

PABNA UNIVERSITY OF SCIENCE AND TECHNOLOGY



Faculty of Engineering & Technology

Department of Information and Communication Engineering

Assignment : Proportions and Variances of Test

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Assignment on Proportions and Variances of Test

Introduction: Statistical analysis plays a crucial role in hypothesis testing, particularly when comparing proportions and variances. This assignment explores different statistical tests used for assessing proportions and variances, their applications, and how to perform them.

Part 1: Proportions Test

A proportions test is used to determine whether the proportion of a particular characteristic in a population differs from a specified value. This is commonly used in fields such as medicine, business, and social sciences.

1.1 Hypothesis for Proportions Test

- Null Hypothesis : The population proportion is equal to a specified value.
- Alternative Hypothesis : The population proportion differs from the specified value.

1.2 Types of Proportions Test

1. **One-Proportion Z-Test:** Used when comparing a sample proportion to a known population proportion.
2. **Two-Proportion Z-Test:** Used when comparing proportions from two independent groups.

1.3 Formula for Z-Test for Proportions

Where:

- \hat{p} = Sample proportion
- p_0 = Population proportion
- n = Sample size

1.4 Example Problem

A company claims that 60% of its customers are satisfied with their service. A survey of 200 customers finds that 120 are satisfied. Conduct a hypothesis test at a 5% significance level.

Solution:

- We are given:
- Claimed proportion (null hypothesis): $p_0=0.60$
- Sample size: $n=200$
- Number of satisfied customers in the sample: $x=120$
- Sample proportion:
- $\hat{p}=120/200= 0.60$
- Significance level: $\alpha=0.05$
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- **Step 1: State the hypotheses**
- We are testing whether the true proportion is different from the claimed 60%. So this is a **two-tailed** test.
- Null Hypothesis: $H_0:p=0.60$
- Alternative Hypothesis: $H_1:p\neq0.60$
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- **Step 2: Check the conditions for a normal approximation**
- We can use the normal approximation since:
- $np_0=200\times0.60=120$
- $n(1-p_0)=200\times0.40=80$
- Both are > 5 , so conditions are satisfied.
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- **Step 3: Compute the test statistic**
- The standard error (SE) is:

- $SE = \sqrt{\frac{p_0(1-p_0)}{n}} = \sqrt{\frac{0.60*0.40}{200}} \approx 0.0346$
- The test statistic (z) is:
- $Z = \frac{\hat{p} - p}{SE} = \frac{0.60 - 0.60}{0.0346} = 0$
- **Step 4: Find the critical z-value**
- For a two-tailed test at $\alpha=0.05$, the critical z-values are:
- $\pm z\alpha/2 = \pm 1.96$
- **Step 5: Make a decision**
- Since the calculated $z = 0$ lies **between** -1.96 and 1.96, we **fail to reject** the null hypothesis.

Conclusion:

There is **not enough evidence** at the 5% significance level to conclude that the true proportion of satisfied customers is different from 60%. The sample result is consistent with the company's claim.

Part 2: Variances Test

Tests for variance help determine if the variability of two or more samples is significantly different. This is essential in quality control, engineering, and finance.

2.1 Hypothesis for Variance Test

- Null Hypothesis : The population variances are equal.
- Alternative Hypothesis : The population variances are not equal.

2.2 Types of Variance Tests

1. **F-Test for Equality of Variances:** Used to compare variances of two independent samples.
2. **Chi-Square Test for a Single Variance:** Used to test if a population variance differs from a specified value.

2.3 Formula for F-Test

Where:

- S_1^2 = Variance of sample 1
- S_2^2 = Variance of sample 2

2.4 Example Problem

Two production lines produce the same product. A sample of 15 items from line A has a variance of 4.5, and a sample of 12 items from line B has a variance of 3.2. Test if the variances are significantly different at a 5% significance level.

Solution:

Given:

- Sample size from line A: $n_1=15$, variance $S_1^2 = 4.5$
- Sample size from line B: $n_2=12$, variance $S_2^2 = 3.2$
- Significance level: $\alpha=0.05$

Step 1: State the hypotheses

This is a **two-tailed** F-test (since we are testing if the variances are *different*).

- $H_0: \sigma_1^2 = \sigma_2^2$ (variances are equal)
- $H_A: \sigma_1^2 \neq \sigma_2^2$ (variances are not equal)

Step 2: Test statistic

We compute the F-statistic as:

$$F = \frac{\text{larger sample variance}}{\text{smaller sample variance}} = \frac{4.5}{35} \approx 1.40625$$

Step 3: Degrees of freedom

- $df_1 = n_1 - 1 = 14$
- $df_2 = n_2 - 1 = 11$

We'll use these to find the **critical values** for a two-tailed test.

Step 4: Find critical F-values

We find the **critical values** $F_{\alpha/2}$ and $F_{1-\alpha/2}$ distribution tables or a calculator.

For $\alpha = 0.05$, $df_1 = 14$, $df_2 = 11$

Lower critical value:

$$F_{0.025}(14,11) \approx 0.300$$

Upper critical value:

$$F_{0.975}(14,11) \approx 3.29$$

Step 5: Decision rule

If F_{FF} is **between** 0.300 and 3.29, we **fail to reject** H_0 .

Since:

$$F = 1.40625 \in (0.300, 3.29)$$

Conclusion:

We **fail to reject** the null hypothesis.

There is **no significant difference** in the variances of the two production lines at the 5% significance level.

Conclusion

Proportion and variance tests are fundamental tools in statistics, allowing researchers to make inferences about population characteristics based on sample data. Understanding these tests enables better decision-making in various fields.