Department of
Information and Communication Engineering

ASSIGNMENT

STAT-2201
Engineering Statistics

Assignment Topic:
Types of statistical errors, Procedure of test of hypothesis

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TYPES OF STATISTICAL ERRORS

When performing a hypothesis test, we make a decision based on sample data. However, there is always a possibility of making an incorrect decision. These incorrect decisions are classified into Type I Error and Type II Error.

1. Type I Error (False Positive - α Error)

Definition:

- Type I error occurs when we reject a true null hypothesis (H_0) incorrectly. In simpler terms, we conclude that there is an effect or difference when there is actually none.

Example:

- Suppose a medical test is designed to detect a disease.
- A healthy person (who does not have the disease) takes the test, but the result incorrectly shows positive for the disease.
- This means the test falsely indicates that the person has the disease when they actually do not.

Probability of Type I Error:

- The probability of committing a Type I error is denoted by α (alpha).
- This is called the Significance Level of the test, commonly set at 0.05 (5%) or 0.01 (1%).
- If $\alpha = 0.05$, it means there is a 5% chance of rejecting (H_0) when it is actually true.

How to Reduce Type I Error:

- Decrease the significance level (e.g., using $\alpha = 0.01$ instead of 0.05).
- Use a larger sample size to improve test accuracy.

2. Type II Error (False Negative - β Error)

Definition:

- Type II error occurs when we fail to reject a false null hypothesis (H_0). In other words, we conclude that there is no effect or difference when there actually is one.

Example:

- Continuing with the medical test example:
    - A sick person (who actually has the disease) takes the test, but the result incorrectly shows negative for the disease.
    - This means the test fails to detect the disease, even though the person actually has it.

Probability of Type II Error:

- The probability of committing a Type II error is denoted by β (beta).
- The power of a test is given by (1 - β), which represents the probability of correctly rejecting a false (H_0).

How to Reduce Type II Error:

- Increase the sample size.
- Use a more sensitive test or a lower significance level.
- Improve measurement accuracy and experimental design.

Procedure of test of hypothesis

The following steps are followed when performing a hypothesis test:

Step 1: State the Hypotheses

- Null Hypothesis (H_0): Assumes no effect or no difference (default assumption).
- Alternative Hypothesis (H_1): What we want to prove (can be one-tailed or two-tailed).

Step 2: Set the Significance Level (α)

- Typically chosen as 0.01, 0.05, or 0.10, representing the probability of a Type I Error

Step 3: Select the Appropriate Test

- Parametric Tests (if data follows a known distribution):
  - Z-test (for large samples, known population variance).
  - t-test (small samples, unknown variance).
  - F-test (ANOVA, comparing variances).
  - Chi-square test (goodness-of-fit, independence).
- Non-parametric Tests (if assumptions are violated):
  - Mann-Whitney U test (non-parametric alternative to t-test).
  - Wilcoxon signed-rank test (paired samples).
  - Kruskal-Wallis test (non-parametric alternative to ANOVA).

Step 4: Calculate the Test Statistic

- ❖ Compute the test statistic (e.g., t-value, z-score, F-statistic, χ^2) based on sample data.

Step 5: Determine the Critical Value or p-value

- ❖ Critical Value Approach: Compare the test statistic with the critical value from tables.
- ❖ p-value Approach:
  1. If p-value $\leq \alpha$, reject H_0.
  2. If p-value $> \alpha$, fail to reject H_0.

Step 6: Make a Conclusion

- ❖ Based on statistical results, we either reject or fail to reject H_0 and interpret the findings.

Example:

Step 1: State the Hypotheses

- Null Hypothesis (H_0): The average weight of the population is 70 kg.

$$H_0:\ \mu = 70$$

- Alternative Hypothesis (H_1): The average weight is not 70 kg (two-tailed test).

$$H_1:\ \mu \neq 70$$

Step 2: Set the Significance Level (α)

- We choose $\alpha = 0.05$, meaning there is a 5% chance of making a Type I Error (rejecting H_0 when it is true).

Step 3: Select the Appropriate Test

- Since the sample size is small (n < 30) and population variance is unknown, we use a t-test.

Step 4: Calculate the Test Statistic

Given Data:

- Sample size: n = 10
- Sample mean: $\bar{x}$ =72 kg
- Sample standard deviation: s = 3
- Population mean(μ_0): 70 kg

The t-test formula for a one-sample t-test:

$$t = \frac{\bar{x} - \mu_0}{\frac{s}{\sqrt{n}}}$$

Substituting the values:

$$t = \frac{72 - 70}{\frac{3}{\sqrt{10}}}$$

$$= \frac{2}{\frac{3}{3.162}}$$

$$= \frac{2}{0.948}$$

$$= 2.11$$

Step 5: Determine the Critical Value or p-value

- Since this is a two-tailed test and degrees of freedom (df) = n - 1 = 10 - 1 = 9, we check a t-table for α = 0.05 (two-tailed).
- The critical t-value for df = 9 at α/2 = 0.025 in each tail is ± 2.262.

Comparing t-Value:

- |t| = 2.11 is less than the critical value 2.262.
- Alternatively, if we check the p-value for t = 2.11 (df = 9), it is around 0.063.

Since p-value > α (0.063 > 0.05), we fail to reject H_0 .

Step 6: Make a Conclusion

- Since the test statistic does not exceed the critical value, we do not have enough evidence to reject H_0.
- There is no significant difference in the average weight from 70 kg.