Signal and System.

B 05103062?

জাতীয় বিশ্ববিদ্যালয়

ক্রমিক নং ঃ ....

অতিরিক্ত উত্তরপত্র

১। .... পরীক্ষা .... সাল

২। বিষয় ঃ..

৩। বিষয়ের শিরোনাম ঃ..

৪। বিষয় কোড ঃ..

৫। পরীক্ষার তারিখ ঃ..

৬। ইনভিজিলেটরের স্বাক্ষর ও তারিখ ঃ ....

(এ স্থান হতে উত্তর লেখা আরম্ভ করতে হবে)

Define signal ** 2019, 2018

A signal is define as a function of one or more variables which conveys information on the nature of a physical phenomenon.

speech, Image Heartbeat, email, Internet.

2020 ** Define one dimension and multi dimension signal.

One-dimension signal.

The function depends only on one independe variable, to represent the signal.

Voice: Amplitude varies with time.

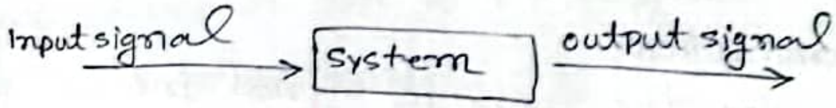
Two-dimension signal.
when a function depends on two independent variables to represent the signal it is said to be a two dimension signal.

Multi-dimension signal.
A function depends on more than one independent variables to represent the signal it is said to be multi-dimension signal.
image.

Input signal:
A signal that enters a system from an external source is ~~de~~ reffered to as an input signal
Voltage from function generator, electrocardiogram from heart, tempert from human body.

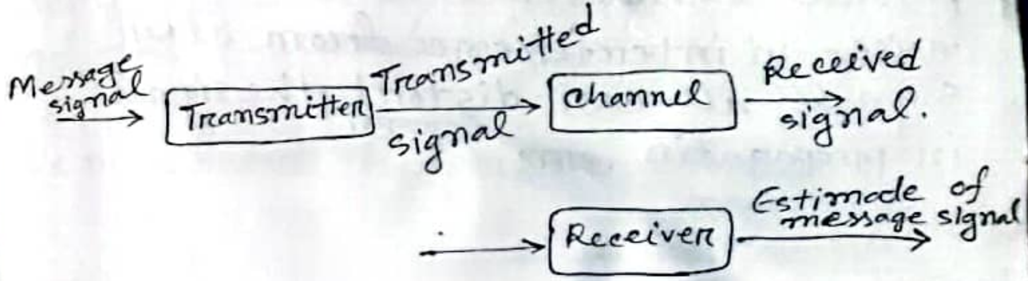
Output signal.
A signal produced by the system in response to the input signal is called

Define system:
System is an entity that manipulates one or more signal to accomplish a function thereby yielding new signal(s)



Overview of Specific systems.
communication systems
control systems
Remote sensing systems.
Biomedical signal Processing systems.
Auditory system.

Overview of communication system.



Basic Elements:

1. Transmitter
2. channel
3. Receiver

Transmitter.

converts the message signal into a form suitable for transmission over the channel.

Channel

The physical medium that connects the ~~th~~ transmitter and the receiver
It can be wired or wireless.
physical characteristics of the channel noise and interference from other signals all can distort (বিকৃত করা) the signal in propagation. প্রচার

Receiver-

Receiver is the portion which receives the corrupted version of the transmitted signal and reconstructs it to a recognizable form of the orginal signal.
It has two responsibilities.

1. Performing the reverse operations of the transmitter
2. Reversing the effect of the channel.

Modulator (in transmitter): Converts message signal into a form that is compatible with transmission characteristic of the channel.

Demodulator: Reverse operation of Modulator.

Digital communication:

* Write the steps of A/D conversion/ Explain the with ex

Sampling: convert the message signal into a sequence of number.

Quantization: Representing the sampled values to the ~~Rea~~ nearest level of pre-selected values.

Coding: Representing each quantized values by a code word.

Modulation: Carrier wave modulation for transmission.

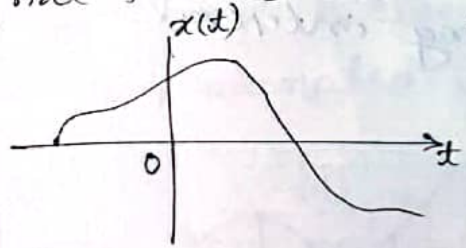
Classification of signal.

1. Continuous time signal and Discreate time signal.
2. Periodic signal and Aperiodic signal
3. Even signal and odd signal.
4. Deterministic signal and Random signal
5. Energy signal and power signal.

1. Continuous and discreate time signal.

Continuous-time signal:

A signal $x(t)$ is said to be continous-time signal if it is defined for all time t.



Arises naturally i.e speech ~~of~~ in voice, conversion of sound or light into electrical signal (by means of transducers, photocell) etc.

Signal Part-2

Discrete-time signal.

A signal that is defined only at discreate instants of time.

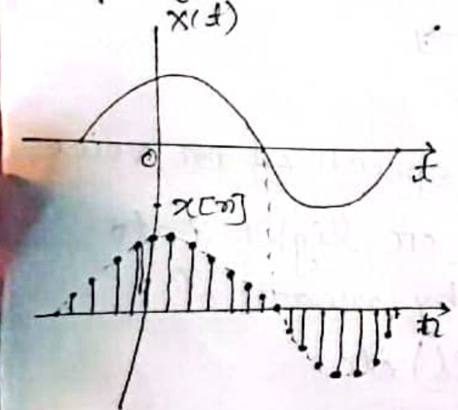
The independent variable only has discreate values which are usually equally placed.

often derived from a continuous time signal by sampling it at a uniform rate.

$x[n] = x(mt)$, $n = 0, \pm1, \pm2 \dots$

m = sampling interval.

Sampling.



Coding:

Sample value are converted to codes.

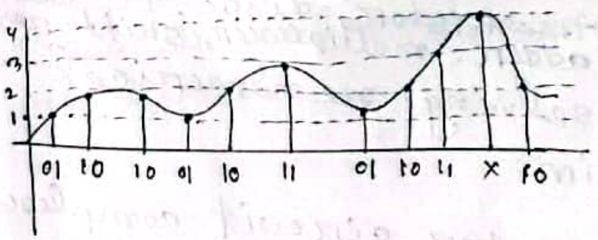
Example: Converted to binarry numbers.

Banary coding

Number of bits for representing one sample value is at first decided e.g 8 bit coding, 16 bit coding etc

Assuming unsigned sampled values (non-negative values), an n bit coding scheme can represent maximum of 2^n number of distinct sampled values.

Example: 2 bit bit coding can represent 0, 1, 2, 3 and 4 sampled values only.



** Analog (continuous time approach) signal processing.

প্রভাবশালী

- It was dominant for many years.

নির্ভরতা

- It Relies on analog circuit elements like resistors, capacitors and Inductors
- Based on solving differential equation that ~~re~~ describe ~~nations~~ natural system.
- Real time solution can be obtain.

* Digital (Discreate time approach) signal

- It is the present time trends in signal processing.
- It relies on digital circuit element like ~~transistor, diode, IC etc.~~ adder, multiplexer, shift resistor,
- Base on solving) ~~re~~ numerical computation
- Requires greater circuit complexity yet no assurance of real time output.

নিশ্চয়তা

* what is the basic difference between discrete-time signal and digital signal

~~Analog signal~~ discreate time sign	digital signal.
A discreate-time signal is a signal that is defined at a discrete points in time.	Digital signal is a specific type of discrete-time signal that has a finite number of quantization levels.
Discrete ~~sim~~ time signals are usually represented as sequences of number where each number represents the amplitude of the signal at a specific time instant.	Here the amplitude values are limited to a finite number of discrete values or levels.
It can be analog or digital	only ~~gi~~ digital.

* Advantage of digital signal over analog signal.

- Flexibility
only change a software can change the functionality of a hardware (e.g filter). But analog signal is hardware dependent.

- Repeatability
Normally does not suffer from external effects like supply voltage or room temperature when repeating ~~again~~ the same task again and again.

(2.1) A continuous signal $x(t) = 5\sin(\pi t)$
sampling period $T = 0.1$ s $\quad 3 \geq t \geq 0$

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জাতীয় বিশ্ববিদ্যালয়

t	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4
$x(t)$	0	2.9	4.7	2.9	0	-2.9	-4.7	-2.8	0	29	4.7	4.7	29

2.6	2.8	3
(		

ক্রমিক নং ঃ ....

অতিরিক্ত উত্তরপত্র

১। .... পরীক্ষা .... সাল

২। বিষয় ঃ $x(t) = 5\sin(\pi t)$

৩। বিষয়ের শিরোনাম ঃ $x(t) = 5\sin(\pi t)$

৪। বিষয় কোড ঃ $x(t) = x(nT)\big|_{t = nT}$

৫। পরীক্ষার তারিখ ঃ $x(t) = x(0.1n)\big|_{t=0.1n}$

৬। ইনভিজিলেটরের স্বাক্ষর ও তারিখ ঃ $x(n) = 5\sin(\pi nT) = 5\sin(\pi n.1)$

(এ স্থান হতে উত্তর লেখা আরম্ভ করতে হবে)

*

t	0	0.2	0.4	0.6	0.8	1	1.2	1.4	1.6	1.8	2	2.2	2.4	2.6	2.8	3
$x(t)$	0	2.9	4.7	4.7	29	0	-2.9	-4.7	-4.7	-2.9	0	29	4.7	47	29	0

For discrete time signal can be obtain by simple calculation.

$$x(n) = 5\sin(\pi nT) = 5\sin(\pi n.1) \quad n = 0, \pm 1, \pm 2$$

n	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
$x(n)$	0	1.5	2.9	4	4.8	5	4.8	4	2.9	1.5	0				

Problem-2

$x(t) = e^{-2t} = \frac{1}{e^{2t}}$

t	-2	-1.5	-1	-.5	0	.5	1	1.5	2
$x(t)$	54.59	20.08	7.38	2.72	1	0.3	0.14	0.05	0.02

$\therefore x(t) = x(nT) \,\big|\, t = nT$

$x(t) = x(0.1n) \,\big|\, t = 0.1n$

$x(n) = e^{-2(0.1n)} = e^{-0.2n}$

n	-4	-3	-2	-1	0	1	2	3	4
$x(n)$	2.23	1.8	1.49	1.2	1	0.8	0.67	0.55	0.44

\# Periodic and aperiodic signal.

A continuous time signal is said to be periodic if

$x(t) = x(t+T)$; $T>0$

where T is period.

A discrete time signal is said to be periodic if

$x(n) = x(n+N)$. for all n

where, N is period.

Test whether the given signals are periodic or not.

(i) $x(t) = e^{\sin(t)}$ (ii) $x(t) = t\, e^{\sin(t)}$

(i) $x(t+T) = e^{\sin(t+T)}$

$= e^{\sin(t+2\pi)}$

$= e^{\sin t}$

$= x(t)$

This is the periodic signal.

(ii) $x(t) = t\, e^{\sin(t)}$

$x(t+T) = (t+T)\, e^{\sin(t+T)}$

$= (t+T)\, e^{\sin(2\pi+t)}$

$= (t+T)\, e^{\sin t} \neq x(t)$

This is not periodic signal.

(iii) $x(t) = e^{j\omega_0 t}$

$x(t+T) = e^{j\omega_0(t+T)} = e^{j\omega_0 t} \cdot e^{j\omega_0 T}$

$= e^{j\omega_0 T} \cdot e^{j\omega_0 t}$

For DC signal $\omega_0 = 0$

$e^{j\omega_0 T} = 1$. $\therefore x(t+T) = 1 \cdot e^{j\omega_0 t} = x(t)$.

For A.

(iv) $x(t) = \cos\left(t + \frac{\pi}{3}\right)$

$x(t+T) = \cos\left(T + t + \frac{\pi}{3}\right)$

$= \cos\left(2\pi + t + \frac{\pi}{3}\right)$

$= \cos\left(t + \frac{\pi}{3}\right) = x(t)$

The fundamental period is

$\omega = 1$

$\omega = \frac{2\pi}{T}$

or, $T = \frac{2\pi}{\omega} = \frac{2\pi}{1} = 2\pi$

The signal is periodic and the periodicity is 2π

(v) $x(t) = \sin\left(\frac{2\pi}{5}\right)t$

$\omega = \frac{2\pi}{5}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\frac{2\pi}{5}} = 2\pi \times \frac{5}{2\pi} = 5$

The fundamental period for which the given signal periodicity is

$T_0 = 5$

(7) $x(t) = \underbrace{\cos\left(\frac{\pi}{3}\right)t}_{x_1(t)} + \underbrace{\sin\left(\frac{\pi}{5}\right)t}_{x_2(t)}$

$\omega_1 = \frac{\pi}{3}$ $\qquad \omega_2 = \frac{\pi}{5}$

$T_1 = \frac{2\pi}{\frac{\pi}{3}} = 6$ $\qquad T_2 = \frac{2\pi}{\frac{\pi}{5}} = 10$

$\frac{T_1}{T_2} = \frac{6}{10} = \frac{3}{5}$

$T_0 = 5T_1 = 3T_2 = 30$

(28) $x(t) = \underbrace{\cos t}_{x_1(t)} + \underbrace{\sin\sqrt{2}\,t}_{x_2(t)}$

$\omega_1 = 1$ $\qquad \omega_2 = \sqrt{2}$

$T_1 = \frac{2\pi}{1}$ $\qquad T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{\sqrt{2}}$

$\frac{T_1}{T_2} = \frac{2\pi}{\frac{2\pi}{\sqrt{2}}} = \sqrt{2}$ which is irrational ratio, this signal is aperiodic.

(2.9) $x(t) = je^{j10t}$

The frequency of the signal is $\omega_0 = 10$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{10} = \frac{\pi}{5}$$

this signal is periodic and fundamental period is $\frac{\pi}{5}$.

(2020)

(i) $x[n] = 5\sin(2n)$

~~$x(n+N) = 5\sin(2n+$~~

Here, the given signal frequency is $\Omega_0 = 2$

The fundamental period $N_0 = \frac{2\pi m}{\Omega_0}$

$$= \frac{2\pi m}{2}$$

$$= \frac{22}{7} m$$

if $m = 7$ $\quad = \frac{22 \times 7}{7}$

$$= 22$$

The signal is periodic and the fundamental period is 22 if $m = 7$.

(ii) $x[n] = \sin(\frac{2\pi}{3})n$

The $f = \Omega_0 = \frac{2\pi}{3}$

if $m = 1$

$$T = \frac{2\pi m}{\Omega_0} = \frac{2\pi m}{\frac{2\pi}{3}} = 3m = 3 \times 1 = 3$$

The signal is periodic and fundamental period is 3 if $m = 1$

(iii) $x[n] = 5\sin(0.2\pi n)$

The frequency of the signal $\Omega_0 = 0.2\pi$

$$T = \frac{2\pi m}{\Omega_0}$$

$$= \frac{2\pi m}{0.2\pi}$$

$= 10$ if $m = 1$

This signal is periodic.

The fundamental period is 10 if m=

(iv) $x[n] = e^{j5\pi n}$

The frequency of the signal is $\Omega_0 =$

$$T = \frac{2\pi m}{\Omega_0} = \frac{2\pi m}{5\pi}$$

$= \frac{2m}{5}$ if $m = 5$

$$= \frac{2 \times 5}{5}$$

$$= 2$$

This signal is periodic. The fundamental period is 2 if

(2.9) $x(t) = je^{j10t}$

The frequency of the signal is $\omega_0 = 10$

$$T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{10} = \frac{\pi}{5}$$

This signal is periodic and fundamental period is $\frac{\pi}{5}$.

2020

(i) $x[n] = 5\sin(2n)$

~~$x(n+N) = 5\sin(2n+$~~

Here, The given signal frequency is $\Omega_0 = 2$

The fundamental period $N_0 = \frac{2\pi m}{\Omega_0}$

$$= \frac{2\pi m}{2}$$

$$= \frac{22}{7} m$$

if $m = 7$

$$= \frac{22 \times 7}{7}$$

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The signal is periodic and the fundamental period is 22 if $m = 7$.

(ii) $x[n] = \sin\left(\frac{2\pi}{3}\right)n$

The $f = \Omega_0 = \frac{2\pi}{3}$

if $m = 1$

$$T = \frac{2\pi m}{\Omega_0} = \frac{2\pi m}{\frac{2\pi}{3}} = 3m = 3 \times 1 = 3$$

The signal is periodic and fundamental period is 3 if $m = 1$

(iii) $x[n] = 5\sin(0.2\pi n)$

The frequency of the signal $\Omega_0 = 0.2\pi$

$$T = \frac{2\pi m}{\Omega_0}$$

$$= \frac{2\pi m}{0.2\pi}$$

$= 10$ if $m = 1$

This signal is periodic.

The fundamental period is 10 if $m = 1$.

(iv) $x[n] = e^{j5\pi n}$

The frequency of the signal is $\Omega_0 = 5\pi$

$$T = \frac{2\pi m}{\Omega_0} = \frac{2\pi m}{5\pi}$$

$$= \frac{2m}{5}$$

if $m = 5$

$$= \frac{2 \times 5}{5}$$

$$= 2$$

This signal is periodic. The fundamental period is 2 if $m = 5$

2.2.3] even and odd signal.

A continuous time signal is said to be even, if

$$x(t) = x(-t) \text{ for all } t$$

odd for all

$$x(t) = -x(-t) \text{ for all } t$$

A discrete time signal is said to be even if

$$x[n] = x[-n] \text{ for all } n$$

odd ~~for~~ if

$$x[n] = -x[-n] \text{ for all } n.$$

2018

* 1.c Develop the even odd decomposit. let us consider a signal, $x(t)$ which can be decomposed as odd and even signal.

$$x(t) = \underbrace{x_o(t)}_{\text{odd signal}} + \underbrace{x_e(t)}_{\text{even signal}} \quad \text{—— (i)}$$

$$x_e(t) = x_e(-t)$$

$$x_o(t) = -x_o(-t)$$

Replace t as $-t$ in equation (i)

$$x(-t) = x_o(-t) + x_e(-t)$$

$$= -x_o(t) + x_e(t)$$

$$= x_e(t) - x_o(t) \quad \text{—— (ii)}$$

Add the eqn (i) and (ii) we get:

$$2x_e(t) = x(t) + x(-t)$$

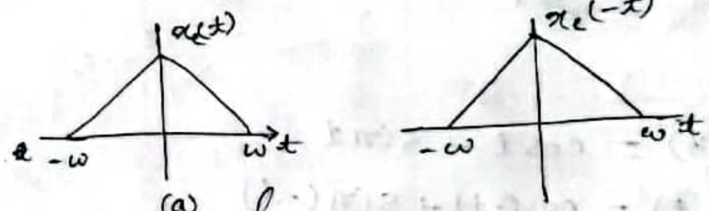
$$x_e(t) = \frac{x(t) + x(-t)}{2}$$

Substruct the eqn (i) and (ii) we get,

$$2x_o(t) = x(t) - x(-t)$$

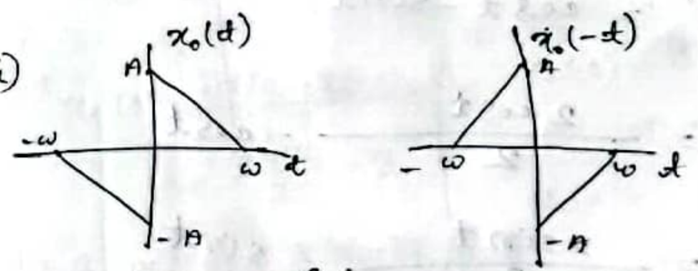
$$x_o(t) = \frac{x(t) - x(-t)}{2}$$

(i)



(a) even signal

(ii)



odd signal.

(2.18) Find the odd and even component of $x(t) = e^{j2t}$

$x(t) = e^{j2t}$

$x(-t) = e^{-j2t}$

even component

$x_e(t) = \frac{x(t)+x(-t)}{2} = \frac{e^{j2t}+e^{-j2t}}{2}$

$= \cos 2t$

odd component.

$x_o(t) = \frac{x(t)-x(-t)}{2} = j\frac{e^{j2t}-e^{-j2t}}{2j} = j\sin 2t.$

$x(t) = x_o(t) + x_e(t)$

$= \cos 2t + j\sin 2t$

$= e^{j2t}.$

2.19.

$x(t) = \cos t + \sin t$

$x(-t) = \cos(-t) + \sin(-t)$

$= \cos t - \sin t.$

$x_e(t) = \frac{2\cos t}{2} = \cos t$

$x_o(t) = \frac{2\sin t}{2} = \sin t.$

$x(t) = x_e(t) + x_o(t)$

$= \cos t + \sin t$

(2.21) Find the even and odd component of $x(n) = \{3, 2, \underset{\uparrow}{1}, 4, 5\}.$

Solⁿ:

n	-2	-1	0	1	2
$x(n)$	3	2	1	4	5

$x_e(n) = \frac{x(n)+x(-n)}{2}$

$n=-2$

$x_e(-2) = \frac{x(-2)+x(2)}{2}$

$= \frac{3+5}{2} = 4$

$n=-1$ $x_e(-1) = \frac{x(-1)+x(1)}{2}$

$= \frac{2+4}{2} = \frac{6}{2}$

$= 3$

$n=0$

$x_e(0) = \frac{x(0)+x(0)}{2}$

$= 2$

$n=1$ $x_e(1) = \frac{x(1)+x(-1)}{2}$

$= \frac{4+2}{2} = 3$

$n=2,\ x_e(2) = \frac{x(2)+x(-2)}{2}$

$= \frac{5+3}{2}$

$= 4$

$x_o(n) = \frac{x(n)-x(-n)}{2}$

$n=-2$

$x_o(n) = \frac{x(-2)-x(2)}{2}$

$= \frac{3-5}{2} = -1$

$n=-1$

$x(-1) = \frac{x(-1)-x(1)}{2}$

$= \frac{2-4}{2} = \frac{-2}{2} = -1$

$x_o(0) = \frac{1-1}{2} = 0$

$n=1$

$x_o(1) = \frac{x(1)-x(-1)}{2}$

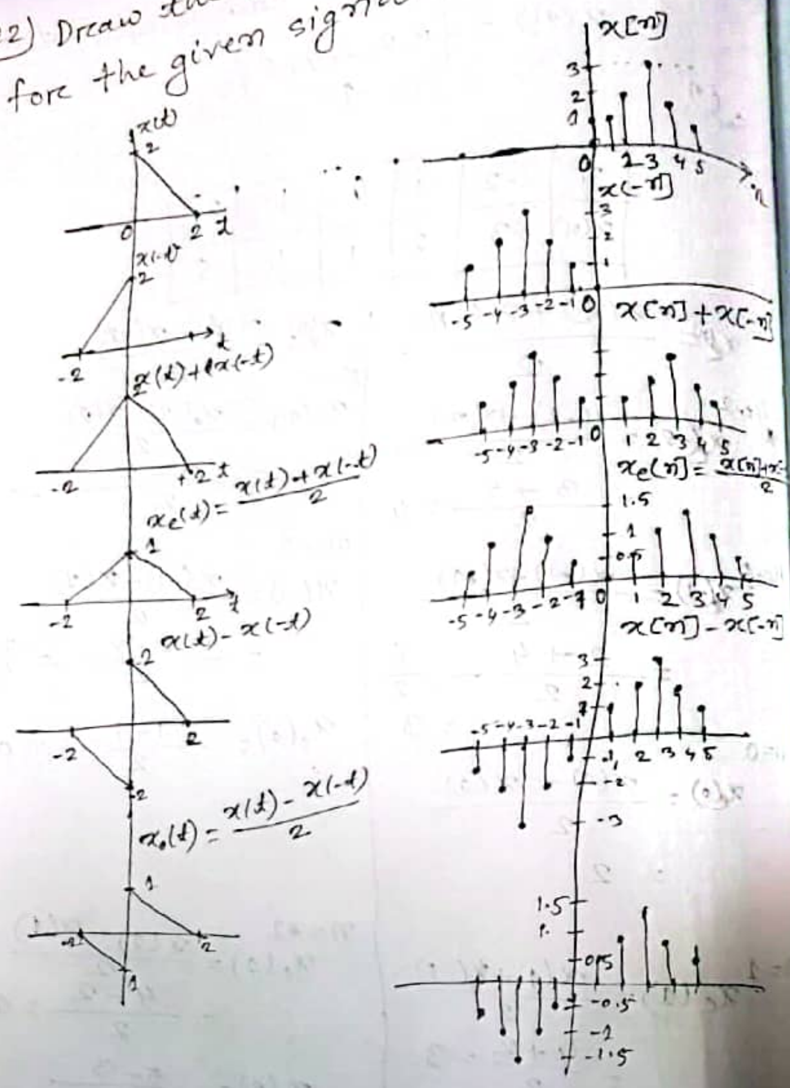
$= \frac{4-2}{2} = 1$

$x_o(2) = \frac{5-3}{2}$

$= \frac{2}{2}$

$= 1.$

2.22) Draw the even and odd representation for the given signal.



$x(n) = u(n) - u(n-5)$

$E = \lim_{N\to\infty} \sum_{n=-N}^{N} |u(n) - u(n-5)|^2 = E \sum_{n=0}^{4} 1 =$

জাতীয় বিশ্ববিদ্যালয়

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ক্রমিক নং ঃ ....

অতিরিক্ত উত্তরপত্র

১। .... পরীক্ষা .... সাল

২। বিষয় ঃ ....

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৬। ইনভিজিলেটরের স্বাক্ষর ও তারিখ ঃ ....

(এ স্থান হতে উত্তর লেখা আরম্ভ করতে হবে)

Complex valued continuous-time signal.

A complex valued signal $x(t)$ is said to be conjugate symmetry if satisfies the condition.

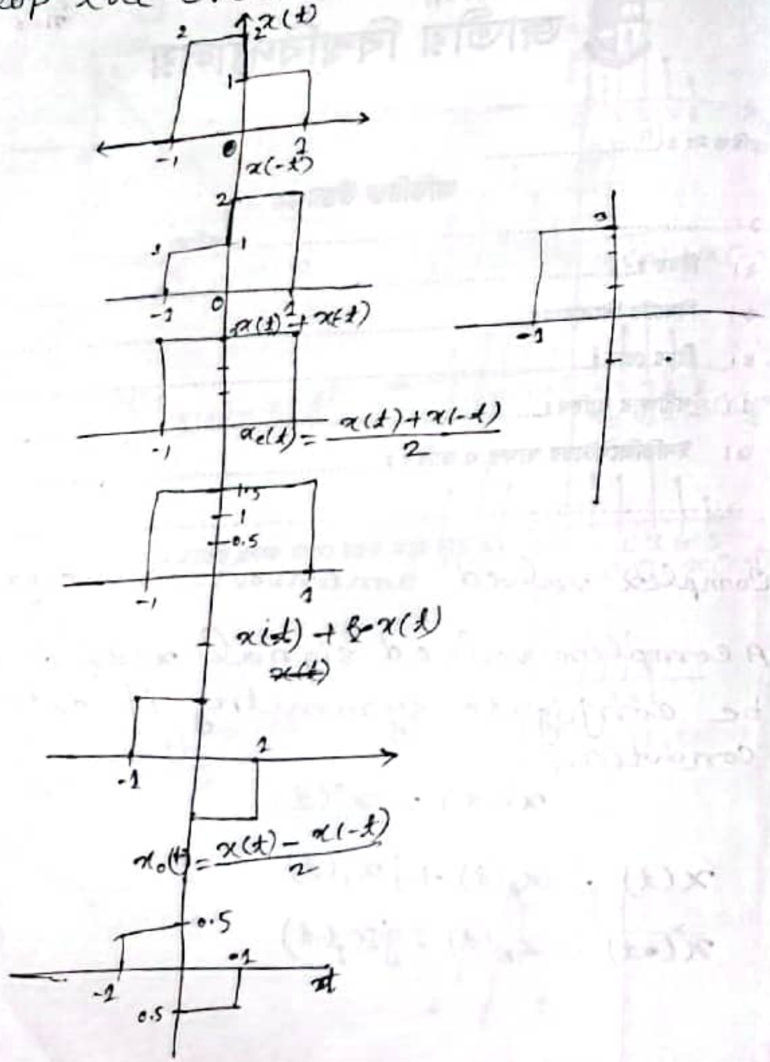
$$x(-t) = x^*(t)$$

$$x(t) = x_R(t) + j x_I(t)$$

$$x^*(-t) = x_R(t) - j x_I(t)$$

~~Energy and power signal.~~

(2018) Develop the even odd decomposition of $x(t)$

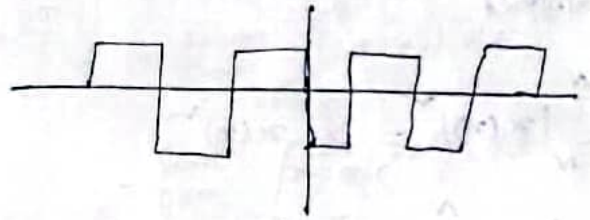


$x(t) = x_e(t) + x_o(t)$

Deterministic and Random signal.

Deterministic signal.

A deterministic signal (continuous and discrete-time) is a signal about which there is certainty with respect to its values at any time. For example ECG, ~~sig~~ sinusoidal signal, square wave, etc.



Random signal.

A random signal (continuous-time ~~or~~ discrete-time) is a signal about which there is uncertainty with respect to ~~time~~ its values at any time. Speech signal, noise, moving object tracking

Energy and power signal.

Continuous-time signal's total energy.

$$E = \int_{-\alpha}^{\alpha} x^2(t)\,dt.$$

$$P_{avg} = \frac{1}{T}\int_{-\alpha}^{\alpha} |x(t)|^2 dt$$

$$P = \lim_{T\to\alpha} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$$

$$P_{Peri} = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$$

total energy.

$$E = \lim_{N\to\alpha} \sum_{n=-N}^{N} |x(n)|^2 = \sum_{n=-\alpha}^{\alpha} x(n)^2$$

$$P_{avg} = \lim_{N\to\alpha} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$$P_{periodic} = \frac{1}{N}\sum_{n=0}^{N-1} x^2[n]$$

Power signal.

A signal that have non-zero. finite average power. i.e $0 < P < \alpha$ infinite energy. periodic signal, random signal.

$P = F$, $E = \alpha$, Power signal.

Energy signal.

A signal that have non-zero finite energy i.e $0 < E < \alpha$.

It has zero average power.

$E = F$, $P = 0$. Energy signal.

(25) Test the signal is power or energy.

$$x(t) = e^{-2t}u(t)$$

Sol^n

$$E = \lim_{T\to\alpha}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$$

$$= \lim_{T\to\alpha}\int_{0}^{\frac{T}{2}} |e^{-2t}|^2 dt$$

$$= \lim_{T\to\alpha}\int_{0}^{\frac{T}{2}} e^{-4t} dt$$

$$= \lim_{T\to\alpha}\left[\frac{e^{-4t}}{-4}\right]_0^{\frac{T}{2}}$$

$$= -\frac{1}{4}\left(e^{-4\times\frac{T}{2}} - 1\right)$$

$$= -\frac{1}{4}\left(e^{\alpha} - 1\right)$$

$$= \frac{1}{4} < \alpha$$

$\frac{1}{\alpha} = 0$

$e^{-\alpha} = 0$

Power signal.

$$P = \lim_{T\to\alpha} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |e^{-2t}u(t)|^2 dt$$

$$= \lim_{T\to\alpha} \frac{1}{T}\int_{0}^{\frac{T}{2}} e^{-4t}\,dt$$

$$= \lim_{T\to\alpha} \frac{1}{T}\left[\frac{e^{-4t}}{-4}\right]_0^{\frac{T}{2}}$$

$$= \lim_{T\to\alpha} \frac{1}{-4T}\left[e^{-4\times\frac{T}{2}} - e^0\right]$$

$$= \lim_{T\to\alpha} -\frac{1}{4T}\left(e^{-\frac{T}{2}} - 1\right)$$

$$= \frac{1}{\alpha}$$

$$= 0$$

Energy is finite and power is zero. then energy signal.

(2b) $x(n) = (-0.5)^n u(n)$

$$E_x = \sum_{n=-\alpha}^{\alpha} |x(n)|^2$$

$$= \sum_{n=-\alpha}^{\alpha} |(-0.5)^n u n|^2 \qquad \sum_{n=0}^{\alpha}(a^n) = \frac{1}{1-a}$$

$$= \sum_{n=0}^{\alpha} |0.25|^n$$

$$= \frac{1}{1-0.25}$$

$$= \frac{1}{0.75}$$

$$E_x = \frac{4}{3} < \alpha \qquad E_x = \text{Finite}$$

Power is $= \sum_{n=-\alpha}^{\alpha} \frac{1}{1}$

$$= \lim_{T\to\alpha} \frac{1}{2N+1} \sum_{n=-N}^{N} [x(n)]^2$$

$$P = \lim_{N\to\alpha} \frac{1}{2N+1} \sum_{n=0}^{N} (0.25)^n$$

$$= \lim_{N\to\alpha} \frac{1}{2N+1} \cdot \frac{4}{3}$$

$$= \frac{1}{\alpha} = 0$$

Energy signal.

(ii) $x(t) = e^{j(2t+\frac{\pi}{4})}$

$$E = \lim_{T\to\alpha} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$$

$$= \lim_{T\to\alpha} \int_{-\frac{T}{2}}^{\frac{T}{2}} |e^{j(2t+\frac{\pi}{4})}|^2 dt$$

$$= \lim_{T\to\alpha} \int_{-\frac{T}{2}}^{\frac{T}{2}} 1^2 dt$$

$$= \lim_{T\to\alpha} [t]_{-\frac{T}{2}}^{\frac{T}{2}}$$

$$= \alpha$$

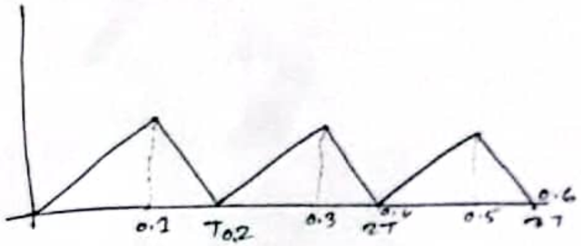
$$P = \lim_{T\to\alpha} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt .$$

$P = 1 < \alpha$

Finite power and infinite energy

~~energy~~ power signal.

(c)



$$E = \lim_{T\to\alpha} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$$

$$= \lim_{T\to\alpha} \int_{0}^{0.2} \left(\frac{1}{0.1}\right)^2 t^2 dt$$

$$= \lim_{T\to\alpha} \frac{1}{10^{-2}} \left[\frac{t^3}{3}\right]_0^{0.1}$$

$$= \lim_{T\to\alpha}$$

Basic operation of on signal.

Operation perform on dependent variables.

- Amplitude scaling of signal.
- Addition of signal
- multiplication of signal.
- Differentiation of signal.
- Integration of signal

Operation perform on independent varia

- Time scaling of signal.
- Reflection of signal.
- Time shifting ~~and scal~~ of signal.

- Amplitude scaling of signal

For continuous-time signal.

Applying amplitude scaling on $x(t)$, yields.

$$y(t) = c\,x(t),$$

where, c is the scalar quantity known as the scaling factor.

For is discrete-time signal

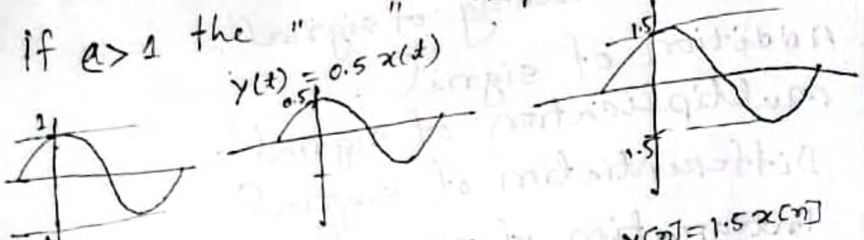
$$y[n] = c\,x[n]$$

CamScanner

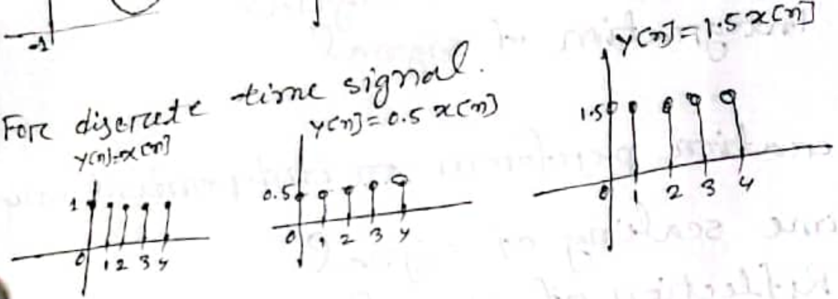
if the scaling factor $c \to 0$

$c < 1$, then the signal attenuates;

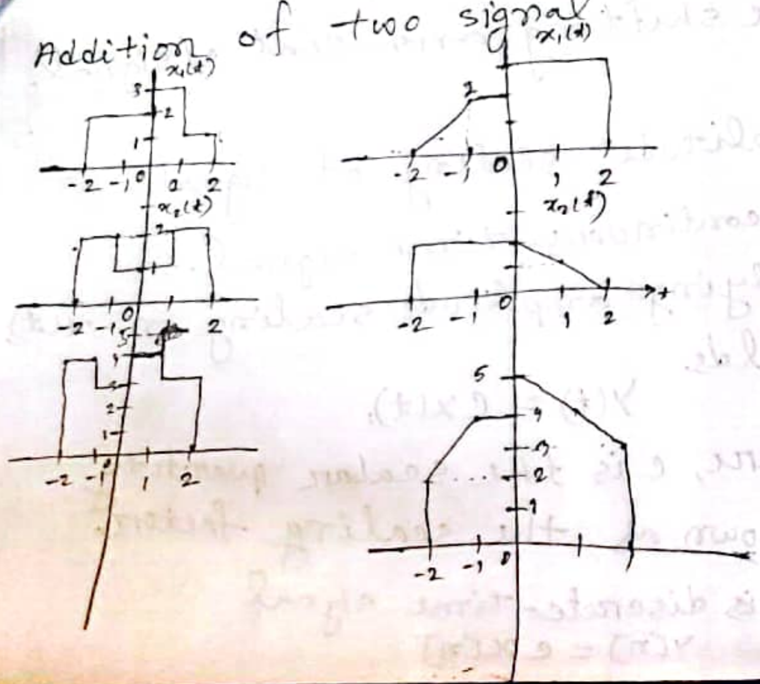
if $c > 1$ the " " amplifies. $y(t) = 1.5x(t)$



For discrete time signal.



Addition of two signal



জাতীয় বিশ্ববিদ্যালয়

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ক্রমিক নং ঃ ....

অতিরিক্ত উত্তরপত্র

১। .... পরীক্ষা .... সাল

২। বিষয় ঃ ....

৩। বিষয়ের শিরোনামঃ ....

৪। বিষয় কোডঃ ....

৫। পরীক্ষার তারিখঃ ....

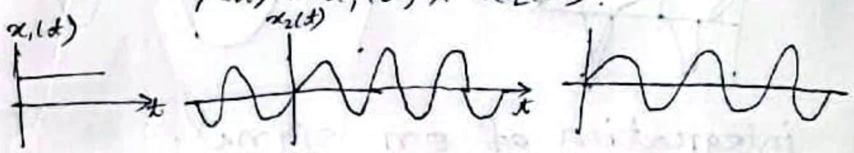
৬। ইনভিজিলেটরের স্বাক্ষর ও তারিখ ঃ ....

(12) Multiplication of Signals.

(এ স্থান হতে উত্তর লেখা আরম্ভ করতে হবে)

Consider a pair of continuous-time signals, $x_1(t)$ and $x_2(t)$ are two signal multiplication of two signal is

$$y(t) = x_1(t) \times x_2(t).$$



Differentiation of an signal.

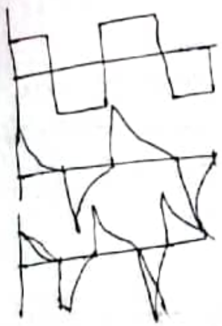
The derivative of an input signal $x(t)$ with respect to time is defined by

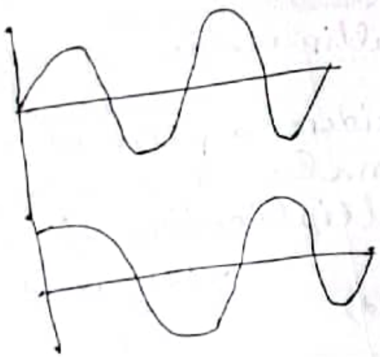
$$y(t) = \frac{d\,x(t)}{dt}$$

example.

Voltage across an inductor of inductor L due to the flow of current $i(t)$.

$$v(t) = L\,\frac{d\,i(t)}{dt}.$$





integration of an signal.

$$y(t) = \int_{-\alpha}^{t} x(t)\,dt.$$

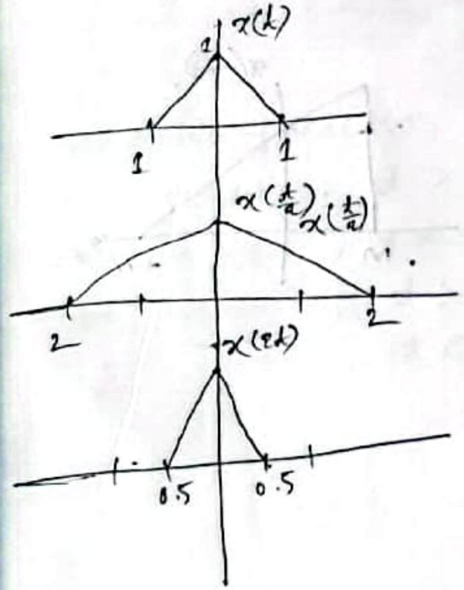
$$y(n) = \sum_{n=-\alpha}^{t} x(n)$$

Time scaling of Signals.

consider a continuous-time signal $x(t)$. Let us introduce a time scaling factor β to the continuous-time signal, i.e

$$y(t) = x(\beta t)$$

where, β = scaling factor (if $\beta < 1$, then the signal expands, $\beta > 1$ the signal is compresses.



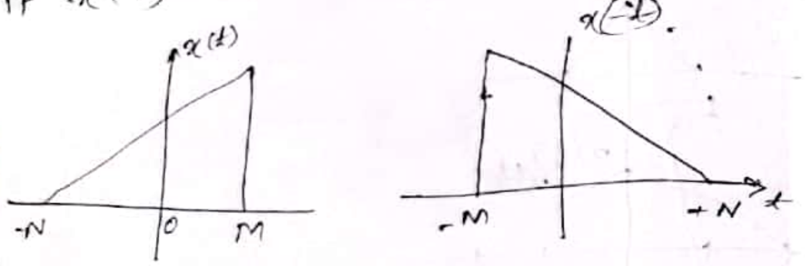
⑦ Reflection of Signals.

Consider a continuous-time signal. Let $y(t)$ denote a signal obtain by replacing t by $-t$ to the continuous time signal.

$y(t) = x(-t)$.

if $x(-t) = x(t)$ even, continuous signal

if $x(-n) = x(n)$ odd, discrete time signal



Time shifting of continuous-time signal.

Consider a signal $x(t)$. Let $y(t)$ denote a signal obtained by shifting the signal $x(t)$ by $(t-t_0)$ that is,

$y(t) = x(t-t_0)$

If the signal $x(t)$ is positive and $t_0 > 0$ for all values of t_0, then the signal is said to be right-shifted signal.

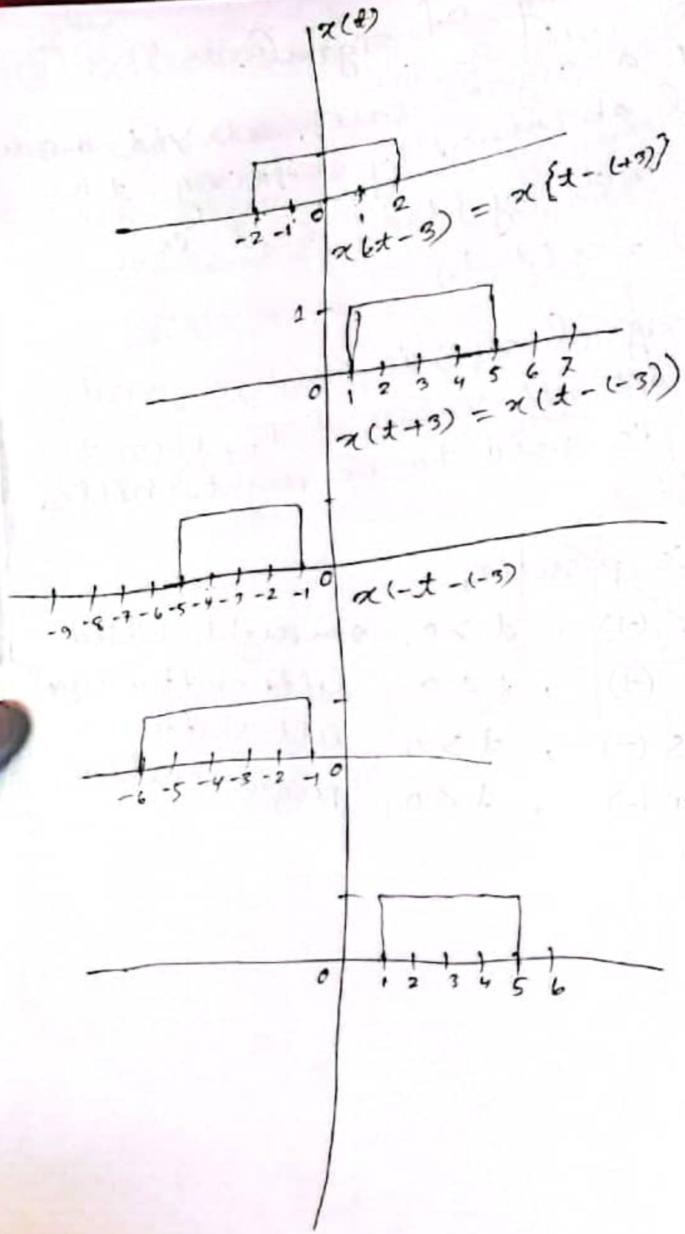
$x(t)$ is possitive,

$x(t)$ is $(+)$, $t > 0$, the right-shifted

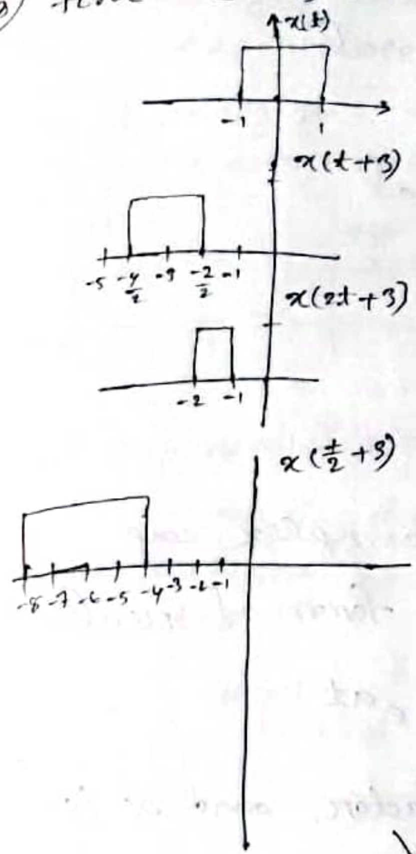
$x(t)$ " $(+)$, $t < 0$, left-shifted signal.

$x(t)$ is $(-)$, $t > 0$, left-shifted "

$x(t)$ " $(-)$, $t < 0$, right-shifted.



(23) find $x(2t+3)$ for a given signal $x(t)$.



LB

$2x+3 = -1$

$x = -2$

UB

$2x+3 = 1$

$x = -1$

$\frac{t}{2}+3 = -2$

$\frac{t}{2} = -4, \; t = -8$

$\frac{t}{2}+3 = 1$

$\frac{t}{2} = -2$

$t = -4$

At First shifting then scaling.

Type of signal.

1. Exponential signal
   (i) Real exp
   (ii) complex exp
2. Sinusoidal signal.
3. Step signal.
4. Impulse signal.
5. Ramp signal.

1. exp signal.
   (i) Real exp and complex exp.

(i) The most general form of real exp signal is

$$x(t) = \beta e^{at}$$

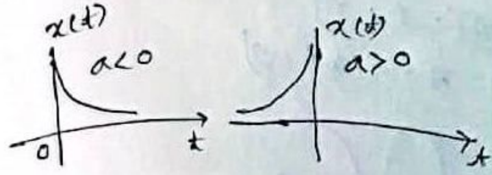
where β is scaling factor, and α is real parameter.

a $\alpha < 0$ the magnitude of real signal decays exponentially.

$\alpha > 0$ the magnitude of a real signal increase exponentially.

example. charging of a capacitor is growing exponential signal.

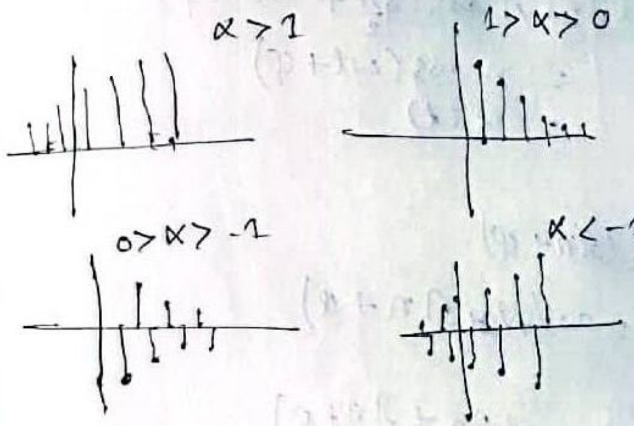
discharging of a capacitor is decaying exponential signal.



Real exponential signal (discrete time signal).

$$x[n] = \beta \alpha^{n}$$

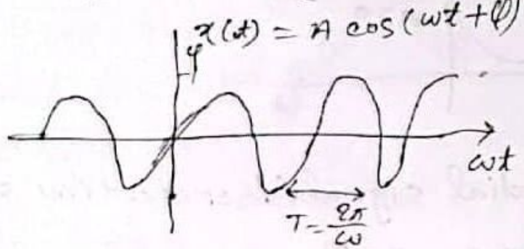
where β is scaling factor and α is real parameter.



Sinusoidal signal. (continuous time)

The continuous version of sinusoidal signal is

$$x(t) = A\cos(\omega t + \phi)$$



Then the equation

$$x(t+T) = A\cos(\omega t + \phi + T)$$
$$= A\cos(2\pi + \omega t + \phi)$$
$$= A\cos(\omega t + \phi)$$
$$= x(t).$$

$T = \frac{2\pi}{\omega}$

$$x[n] = \cos(\Omega n + \phi)$$
$$\Rightarrow x(n+N) = \cos(N\Omega + \Omega n + \phi)$$

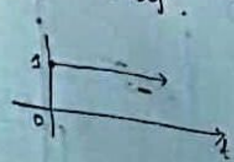
$N = \frac{2\pi m}{\Omega}$

$$= \cos(2\pi + \Omega n + \phi)$$
$$= \cos(\Omega n + \phi).$$

③ Step function.

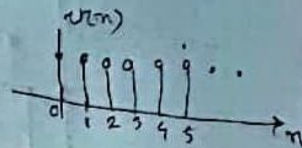
A continuous step function is commonly denoted by $u(t)$ and define as.

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

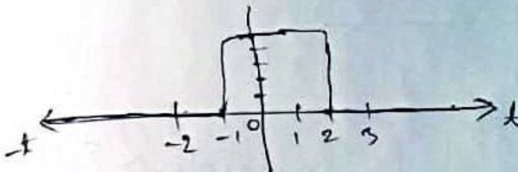


For discrete

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$



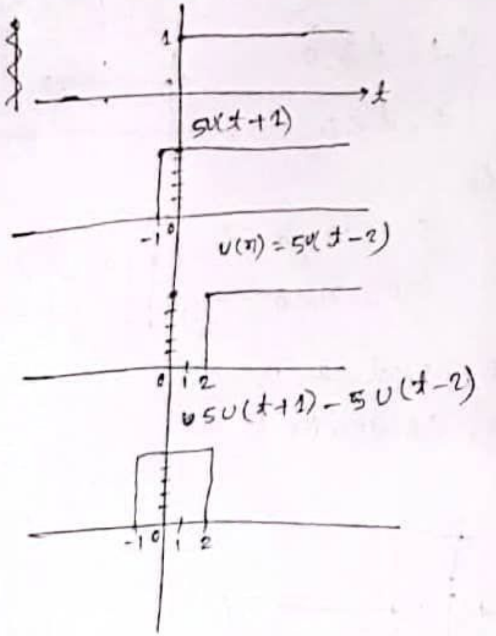
$$x(t) = \begin{cases} 5, & -1 \leq t \leq 2 \\ 0, & \text{Otherwise} \end{cases}$$



$u(t) =$

$= 5 * u(t+1) - 5 * u(t-2)$

$$u(t) = \begin{cases} 1, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



Exponentially damped sinusoidal signal

when a real value decayed exponential signal is multiplied with the sinusoidal signal is defined as exponentially damped sinusoidal signal.

$x(t) = A e^{-at} \sin(\omega t + \theta)$

$x[n] = A e^{-an} \sin(\Omega n + \theta)$

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জাতীয় বিশ্ববিদ্যালয়

ক্রমিক নং ঃ ....

অতিরিক্ত উত্তরপত্র

১। .... পরীক্ষা .... সাল

২। বিষয় ঃ..

৩। বিষয়ের শিরোনাম ঃ..

৪। বিষয় কোড ঃ..

৫। পরীক্ষার তারিখ ঃ..

৬। ইনভিজিলেটরের স্বাক্ষর ও তারিখ ঃ ....

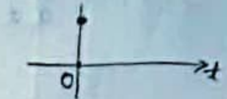
Impulse function. Dirac delta function

(এ স্থান হতে উত্তর লেখা আরম্ভ করতে হবে)

A function that has zero duration, infinite amplitude and unit area under it. is define of impulse function.

Designated by $\delta(t)$ where,

$\delta(t) = 0, \; t \neq 0$



and $\int_{-\infty}^{\infty} \delta(t)\, dt = 1.$

The shifting properties of Impulse func

If a continuous signal ~~signa~~ $x(t)$ is multiplied by an impulse $\delta(t)$ at $t=0$ we get,

$$x(0)\,\delta(t)$$

Integrating,

$$\int_{-\infty}^{\infty} x(0)\,\delta(t)\,dt$$

$$= x(0)\int_{-\infty}^{\infty} \delta t\,dt$$

$$= x(0) \times 1$$

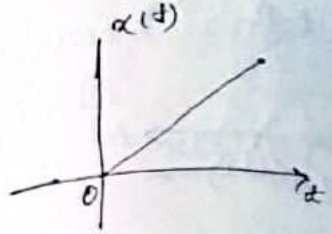
$$= x(0)$$

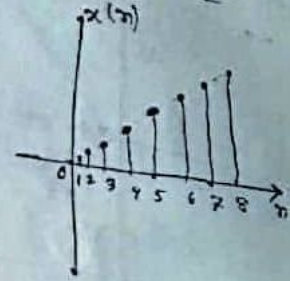
Properties of impulse function.

1. $\delta(t) = \delta(-t)$
2. $\delta(at) = \frac{1}{a}\delta(t)$, $a>0$
3. $\delta(t) = \frac{d}{dt}u(t)$, $t \neq 0$

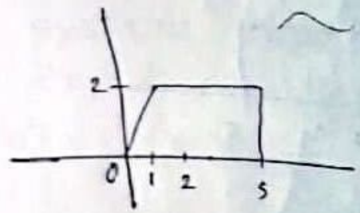
Ramp function.

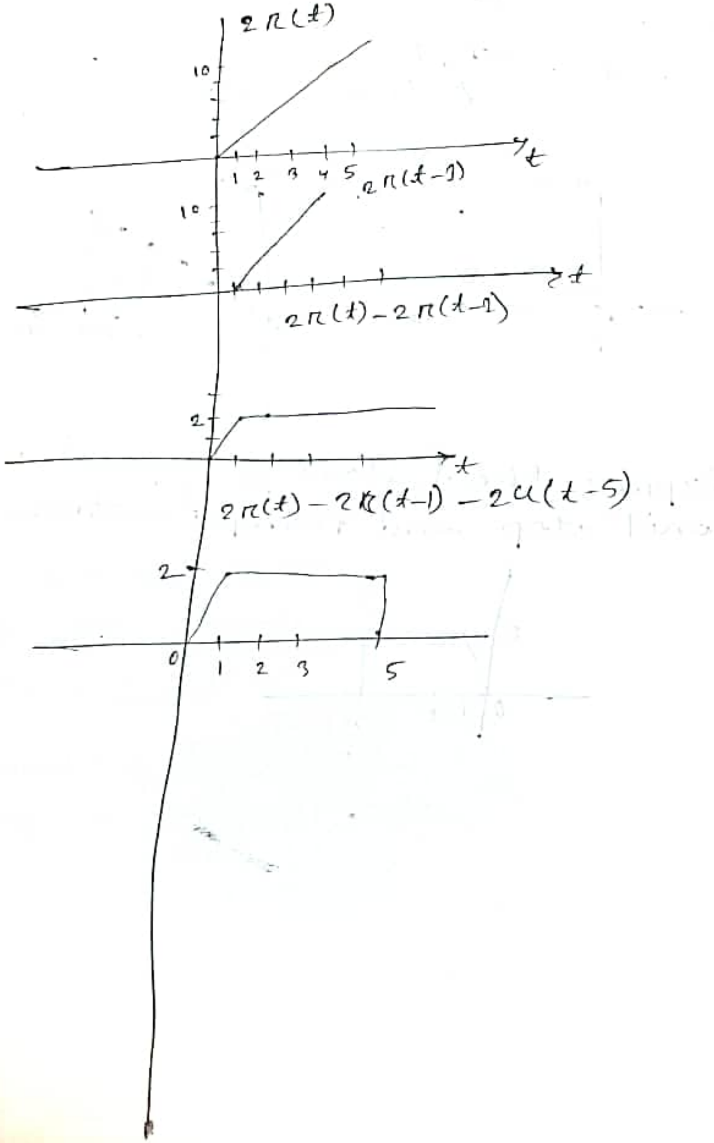
$$x(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases}, \quad r(n) = \begin{cases} n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$





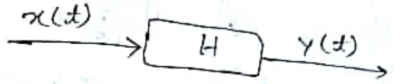
*Express the following signal with unit step and ramp function.





System.

A system can be viewed as interconnections of operations that transform input signal into an output signal having different properties than the input signal.

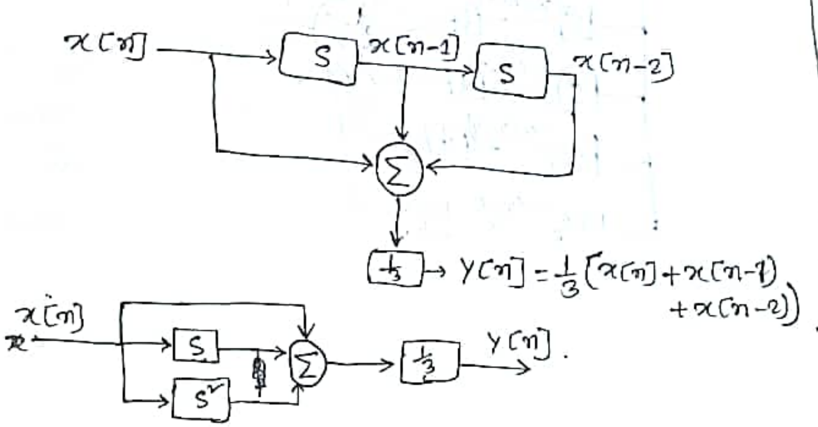


H is the over all operator.

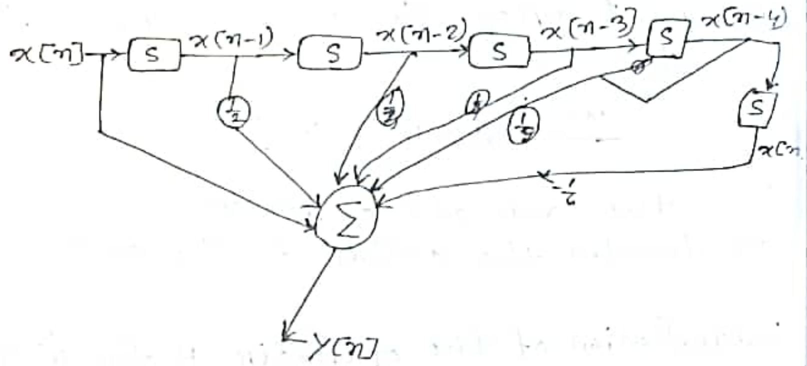
It denotes the action of the system.

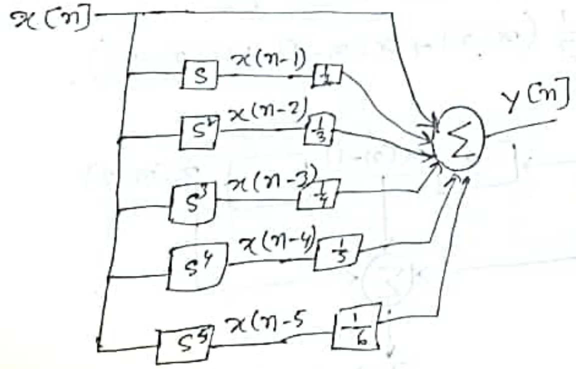
Formulation of the operator H for discrete-time system that has the following output input relationship.

$$y[n] = \frac{1}{3}\left(x[n] + x[n-1] + x[n-2]\right).$$

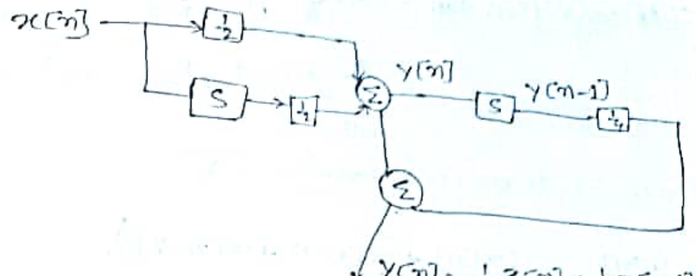


$$y[n] = x[n] + \frac{1}{2}x(n-1) + \frac{1}{3}x[n-2] + \frac{1}{4}x(n-3) + \frac{1}{5}x(n-4) - \frac{1}{6}x[n-5].$$





$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{2}x[n] + \frac{1}{2}x[n-1].$$



$$y[n] = \frac{1}{2}x[n] + \frac{1}{2}x[n-1] + \frac{1}{4}y[n-1].$$

Properties of system:

(1) stability (2) Memory (3) Invertibility
(4) Time variant (5) Causality (6) linearity.

(1.) Stability.

A given system is (BIBO) bounded input and bounded output is said to be stable if and only if every bounded input produces a bounded output.

if $|x(t)| \le M_x < \alpha$ for all t

$|y(t)| \le M_y < \alpha$ for all t.

(i) $h_1(n) = 2^n u(n-3)$

This condition for stability is $\sum_{n=3}^{\infty} |2^n| < \infty$

$$\sum_{n=3}^{\infty} 2^n = 2^3 + 2^4 + \cdots + 2^{\infty} = \infty$$

Hence system is stable.

(ii) $y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$.

$= \frac{1}{3}x[n] + \frac{1}{3}x(n-1) + \frac{1}{3}x[n-2]$

$= \frac{1}{3}x[n] + \frac{1}{3}$

$\sum_{n=0}^{\infty} \frac{1}{3}x[n] = \frac{1}{3}(0+1+2+\cdots+\infty) = \infty$

$\frac{1}{3}\sum_{n=1}^{\infty} x[n-1] = \frac{1}{3}(0+1+2+3+\cdots+\infty) = \infty$

$\frac{1}{3}\sum_{n=2}^{\infty} x[n-1] = \frac{1}{3}(0+1+2+3\cdots+\infty) = \infty$

This system is stable.

(iii) $y[n] = r^n x[n] \quad r > 1$

*** Memory system:

A system is said to be ~~pre~~ possess memory if the output of the system past values of input system. also known as dynamic system.

An inductor or ~~volt~~ capacitor has a memory. voltage across the capacitor or ~~co~~ current through the inductor is called the memory ~~system~~ system.

$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = C\frac{dv(t)}{dt}.$$

$y[n] = \frac{1}{3}(x[n] + x[n-1] + x[n-2])$
output is depend on present input and past input. This system is called memory system.

$y[n] = x[n]$ only dependent on the present input. memory less system.

Causal and Non causal system.

The output response of the system at any time depends only on the present input or past input not future input the system is called the causal system.

$y[n] = \frac{1}{3}(x[n] + x(n-1) + x[n-2])$ ~~causal~~
Causal system.

$y[n] = \frac{1}{3}(x[n] + x(n+1) + x[n-2])$
non-causal system.

(i) $y(t) = t\,x(t)$

$t=0$, $y(0) = 0$

$t=1$, $y(1) = x(1)$

$t=-1$, $y(-1) = x(-1)$

causal system.

(ii) $y(t) = x(t^2)$

let, $t=0$, then $y(0) = x(0)$

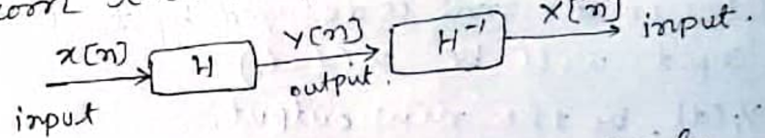
$t=1$, then $y(1) = x(1)$

$t=-1$, then $y(-1) = x(+1)$

non-causal system.

Invertible or non-invertible.

A system is said to be invertable if the input can be reconstructed from the output.



$y[n] = x^2[n]$ - Non invertible

$y(t) = \frac{d}{dt}\{x(t)\}$ Non "

$y[n] = x^3[n]$ Invertible.

$H^{-1}\{y[n]\} = H^{-1}\{H(x[n])\}$

$= HH^{-1}(x(n))$ $\quad HH^{-1} = I$

$= x(n)$

~~Time~~ Time-invariant and Time Varia

Time invariant.

A system is time invariant if a time shift in the input signal leads to an ~~indical~~ identical time shift in the output signal.

Stated: The characteristics of a time-invariant system do not change with time.

Consider the system where $y(t) = H\{x(t)\}$

No if the input signal is delayed by t_0 then new input will be $x(t-t_0)$

let $y_i(t)$, be the new output.

$$y_i(t) = H\{S^{t_0}\{x(t)\}\}$$

$$x(t) \rightarrow \boxed{S^{t_0}} \xrightarrow{x(t-t_0)} \boxed{H} \rightarrow y_i(t)$$

$$y_i(t) = HS^{t_0}\{x(t)\}$$

Now let's assume $y_0(t)$ is the output of the orginal system shifted by t_0.

$$y_0(t) = S^{t_0}\{y(t)\}$$

$$y_0(t) = S^{t_0}\{H\{x(t)\}\}.$$

$$y_0(t) = S^{t_0}\{H\{x(t)\}\}.$$

To be time invariant,

$HS^{t_0} = S^{t_0}H$. H and S^{t_0} must commute.

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জাতীয় বিশ্ববিদ্যালয়

ক্রমিক নং ঃ ....

অতিরিক্ত উত্তরপত্র

১। .... পরীক্ষা .... সাল

২। বিষয় ঃ..............................

৩। বিষয়ের শিরোনামঃ..............................

৪। বিষয় কোডঃ..............................

৫। পরীক্ষার তারিখঃ..............................

৬। ইনভিজিলেটরের স্বাক্ষর ও তারিখ ঃ ....

~~Linearity~~

(এ স্থান হতে উত্তর লেখা আরম্ভ করতে হবে)

the input output relationship is given by $y(t) = \sin[x(t)]$. Determine the system is time invariant or not.

$$y(t) = \sin[x(t)] \quad \text{—— (i)}$$

let us assume the signal of the form.

$$y_1(t) = \sin[x_1(t)] \quad \text{—— (1)}$$

let us introduce time delay t_0 in the input signal in eq^n (i).

$$x_2(t) = x_1(t-t_0)$$

the delay input therefore results in the output

$$y_2(t) = \sin\{x_1(t-t_0)\}. \quad \text{—— (2)}$$

let us introduce the same delay t_0 in the output of the equation.

$$Y_1(t-t_0) = \sin[x(t-t_0)] \quad \text{--- (3)}$$

Compare eqn (2) and (3)

$$Y_2(t) = Y_1(t-t_0)$$

Hence, system is time-invariant.

(ii) $y(t) = t\,x_1(t)$

let us introduce the time delay input signal.

$$x_1(t) = x_1(t-t_0)$$

$$Y_1(t) = t\,x_1(t) = t\,x_1(t-t_0). \quad \text{--- (1)}$$

Again
let us introduce the time delay in output signal.

$$Y_2(t-t_0) = (t-t_0)\,x_1(t-t_0) \quad \text{--- (ii)}$$

compare the eqn (i) and (ii) we get.

$$Y_1(t) \neq Y_2(t-t_0)$$

Hence the system is not time invariant.

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(iii) $y(t) = x(4t)$

let us assume the signal is

$$Y_1(t) = x_1(4t)$$

let us introduce the time delay t_0 in the input signal

$$x_1(t-t_0) = x_1[4(t-t_0)].$$

$$Y_1(t) = x_1[4(t-t_0)] \quad \text{--- (1)}$$

Again
let us introduce the time ~~vari~~ delay t_0 in the output signal.

$$Y_2(t-t_0) = x[4(t-t_0)] \quad \text{--- (ii)}$$

Compare the equation (1) and (2)

$$Y_1(t) = Y_2(t-t_0)$$

Hence the signal is time invariant.

(iii) $y(t) = e^{x(t)}$

$$y(t) = T[x(t)] = e^{x(t)}$$

Introduce time delay t_0 in the input i.e

$$x_2(t) = x(t-t_0)$$

$$y_2(t) = e^{x_1(t)} = e^{x(t-t_0)} \text{ --- (i)}$$

Again Introduce time delay t_0 in the output i.e

$$y_2(t-t_0) = e^{x(t-t_0)} \text{ --- (ii)}$$

compare eqn (i) and (ii).

$$y_1(t) = y_2(t-t_0)$$

Hence, The system is time invariant.

Linear and nonlinear system.

A system is linear if it satisfies the principle of superposition.

Superposition principle is that the responses of a linear system to a sum of inputs is equal to the sum of the responses of the system to each individual input.

In mathematically,

let us consider, a signal

$$y(t) = x(t)$$

If the input is $x_1(t)$ the output is $y_1(t)$

$$y_1(t) = x_1(t)$$

Similarly $y_2(t) = x_2(t)$

Above signal $x_1(t)$ and $x_2(t)$ are related as

$$x_3(t) = a x_1(t) + b x_2(t)$$

where a, b are constant.

The the output $y_3(t)$ is define.

$$y_3(t) = x_3(t)$$

$$= [a x_1(t) + b x_2(t)]$$

$$y_3(t) = a y_1(t) + b y_2(t) = a x_1(t) + b x_2(t) \quad \text{linear}$$

(2.48) $y(t) = t\,x(t) + k$

let us ~~considere~~ define the input signal $x_1(t)$ where result the output signal is $y_1(t)$

$$y_1(t) = x_1(t) + k$$

similarly let us define the input signal $x_2(t)$ whe result the output signal is $y_2(t)$.

$$y_2(t) = x_2(t) + k$$

The above signal $x_1(t)$ and $x_2(t)$ are related by

$$x_3(t) = a x_1(t) + b x_2(t)$$

where a, b are constant.

Then the output $y_3(t)$ is define as,

$$y_3(t) = x_3(t) + k$$
$$= [a x_1(t) + b x_2(t)] + k$$

Again $y_3'(t) = y_1(t) + y_2(t)$
$$= x_1(t) + k + x_2(t) + k$$

$y_3(t) \neq y'(t)$ are not equal.

The system is not linear.

(51) $y(t) = e^{x(t)}$

let us ~~considere~~ define the signal input $x_1(t)$ which produce the output signal $y_1(t)$.

$$y_1(t) = x_1(t) = e^{x_1(t)}$$

Similarly Again let us define input signal $x_2(t)$ which ~~prod~~ result $y_2(t)$.

$$y_2(t) = x_2(t) = e^{x_2(t)}$$

The input $x_1(t)$ and $x_2(t)$ are related as

$$x_3(t) = a x_1(t) + b x_2(t)$$

where a, b are constant.

The the output is define as.

$$y_3(t) = \cancel{x_3(t)}\, e^{x_3(t)}$$
$$= e^{a x_1(t) + b x_2(t)}$$

$$y_3'(t) = a y_1(t) + b y_2(t)$$
$$= a e^{x_1(t)} + b e^{x_2(t)}$$

$$y_3(t) \neq y_3'(t)$$

The system is not linear.

(ii) $y(n) = x^2(n)$ —— (1)

let us define input signal $x_1(n)$ which result output signal is $y_1(n)$.

$$y_1(n) = x_1^2(n)$$

Similarly $x_2(n)$ which result $y_2(n)$

$$y_2(n) = x_2^2(n)$$

The two input signal $x_1(n)$ and $x_2(n)$ related of

$$x_3(n) = a\,x_1(n) + b\,x_2(n) \quad a, b \text{ are const}$$

Then the output al

$$y_3(n) = [x_3(n)]^2$$

$$= [a x_1(n) + b x_2(n)]^2$$

$$= a^2 x_1^2(n) + 2 a x_1(n) \cdot b x_2(n) + b^2 x_2^2(n)$$

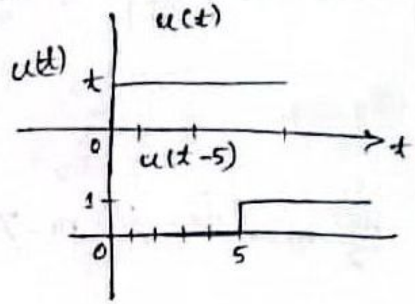
$$y_3'(n) = a y_1(n) + b y_2(n)$$

$$= a x_1^2(n) + b x_2^2(n)$$

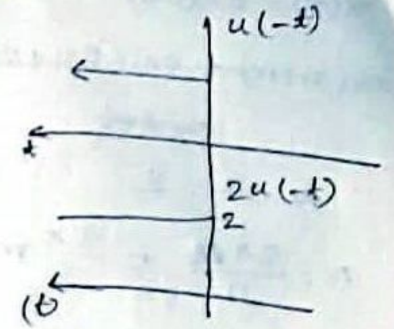
$$y_3(n) \neq y_3'(n)$$

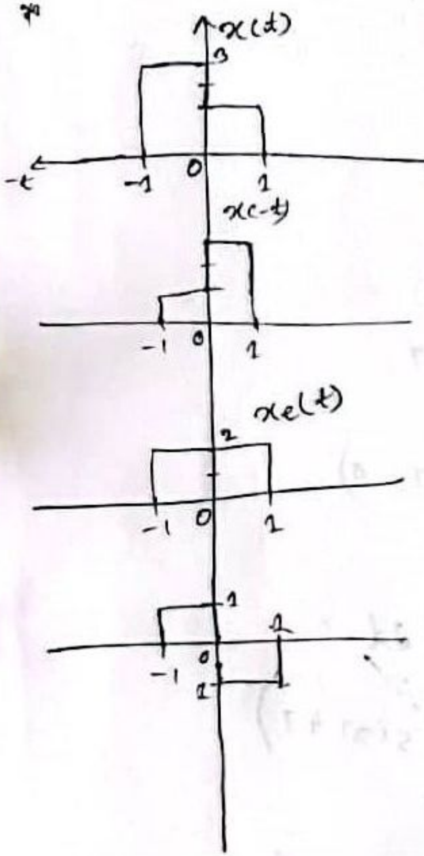
The system is non linear.

plot $u(t-5)$



plot $(2u(-t))$.





$x(n) = \cos(2n)$ $\qquad \cos(2n\Omega)$

$~~x(n+N) = \cos(2n+2\pi)~~$

$~~= \cos~~$

$\Omega = 2$

$N = \frac{2\pi}{\Omega} = \frac{2\pi}{2} m = \frac{22}{7} m = 22$ if $m = 7$

2. $\sin(2t)$ Energy or power signal.

$$E = \int_{-\infty}^{\infty} x^2(t)\,dt$$

$$\lim_{T\to\infty} = \frac{1}{2}\int_0^{T} 2\sin^2(2t)\,dt$$

$$= \frac{1}{2}\int_0^{T}(1-\cos 4t)\,dt$$

$$= \frac{1}{2}(t)_0^{T} - \left[\frac{\sin 4t}{4}\right]_0^{T}$$

$$\lim_{T\to\infty}\left(\frac{1}{2}(T-0) - \frac{1}{8}(\sin 4T - 0)\right)$$

$$= \infty$$

$$P = \lim_{T\to\infty} \frac{1}{T}\int_0^{T} \sin^2(2t)\,dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\left(\frac{1}{2}\times T - \frac{1}{8}\sin 4T\right)$$

$$= \frac{1}{2} - 0$$

power signal.

response

$$P_{av} = \int_0^{T} \frac{1}{T}\sin^2(2t)\,dt$$

$$= \frac{1}{2T}\int_0^{T}(1-\cos 4t)\,dt$$

$$= \frac{1}{2T}(t)_0^{T} - \frac{1}{2T}\cdot\left[\frac{\sin 4t}{4}\right]_0^{T}$$

$$= \frac{1}{2} - \frac{1}{8T}\sin 4T \qquad T = 2\pi$$

$$= \frac{1}{2} - \frac{1}{8\cdot 2\pi}\sin 8\pi$$

$$= \frac{1}{2}$$

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জাতীয় বিশ্ববিদ্যালয়

ক্রমিক নং : ....

অতিরিক্ত উত্তরপত্র

১। .... পরীক্ষা .... সাল

২। বিষয় : ....

৩। বিষয়ের শিরোনাম: ....

৪। বিষয় কোড: ....

৫। পরীক্ষার তারিখ: ....

৬। ইনভিজিলেটরের স্বাক্ষর ও তারিখ : ....

(এ স্থান হতে উত্তর লেখা আরম্ভ করতে হবে)

Define impulse respone.

The impulse response is the output of the linear Time-invariant system when the impulse $\delta(n)$ is applied to it. $h(n)$

$$\delta(n) = \begin{cases} 1, & n = 0 \\ 0, & \text{otherwise} \end{cases}$$

It is the characteristic behavior of LTI system.

* What do you understand by LTI syst.

LTI stands for Linear Time invariant system. In control theory and signal processing an LTI system is a system that has two fundamental properties linearity and time-invariance.

I Linearity means the system satisfies the principle of superposition.

II Time invariance the system response to an input signal is the same regardless of when the input signal is applied.

3.1.1) Representation of Discrete-time signal.

Let us consider the product of signal $x(n)$ and the impulse sequence $\delta(n)$, written as

$$x(n)\delta(n) = x(n)\delta(0)$$

It is clear that the impulse sequence exists only at $n=0$, the input signal $x(n)$ exists in the remaining samples.

Similarly, $x(-2)\delta(n+2)$

$x(-1)\delta(n+1)$

$x(0)\delta(n)$

$x(1)\delta(n-1)$

$x(2)\delta(n-2)$

Therefore the generalized relationship between the input signal $x(n)$ and the shifted

$$x(n)\delta(n-k) = x(k)\delta(n-k) \quad \text{--- (i)}$$

input signal

The magnitude of impulse time k.

Product of $x[n]$ and time-shifted impulse $\delta(n-k)$ results in the

Let us analysis the statement graphically the signal can be decomposed into the product of time shifted impulse and signal $x(n)$ at k.

$$x[n] = \ldots x[-2]\delta(n+2) + x[-1]\delta(n+1) + x[0]\delta[n] + x[1]\delta(n-1) + x[2]\delta(n-2) \ldots$$

$$= \sum_{k=-\infty}^{\infty} x[k]\delta(n-k)$$

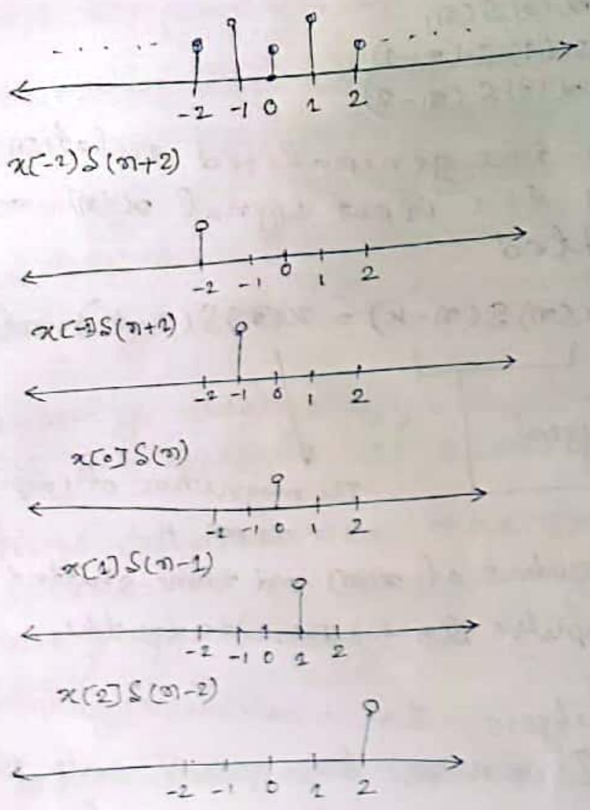


Fig: signal decomposition.

Convocation sum.

From the shifted properties of unit impulse, we can write

$$x[n]\,\delta(n) = \cancel{\delta(0}\; x[0]\,\delta(n)$$

$$\Rightarrow x(n)\,\delta(n-k) = x[k]\,\delta(n-k)$$

we we can write,

$$x(n) = \dots\; x(-2)\,\delta(n+2) + x[-1]\,\delta(n-1) + x(0)\,\delta(n) + x(1)\,\delta(n-1) + x(2)\,\delta(n-2) + \dots$$

$$= \sum_{n=-\alpha}^{\alpha} x(k)\,\delta(n-k)$$ Discrete time signal impulse.

Convocation sum.

Let us consider a system $H[\cdot]$, to which the input signal $x(n)$ and the output signal $Y(n) = H[x(n)]$

$$= H\left[\sum_{n=-\alpha}^{\alpha} x(k)\,\delta(n-k)\right]$$

$$x[n] \longrightarrow H[\cdot] \longrightarrow Y[n]$$

$$= \sum_{n=-\alpha}^{\alpha} x(k)\,\delta H[\delta(n-k)]$$

$$h_k(n) = H[\delta(n-k)] = \sum_{n=-\alpha}^{\alpha} x(k)\, h_k(n)$$

$$= \sum_{n=-\alpha}^{\alpha} k x(k)\, h(n-k) = x(n) * h(n).$$

2.1 $x(n) = \{1, -2, 8, \underset{\uparrow}{4}, 5, -3, 7\}$.

express the given signal as time shifted implse

n	-3	-2	-1	0	1	2	3
$x(n)$	1	-2	8	4	5	-3	7

we know that

$$x[n] = \sum_{k=-\alpha}^{\alpha} x[k]\,\delta(n-k)$$

$$= \sum_{k=-3}^{3} x(k)\,\delta(n-k)$$

$$= x(-3)\delta(n+3) + x(-2)\delta(n+2) + x(-1)\delta(n+1) + x(0)\delta(n) + x(1)\delta(n-1) + x(2)\delta(n+2) + x(3)\delta(n+3)$$

$$= \delta(n+3) - 2\delta(n+2) + 8\delta(n+1) + 4\delta(n) + 5\delta(n-1) + -3\delta(n+2) + 7\delta(n-3).$$

Write properties of convocation sum.

distributive properties.

$x[n] * [h_1(n) + h_2(n)] = x[n]*h_1[n] + x[n]*h_2[n]$

Associative properties.

$[x[n]*h_1(n)]*h_2(n) = x[n]*[h_1(n)*h_2(n)]$

Commulative property.

$h_1(n)*h_2(n) = h_2(n)*h_1(n)$

step of linear convocation.

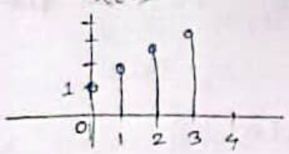
$y[n] = \sum_{k=-\alpha}^{\alpha} x[k]\,h(n-k)$ can be define using algorithm.

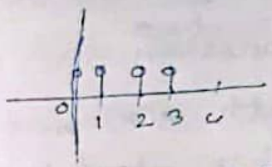
1. Plot both $x(k)$ and $h(k)$
2. Reflect $h(k)$ about $k=0$ to obtain $h(-k)$
3. shifted $h(-k)$ by n,
4. let the initial value of n be negative.
5. Multiply the each element of $x(k)$ and $h(n-k)$ and add the product of the obtain term. $y(n)$.
6. Shift $h(n-k)$ by

data in the memory

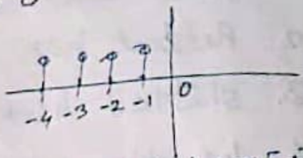
(3.5) Perform the convocation of two sequence

$x[n] = \{1, 2, 3, 4\}; \quad h(n) = \{1, 1, 1, 1\}$



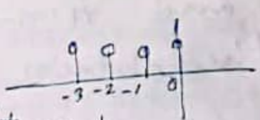


$$y[n] = \sum_{k=-\alpha}^{\alpha} x[k]\, h(n-k)$$



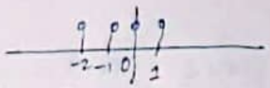
$y(-1) = x[-4]\,h(-4) + x[-3]\,h[-3] + x[-2]\,h(-2) + x[-1]\,h[-1] + x[0]\,h[0] + x[1]\,h[1] + x[2]\,h[-2] + x[3]\,h[3] + x[4]\,h[4]$

$= 0$



$y[0] = x[3]\,h[-3] + \cdots + x[0]\,h[0] + \cdots +$

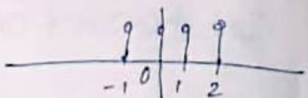
$= 0 + 1 \times 1 + 0 = 1$



$y[1] = x[-2]\,h[-2] + \cdots + x[0]\,h[0] + x[1]\,h[1] + \cdots$

$= 1 + 2$

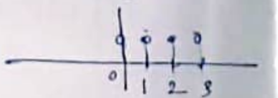
$= 3$



$y[2] = x[-1]\,h[-1] + x[0]\,h[0] + x[1]\,h[1] + x[2]\,h[2] + \cdots$

$= 0 + 1 \times 1 + 2 \times 1 + 3 \times 1 + 0$

$= 1 + 2 + 3 = 6$



$y[3] = x[0]\,h[0] + x[1]\,h[1] + x[2]\,h[2] + x[3]\,h[3] + x[4]\,h[4]$

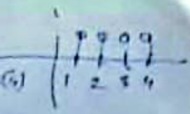
$= 1 \times 1 + 1 \times 2 + 1 \times 3 + 1 \times 4 + 0$

$= 10$

response

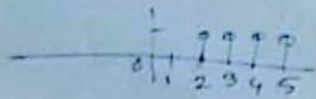
$x[n]\ h[n]$

$y[4] = x[1]\,h[1] + x[2]\,h[2] + x[3]\,h[3] + x[4]\,h[4]$



$= 0 + 2 \times 1 + 3 \times 1 + 4 \times 1 + 0$

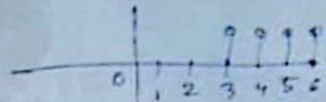
$= 2 + 3 + 4 = 9$



$y[5] = x[0]\,h[0] + \cdots + x[2]\,h[2] + x[3]\,h[3] + \cdots + x[5]\,h[5]$

$= 0 + 3 \times 1 + 4 \times 1 + 0$

$= 7$



$y[6] = x[0]\,h[0] + \cdots + x[3]\,h[3] + \cdots$

$= 0 + 4 \times 1 + 0$

$= 4$

$y[7] = 0$

$y[n] = x[n]\ h[n] = \{1, 3, 6, 10, 9, 4\}$

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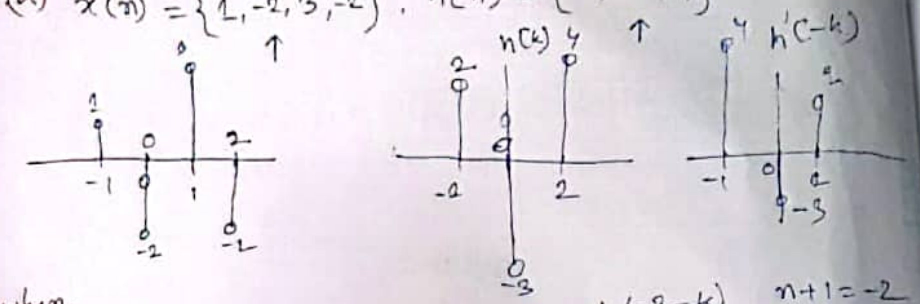
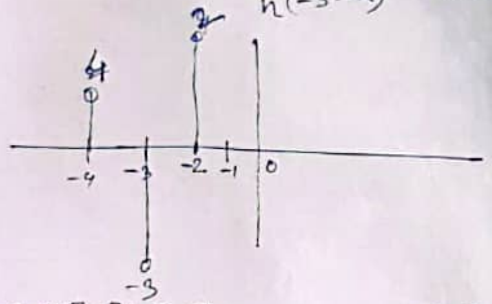
4(a) $x(n) = \{1, -2, 3, -2\}$, $h[n] = \{2, -3, 4\}$



when, $n = -3$, $n+1 = -2$

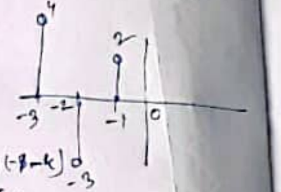


$y(-3) = x[-4]h[-4] + x[-3]h[-3] + \dots + x[2]h[2]$

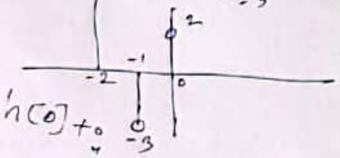
$= 0 + 0 + 0 + \dots + 0 = 0$

when $n+1 = 0-1$

$y(-2) = 2$



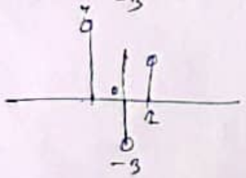
when, $n+1 = 0$



$y[-1] = x[-1]h[-1] + x[0] + h[0] + 0$

$= 1 \times -3 + (-2) \times 2$

$= -3 - 4 = -7$



when $n+1 = 1$

$y[0] =$

Linear Convolution using matrix

$x(n) = \{1, 2, 3, 4\}$ $h(n) = \{1, 1, 1, 1\}$

$$x[n] = \begin{vmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 0 \\ 0 \\ 0 \end{vmatrix}$$

3.5 Linear convolution using cross table method.

Let us consider the convolution,

$$y[n] = x[n] * h[n]$$

where,

$x[n] = \{x_1[n], x_2[n], x_3[n]\}$ and $h[n] = \{h_1[n], h_2[n], h_3[n]\}$

can be performed as.

	$x_1(n)$	$x_2(n)$	$x_3(n)$
$h_1(n)$	$x_1(n)h_1(n)$		
$h_2(n)$	$x_1(n)h_1(n) + x_2[n]h_1[n]$		
$h_3(n)$	$x_1(n)h_1(n) + x_2[n]h_2(n) + x_3(n)h_3(n)$		

	a	b	c
x	ax	bx	cx
y	ay	by	cy
z	az	bz	cz

(3.8) $x(n) = \{1, 2, 3, 4\}$ $h(n) = \{1, 1, 1, 1\}$

using cross table method.

$h(n)$ \ $x(n)$	1	2	3	4
1	1	2	3	4
1	1	2	3	4
1	1	2	3	4
1	1	2	3	4

$y(n) = \{1, 3, 6, 10, 9, 7, 4\}$.

(3.9)

$x(n) = \{1, -2, 3, -4\}$, $h(n) = \{4, -3, 2, -1\}$.

	1	-2	3	-4
4	4	-8	12	-16
-3	-3	6	-9	12
2	2	-4	6	-8
-1	-1	2	-3	4

$y[n] = \{4, -11, 20, -30, 20, -11, 4\}$

linear convolution using matrix.

$x(n) \times H(n)$

$$x(n) = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 2 & 1 & 0 \\ 4 & 3 & 2 & 1 \\ 0 & 4 & 3 & 2 \\ 0 & 0 & 4 & 3 \\ 0 & 0 & 0 & 4 \end{vmatrix}_{7\times4} \begin{vmatrix} 1 \\ 1 \\ 1 \\ 1 \end{vmatrix}_{4\times1} \quad 7\times1$$

$$= \begin{vmatrix} 1 \\ 2+1 \\ 3+2+1 \\ 4+3+2+1 \\ 4+3+2 \\ 4+3 \\ 4 \end{vmatrix} = \begin{vmatrix} 1 \\ 3 \\ 6 \\ 10 \\ 9 \\ 7 \\ 4 \end{vmatrix}$$

De composition.

$$x[0] = \frac{y[0]}{h[0]}$$

$$x[1] = \frac{y[1] - x[0]\,h[1]}{h(0)}$$

$$x(2) = \frac{y[2] - x[0]\,h[2] - x[1]\,h[1]}{h(0)}$$

$$x[n] = \frac{y[n] - \sum_{m=0}^{n-1} x(m)\,h(n-m)}{h(0)}.$$

3.21 $h(n) = \{\underset{\uparrow}{1}, 2, 1\}$

$y[n] = \{\underset{\uparrow}{1}, 5, 10, 11, 8, 4, 1\}$.

$x[n] = ?$

$n_1 + 3 - 1 = 7$

$n_1 = 5$

$$x(0] = \frac{y[0]}{h[0]} = \frac{1}{1} = 1$$

$$x[1] = \frac{y[1] - x[0]\,h[1]}{h[0]} = \frac{5 - 1\times 2}{1}$$

$= 3$

$$x[2] = \frac{y[2] - x[0]\,h[2] - x[1]\,h[1]}{h[0]}$$

$$= \frac{10 - 1\times 1 - 3\times 2}{1}$$

$= 3$

$$x[3] = \frac{y[3] - x[0]\,h[3] - x[1]\,h[2] - x[2]\,h[1]}{h[0]}$$

$$= \frac{11 - 1\times 0 - 3\times 1 - 3\times 2}{1}$$

$$= \frac{11 - 3 - 6}{1} = 2$$

$$x[4] = \frac{y[4] - x[0]\,h[4] - x[1]\,h[3] - x[2]\,h[2] - x[3]\,h[1]}{h[0]}$$

$$= \frac{8 - 1\times 0 - 0 - 3\times 1 - 2\times 2}{1}$$

$$= \frac{8 - 3 - 4}{1} = 1.$$

$$x[n] = \{\underset{\uparrow}{1}, 3, 3, 2, 1\}$$

For linear: circular Convolution

$\mathrm{len}(y[n]) = \mathrm{len}(x[n]) + \mathrm{len}(h[n]) - 1$.

The circular convolution is given by

$y[n] = x_1(n) \circledast x_2(n) = x_1[n] * x_2(n)$.

Steps of circular convolution.

1. Draw a circle called outer circle and place all the data point of $x_1(n)$ at equidistance around the circle in counter clockwise direction.

2. Draw another circle in the outer-circle called inner circle and place all the data element of $x_2(n)$ place in the inner circle at equi~~din~~distance in clockwise direction. (Data value both circle are equal)

3. Multiply corresponding sample on the two circle and add the product term which the first data value of convocation of circular.

4. Rotate the inner circle anticlockwise one step apply step the (3) then data value are next convolution.

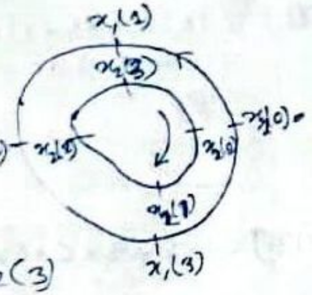
5. Repeat step 4 again and again and complete one ~~side~~. cycle.

5.36.) $x_1(n) = \{1,3,5,7\}$ and $x_2(n) = \{2,4,6,8\}$

Find the circular convolution using circle method.

Solⁿ:

Multiply corresponding data values. ie



$Y[0] = x_1[0]\, x_2[0] + x_1[3]\, x_2(1) + x_1[2]\, x_2[2] + x_1(3)\, x_2(3)$

$= 1\times2 + 3\times4 + 5\times6 + 7\times8$

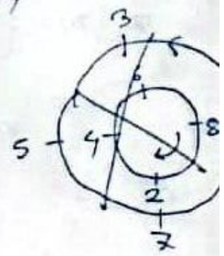
$= 2 + 12 + 30 + 56$

$= 100$

$Y[0] = x_1(0)\, x_2[0] + x_1[1]\, x_2[3] + x_1[2]\, x_2[2] + x_1(3)$

$= 1\times2 + 3\times8 + 5\times6 + 7\times4$

$= 2 + 24 + 30 + 28 = 084$



$Y[3]$ ~~$= 1\times8 + 3\times6 +$~~ $5\times4 + 7\times2$

~~$= 8 + 18 + 20 + 14$~~

$=$

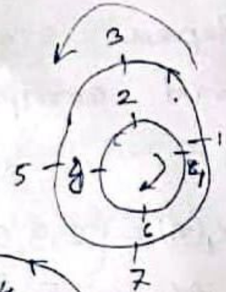
$y[1] = \cancel{1\times8+2\times2+5\times4+7\times6}$

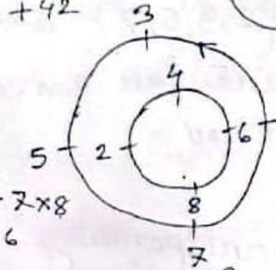
$= \cancel{8+6+20+42}$

$= 1\times4+3\times2+5\times8+6\times7$

$= 4+6+40+42$

$= 92$

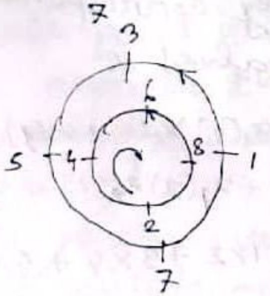




$y[2] = 1\times6+3\times4+5\times2+7\times8$

$= 6+12+10+56$

$= 84$



$y[3] = 1\times8+3\times6+5\times4+7\times2$

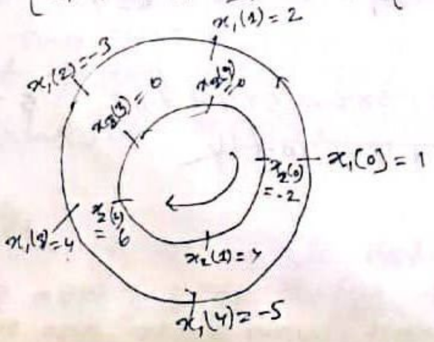
$= 8+18+20+14$

$= 60$

$y[n] = \{\cancel{100}, 84, 92, 84, 60\}$.

② Find the convocation using circular method.

$x_1(n) = \{1,2,-3,4,-5\}$, $x_2(n) = \{-2,4,6,0,0\}$



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জাতীয় বিশ্ববিদ্যালয়

ক্রমিক নং ঃ ....

অতিরিক্ত উত্তরপত্র

১। .... পরীক্ষা .... সাল

২। বিষয় ঃ..

৩। বিষয়ের শিরোনাম ঃ..

৪। বিষয় কোড ঃ..

৫। পরীক্ষার তারিখ ঃ..

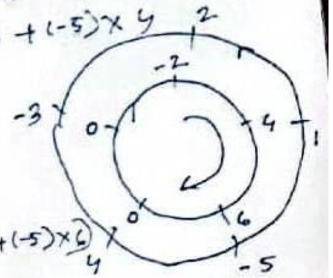
৬। ইনভিজিলেটরের স্বাক্ষর ও তারিখ ঃ ....

(এ স্থান হতে উত্তর লেখা আরম্ভ করতে হবে)

$y[0] = 1\times-2+2\times0+(-3)\times0+4\times6+(-5)\times4$

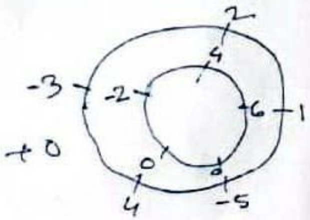
$= -2+0+0+24-20$

$= 2$



$y[1] = 1\times4+2\times(-2)+(-3)\times0+4\times(0)+(-5)\times6$

$= 4-4-30 = -30$

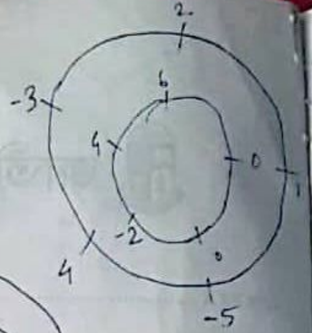


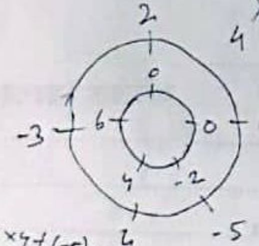
$y(2) = 1\times6+2\times4+(-3)\times(-2)+0+0$

$= 6+8+6+0+0$

$= 20$

$$Y(3) = 0 + 2\times 6 + (-3)\times 4 + 4\times(-2) + 0$$
$$= 12 - 12 - 8$$
$$= -8$$





$$Y(4) = 0 + 0 + (-3)\times 6 + 4\times 4 + (-5)\times -2$$
$$= -18 + 16 + 10$$
$$= 8$$

one cycle is complete.

$$Y(n) = \{2, -30, 20, -8, 8\}$$

Find the convocation of two given data sequence using matrix method.

$$x_1(n) = \{1, 3, 5, 7\}, \quad x_2(n) = \{2, 4, 6, 8\}$$

Sol^n:

$$\begin{array}{c} x_2(n) \\ \begin{bmatrix} 2 & 8 & 6 & 4 \\ 4 & 2 & 8 & 6 \\ 6 & 4 & 2 & 8 \\ 8 & 6 & 4 & 2 \end{bmatrix} \end{array} \begin{bmatrix} 1 \\ 3 \\ 5 \\ 7 \end{bmatrix} = \begin{bmatrix} 2+24+30+28 \\ 4+6+40+42 \\ 6+12+10+56 \\ 8+18+20+14 \end{bmatrix}$$

$$= \begin{bmatrix} 84 \\ 92 \\ 87 \\ 60 \end{bmatrix}$$

* what do you understand by Fourier representation of signals.

Fourier representation also known as fourier analysis. To study of signals and systems using sinusoidal representation is called fourier analysis.

There are four distinct fourier representation.

(1) continuous time periodic f series (FS)

(2) " " Non " f transform (FT)

(3) Discrete time periodic f series (DTFS)

(4) " " Non " f transform (DTFT).

Explain DTFS. Deduce its pair expression.

If $x[n]$ is discrete-time signal with fundamental period N. (i.e fundamental frequency $\Omega_0 = \frac{2\pi}{N}$) then we can represent $x[n]$ by the DTFS (Discrete time fourier series) as

$$\hat{x}[n] = \sum_k A[k] e^{jk\Omega_0 n}$$

Deduce its pair expression.

Complex sinusoids with discrete frequencies are not always distinct.

$$e^{j(N+k)\Omega_0 n} = e^{jN\Omega_0 n} \cdot e^{jk\Omega_0 n}$$
$$= e^{jN \cdot \frac{2\pi}{N} n} e^{jk\Omega_0 n}$$
$$= e^{j2\pi n} e^{jk\Omega_0 n}$$
$$= e^{jk\Omega_0 n}$$

Hence $k =$ ~~$-(N-1)$~~ 0 to $(N-1)$ is sufficient.

Again for even or odd signal

$k = -\frac{(N-1)}{2}$ to $+\frac{(N-1)}{2}$ is ok. if N is odd.

DTFS weights or coefficients are calculated using mse method.

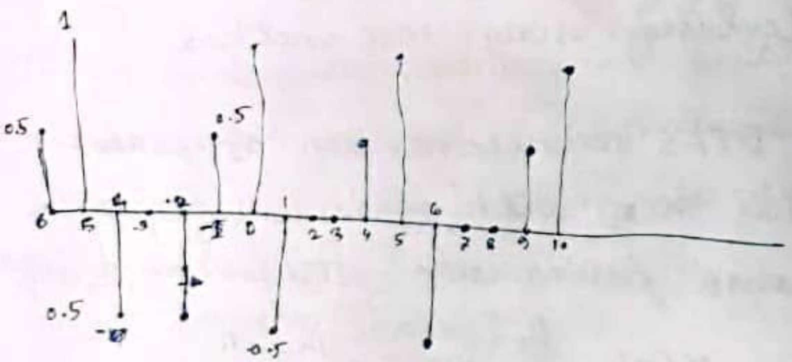
The DTFS representation of periodic signal $x[n]$ with fundamental period N and fundamental frequency $\Omega_0 = \frac{2\pi}{N}$

$$x[n] = \sum_{k=0}^{N-1} x[k] e^{jk\Omega_0 n}$$

$$\text{where, } x[k] = \frac{1}{N} \sum_{k=0}^{N-1} x[n] e^{-jk\Omega_0 n}$$

are the DTFS coefficients of $x[n]$. ~~$x[n]$~~

$x[n]$ and $x[k]$ are known as DTFS pair.



$N = 5,$

$\Omega_0 = \frac{2\pi}{5}$

$-2\pi + \frac{2\pi}{5} = \frac{-10\pi + 2\pi}{5} = -\frac{8\pi}{5}$

Here,

$$x[k] = \frac{1}{N} \sum_{n=0}^{N-1} x[n] e^{-jk\frac{2\pi n}{5}}$$

$$= \frac{1}{5} \sum_{n=0}^{4} x[n] e^{-jk\frac{2\pi n}{5}}$$

$$= \frac{1}{5} \left(x[0] e^{0} + x[1] e^{-jk\frac{2\pi}{5}} + x[2] e^{-jk\frac{4\pi}{5}} + x[3] e^{-jk\frac{6\pi}{5}} + x[4] e^{-jk\frac{8\pi}{5}} \right)$$

$$= \frac{1}{5} \left(\cancel{0.5}\, 1 - 0.5\, e^{-jk\frac{2\pi}{5}} + 0.5\, e^{-jk\frac{8\pi}{5}} \right)$$

$$= \frac{1}{5} \left(1 - 0.5\, e^{-jk\frac{2\pi}{5}} + 0.5\, e^{-j2\pi k} \cdot e^{jk\frac{2\pi}{5}} \right)$$

$$= \frac{1}{5} \left(1 - 0.5\, e^{-jk\frac{2\pi}{5}} + 0.5\, (\cos(k2\pi) - j\sin(k2\pi))\, e^{jk\frac{2\pi}{5}} \right.$$

$$= \frac{1}{5} \left(1 - 0.5\, e^{-jk\frac{2\pi}{5}} + 0.5\, e^{jk\frac{2\pi}{5}} \right.$$

$$= \frac{1}{5} \left\{ 1 + \cancel{0}\, j\sin(2\pi k/5) \right\}$$

$$x[k] = \frac{1}{5} \left\{ 1 + j\sin(2\pi k/5) \right\}$$

$$x[0] = \frac{1}{5} (1 + j\sin 0) = 0.2$$

$$x[1] = \frac{1}{5} \left(1 + j\sin \frac{2\pi}{5}\right) = 0.2 + j$$

$$x[2] = \frac{1}{5} \left(1 + j\sin \frac{4\pi}{5}\right)$$

$$x[3] = \frac{1}{5} \left(1 + j\sin \frac{6\pi}{5}\right)$$

$$x[4] = \frac{1}{5} \left(1 + j\sin \frac{8\pi}{5}\right).$$

$a + ib =$

(b)

$x(n)$	0.5	1	-4	0	0
n	0	1	2	3	4

$N = 5$

$\Omega_0 = \frac{2\pi}{5}$

Here,

$$x[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\Omega_0 n}$$

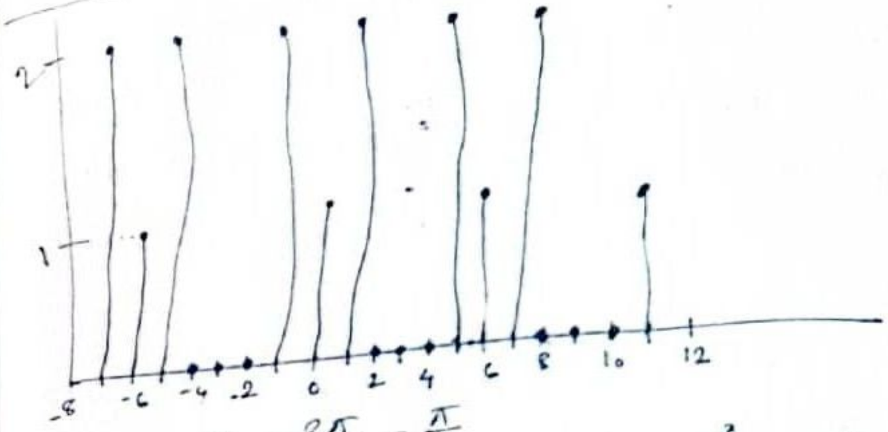
$$x[k] = \frac{1}{5}\sum_{n=0}^{4} x[n]\, e^{-jk\frac{2\pi}{5}\cdot n}$$

$$= \frac{1}{5}\left(x[0]e^{0} + x[1]e^{-jk\frac{2\pi}{5}} + x[2]e^{-jk\frac{4\pi}{5}} + x[3]e^{-jk\frac{6\pi}{5}} + x[4]e^{-jk\frac{8\pi}{5}}\right)$$

$$= \frac{1}{5}\left(0.5 + 1e^{-jk\frac{2\pi}{5}} + (-4)e^{-jk\frac{4\pi}{5}} + 0 + 0\right)$$

$$= \frac{1}{5}\left(0.5 + e^{-jk\frac{2\pi}{5}} - 4e^{-\frac{2\pi}{5}}\right.$$

example.2



$N = 6$, $\Omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$

$$x[k] = \frac{1}{N}\sum_{n=0}^{N-1} x[n]\, e^{-jk\Omega_0 n}$$

$$x[k] = \frac{1}{6}\sum_{n=0}^{5} x[n]\, e^{-jk\frac{\pi}{3}n}$$

$$= \frac{1}{6}\left(x[0]e^{0} + x[1]e^{-jk\frac{\pi}{3}} + x[2]e^{-jk\frac{2\pi}{3}} + x[3]e^{-jk\frac{3\pi}{3}} + \ldots + x[5]e^{\ldots}\right.$$

$$= \frac{1}{6}\left(1 + 2e^{-jk\frac{\pi}{3}} + 0 + 0 + 0 + 2e^{-jk\frac{5\pi}{3}}\right)$$

$$= \frac{1}{6}\left(1 + 2e^{-jk\frac{\pi}{3}} + 2e^{-jk2\pi}\cdot e^{jk\frac{\pi}{3}}\right)$$

$$= \frac{1}{6}\left(1 + 2e^{-jk\frac{\pi}{3}} + 2(\cos(2\pi k) - j\sin(2\pi k))e^{j\ldots}\right.$$

$$= \frac{1}{6}\left(1 + 2e^{-jk\frac{\pi}{3}} + 2e^{jk\frac{\pi}{3}}\right)$$

$$= \frac{1}{6}\left(1 + 4\left(\frac{e^{jk\frac{\pi}{3}} + e^{-jk\frac{\pi}{3}}}{2}\right)\right)$$

$$= \frac{1}{6}\left(1 + 4\cos k\frac{\pi}{3}\right.$$

$\frac{\pi}{3} \cdot x =$

$x = \ldots$

$\cos(x) = \frac{e^{jx} \ldots}{}$

$\sin(x) = \ldots$

DTFS Determination.

- Determination of $x[k]$ by inspection.
- Applicable when $x[n]$ i.e the original time domain signal is a real or complex sinusoids.

Method

Step 1: Expand $x[n]$ in terms of complex sinusoids.

Step 2: Compare outcome of step 2 with each term of the following equation.

$$x[n] = \sum_{k=0}^{N-1} x[k]\, e^{jk\Omega_0 n}$$

example:

$$x[n] = \cos\left(\frac{\pi n}{3} + \varphi\right)$$

Here, $\Omega_0 = \frac{2\pi}{6} = \frac{\pi}{3}$ & $N = \frac{2\pi}{\Omega_0} = \frac{2\pi}{\frac{\pi}{3}} = 6$

$$x[n] = \frac{1}{2}\left(e^{j\left(\frac{n\pi}{3}+\varphi\right)} + e^{-j\left(\frac{n\pi}{3}+\varphi\right)}\right)$$

$$= \frac{1}{2} e^{j\varphi} e^{\frac{n\pi}{3}} + \frac{1}{2} e^{-j\varphi} \cdot e^{-\frac{n\pi}{3}}$$

$$x[n] = \sum_{n=0}^{5} x[k]\, e^{j\Omega_0 k n}$$

জাতীয় বিশ্ববিদ্যালয়

ক্রমিক নং : ....

অতিরিক্ত উত্তরপত্র

১। .... পরীক্ষা .... সাল

২। বিষয় : ....

৩। বিষয়ের শিরোনাম : ....

৪। বিষয় কোড : ....

৫। পরীক্ষার তারিখ : ....

৬। ইনভিজিলেটরের স্বাক্ষর ও তারিখ : ....

(এ স্থান হতে উত্তর লেখা আরম্ভ করতে হবে)

$$= x[0]e^{0} + x[1]e^{jn\frac{\pi}{3}} + x[2]e^{jn\frac{2\pi}{3}} + x[3]e^{jn\pi} + x[4]e^{jn\frac{4\pi}{3}} + x[5]e^{jn\frac{5\pi}{3}}$$

$x[0] = 0$

$x[1] = \frac{1}{2} e^{j\varphi}$

$x[2] = 0$

$x[3] = 0$

$x[4] = 0$

$x[5] = \frac{1}{2} e^{-j\varphi}$.