Pabna University of Science & Technology

Department of

INFORMATION AND COMMUNICATION ENGINEERING

COURSE CODE: STAT-2201
COURSE TITLE : ENGINEERING STATISTICS

ASSIGNMENT

- **Submitted by:**

  Jannatul Azmi
  Roll : 220623
  Session :2021-2022
  2nd year 2nd semester
  Dept. of Information and Communication Engineering

- **Submitted to:**

  Dr. Md. Sarwar Hossain
  Associate Professor
  Dept. of Information and Communication Engineering
  Pabna University of Science and Technology

Submission date: 23/04/2025

Statistical Decision, Statistical Hypothesis: Critical Region, Best Critical Region

Statistical Decision

A statistical decision refers to making a conclusion or choice based on data-driven evidence rather than assumptions or intuition. It plays a crucial role in various fields—engineering, medicine, business—where decisions must be justified with empirical proof. These decisions rely on statistical procedures like hypothesis testing, estimation, and confidence intervals.

In essence, it helps answer questions like:
• Is this new drug more effective than the old one?
• Is the defect rate in the production line acceptable?
• Has the average income in a region significantly changed?

Such decisions aim to minimize the risk of errors (Type I and Type II) and to maximize the validity of conclusions based on sample data.

For example:

Scenario:

A manufacturer wants to decide if a new machine produces fewer defective items than the old one.

Action:

Collect sample data from both machines. Perform a statistical test (e.g., proportion test).

Conclusion:

If the defect rate of the new machine is significantly lower, the company decides to switch—this is a **statistical decision** based on data, not intuition.

Statistical Hypothesis

A statistical hypothesis is a formal claim or assumption about a population parameter (such as the mean, proportion, or variance), which is testable using sample data.

There are two main hypotheses in any test:
• Null Hypothesis (H_0): This is the default or "no effect" assumption. It represents the status quo or the claim to be tested.
• Alternative Hypothesis (H_1): This is what you want to prove. It suggests a significant effect, difference, or change.

Example: A snack company claims that its chips packets contain 500 grams of chips.
H_0: $\mu = 500$g (Mean weight is 500g)
H_1: $\mu \neq 500$g (Mean weight is not 500g)

The hypothesis test helps determine whether the observed difference in sample mean is due to random chance or a genuine shift in the population mean.

Critical Region

The critical region (also called the rejection region) is the range of values of the test statistic that leads to rejection of the null hypothesis (H_0). It is determined based on a pre-selected significance level (α), which represents the probability of rejecting H_0 when it is actually true (Type I error).

The location and size of the critical region depend on:
• The nature of the test (one-tailed or two-tailed)
• The α level (common values: 0.01, 0.05, 0.10)

Decision Rule:

If the test statistic falls within the critical region → Reject H_0
If the test statistic falls outside the critical region → Fail to reject H_0

Z-Test Critical Region Table

Test Type	α Level	Critical Value(s)	Decision Rule
One-tailed (Right)	0.05	$Z > 1.645$	Reject H_0 if $Z > 1.645$
One-tailed (Left)	0.05	$Z < -1.645$	Reject H_0 if $Z < -1.645$
Two-tailed	0.05	$Z < -1.96$ or $Z > 1.96$	Reject H_0 if $Z < -1.96$ or $Z > 1.96$

Best Critical Region

The best critical region is one that maximizes the probability of correctly rejecting a false null hypothesis (H_0), while keeping the probability of a Type I error fixed at α. This concept is rooted in the Neyman-Pearson Lemma, which provides a systematic way to identify the most powerful test for simple hypotheses.

Key idea: Among all tests with the same significance level, the best critical region is the one with the highest power, meaning it has the greatest ability to detect the alternative hypothesis (H_1) when it's true.

This is done by using the likelihood ratio test:

If $L_1(x)/L_0(x) > k$, then the value x lies in the best critical region.

Where:
$L_1(x)$: Likelihood under H_1
$L_0(x)$: Likelihood under H_0
k: Critical constant based on α

Example: Best Critical Region in Practice

Test:
$H_0: \mu = 50$
$H_1: \mu \neq 50$

Given:
Sample mean $\bar{x} = 53$
Standard deviation $\sigma = 4$
Sample size $n = 36$
Significance level $\alpha = 0.05$ (two-tailed)

Step 1: Compute Z-value
$Z = (\bar{x} - \mu) / (\sigma / \sqrt{n}) = (53 - 50) / (4 / \sqrt{36}) = 3 / 0.6667 \approx 4.5$

Step 2: Compare with Critical Values
For a two-tailed test at $\alpha = 0.05$, the critical values are ± 1.96.

Since $Z = 4.5 > 1.96$, it lies in the critical region.

Conclusion:
Reject H_0. There is strong evidence that the population mean is not 50.
This test used the best critical region under Neyman-Pearson Lemma to ensure maximum detection power for the alternative.