



Topic Name: Bartlett's test for homogeneity of variances

Course Name: Engineering Statistics Course Code: STAT - 2201

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Session :2021 – 2022

2nd Year 2nd Semester

Department of ICE, PUST

Introduction

What is Bartlett's Test?

- Bartlett's test is a parametric statistical test.
- A statistical test used to check if multiple samples have equal variances.
- Used to test homogeneity of variances across multiple groups.

What is Homogeneity of Variances?

- The assumption that different samples have the same variance.
- Importance in statistical analysis.

Why Test for Equal Variance?

- •Many statistical methods assume equal variance across groups.
- •Unequal variances can affect the validity of results.
- •Bartlett's test provides a way to verify this assumption before proceeding.

When to Use Bartlett's Test

- 1. Comparing three or more groups.
- 2. When data is normally distributed.
- 3.Use Cases:
 - When comparing variances across multiple groups.
- 4. Assumptions:
 - Samples are independent.
 - Samples are normally distributed.
- 5. Alternative tests for non-normal data:
 - Levene's Test
 - Brown-Forsythe Test

Hypotheses in Bartlett's Test

- Null Hypothesis (H₀):
 - All group variances are equal.
- Alternative Hypothesis (H_1) :
 - At least one group variance is different.
- Significance:
 - If the p-value of the test is less than the chosen significance level (e.g., 0.05), the null hypothesis is rejected, indicating that the variances are not all equal.
- Use in ANOVA:
 - Bartlett's test is often used as a prerequisite to one-way ANOVA, which assumes that the variances of the groups being compared are equal.

Test Statistic (χ^2)

The Bartlett's test statistic is:

$$\chi^2 = \frac{(N-k)\ln(S_p^2) - \sum_{i=1}^k (n_i - 1)\ln(S_i^2)}{1 + \frac{1}{3(k-1)} \left(\sum_{i=1}^k \left(\frac{1}{n_i - 1}\right) - \frac{1}{N-k}\right)}$$

Where:

 s_p^2 : pooled variance

 s_i^2 : variance of the i-th group

 n_i : sample size of the i-th group

k:number of groups

N:total number of observations

How It Works – Steps

- •Calculate variances of all groups.
- •Compute pooled variance.
- •Plug into test statistic formula.
- •Compare test statistic with chi-square distribution.
- •Determine p-value and make decision.

Example Problem

Data:

- Group A: 5, 7, 6 (Variance = 1)
- Group B: 10, 12, 11 (Variance = 1)
- Group C: 20, 18, 19 (Variance = 1)
- Since all variances are equal:
- Bartlett's test should give high p-value.
- Fail to reject $H_0 \rightarrow Variances$ are equal
- Now change Group C to: 20, 25, 30 (Variance = 25)
 - Now you may get low p-value → Reject H₀

• Degrees of Freedom:

- (df = k 1)
- Where (k) is the number of groups

Critical Value:

Compare the test statistic to the chi-squared distribution.

Decision Rule:

• If chi-squared > critical value, reject H₀ (evidence of unequal variances).

Interpretation of Results

- $\mathbf{p} > 0.05 \rightarrow$ Fail to reject $H_0 \rightarrow$ No significant difference in variances.
- $\mathbf{p} < \mathbf{0.05} \rightarrow \text{Reject H}_0 \rightarrow \text{At least one group has a}$ different variance.

Limitations of Bartlett's Test

- Very sensitive to departures from normality.
- Outliers can make the test misleading.
- Not robust with small sample sizes.

Alternatives to Bartlett's Test

Alternative Tests:

- Levene's Test: More robust to non-normality.
- Brown-Forsythe Test: Variation of Levene's Test.

Conclusion

- Bartlett's test checks if variances are equal.
- Importance of testing for homogeneity of variances.
- Bartlett's Test as a tool for statistical analysis.
- Sensitive to non-normality and outliers.
- Consideration of assumptions and limitations.
- Know when to use alternatives.

