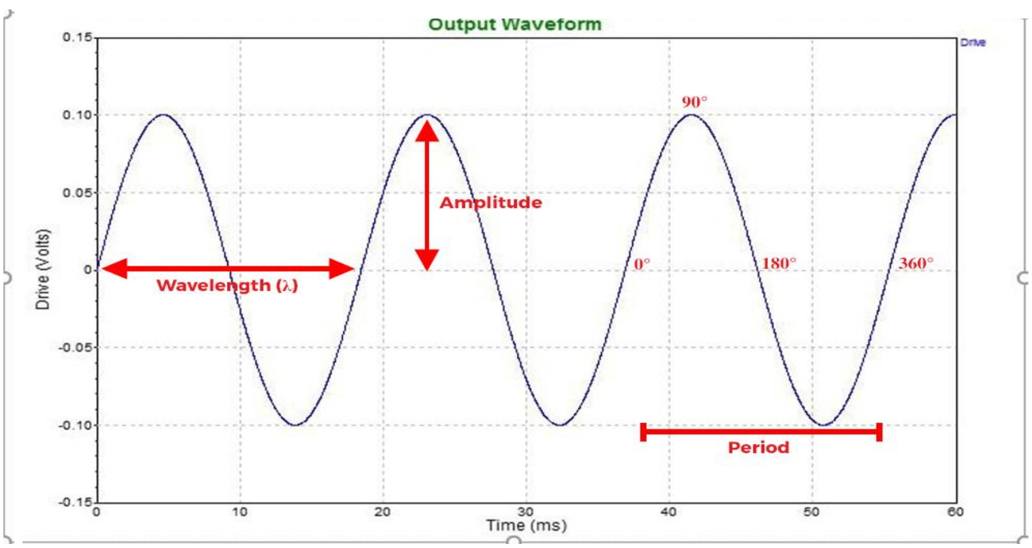
What are Deterministic Signals?

Deterministic signals are those that can be completely predicted and described by a mathematical equation or rule. They have a fixed pattern and behave consistently over time.

Another definition is, a deterministic signal is a signal that has no uncertainty with respect to its value at any instant of time.

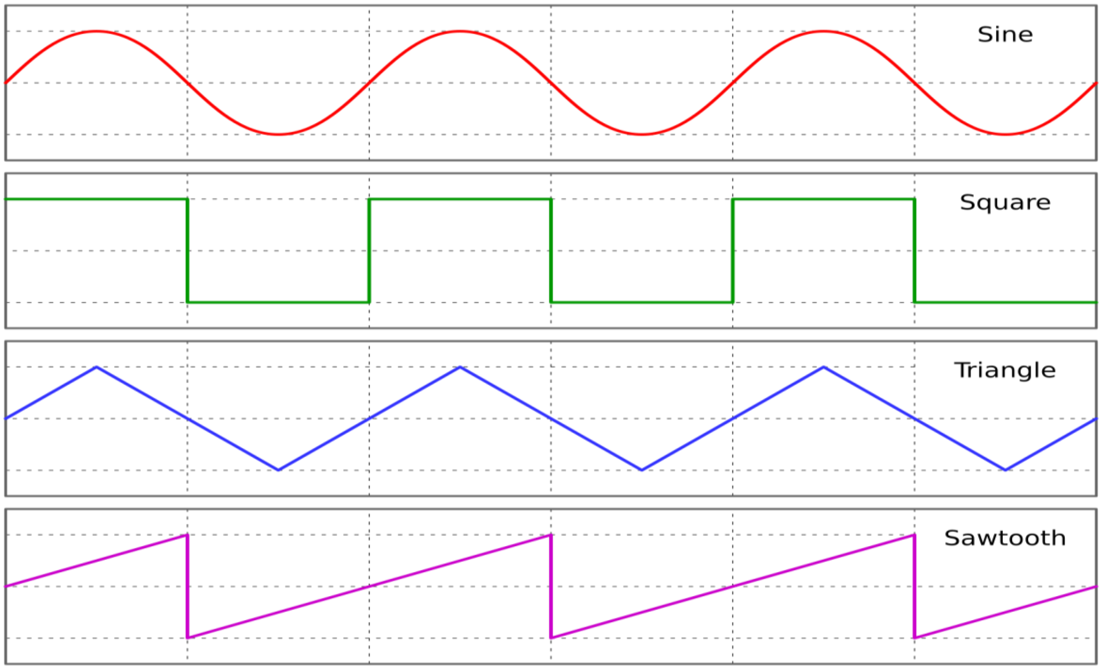
Examples of deterministic signals:

- Sine wave: A fundamental waveform with a smooth, periodic oscillation.



Sine Wave - Credit: VibrationResearch

- Square wave: A periodic signal with abrupt transitions between high and low values.



Square wave and other waves - Credit: Wikipedia

- Music signal: When played repeatedly, the music follows a specific pattern and can be precisely reproduced.

What are Random Signals?

Random signals also known as non-deterministic signals, on the other hand, are unpredictable and exhibit randomness in their values.

They cannot be precisely described by a single equation but are characterized by their statistical properties like mean, variance, and probability distribution.

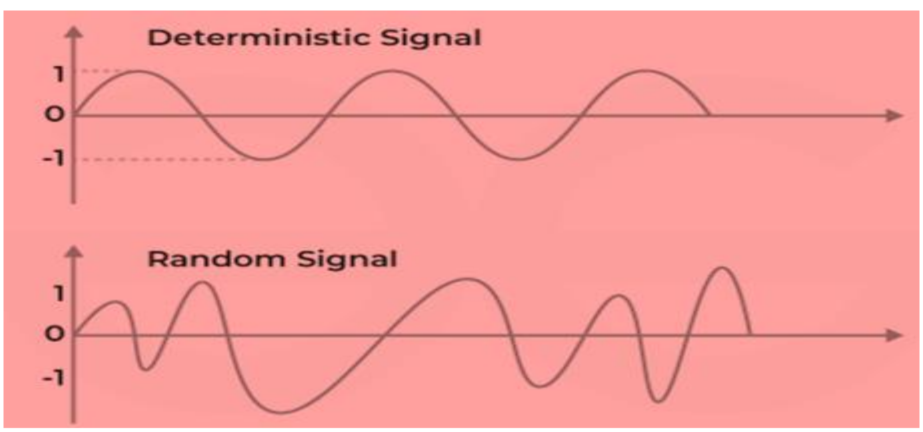


Fig. Deterministic and Random Signal Graph

Examples of random signals:

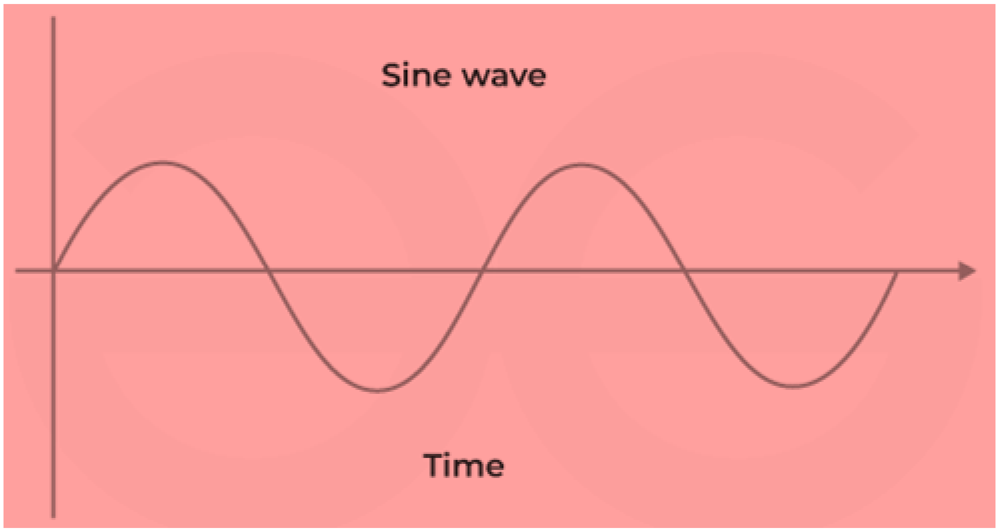
- Noise: Unwanted electrical or acoustic signals with unpredictable fluctuations.
- Thermal fluctuations: Tiny, random variations in temperature due to the movement of atoms and molecules.
- ECG signal: The electrical activity of the heart, exhibiting some randomness due to biological variations.

ifferences between Deterministic and Random Signals

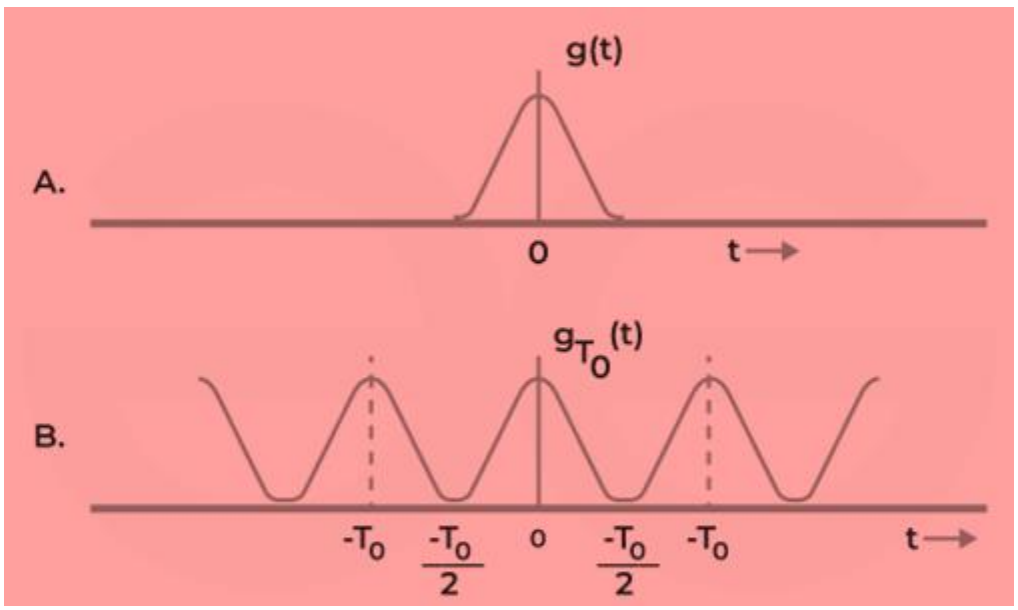
	Deterministic Signals	Random Signals
Predictability	Completely predictable	Unpredictable
Description	Defined by a mathematical equation or rule	Characterized by statistical properties
Pattern	Fixed and consistent	Random and irregular
Examples	Sine wave, square wave, music signal (when played repeatedly)	Noise, thermal fluctuations, ECG signal

5. Periodic and Non-Periodic Signals

A continuous signal is a signal of infinite duration that repeats the same pattern over and over again is called periodic signal. One-sided or time-limited signals can never be periodic. Any continuous-time signal which is not periodic signal is known as non-periodic (or aperiodic) signal.



Periodic Signal Graph



Aperiodic signal Graph

Signal Parameters

Some of the signal parameters are:

- Amplitude
- Frequency
- Phase
- Wavelength

2.2.1 Continuous-time Signal and Discrete-time Signal

Signal can be represented either by continuous or discrete values.

Continuous-time signal A signal $x(t)$ is said to be a continuous-time signal if it is defined for all time t. The amplitude of the signal varies continuously with time. In general, all signals by nature are continuous-time signals.

The speech signal is a continuous-time signal, that is, conversation between persons is continuous with respect to time (Fig. 2.6a).

Discrete-time signal Most of the signals that are obtained from their sources are continuous in time. Such signals have to be discretised since the processing done on the digital computer is digital in nature. A signal $x(n)$ is said to be discrete-time signal if it can be defined for a discrete instant of time (say n). For a discrete-time signal, the amplitude of the signal varies at every discrete value n, which is generally uniformly spaced. A discrete-time signal $x(n)$ is often obtained by sampling the continuous-time signal $x(t)$ at a uniform or nonuniform rate. The discrete-time representation of speech signal, electrocardiogram and sinusoidal signal is shown in Fig. 2.6(b), 2.7(b) and 2.8(b) respectively.

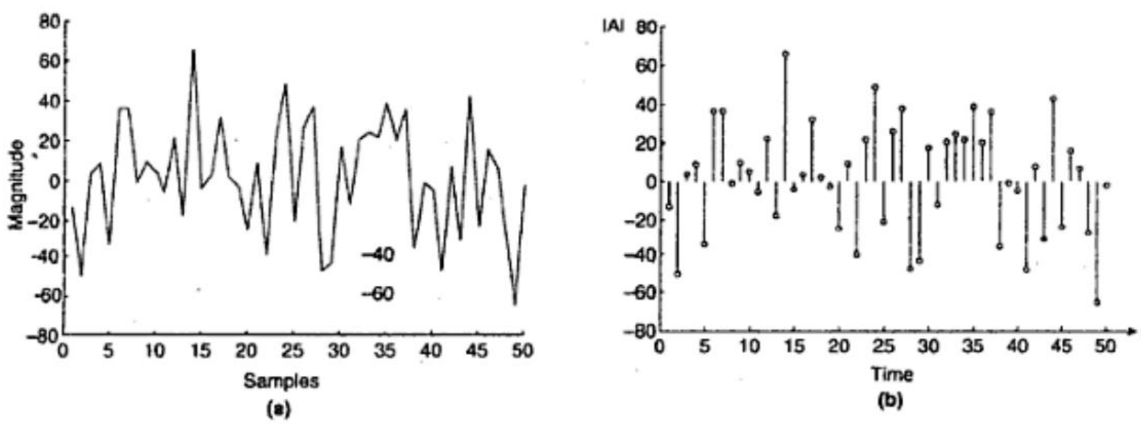


***Fig. 2.6** (a) Continuous-time Signal Representation of Speech Signal*
(b) Discrete-time Signal Representation of Speech Signal

The electrocardiogram, which is the electrical representation of the cardiac muscle, is continuous with respect to time (Fig. 2.7(a)).

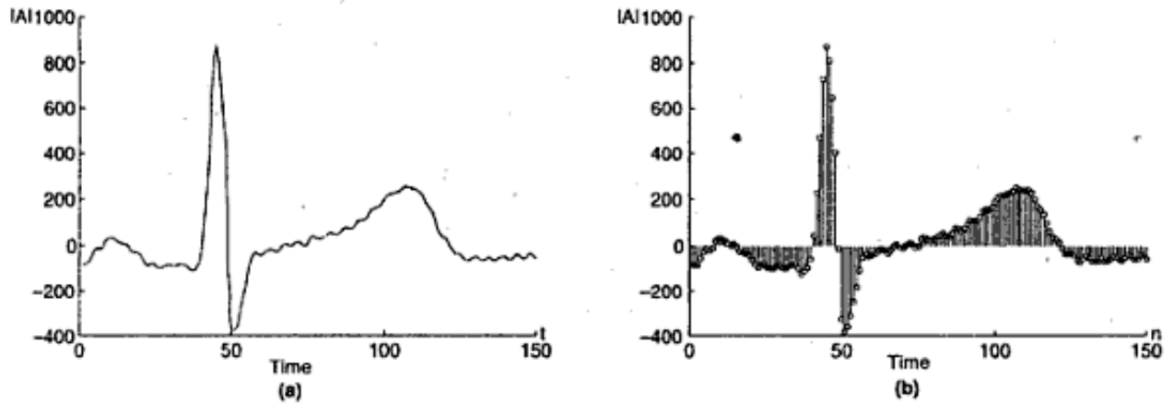


Fig. 2.7 (a) Continuous-time Signal Representation of Electrocardiogram
(b) Discrete-time Signal Representation of Electrocardiogram

- **Define periodic and aperiodic signal.**

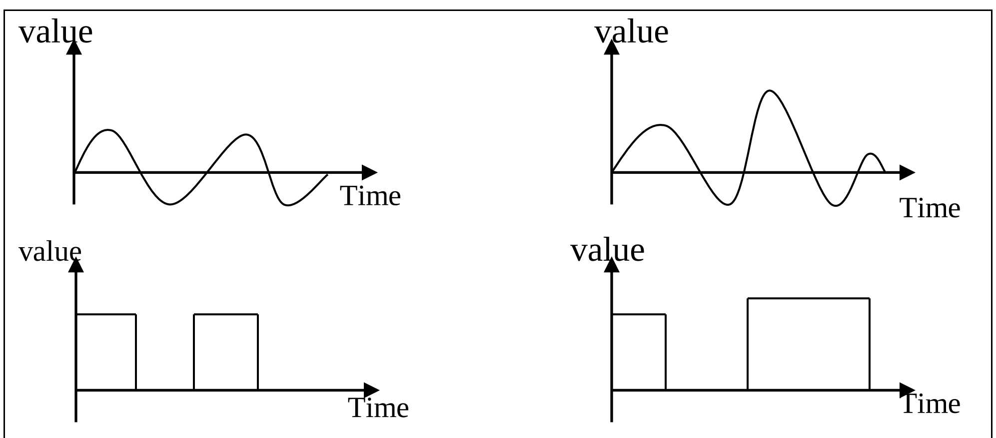
A signal completes a pattern within a measurable time frame and repeats that pattern over subsequent identical periods. This signal is called **periodic signal**.

A continuous-time signal $x(t)$ is said to be periodic if

$$x(t) = x(t + T), \ T > 0$$

for all values of t,

where T = period of a cycle, which is an integer value



a. Periodic Signal   b. Aperiodic Signal

Figure 02. Periodic & Aperiodic Signal

A signal, it changes without exhibiting a pattern or cycle that repeats over time, called **aperiodic signal**.

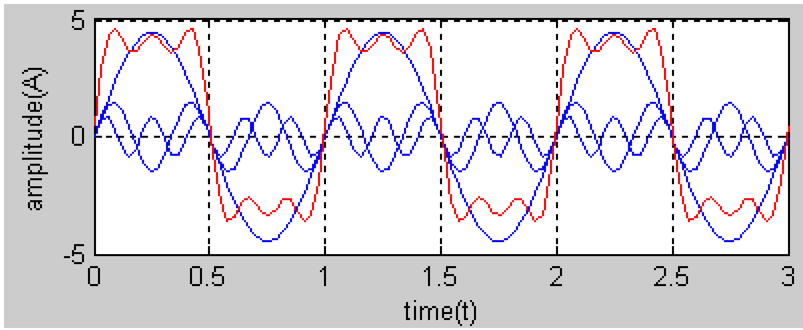


Fig 2.2(d): composite signal with several harmonics

Problem 2.3 Test whether the given signals are periodic or not.

(i) $x(t) = e^{\sin(t)}$ (ii) $x(t) = te^{\sin(t)}$

Solution

(i) $x(t) = e^{\sin(t)}$

From the definition of periodicity, $x(t) = x(t + T)$ for $T > 0$

Substitute $t = (t + T)$,

$$x(t+T) = e^{\sin(t+T)}$$

Since $T = 2\pi$, $\sin(t + T) = \sin(t + 2\pi) = \sin(t)$

Therefore, $x(t+T) = e^{\sin(t+T)} = e^{\sin(t)} = x(t)$

Hence, the signal $x(t) = e^{\sin(t)}$ is periodic.

(ii) $x(t) = te^{\sin(t)}$

From the definition of periodicity, $x(t) = x(t+T)$ for $T > 0$

Substitute $t = (t + T)$,

$$x(t+T) = (t+T)\, e^{\sin(t+T)}$$

Since $T = 2\pi$, $\sin(t + T) = \sin(t + 2\pi) = \sin(t)$

Therefore, $x(t + T) = (t + T)\, e^{\sin(t+T)} = (t + T)e^{\sin(t)} \neq x(t)$

Hence, the signal $x(t) = te^{\sin(t)}$ is aperiodic.

- **Define period and frequency. What is the relationship between period and frequency?**

The amount of time a signal needs to complete one cycle, in seconds, is called **period**.

The number of periods in one second is called **frequency**.

Frequency and **period** are inverses of each other.

$f = \frac{1}{T}$ and $T = \frac{1}{f}$

Where, f = frequency in Hz, and
T = period in second

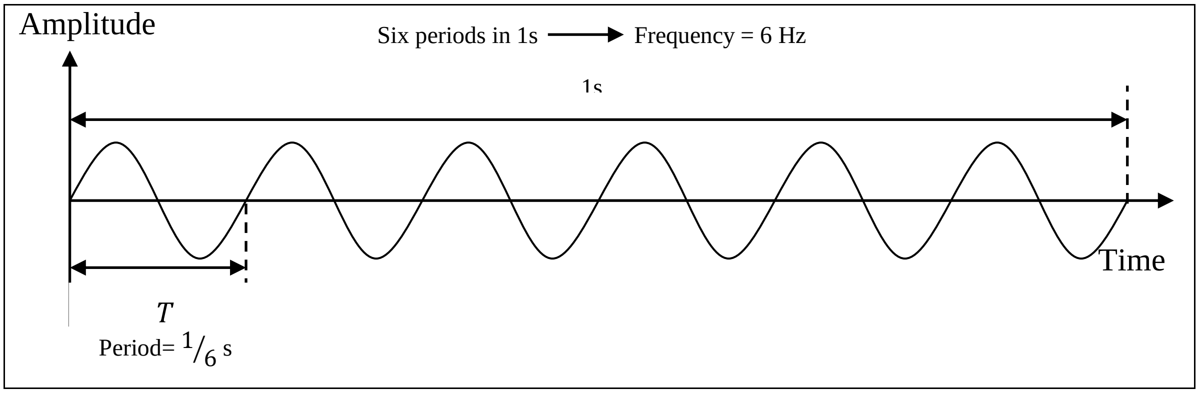


Figure 05. The concept of period and frequency

- **What do you mean by phase?**

The **phase** describes the position of the waveform relative to the time zero. If we think of the wave as something that can be shifted backward or forward *along the time axis*, **phase** describes the amount of that shift.

Phase is measured in degrees or radians. As shown in figure 06,

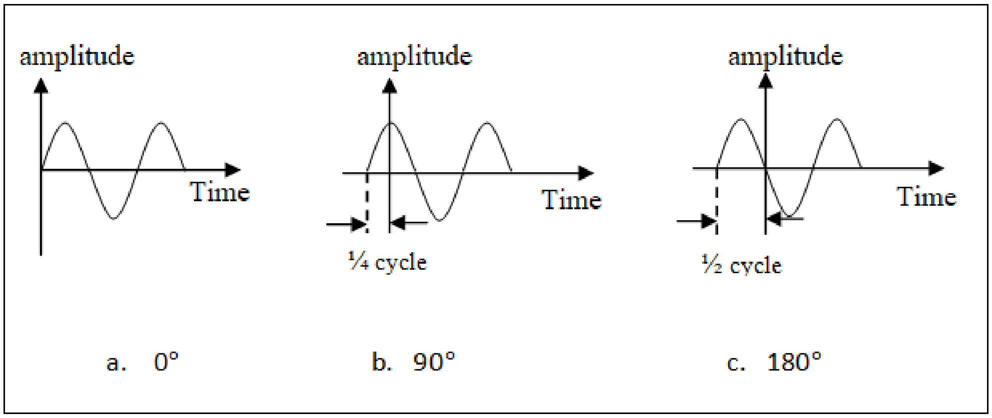


Figure 06. Relationships between different phases

A **phase** shift of 0° corresponds to no shift of a period; a **phase** shift of 90° corresponds to a shift of one quarter of a period; a **phase** shift of 180° corresponds to a shift of one-half of a period.

- **What is time domain and frequency domain? Give example.**

The **time domain** is instantaneous amplitude with respect to time. As shown in figure 07,

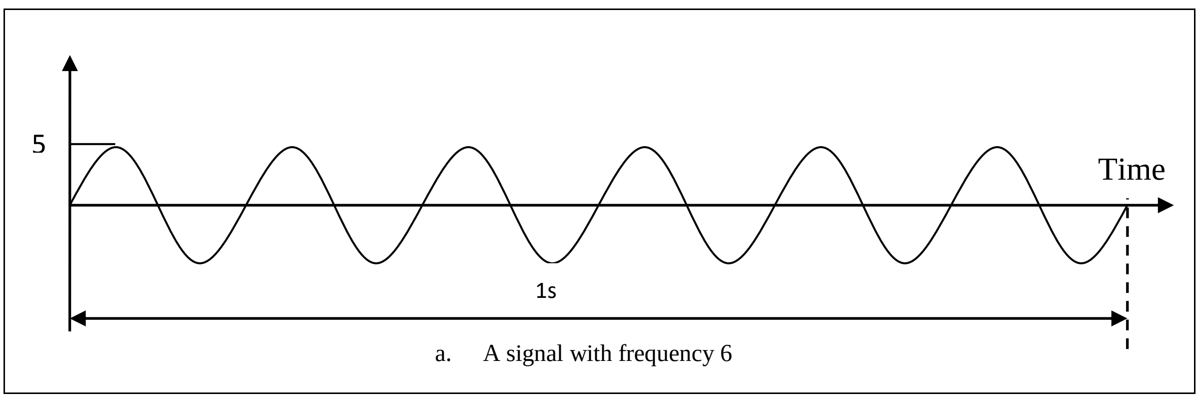
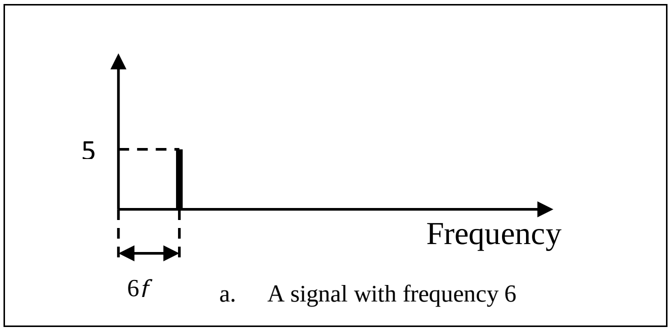


Figure 07. Time domain

The **frequency domain** is peak amplitude with respect to frequency. As shown in figure 08,



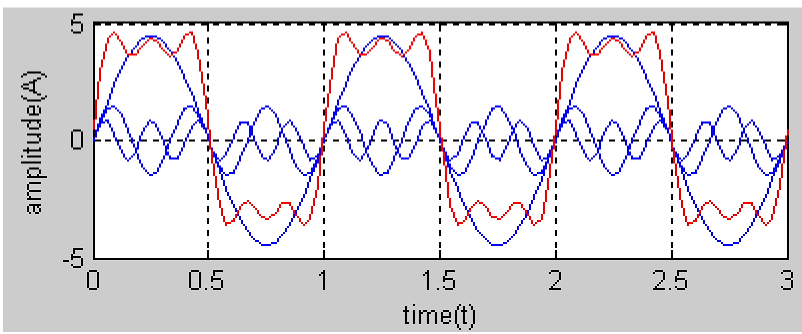
a. A signal with frequency 6

Figure 08. Frequency domain

- **What is composite signal?**

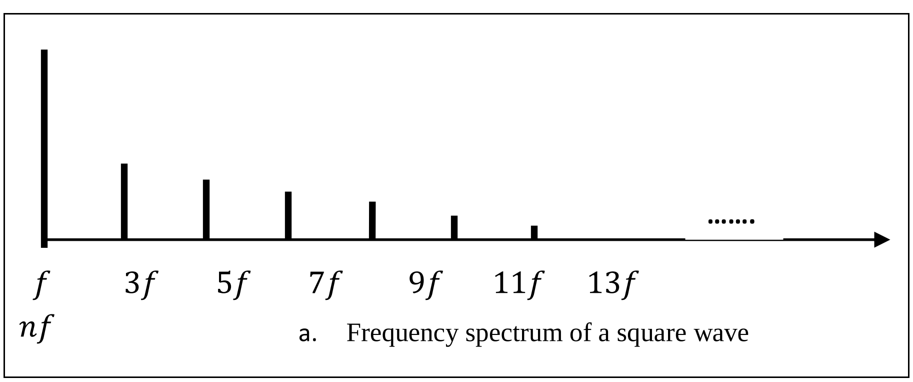
A signal, in which more than one frequency exists, called **composite signal**.

When we change one or more characteristics of a signal-frequency signal, it becomes a **composite signal** made of many frequencies.



- **What is the spectrum of a signal?**

The description of a signal using the frequency domain and containing all its components is called **frequency spectrum** of that signal. For *example*, figure 09 shows the **frequency spectrum** of a square wave;



a. Frequency spectrum of a square wave

Figure 09. Frequency spectrum

➢ What is bandwidth?

The range of frequencies that a medium can pass is called its **bandwidth**. The **bandwidth** is a property of medium. It is the difference between the highest and the lowest frequencies that the medium can satisfactorily pass.

$$B = f_h - f_l$$

Where, B = bandwidth
f_h = highest frequency, and
f_l = lowest frequency

Example:

If a periodic signal is decomposed into five sine waves with frequencies of 100, 300, 500, 700, and 900 Hz, what is the *bandwidth*? Draw the *spectrum.* Assuming all components have a maximum amplitude of 10 V.

<u>*Solution:*</u>

Let f_h be the highest frequency, f_l the lowest frequency, and B the bandwidth. Then

$$B = f_h - f_l = 900 - 100 = 800\ Hz$$

The spectrum has only five spikes, at 100, 300, 500, 700 and 900. As shown in figure 10;

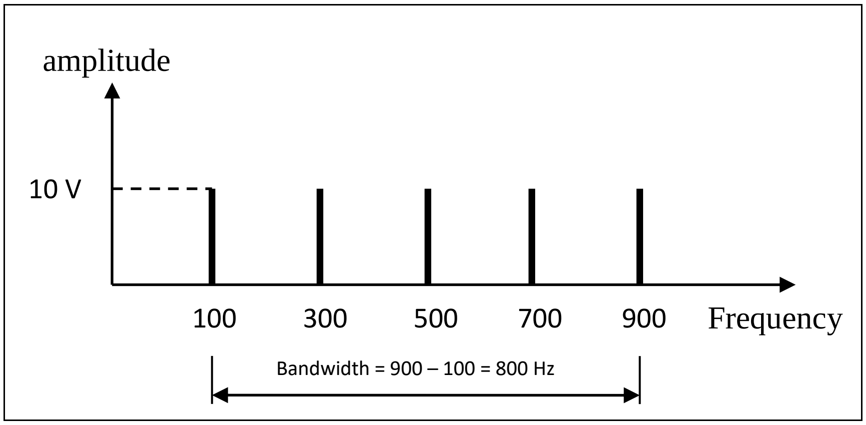


Figure 10. Spectrum

➢ What is the bit interval and bit rate?

The **bit interval** is the time required to send one single bit.

The **bit rate** is the number of *bit intervals* per second, usually expressed in *bit per second (bps).*

As shown in figure 11;

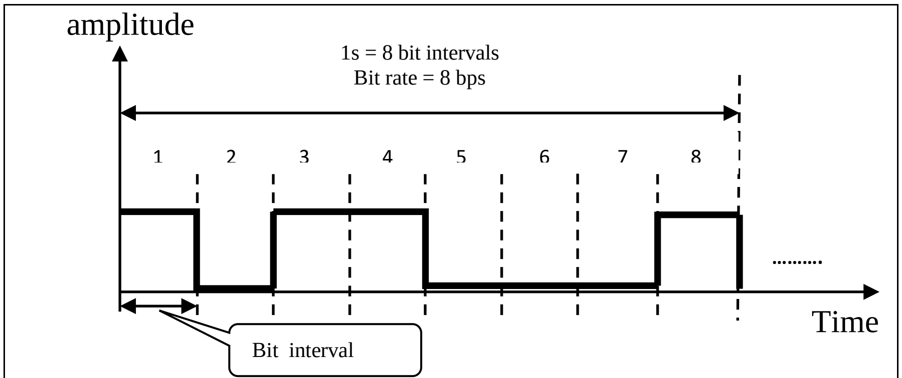


Figure 11. Bit rate and Bit interval

Example:

A digital signal has a bit rate of 2000 bps. What is duration of each bit(bit interval)?

Solution:

The bit interval is the inverse of the bit rate.

Bit interval = $\frac{1}{bit\ rate} = \frac{1}{2000} = 0.000500 \times 10^6 \mu s = 500\ \mu s$

- **What do you mean by low-pass and band-pass channel?**

A **low-pass channel** has a bandwidth with frequencies between 0 and f. The lower limit is 0, the upper limit can be any frequency (including infinity).

A **band-pass channel** has a bandwidth with frequencies between f_1 and f_2. Figure 12. shows the bandwidth of a **low-pass channel** and **band-pass channel**.

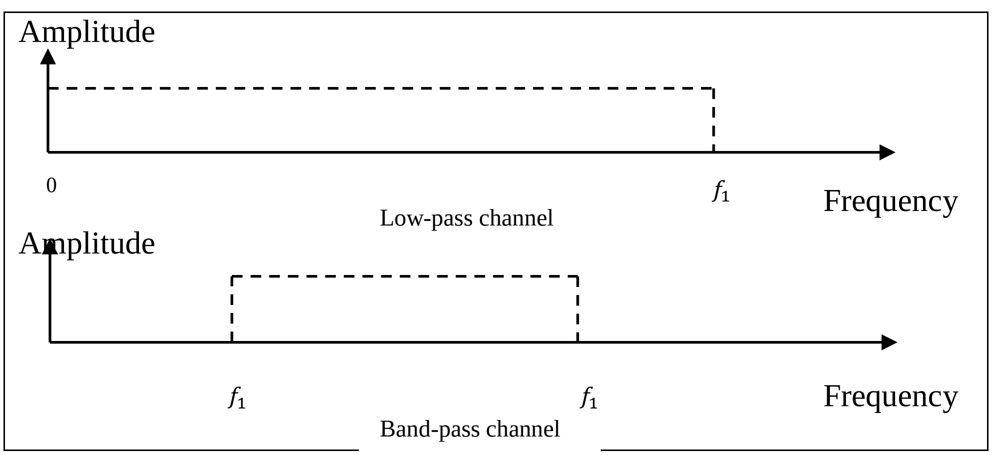


Figure 12. Low-pass and Band-pass

Multichannel and Multidimensional Signals

2.1.2 One-dimensional Signal

When a function depends on a single independent variable to represent the signal, it is said to be a one-dimensional signal.

The ECG signal and speech signal shown in Fig. 2.1(a) and 2.1(b) respectively are examples of one-dimensional signals where the independent variable is time. The magnitude of the signals is dependent variable.

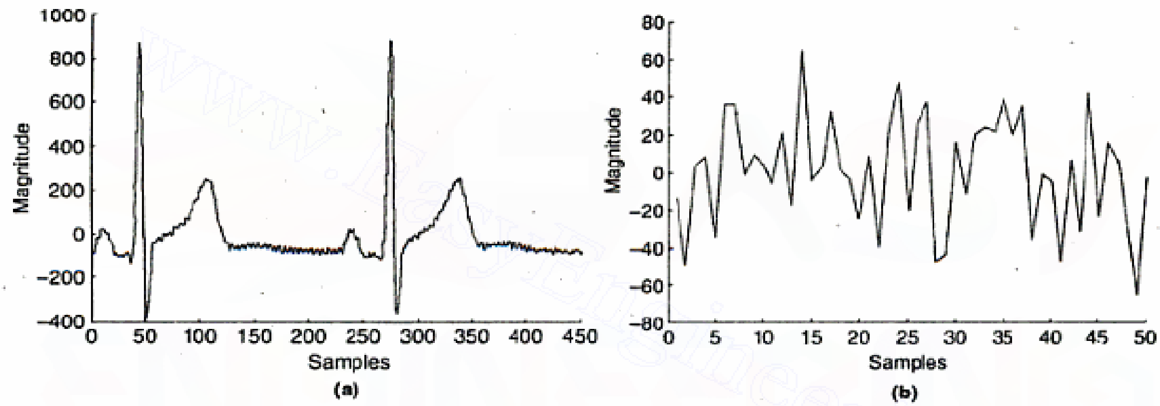


Fig. 2.1 One-dimensional Signal

(a) ECG Signal (b) Speech Signal

2.1.3 Two-dimensional Signal

When a function depends on two independent variables to represent the signal, it is said to be a two-dimensional signal. For example, photograph shown in Fig. 2.2 is an example of two-dimensional signal wherein the two independent variables are the two spatial coordinates which are usually denoted by x and y.

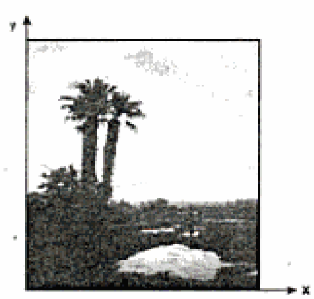


Fig. 2.2 Two-dimensional Photograph

2.1.4 Multi-dimensional Signal

When a function depends on more than one independent variables to represent the signal, it is said to be a multi-dimensional signal. For example, space missile shown in Fig. 2.3 is an example of three-dimensional image.

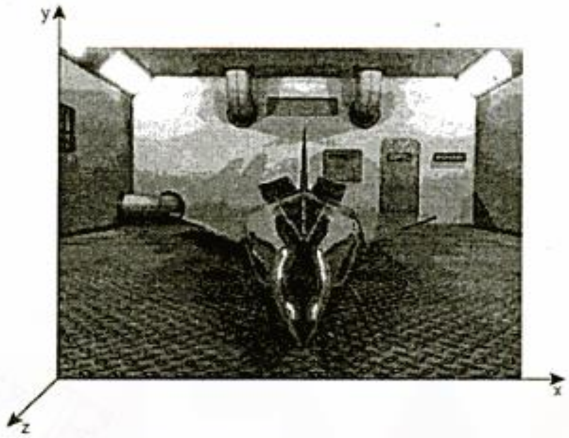


Fig. 2.3 3D-Space Missile

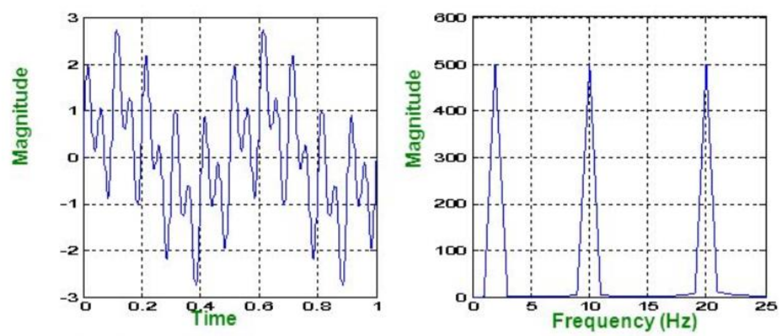
Stationary and Non-stationary Signal

Stationary signals consist of spectral components that do not change in time

- All spectral components exist at all time
- No need to know any time information
- FT works well for stationary signals

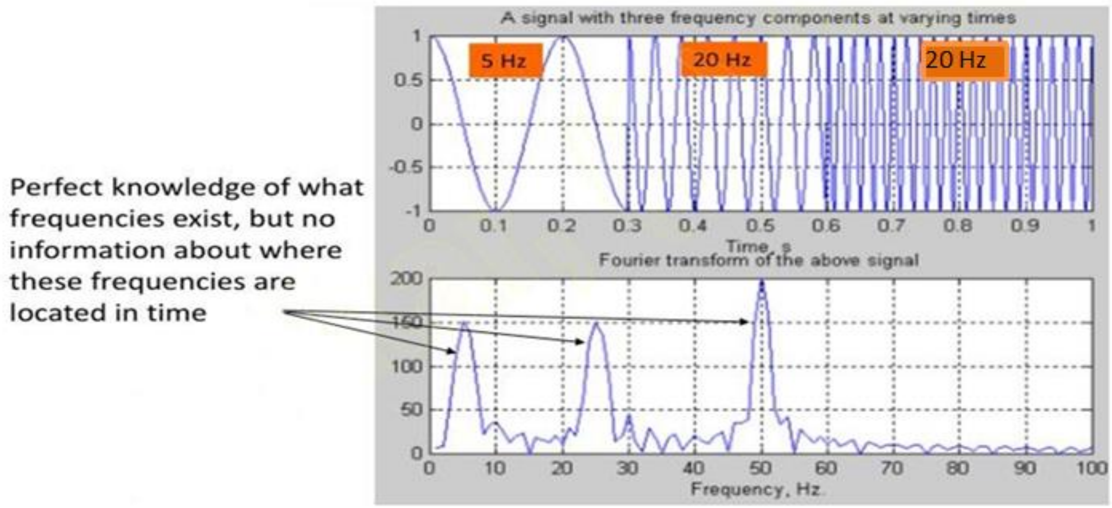
2 Hz + 10 Hz + 20Hz

Stationary



- However, most of carrying signals are non-stationary, so we need to know whether and also when an incident was happened.

Non-stationary signals consists of time varying spectral components



Symmetric (Even) and Antisymmetric (Odd) signals

Symmetric (even) and antisymmetric (odd) signals. A real-valued signal $x(n)$ is called symmetric (even) if

$$x(-n) = x(n) \tag{2.1.24}$$

On the other hand, a signal $x(n)$ is called antisymmetric (odd) if

$$x(-n) = -x(n) \tag{2.1.25}$$

We note that if $x(n)$ is odd, then $x(0) = 0$. Examples of signals with even and odd symmetry are illustrated in Fig. 2.8.

We wish to illustrate that any arbitrary signal can be expressed as the sum of two signal components, one of which is even and the other odd. The even signal component is formed by adding $x(n)$ to $x(-n)$ and dividing by 2, that is,

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)] \tag{2.1.26}$$

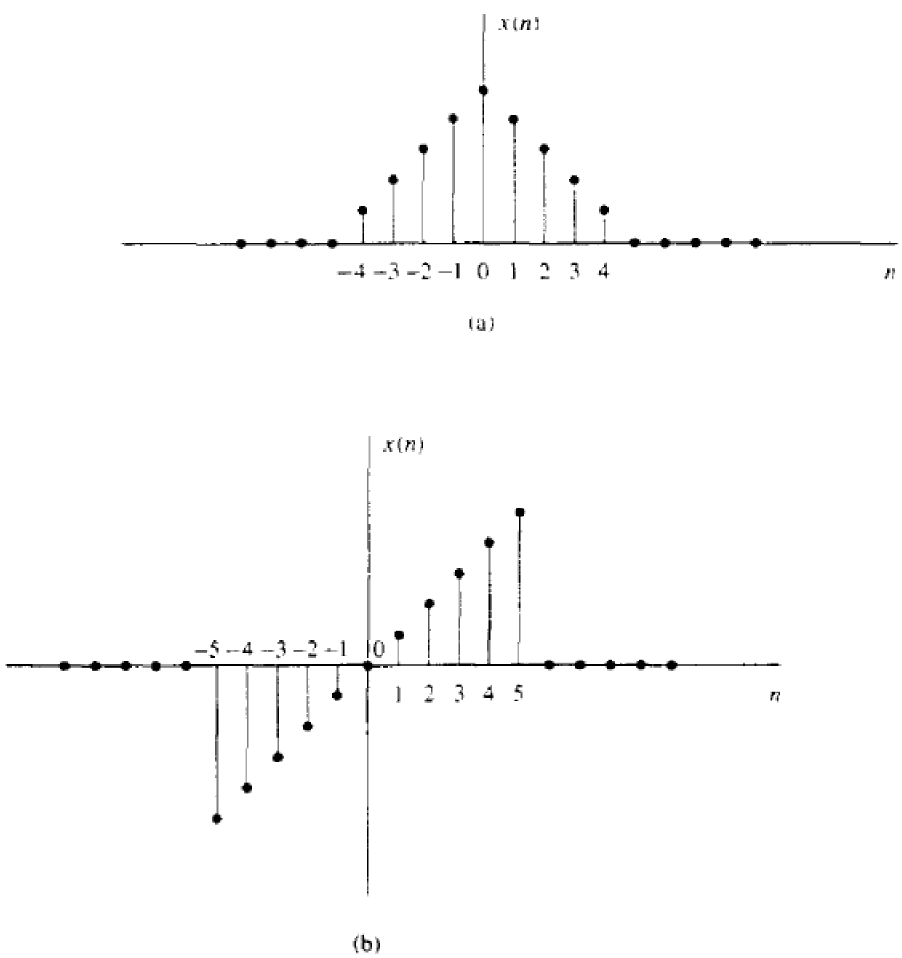


Figure 2.8 Example of even (a) and odd (b) signals.

Clearly, $x_e(n)$ satisfies the symmetry condition (2.1.24). Similarly, we form an odd signal component $x_o(n)$ according to the relation

$$x_o(n) = \tfrac{1}{2}[x(n) - x(-n)] \tag{2.1.27}$$

Again, it is clear that $x_o(n)$ satisfies (2.1.25); hence it is indeed odd. Now, if we add the two signal components, defined by (2.1.26) and (2.1.27), we obtain $x(n)$, that is,

$$x(n) = x_e(n) + x_o(n) \tag{2.1.28}$$

Thus any arbitrary signal can be expressed as in (2.1.28).

Problem 2.18 Find the odd and even components of $x(t)=e^{j2t}$

Solution We know that, any signal comprises of even and odd parts, i.e.

$$x(t) = x_e(t) + x_o(t) = e^{j2t}$$

The even signal is given by

$$x_e(t) = \frac{x(t)+x(-t)}{2} = \frac{e^{j2t}+e^{-j2t}}{2} = \cos 2t$$

The odd signal is given by

$$x_o(t) = \frac{x(t)-x(-t)}{2} = \frac{e^{j2t}-e^{-j2t}}{2}$$

$$x_o(t) = j\left[\frac{e^{+j2t}-e^{-j2t}}{2j}\right] = j\sin 2t$$

$$x(t) = x_e(t) + x_o(t) = \cos 2t + j\sin 2t$$

Summary Even × Even = Even; Odd × Odd = Even; Even × Odd = Odd

Problem 2.20 Find the even and odd components of $x(t) = \cos t + \sin t$.

Solution

$$x(t) = \cos t + \sin t$$

$$x(-t) = \cos(-t) + \sin(-t) = \cos t - \sin t$$

The even part is given by

$$x_e(t) = \frac{x(t)+x(-t)}{2} = \frac{(\cos t + \sin t)+(\cos t - \sin t)}{2} = \cos t$$

The odd part is given by

$$x_o(t) = \frac{x(t)-x(-t)}{2} = \frac{(\cos t + \sin t)-(\cos t - \sin t)}{2} = \sin t$$

$$x(t) = x_e(t) + x_o(t) = \cos t + \sin t$$

Problem 2.21 Find the even and odd components of $x(n) = \{3, 2, 1, 4, 5\}$.

Note The arrow mark always shows the value for 0th position i.e. ↑

Position	−2	−1	0	1	2
$x(n)$	3	2	1	4	5

Solution The even part is given by

$$x_e(n) = \frac{x(n)+x(-n)}{2}$$

For $n=-2$, $$x_e(-2) = \frac{x(-2)+x(2)}{2} = \frac{3+5}{2} = 4$$

For $n=-1$, $$x_e(-1) = \frac{x(-1)+x(1)}{2} = \frac{2+4}{2} = 3$$

For $n=0$, $$x_e(0) = \frac{x(0)+x(0)}{2} = \frac{1+1}{2} = 1$$

For $n=1$, $$x_e(1) = \frac{x(1)+x(-2)}{2} = \frac{4+2}{2} = 3$$

For $n=2$, $$x_e(2) = \frac{x(2)+x(-2)}{2} = \frac{5+3}{2} = 4$$

$$x_e(n) = \{4, 3, 1, 3, 4\}$$

The odd part is given by

$$x_o(n) = \frac{x(n)-x(-n)}{2}$$

For $n=-2$, $$x_e(-2) = \frac{x(-2)-x(2)}{2} = \frac{3-5}{2} = -1$$

For $n=-1$, $$x_e(-1) = \frac{x(-1)-x(1)}{2} = \frac{2-4}{2} = -1$$

For $n=0$, $$x_o(0) = \frac{x(0)-x(0)}{2} = 0$$

For $n=1$, $$x_o(1) = \frac{x(1)-x(-1)}{2} = \frac{4-2}{2} = 1$$

For $n=2$, $$x_o(2) = \frac{x(2)-x(-2)}{2} = \frac{5-3}{2} = 1$$

$$x_o(n) = \{-1, -1, 0, 1, 1\}$$

Adding $x_e(n)$ and $x_o(n)$ position-wise, we obtain the original signal $x(n)$.

1.4 ANALOG-TO-DIGITAL AND DIGITAL-TO-ANALOG CONVERSION

Most signals of practical interest, such as speech, biological signals, seismic signals, radar signals, sonar signals, and various communications signals such as audio and video signals, are analog. To process analog signals by digital means, it is first necessary to convert them into digital form, that is, to convert them to a sequence of numbers having finite precision. This procedure is called *analog-to-digital (A/D) conversion*, and the corresponding devices are called *A/D converters* (*ADCs*).

Conceptually, we view A/D conversion as a three-step process. This process is illustrated in Fig. 1.14.

1. *Sampling*. This is the conversion of a continuous-time signal into a discrete-time signal obtained by taking "samples" of the continuous-time signal at discrete-time instants. Thus, if $x_a(t)$ is the input to the sampler, the output is $x_a(nT) \equiv x(n)$, where T is called the *sampling interval*.
2. *Quantization*. This is the conversion of a discrete-time continuous-valued signal into a discrete-time, discrete-valued (digital) signal. The value of each

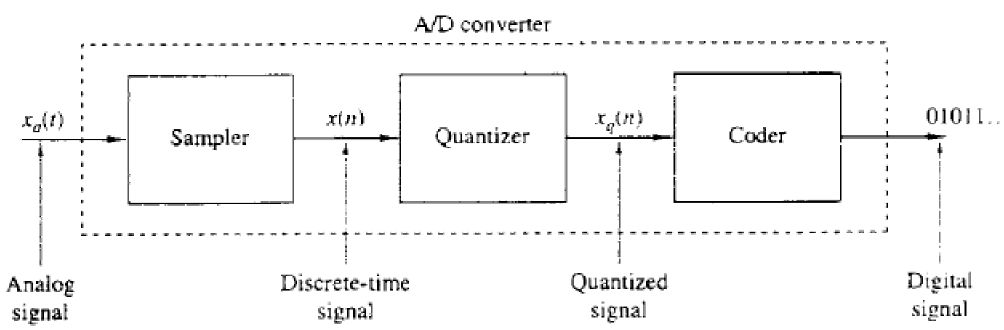


Figure 1.14 Basic parts of an analog-to-digital (A/D) converter.

signal sample is represented by a value selected from a finite set of possible values. The difference between the unquantized sample $x(n)$ and the quantized output $x_q(n)$ is called the quantization error.

3. *Coding*. In the coding process, each discrete value $x_q(n)$ is represented by a b-bit binary sequence.

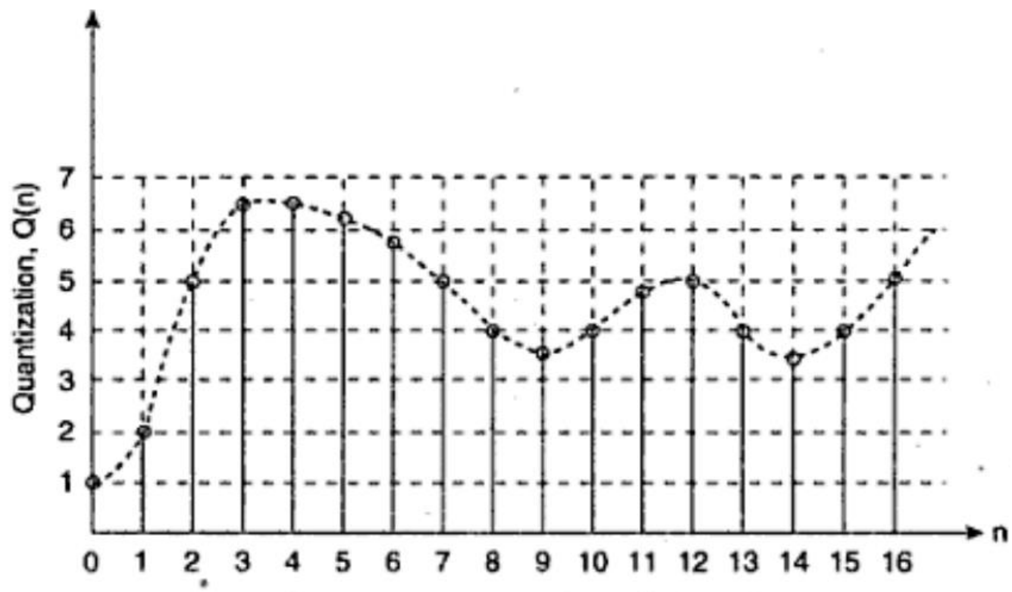


Fig. 2.5 Quantization of Signal

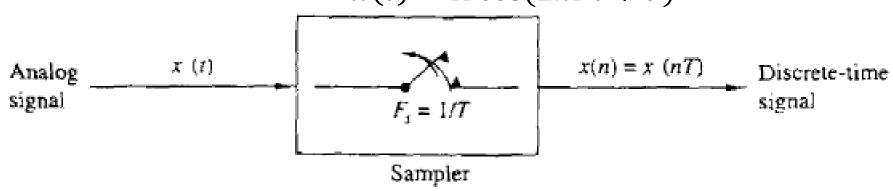
The time interval *T* between successive samples is called the *sampling period* or *sample interval* and its reciprocal $1/T = F_s$ is called the *sampling rate* (samples per second) or the sampling frequency (hertz).

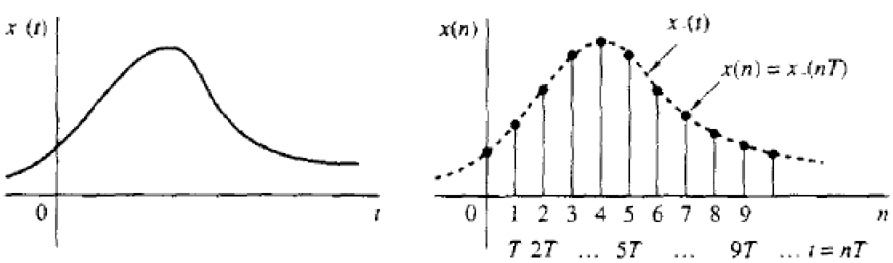
To establish this relationship between analog and digital signal

$$t = nT = \frac{n}{F_s}$$

Consider an analog signal sinusoidal signal of the form

$$x(t) = A\cos(2\pi Ft + \theta)$$





Which, when sampled periodically at a rate $F_s = 1/T$ samples per second, yields

$$x(nT) = x(n) = A\cos(2\pi FnT + \theta)$$
$$= A\cos\left(\frac{2\pi Fn}{F_s} + \theta\right)$$

$$f = \frac{F}{F_s}$$

Example 1: Consider the analog signal

$$x(t) = 3\cos 100\pi t$$

(a) Determine the minimum sampling rate required to avoid aliasing.

(b) Suppose that the signal is sampled at the rate $F_s = 200\ Hz$. What is the discrete-time signal obtained after sampling?

(c) Suppose that the signal is sampled at the rate $F_s = 75\ Hz$. W hat is the discretetime signal obtained after sampling?

Solution

(a) The frequency of the analog signal is $F = 50$ Hz. Hence the minimum sampling rate required to avoid aliasing is $F_s = 100$ Hz.

(b) If the signal is sampled at $F_s = 200$ Hz, the discrete-time signal is

$$x(n) = 3\cos\frac{100\pi}{200}n = 3\cos\frac{\pi}{2}n$$

(c) If the signal is sampled at $F_s = 75$ Hz, the discrete-time signal is

$$\begin{aligned} x(n) &= 3\cos\frac{100\pi}{75}n = 3\cos\frac{4\pi}{3}n \\ &= 3\cos\left(2\pi - \frac{2\pi}{3}\right)n \\ &= 3\cos\frac{2\pi}{3}n \end{aligned}$$

Example 2: Consider the analog signal

$$x(t) = 3\cos 50\pi t + 10\ \sin 300\pi t - \cos 100\pi t$$

What is the Nyquist rate for this signal?

Solution The frequencies present in the signal above are

$$F_1 \ = \ 25\ Hz.\ F_2 \ = \ 150\ Hz.\ F_3 \ = \ 50\ Hz$$

Thus $F_{max} \ = \ 150\ Hz,$

$$F_s \ > \ F_{max} \ = \ 300\ Hz$$

The Nyquist rale is $F_N = 2\ F_{max}$. Hence

$F_N == \ 300\ Hz$

Example 3: Consider the analog signal

$$x(t) = 3\cos 2000\pi t + 5\sin 6000\pi t + 10\cos 12000\ \pi t$$

(a) What is the Nyquist rate for this signal?

(b) Assume now that we sample this signal using a sampling rate Fs = 5000 samples/s. What is the discrete-time signal obtained after sampling?
(c) What is the analog signal $y_a(t)$ we can reconstruct from the samples if we use ideal interpolation?

Solution

(a) The frequencies existing in the analog signal are

$$F_1 = 1 \text{ kHz}, \qquad F_2 = 3 \text{ kHz}, \qquad F_3 = 6 \text{ kHz}$$

Thus $F_{max} = 6$ kHz, and according to the sampling theorem,

$$F_s > 2F_{max} = 12 \text{ kHz}$$

The Nyquist rate is

$$F_N = 12 \text{ kHz}$$

(b) Here $F_s = 5\ KHz$

$$x(n) = x_a(nT) = x_a\left(\frac{n}{F_s}\right)$$

$$= 3\cos 2\pi(\tfrac{1}{5})n + 5\sin 2\pi(\tfrac{3}{5})n + 10\cos 2\pi(\tfrac{6}{5})n$$

$$= 3\cos 2\pi(\tfrac{1}{5})n + 5\sin 2\pi(1-\tfrac{2}{5})n + 10\cos 2\pi(1+\tfrac{1}{5})n$$

$$= 3\cos 2\pi(\tfrac{1}{5})n + 5\sin 2\pi(-\tfrac{2}{5})n + 10\cos 2\pi(\tfrac{1}{5})n$$

Finally, we obtain

$$x(n) = 13\cos 2\pi(\tfrac{1}{5})n - 5\sin 2\pi(\tfrac{2}{5})n$$

(c) Since only the frequency components at 1 kHz and 2 kHz are present in the sampled signal, the analog signal we can recover is

$$y_a(t) = 13\cos 2000\pi t - 5\sin 4000\pi t$$

2.1.1 Some Elementary Discrete-Time Signals

In our study of discrete-time signals and systems there are a number of basic signals that appear often and play an important role. These signals are defined below.

1. The *unit sample sequence* is denoted as $\delta(n)$ and is defined as

$$\delta(n) \equiv \begin{cases} 1, & \text{for } n = 0 \\ 0, & \text{for } n \neq 0 \end{cases} \tag{2.1.6}$$

In words, the unit sample sequence is a signal that is zero everywhere, except at $n = 0$ where its value is unity. This signal is sometimes referred to as a *unit impulse*. In contrast to the analog signal $\delta(t)$, which is also called a unit impulse and is defined to be zero everywhere except $t = 0$, and has unit area, the unit sample sequence is much less mathematically complicated. The graphical representation of $\delta(n)$ is shown in Fig. 2.2.

2. The *unit step signal* is denoted as $u(n)$ and is defined as

$$u(n) \equiv \begin{cases} 1, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases} \tag{2.1.7}$$

Figure 2.3 illustrates the unit step signal.

3. The *unit ramp signal* is denoted as $u_r(n)$ and is defined as

$$u_r(n) \equiv \begin{cases} n, & \text{for } n \geq 0 \\ 0, & \text{for } n < 0 \end{cases} \tag{2.1.8}$$

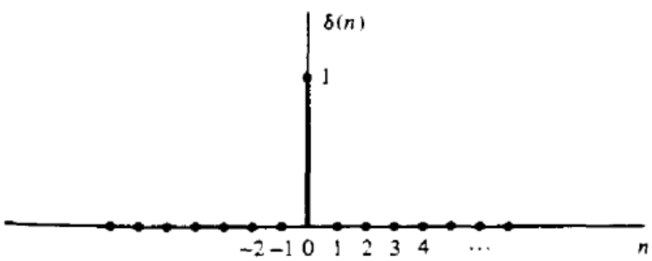


Figure 2.2 Graphical representation of the unit sample signal.

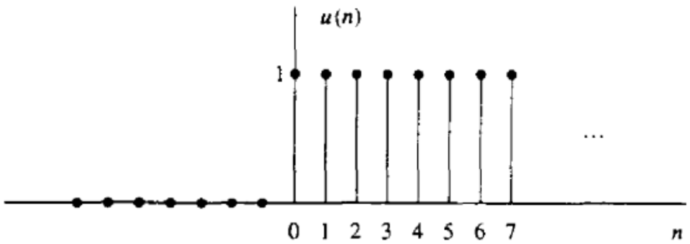


Figure 2.3 Graphical representation of the unit step signal.

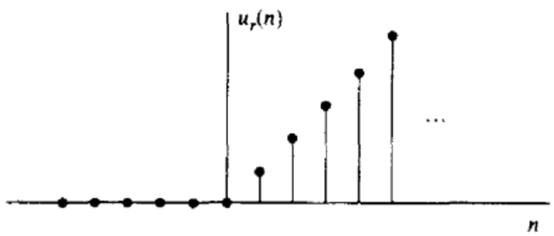


Figure 2.4 Graphical representation of the unit ramp signal.

4. The *exponential signal* is a sequence of the form

$$x(n) = a^n \qquad \text{for all } n \tag{2.1.9}$$

If the parameter a is real, then $x(n)$ is a real signal. Figure 2.5 illustrates $x(n)$ for various values of the parameter a.

When the parameter a is complex valued, it can be expressed as

$$a \equiv re^{j\theta}$$

where r and θ are now the parameters. Hence we can express $x(n)$ as

$$\begin{aligned} x(n) &= r^n e^{j\theta n} \\ &= r^n(\cos\theta n + j\sin\theta n) \end{aligned} \tag{2.1.10}$$

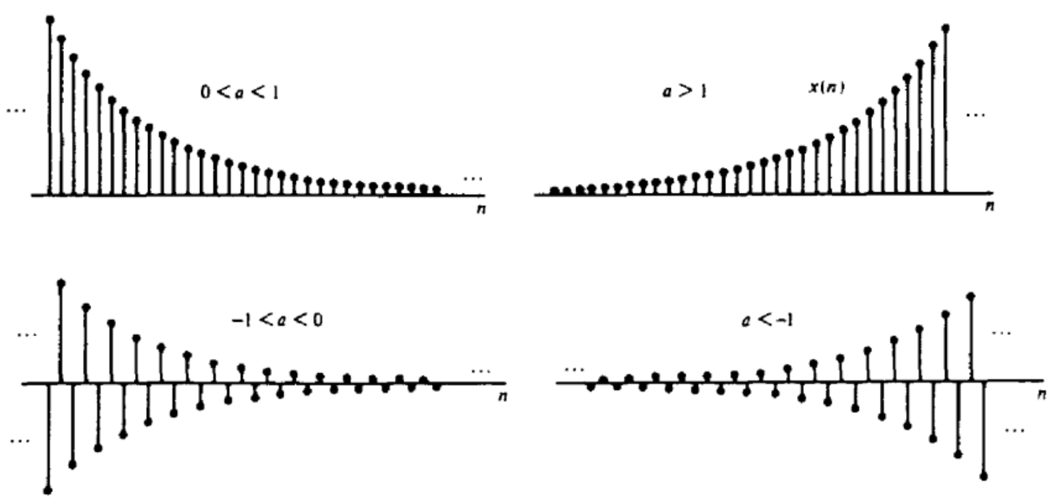


Figure 2.5 Graphical representation of exponential signals.

2.2.2 Block Diagram Representation of Discrete-Time Systems

It is useful at this point to introduce a block diagram representation of discrete-time systems. For this purpose we need to define some basic building blocks that can be interconnected to form complex systems.

An adder. Figure 2.13 illustrates a system (adder) that performs the addition of two signal sequences to form another (the sum) sequence, which we denote

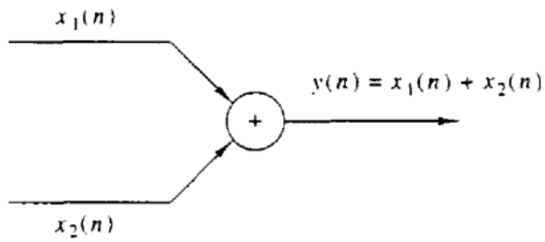


Figure 2.13 Graphical representation of an adder.

as $y(n)$. Note that it is not necessary to store either one of the sequences in order to perform the addition. In other words, the addition operation is *memoryless*.

A constant multiplier. This operation is depicted by Fig. 2.14, and simply represents applying a scale factor on the input $x(n)$. Note that this operation is also memoryless.

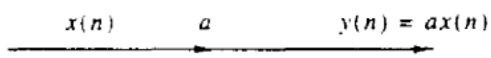


Figure 2.14 Graphical representation of a constant multiplier.

A signal multiplier. Figure 2.15 illustrates the multiplication of two signal sequences to form another (the product) sequence, denoted in the figure as $y(n)$. As in the preceding two cases, we can view the multiplication operation as memoryless.

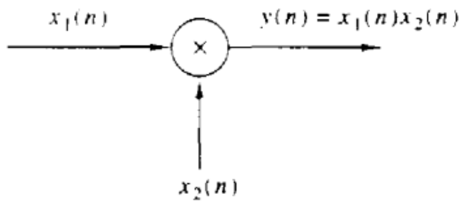


Figure 2.15 Graphical representation of a signal multiplier.

A unit delay element. The unit delay is a special system that simply delays the signal passing through it by one sample. Figure 2.16 illustrates such a system. If the input signal is $x(n)$, the output is $x(n-1)$. In fact, the sample $x(n-1)$ is stored in memory at time $n-1$ and it is recalled from memory at time n to form

$$y(n) = x(n-1)$$

Thus this basic building block requires memory. The use of the symbol z^{-1} to denote the unit of delay will become apparent when we discuss the z-transform in Chapter 3.

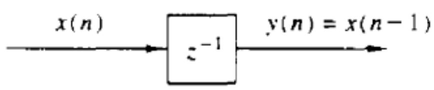


Figure 2.16 Graphical representation of the unit delay element.

A unit advance element. In contrast to the unit delay, a unit advance moves the input $x(n)$ ahead by one sample in time to yield $x(n+1)$. Figure 2.17 illustrates this operation, with the operator z being used to denote the unit advance.

Figure 2.17 Graphical representation of the unit advance element.

We observe that any such advance is physically impossible in real time, since, in fact, it involves looking into the future of the signal. On the other hand, if we store the signal in the memory of the computer, we can recall any sample at any time. In such a nonreal-time application, it is possible to advance the signal $x(n)$ in time.

Example 2.2.3

Using basic building blocks introduced above, sketch the block diagram representation of the discrete-time system described by the input–output relation.

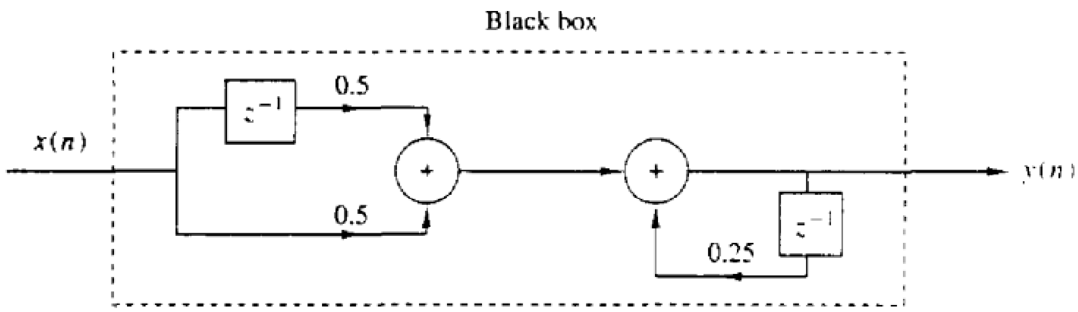
$$y(n) = \tfrac{1}{4}y(n-1) + \tfrac{1}{2}x(n) + \tfrac{1}{2}x(n-1) \tag{2.2.5}$$

where $x(n)$ is the input and $y(n)$ is the output of the system.

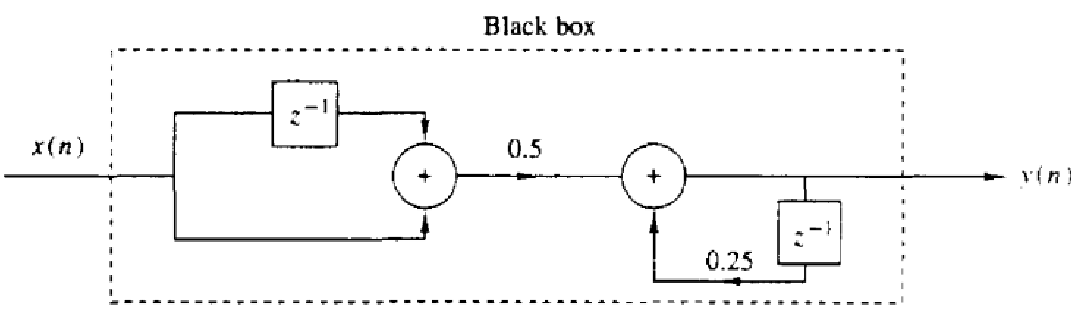
Solution According to (2.2.5), the output $y(n)$ is obtained by multiplying the input $x(n)$ by 0.5, multiplying the previous input $x(n-1)$ by 0.5, adding the two products, and then adding the previous output $y(n-1)$ multiplied by $\frac{1}{4}$. Figure 2.18a illustrates this block diagram realization of the system. A simple rearrangement of (2.2.5), namely,

$$y(n) = \tfrac{1}{4}y(n-1) + \tfrac{1}{2}[x(n) + x(n-1)] \tag{2.2.6}$$

leads to the block diagram realization shown in Fig. 2.18b. Note that if we treat "the system" from the "viewpoint" of an input–output or an external description, we are not concerned about how the system is realized. On the other hand, if we adopt an



(a)



(b)

Figure 2.18 Block diagram realizations of the system $y(n) = 0.25y(n-1) + 0.5x(n) + 0.5x(n-1)$.

2.3.8 Time Shifting of Signals

Consider a continuous-time signal $x(t)$. Let $y(t)$ denote a signal obtained by shifting the signal $x(t)$ by $(t - t_0)$, that is,

$$y(t) = x(t - t_0) \tag{2.42}$$

If the signal $x(t)$ is positive, and $t_0 > 0$ for all values of t_0, then the signal is said to be right-shifted signal. In the example shown in Fig. 2.33(b), the signal is shifted to right side by 3 units.

If the signal $x(t)$, is positive, and $t_0 < 0$ for all values of t_0 then the signal is said to be left-shifted signal. In the example shown in Fig. 2.33(c), the signal is shifted to left side by 4 units.

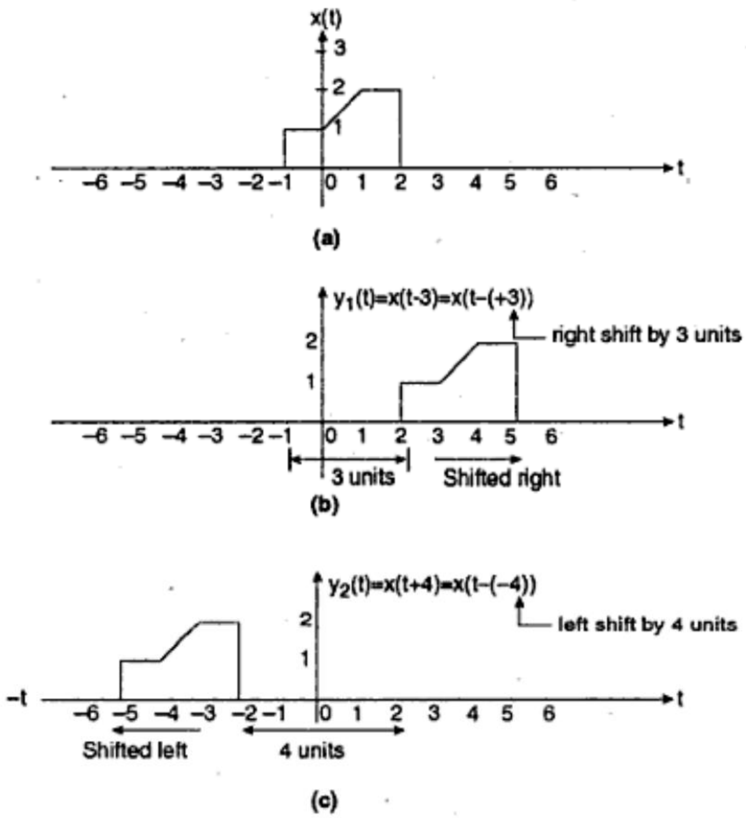


Fig. 2.33 (a) Original Positive Signal x(t) (b) Right-shifted Signal x(t-3) (c) Left-shifted Signal x(t+4)