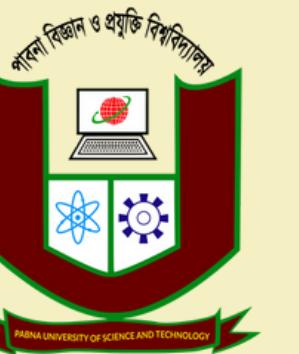


PRESENTATION

Fisher's Lemma and Study of Chi-Square (χ^2) Distribution



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OVERVIEW:

Fisher's Lemma

Understanding the χ^2 Distribution

Properties & Applications

Visual Aids



FISHER'S LEMMA

Fisher's Lemma (also called Fisher's Information Lemma) is a key result in the theory of statistical estimation, especially in Maximum Likelihood Estimation (MLE). Here's a simplified version:

$$\mathbb{E} \left[\left(\frac{\partial}{\partial \theta} \ln f(X; \theta) \right)^2 \right] = -\mathbb{E} \left[\frac{\partial^2}{\partial \theta^2} \ln f(X; \theta) \right]$$

👉 Meaning:

- The left side is the variance of the score function (first derivative of the log-likelihood).
- The right side is the expected negative second derivative of the log-likelihood (also called the observed information).
- This quantity is called the Fisher Information: $I(\theta)$.

FISHER'S LEMMA

📌 Use:

- It tells you how much information your sample gives you about the parameter $\theta\backslash\theta$.
- It's crucial in building confidence intervals and hypothesis tests using MLE.

📌 Importance of Fisher's Lemma

- Measures the amount of information a sample carries about the unknown parameter
- Crucial in deriving the Cramér-Rao Lower Bound
- Helps evaluate the efficiency of estimators

CHI-SQUARE (χ^2) DISTRIBUTION

The Chi-Square distribution is widely used in statistics, especially in hypothesis testing (like goodness-of-fit, independence tests) and in confidence intervals.

👉 Definition:

If Z_1, Z_2, \dots, Z_k are independent standard normal variables (i.e., $Z_i \sim N(0, 1)$), then:

$$\chi_k^2 = Z_1^2 + Z_2^2 + \cdots + Z_k^2$$

📌 Applications:

- **Goodness-of-fit test:** to check if data fits a particular distribution.
- **Test of independence:** like in contingency tables.
- **Confidence intervals for variance:** for a normal distribution.
- **Likelihood ratio tests:** in comparing models.

VISUAL AIDS

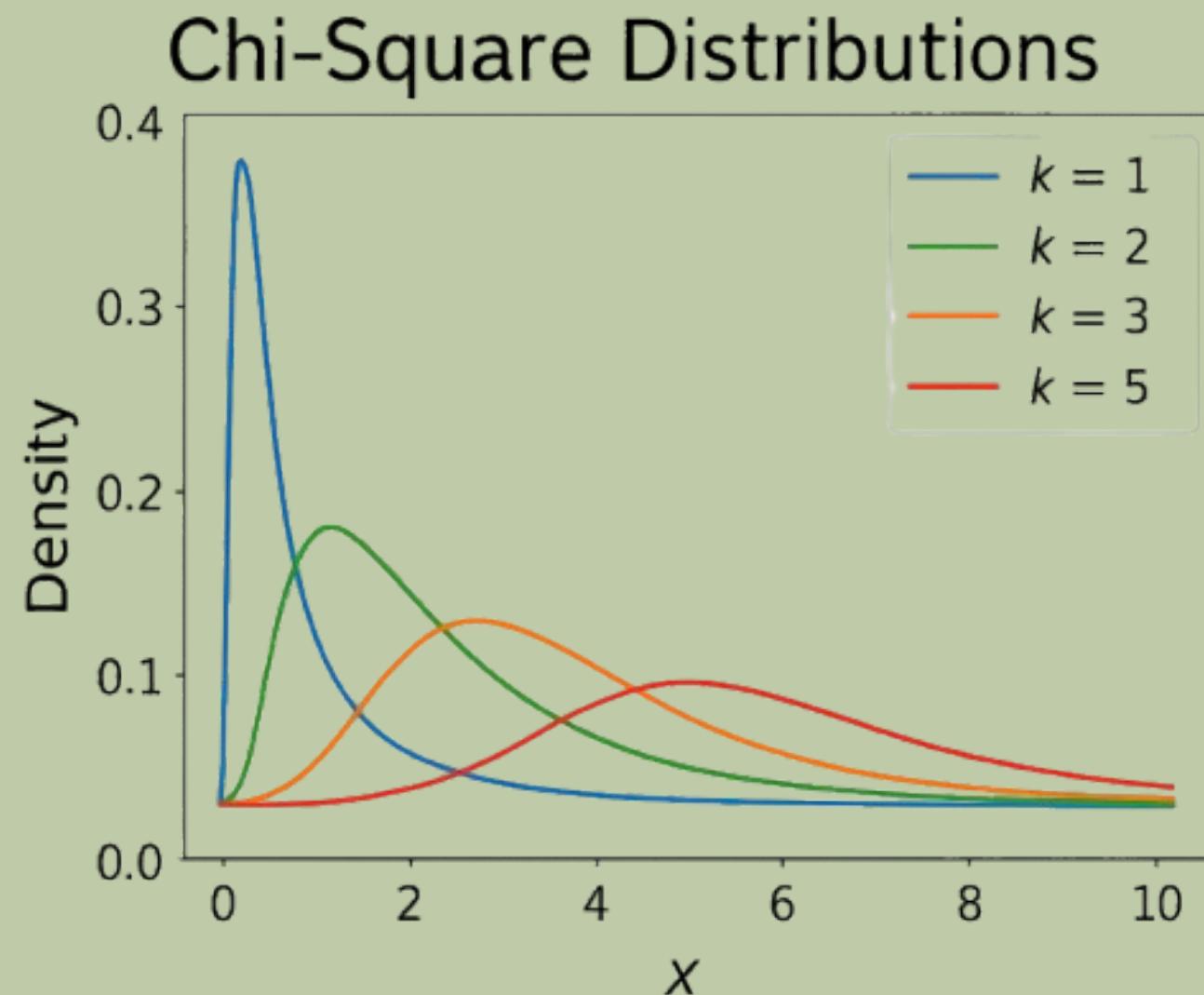


Diagram showing how the sum of squares forms χ^2

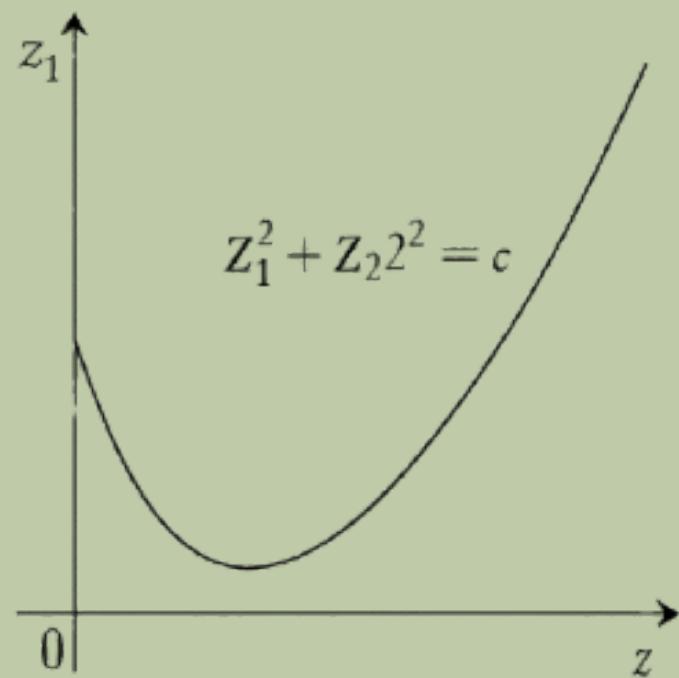
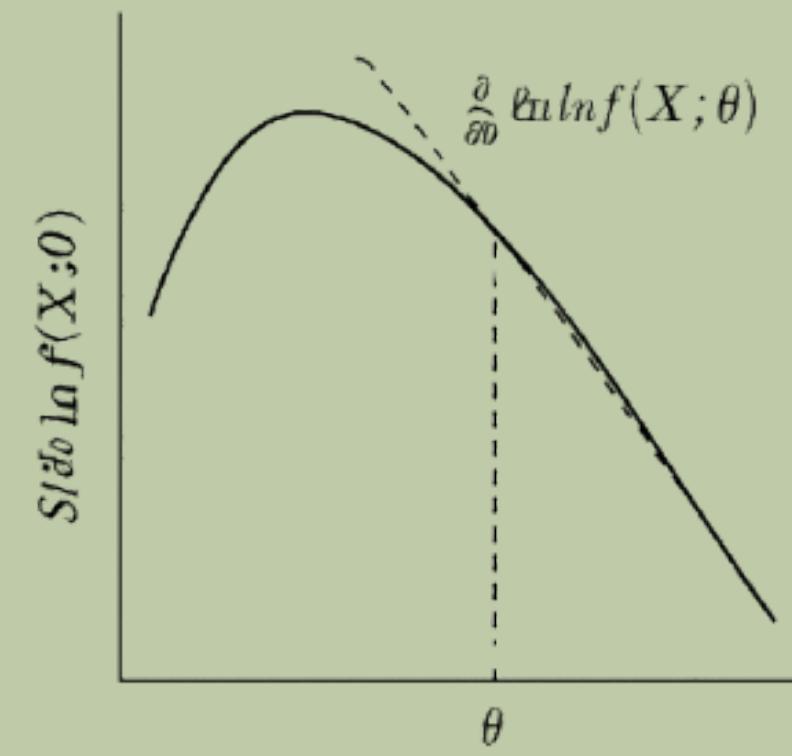
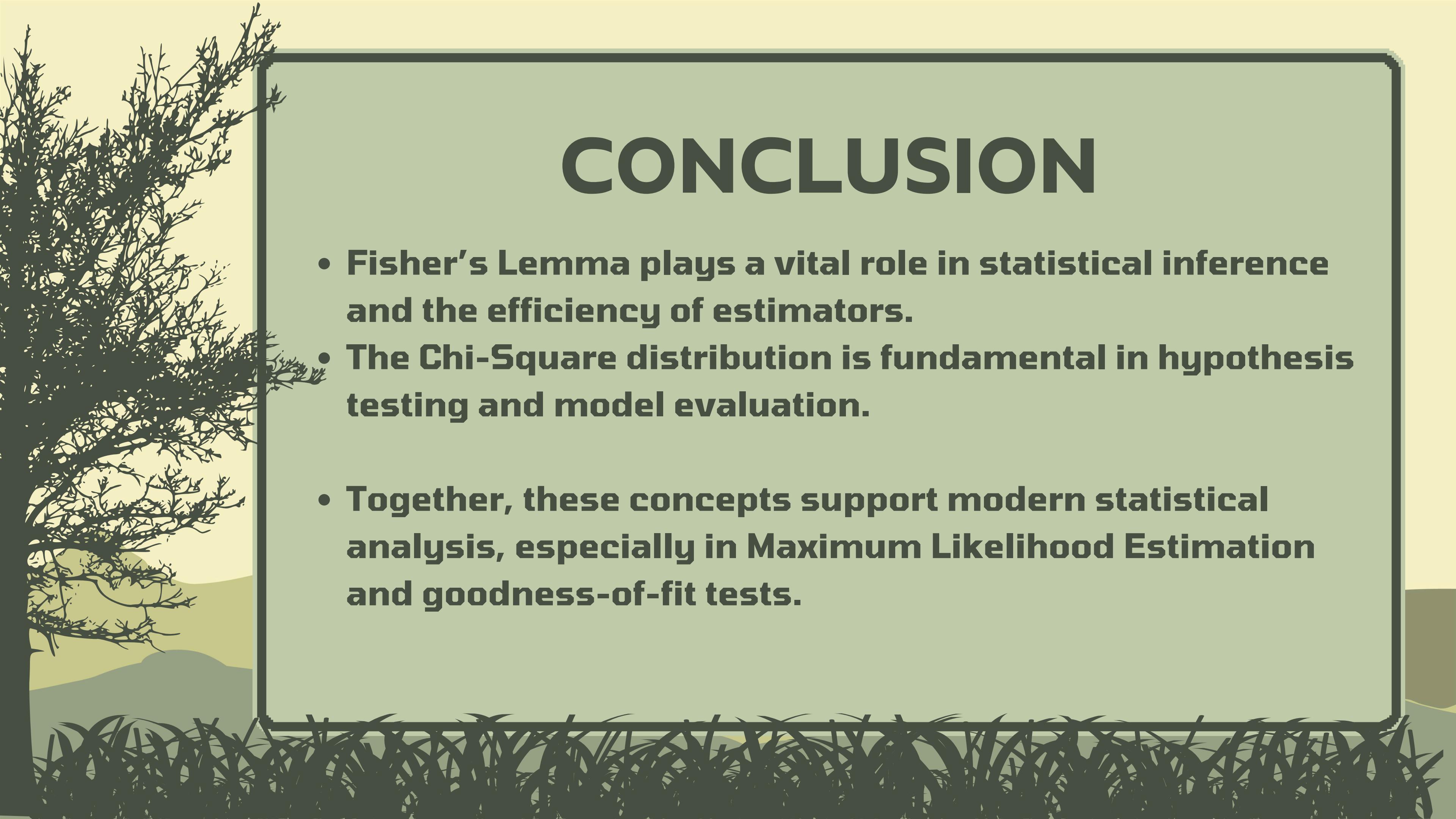


Illustration of score function in Fisher's Lemma





CONCLUSION

- Fisher's Lemma plays a vital role in statistical inference and the efficiency of estimators.
- The Chi-Square distribution is fundamental in hypothesis testing and model evaluation.
- Together, these concepts support modern statistical analysis, especially in Maximum Likelihood Estimation and goodness-of-fit tests.



THANK YOU