Input-Output Analysis in Economics

Input-Output (I-O) analysis is a quantitative economic technique used to study the interdependencies between different sectors of an economy. Developed by **Wassily Leontief** (who won the Nobel Prize in Economics in 1973 for this work), it provides a detailed framework for understanding how the output of one industry is an input for another, creating a network of economic activities.

Core Concepts

1. **Economic Sectors:**
   - The economy is divided into various sectors (e.g., agriculture, manufacturing, services).
   - Each sector produces goods or services, part of which may be used by other sectors or consumed directly.
2. **Interdependencies:**
   - One sector's output becomes another's input. For example:
     - Agriculture produces wheat, which the food processing industry uses to make bread.
     - The construction industry uses steel from manufacturing.
3. **Transactions Table:**
   - Input-Output analysis begins with a transactions table (or matrix), which shows:
     - Rows: Output distribution of a sector to other sectors or final demand.
     - Columns: Inputs required by a sector from other sectors or factors of production.

Key Components of Input-Output Analysis

1. **Input-Output Table:**
   - A matrix that captures how the output of each sector is distributed across other sectors and final consumers.

- It includes:
  - **Intermediate consumption:** Goods and services used in production by other sectors.
  - **Final demand:** Consumption by households, governments, exports, and investments.

2. **Technical Coefficients Matrix (A):**
   - This matrix represents the **direct input requirements** per unit of output for each sector.
   - Each element a_{ij} shows how much output from sector i is needed to produce one unit of output in sector j.
3. **Leontief Inverse Matrix $(I - A)^{-1}$:**
   - Shows **total input requirements** (direct + indirect) to meet a unit of final demand.
   - Useful for understanding the ripple effects of changes in demand across the economy.

How It Works

1. **Equations of Balance:**
   - For each sector: $X_i = \sum_{j=1}^{n} a_{ij}X_j + F_i$

     Where:
     - X_i: Total output of sector i.
     - a_{ij}: Input required from sector i per unit of output in sector j.
     - F_i: Final demand for sector i's output.
2. **Modeling Economic Impacts:**
   - Solving these equations helps estimate the total output needed to meet a given level of final demand.

Applications of Input-Output Analysis

1. **Economic Impact Analysis:**
   - Estimate the effects of changes in demand or supply in one sector on others.
   - Example: Studying how increased demand for electric vehicles affects the mining, battery manufacturing, and energy sectors.
2. **Policy Planning:**
   - Assess the outcomes of government policies like infrastructure investments or subsidies.
3. **Environmental Analysis:**
   - Calculate resource use, emissions, or energy consumption by linking economic outputs to environmental factors.
4. **Trade Analysis:**
   - Understand how changes in international trade affect domestic industries.

Limitations

1. **Static Framework:**
   - Assumes fixed technical coefficients, ignoring technological changes over time.
2. **Linear Relationships:**
   - Assumes proportional relationships between inputs and outputs, which may not hold for all industries.
3. **Data Intensity:**
   - Requires detailed and accurate economic data, which can be challenging to obtain.
4. **No Price Dynamics:**
   - Ignores price changes, which may influence production and consumption decisions.

Example: A Simple Economy

Consider an economy with three sectors:

1. **Agriculture (A)**
2. **Manufacturing (M)**
3. **Services (S)**

Step 1: Input-Output Table (Transactions Table)

The I-O table shows the monetary value of goods/services exchanged between sectors and for final demand (consumption, exports, etc.).

Sector →	**Agriculture (A)**	**Manufacturing (M)**	**Services (S)**	**Final Demand**	**Total Output**
Agriculture	20	40	10	30	100
Manufacturing	30	20	20	50	120
Services	10	30	20	40	100

- **Final Demand:** Consumption by households, government, or exports.
- **Total Output:** Sum of all rows for each sector (how much is produced by each sector).

Step 2: Technical Coefficients Matrix (A)

The technical coefficients are calculated as:

$$a_{ij} = \frac{\text{Input from sector i to sector j}}{\text{Total output of sector j}}$$

For Agriculture (A → M):

$$a_{AM} = \frac{40}{120} = o.33$$

For all sectors:

Sector →	Agriculture (A)	Manufacturing (M)	Services (S)
Agriculture	$\frac{20}{100} = 0.20$	$\frac{40}{120} = 0.33$	$\frac{10}{100} = 0.10$
Manufacturing	$\frac{30}{100} = 0.30$	$\frac{20}{120} = 0.17$	$\frac{20}{100} = 0.20$
Services	$\frac{10}{100} = 0.10$	$\frac{30}{120} = 0.25$	$\frac{20}{100} = 0.20$

Technical coefficients matrix A:

$$\begin{bmatrix} 0.20 & 0.33 & 0.10 \\ 0.30 & 0.17 & 0.20 \\ 0.10 & 0.25 & 0.20 \end{bmatrix}$$

Step 3: Leontief Equation

The Leontief equation:

$$X = (I - X)^{-1} F$$

Where:

- X: Total output vector.
- F: Final demand vector.
- $(I - X)^{-1}$: Leontief inverse matrix.

Step 4: Leontief Inverse

First, calculate I−A:

$$I - A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.20 & 0.33 & 0.10 \\ 0.30 & 0.17 & 0.20 \\ 0.10 & 0.25 & 0.20 \end{bmatrix} = \begin{bmatrix} 0.80 & -0.33 & -0.10 \\ -0.30 & 0.83 & -0.20 \\ -0.10 & -0.25 & 0.80 \end{bmatrix}$$

Now, calculate the inverse of I−A

Assume the Leontief inverse is:

$$(I - A)^{-1} = \begin{bmatrix} 1.2 & 0.4 & 0.1 \\ 0.3 & 1.1 & 0.2 \\ 0.1 & 0.3 & 1.2 \end{bmatrix}$$

Step 5: Final Demand and Total Output

Assume the final demand vector:

$$F = \begin{bmatrix} 30 \\ 50 \\ 40 \end{bmatrix}$$

Calculate *X*:

$$X = (I - X)^{-1}F$$

$$X = \begin{bmatrix} 1.2 & 0.4 & 0.1 \\ 0.3 & 1.1 & 0.2 \\ 0.1 & 0.3 & 1.2 \end{bmatrix} \begin{bmatrix} 30 \\ 50 \\ 40 \end{bmatrix}$$

Performing the matrix multiplication:

$$X = \begin{bmatrix} 1.2 \times 30 + 0.4 \times 50 + 0.1 \times 40 \\ 0.3 \times 30 + 1.1 \times 50 + 0.2 \times 40 \\ 0.1 \times 30 + 0.3 \times 50 + 1.2 \times 40 \end{bmatrix}$$

$$\text{X=} \begin{bmatrix} 60 \\ 72 \\ 66 \end{bmatrix}$$

So, the answer is total output for Agriculture, Manufacture and Service sector is 60, 72 and 66 units respectively.