Pabna University of Science and Technology

Faculty of Engineering and Technology

Department of Information and Communication Engineering

Assignment on Basic concepts of Estimation, Consistent of Estimates, Unbiased Estimates

Course Code: **STAT-2201**

Course title: **Engineering Statistics**

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1. Basic Concepts in Estimation

Estimation involves using observed data to approximate unknown population parameters. Key components include:

a. Parameters vs. Estimators

- Parameter (θ): A fixed, unknown numerical characteristic of a population (e.g., mean μ, variance σ^2).
- Estimator ($\hat{\theta}$): A rule or formula (e.g., sample mean $\bar{X}$, sample variance s^2) used to estimate θ.
- Estimate: The computed value of $\hat{\theta}$ from a given sample.

b. Properties of a Good Estimator

An ideal estimator should have:

1. Unbiasedness – On average, it equals the true parameter.
2. Consistency – Improves with larger sample sizes.
3. Efficiency – Has the smallest possible variance among competing estimators.
4. Sufficiency – Uses all available information in the sample.

c. Common Estimation Methods

- Method of Moments (MoM): Equates sample moments to population moments.
- Maximum Likelihood Estimation (MLE): Finds θ that maximizes the likelihood function.
- Least Squares Estimation (LSE): Minimizes the sum of squared errors (common in regression).

2. Unbiased Estimates

An estimator is unbiased if its expected value equals the true parameter.

a. Mathematical Definition

$$E[\hat{\theta}] = \theta$$

b. Examples

- Sample Mean ($\bar{X}$) is unbiased for the population mean (μ):

$$E[\bar{X}] = \mu, \quad \text{where } \bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$$

- Sample Variance (s^2) is unbiased for σ^2:

$$E[s^2] = \sigma^2, \quad \text{where } s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$$

c. Importance of Unbiasedness

- Ensures no systematic over- or underestimation.
- Critical in hypothesis testing and confidence intervals.

d. Limitations

- An unbiased estimator may have high variance, leading to poor precision.
- Some biased estimators (e.g., ridge regression) perform better in practice.

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3. Consistent Estimates

An estimator is consistent if it converges to the true parameter as the sample size grows.

a. Mathematical Definition (Convergence in Probability)

$$\lim_{n\to\infty} P\left(\left|\hat{\theta} - \theta\right| \geq \epsilon\right) = 0 \quad \text{for any } \epsilon > 0$$

Or, equivalently:

$$\hat{\theta} \sim \ P0$$

b. Examples

- Sample Mean ($\bar{X}$) is consistent for μ (by the Law of Large Numbers).
- Maximum Likelihood Estimators (MLEs) are typically consistent under regularity conditions.

c. Importance of Consistency

- Ensures that with enough data, estimates become arbitrarily close to the truth.
- A minimal requirement for reliable inference in large samples.

d. Relationship with Unbiasedness

- Unbiasedness does not imply consistency:
  - Example: Using only the first observation (X_1) to estimate μ is unbiased but inconsistent.
- Consistency does not require unbiasedness:
  - Example: A biased estimator like $\hat{\theta} = \bar{X} + 1/n$ is consistent because the bias vanishes as $n \to \infty$.

4. Practical Implications and Trade-offs

a. Bias-Variance Trade-off

- Mean Squared Error (MSE) balances bias and variance:
- $\mathrm{MSE}(\hat{\theta}) = E\left[(\hat{\theta} - \theta)^2\right] = Var(\hat{\theta}) + \left(Bias(\hat{\theta})\right)^2$
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- Sometimes, accepting small bias reduces variance, improving overall accuracy.

b. Choosing Between Estimators

- For small samples: Unbiasedness may be prioritized.
- For large samples: Consistency becomes crucial.

c. Applications

- Economics: Consistent estimators ensure policy evaluations improve with more data.
- Machine Learning: Regularized (biased) estimators prevent overfitting.
- Quality Control: Unbiased measurements ensure accurate process monitoring.