Volatility Targeting Strategy Using Options and Futures with Synthetic Daily Straddle Hedging

Contents

1 Introduction

This guideline provides a comprehensive framework for implementing a volatility-targeting strategy using options and futures to replicate an equity portfolio, combined with a synthetic daily straddle hedge. The strategy aims to maintain a target level of portfolio volatility while hedging against fluctuations by dynamically adjusting positions in options and futures. This approach allows the portfolio to benefit from volatility spikes and mitigate risks associated with large market movements.

2 Strategy Specifications

2.1 Scope of the Strategy

The core portfolio replicates equity exposure using options and futures contracts instead of directly buying stocks or ETFs. To hedge against sudden volatility changes, a synthetic straddle is created daily by rebalancing positions in options and futures, rather than purchasing additional options. This dual approach allows the portfolio to:

- Achieve desired equity exposure using derivatives.
- Maintain a target volatility level through dynamic adjustments.
- Hedge against volatility spikes by creating a synthetic straddle.

2.2 Configuration Parameters

- **target_volatility** (σ_{target}): Desired portfolio volatility.
- **target_gamma** (γ_{target}): Desired portfolio gamma.
- **lookback_window** (N): Period for calculating realized volatility.
- **underlying_assets**: List of equity indices or benchmarks to replicate.
- **start_date**: Strategy initiation date.
- **option_and_future_contracts**: Available contracts for replication and hedging.
- **rebalance_frequency**: Frequency of portfolio rebalancing (daily).

2.3 Synthetic Asset

To replicate an asset using options, we can leverage the concept of put-call parity, which defines a fundamental relationship between the prices of European call and put options, the underlying asset, and a risk-free bond. This relationship provides a basis for creating synthetic positions that replicate the payoff of holding the underlying asset.

2.3.1 Put-Call Parity

Put-call parity states that for a given underlying asset, a call option (C) and a put option (P) with the same strike price (K) and expiration date satisfy the following relationship:

$$C + Ke^{-rT} = P + S$$

where:

- C is the price of the call option,
- P is the price of the put option,
- S is the spot price of the underlying asset,
- K is the strike price,
- r is the risk-free interest rate, and
- T is the time to expiration.

This equation implies that holding a call option and a risk-free bond (with face value equal to the strike price) is equivalent to holding a put option and the underlying asset. This relationship allows us to construct synthetic positions.

2.3.2 Synthetic Long and Short Positions

Using put-call parity, we can create synthetic long and short positions in the underlying asset:

- **Synthetic Long Position**: To create a synthetic long position in the asset, we can purchase a call option and sell a put option with the same strike price and expiration date. This replicates the payoff of a long position in the underlying asset:

  $$\text{Synthetic Long} = C - P + Ke^{-rT}$$

  where the combined position will move similarly to holding the actual asset as the price fluctuates.

- **Synthetic Short Position**: Conversely, to create a synthetic short position in the asset, we can sell a call option and buy a put option with the same strike price and expiration date:

  $$\text{Synthetic Short} = P - C - Ke^{-rT}$$

  This position replicates the payoff of a short position in the underlying asset, allowing us to profit from downward price movements.

2.4 Benefits of Synthetic Asset Replication

Using options to create synthetic long and short positions provides flexibility in portfolio construction and risk management. Key advantages include:

- **Capital Efficiency**: Synthetic positions require less capital than directly holding or shorting the asset.
- **Risk Management**: By adjusting option positions, we can control delta, gamma, and vega exposure, making it easier to manage risk under changing market conditions.
- **Volatility Exposure**: Synthetic replication allows us to take advantage of implied volatility movements in addition to directional price changes.

3 Equity Replication Using Options

3.1 Daily Return Calculation Using Log Returns

The daily return (R_t) of the replicated portfolio is calculated using the log return formula:

$$R_t = \ln\left(\frac{V_t}{V_{t-1}}\right)$$

where:

- V_t: Portfolio value at time t.
- V_{t-1}: Portfolio value at time $t-1$.

Log returns are preferred as they are time additive, simplifying the computation of cumulative returns over a given period. Additionally, they provide a more accurate representation of returns for portfolio performance analysis in volatile markets.

3.2 Portfolio Volatility Calculation

3.2.1 Realized Volatility (σ_{realized})

Compute the realized volatility over the lookback window N:

$$\sigma_{\text{realized}} = \sqrt{\frac{252}{N}\sum_{i=t-N+1}^{t}(R_i - \bar{R})^2}$$

where $\bar{R}$ is the average daily return.

3.2.2 Volatility Adjustment Factor

Determine the adjustment factor to align with the target volatility:

$$\text{Adjustment Factor} = \frac{\sigma_{\text{target}}}{\sigma_{\text{realized}}}$$

3.3 Position Construction

Replicate the desired equity exposure using a combination of options and futures:

1. **Select Contracts**: Choose options (calls and puts) on the underlying assets.

   General formula for n (amount of synthetical positions)

   The amount of synthetical positions n is calculated with the following formula:

   $$n_0 = \frac{\text{Invested amount}}{C_0 - P_0}$$

   Where :

   - C_0 is the call option price.
   - P_0 is the put option price.

2. **Calculate Deltas** (Δ) , **Gammas** (Γ) and **Vegas** (ν) using the Black-Scholes model for options.

   For a portfolio consisting of multiple options, the total Delta and Gamma can be obtained by aggregating the individual Deltas and Gammas of each position.

Portfolio Delta

The **Delta** (Δ) measures the sensitivity of the portfolio's value to changes in the price of the underlying asset. For a portfolio containing n options, the total Delta is calculated as:

$$\Delta_{\text{portfolio}} = \sum_{i=1}^{n} n_i \times \Delta_i$$

Where:

- n_i: Number of contracts for the i-th option.
- Δ_i: Delta of the i-th option, calculated using the Black-Scholes model:

$$\Delta_{\text{call}} = N(d_1), \quad \Delta_{\text{put}} = N(d_1) - 1$$

- $N(d_1)$: Cumulative distribution function of the standard normal distribution.
- $d_1 = \frac{\ln(S/K)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$

Portfolio Gamma

The **Gamma** (Γ) measures the sensitivity of the portfolio's Delta to changes in the underlying asset's price. The total Gamma of the portfolio is given by:

$$\Gamma_{\text{portfolio}} = \sum_{i=1}^{n} n_i \times \Gamma_i$$

Where:

- Γ_i: Gamma of the i-th option, calculated as:

$$\Gamma_i = \frac{N'(d_1)}{S\sigma\sqrt{T}}$$

- $N'(d_1)$: Probability density function of the standard normal distribution:

$$N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$$

- S: Current price of the underlying asset.
- σ: Volatility of the underlying asset.
- T: Time to expiration.

Portfolio Vega

The Vega (ν) of an option quantifies its sensitivity to changes in implied volatility. It is computed as follows:

$$\nu = S\sqrt{T}N'(d_1)$$

Where:

- S: Current price of the underlying asset.
- T: Time to expiration.
- $N'(d_1)$: Probability density function of the standard normal distribution:

$$N'(d_1) = \frac{1}{\sqrt{2\pi}} e^{-\frac{d_1^2}{2}}$$

- $d_1 = \frac{\ln(S/K)+\left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$

Portfolio vega

- the portfolio Vega is the sum of the Vegas of individual positions:

$$\nu_{\text{portfolio}} = \sum_{i=1}^{n} n_i \times \nu_i$$

Where n_i is the number of contracts for the i-th option.

Interpretation

- **Delta**: Represents the portfolio's exposure to small changes in the underlying asset price.
- **Gamma**: Captures how the Delta changes as the price of the underlying asset moves, which is crucial for managing non-linear risk.
- **Vega** : reflects how much the option's value changes when the implied volatility changes by 1 percent .

3. **Portfolio value at time t**:

$$V_t = \sum_i n_i \times \Delta_i \times \text{Contract Size}_i \times \text{Price}_i$$

Solve for n_i to achieve the desired exposure. Contract size specifies the amount of underlying the option is about

4 Daily Portfolio Risk Adjustment

4.1 Adjusting Position Sizes

Scale the number of contracts to maintain target volatility:

$$n_i^{\text{new}} = n_i^{\text{old}} \times \text{Adjustment Factor}$$

4.2 Maintaining Delta Neutrality

After adjusting for volatility:

1. **Recalculate Deltas** for all positions.
2. **Adjust Positions** to ensure:

$$\Delta_{\text{portfolio}} = \sum_i n_i^{\text{new}} \times \Delta_i = 0$$

5 Synthetic Straddle Hedging Strategy

5.1 Concept of Synthetic Straddle

A straddle involves holding both a call and a put option at the same strike price, benefiting from large price movements in either direction. Instead of buying options, a synthetic straddle is created by adjusting existing positions to achieve similar payoff characteristics.

5.2 Delta neutrality

Aim for a portfolio with:

- **Delta Neutrality**: Zero sensitivity to small price movements.

Delta neutralization involves adjusting the positions in a portfolio to make its sensitivity to changes in the underlying asset zero. Mathematically, this means setting the **total Delta** of the portfolio to zero:

$$\Delta_{\text{portfolio}} = \sum_{i=1}^{n} n_i \times \Delta_i = 0$$

where:

- n_i: Number of contracts for position i,
- Δ_i: Delta of the instrument i,
- n: Total number of positions.

To achieve Delta neutrality:

$$n_\Delta = -\frac{\Delta_{\text{portfolio}}^{\text{initial}}}{\Delta_{\text{underlying}}}$$

where n_Δ is the number of underlying units to buy or sell. After this adjustment:

$$\Delta_{\text{portfolio}}^{\text{new}} = 0$$

5.3 Mathematical framework for increasing gamma (rajouter à notre boite à outils)

- **Positive Gamma**: Gains from large price movements in either direction.

The Gamma (Γ) of a derivatives portfolio measures the rate of change of Delta (Δ) with respect to changes in the underlying price S. Mathematically:

$$\Gamma = \frac{\partial \Delta}{\partial S} = \frac{\partial^2 V}{\partial S^2}$$

where V is the value of the derivative.

Here's how the proposed methods relate to this mathematical framework:

Buy At-The-Money (ATM) Options

ATM options maximize Gamma because they are most sensitive to small price changes near the strike price:

$$\Gamma_{\text{ATM}} = \max\left(\frac{\partial^2 C}{\partial S^2}, \frac{\partial^2 P}{\partial S^2}\right)$$

where C and P are the call and put prices, respectively.

Sell In-The-Money (ITM) or Out-of-The-Money (OTM) Options

ITM and OTM options have lower Gamma:

$$\Gamma_{\text{ITM/OTM}} < \Gamma_{\text{ATM}}$$

Selling these reduces the portfolio's exposure to low-Gamma positions, thereby increasing the average Gamma.

Synthetic Straddle or Strangle

A synthetic straddle or strangle involves buying both a call and a put:

$$\Gamma_{\text{straddle}} = \Gamma_{\text{call}} + \Gamma_{\text{put}}$$

These strategies increase the portfolio's overall Gamma, especially near the strike prices, benefiting from large price movements in either direction.

Reduce Low-Gamma Positions

Certain spreads (e.g., vertical or diagonal) have a flatter Gamma profile:

$$\Gamma_{\text{spread}} = \frac{\partial^2 (V_{\text{long}} - V_{\text{short}})}{\partial S^2}$$

Reducing or closing these positions increases the net Gamma.

Use Short-Maturity Options

Gamma is inversely related to time to maturity:

$$\Gamma \propto \frac{1}{\sqrt{T}}$$

Short-maturity options have higher Gamma, as their Delta changes more rapidly with price movements.

Dynamic Adjustments (Gamma Scalping)

Gamma scalping involves profiting from Gamma by dynamically adjusting Delta:

$$\text{Profit} \approx \frac{1}{2}\Gamma(S_{\text{new}} - S_{\text{old}})^2$$

Here, Γ ensures gains from rebalancing as the underlying price fluctuates.

5.4 Adjustment of Synthetic Straddles gamma for portfolio 2

Adjusting the number of **synthetic straddles** in portfolio 2 depends on several factors, primarily realized volatility, target volatility, and the desired Gamma level to capture market movements. Here is the methodology:

Step 1 : Evaluation of Current and Target Gamma

- **Current Gamma** (Γ_{current}): Calculate the current Gamma of portfolio 2.
- **Target Gamma** (Γ_{target}): Determine the Gamma level you aim to achieve.

Step 2 : Calculation of the Exposure to Adjust

$$\Delta\Gamma = \Gamma_{\text{target}} - \Gamma_{\text{current}}$$

This $\Delta\Gamma$ represents the Gamma gap that needs to be covered.

Step 3 : Gamma of a Synthetic Straddle

The **Gamma of a synthetic straddle** (Γ_{straddle}) is given by the sum of the Gammas of the call and put options:

$$\Gamma_{\text{straddle}} = \Gamma_{\text{call}} + \Gamma_{\text{put}}$$

Step 4 : Number of Straddles to Add

The number of synthetic straddles needed ($n_{\text{straddles}}$) to reach the target Gamma is calculated as:

$$n_{\text{straddles}} = \frac{\Delta\Gamma}{\Gamma_{\text{straddle}}}$$

- If $\Delta\Gamma > 0$: Add straddles to increase Gamma exposure.
- If $\Delta\Gamma < 0$: Reduce the number of straddles to decrease Gamma exposure.

Step 5 : Dynamic Adjustment Based on Volatility

Volatility Factor:

$$\text{Volatility Factor} = \frac{\sigma_{\text{realized}}}{\sigma_{\text{target}}}$$

Application to the number of straddles:

$$n_{\text{new straddles}} = n_{\text{current straddles}} \times \text{Volatility Factor}$$

Practical Example

- **Current Gamma**: 0.5
- **Target Gamma**: 1.0
- **Gamma per synthetic straddle (ATM, short-term)**: 0.1

Calculate $\Delta\Gamma$:

$$\Delta\Gamma = 1.0 - 0.5 = 0.5$$

Number of required straddles:

$$n_{\text{straddles}} = \frac{0.5}{0.1} = 5$$

Add **5 synthetic straddles** to reach the target Gamma.

5.5 Dynamic Risk Allocation Between Portfolios

In this strategy, dynamic risk management ensures that both Portefeuille 1 (Volatility Targeting) and Portefeuille 2 (Gamma Targeting) operate effectively without conflicting adjustment factors. This is achieved through a coordinated allocation of risk.

5.5.1 Step 1: Risk Allocation Calculation

The total portfolio risk (σ_{global}) is determined by the combined risks of both portfolios:

$$\sigma^2_{\text{global}} = \sigma^2_{\text{P1}} + \sigma^2_{\text{P2}}$$

Based on this, the risk allocation for each portfolio is calculated as follows:

$$\text{Allocation}_{P1} = \frac{\sigma^2_{\text{P1}}}{\sigma^2_{\text{global}}}, \quad \text{Allocation}_{P2} = \frac{\sigma^2_{\text{P2}}}{\sigma^2_{\text{global}}}$$

5.5.2 Step 2: Adjusting Portfolio 1 (Volatility Targeting)

If the realized volatility of Portefeuille 1 exceeds the target volatility ($\sigma_{\text{P1}} > \sigma_{\text{target}}$), positions are scaled down:

$$n_i^{\text{new}} = n_i^{\text{old}} \times \frac{\sigma_{\text{target}}}{\sigma_{\text{P1}}}$$

A capital buffer is maintained to ensure sufficient allocation for Portfolio 2.

5.5.3 Step 3: Adjusting Portfolio 2 (Gamma Targeting)

To maintain or achieve the Gamma target (Γ_{target}), the number of synthetic straddles in Portefeuille 2 is adjusted:

$$n^{\text{new}}_{\text{straddles}} = \min\left(\frac{\Gamma_{\text{target}} - \Gamma_{\text{current}}}{\Gamma_{\text{straddle}}}, \text{Max Allocation}_{P2}\right)$$

Here, Max Allocation$_{P2}$ represents the maximum allowable capital for Portfolio 2 based on its risk contribution.

Max Allocation for Portfolio 2 (Max Allocation$_{P2}$)

The maximum capital allocation for Portfolio 2 (Max Allocation$_{P2}$) ensures that the portfolio does not exceed its allocated risk within the global volatility target. It is calculated as:

$$\text{Max Allocation}_{P2} = \text{Total Capital} \times \text{Allocation}_{P2}$$

Where:

- Total Capital is the total amount of capital available for both portfolios.
- $\text{Allocation}_{P2} = \frac{\sigma^2_{\text{P2}}}{\sigma^2_{\text{global}}}$ is the risk allocation for Portfolio 2, as defined in the risk allocation step.

This ensures that Portfolio 2's operations stay within its risk limits while maintaining flexibility to adjust for Gamma targets.

5.5.4 Dynamic Coordination

This dynamic framework ensures:

- Portfolio 1 operates within its volatility limits without sacrificing the Gamma optimization of Portfolio 2.
- Portfolio 2 maximizes Gamma exposure while adhering to global risk constraints.

Practical Implementation

For example, if $\sigma_{\text{target}} = 10\%$, $\sigma_{\text{P1}} = 12\%$, and $\Gamma_{\text{current}} = 0.5$ while $\Gamma_{\text{target}} = 1.0$:

1. Adjust positions in Portfolio 1:

$$\text{Adjustment Factor}_{P1} = \frac{10\%}{12\%} \approx 0.833$$

2. Calculate $\Delta\Gamma$ for Portfolio 2:

$$\Delta\Gamma = 1.0 - 0.5 = 0.5$$

3. Adjust the number of straddles:

$$n^{\text{new}}_{\text{straddles}} = \min\left(\frac{0.5}{0.1}, \text{Max Allocation}_{P2}\right)$$

This approach ensures a balanced risk-adjusted performance across both portfolios.

5.6 Scenarios where Vega management might matter

There are specific cases where managing Vega could become crucial. Below are the key scenarios explained mathematically:

a) Sudden Implied Volatility Shocks

A sharp change in implied volatility (σ_{implied}) can significantly impact the price of options, even if the underlying asset's price (S) remains constant.

If implied volatility drops significantly, the total portfolio value decreases because both options lose value proportionally to their Vega:

$$\Delta V_{\text{portfolio}} \approx \text{Portfolio Vega} \times \Delta\sigma_{\text{implied}}$$

Even if realized volatility (σ_{realized}) remains high, this decrease in implied volatility will lead to losses.

b) Discrepancy Between Implied and Realized Volatility

Options are priced using implied volatility, but your strategy focuses on realized volatility. The value of an option under the Black-Scholes model is:

$$C = SN(d_1) - Ke^{-rT}N(d_2)$$

Where the volatility term σ in d_1 and d_2 is based on implied volatility. If implied volatility (σ_{implied}) significantly differs from realized volatility (σ_{realized}), the expected option value can diverge from its actual hedging performance.

The difference in portfolio performance due to volatility mismatch can be approximated by:

$$\Delta V_{\text{portfolio}} \propto (\sigma_{\text{implied}} - \sigma_{\text{realized}})$$

The symbol $\propto$ means "proportional to." It indicates a linear relationship between the two terms, but without specifying an explicit constant of proportionality.

A large gap between the two volatilities can result in suboptimal hedging or inefficient use of capital.

c) Short Volatility Exposure

The strategy implicitly involves short Vega positions (e.g., selling straddles or strangles), it becomes highly sensitive to spikes in implied volatility. The loss due to a spike in implied volatility is proportional to Vega:

$$\Delta V_{\text{portfolio}} \approx -\nu_{\text{portfolio}} \times \Delta\sigma_{\text{implied}}$$

For short straddles, the total portfolio Vega is negative:

$$\text{Portfolio Vega} = \nu_{\text{call}} + \nu_{\text{put}} < 0$$

A sudden increase in σ_{implied} can lead to significant losses, even if the underlying asset price remains unchanged.

6 Optimal Strike Price Determination with the ϵ Parameter

The goal is to determine optimal strike prices using the ϵ parameter, which adjusts the relative distance between the strike price and the spot price (S_0) based on the desired sensitivities.

6.1 General Problem Definition

For a portfolio consisting of n options, the objective is to maximize an objective function $f(K)$, defined as a combination of sensitivities (Δ, Γ, ν) and costs $C(K)$:

$$\max_K \; f(K) = w_1\Gamma(K) + w_2\nu(K) - w_3C(K)$$

Where:

- w_1, w_2, and w_3 are weights reflecting the relative importance of each component.
- $C(K)$ is the cost of the option for a given strike K.
- $\Gamma(K)$ represents the sensitivity of Delta to the underlying asset's price.
- $\nu(K)$ represents the sensitivity to implied volatility.

6.2 General Definition of ϵ

The ϵ parameter is defined as follows:

$$K = S_0 \times (1 + \epsilon)$$

where:

- K is the strike price of the option.
- S_0 is the current spot price.
- ϵ is the parameter that adjusts the distance between the strike and the spot.

6.3 Strike Optimization with Gamma

To maximize $\Gamma(K)$, the optimal strike is given by:

$$K^*_{\text{straddle}} = \arg\max_K \Gamma(K)$$

arg max means the x value that maximizes the function This optimization is achieved when K is close to the current spot price S_0:

$$K^*_{\text{straddle}} \approx S_0$$

6.4 Formulas for the Portfolios

The optimal values of ϵ depend on the objectives of each portfolio.

a) Portfolio 1: Synthetic Long Positions To maximize directional exposure while maintaining limited cost, ϵ is chosen slightly negative (ITM):

$$K_{\text{synthetic long}} = S_0 \times (1 - \epsilon_{\text{long}}), \quad \epsilon_{\text{long}} > 0$$

b) Portfolio 2: Synthetic Straddles To maximize Γ, straddles are centered around the spot ($\epsilon \approx 0$):

$$K_{\text{straddle}} = S_0 \times (1 + \epsilon_{\text{straddle}}), \quad \epsilon_{\text{straddle}} \approx 0$$

c) Protective Put Options To minimize costs while maintaining effective downside protection:

$$K_{\text{put}} = S_0 \times (1 - \epsilon_{\text{put}}), \quad \epsilon_{\text{put}} > 0$$

6.5 Optimizing ϵ

The optimization of ϵ is based on achieving the target sensitivities (Δ, Γ, ν):

$$\epsilon = \arg\min_{\epsilon} |\text{Greeks}(K) - \text{Targets}|$$

For each portfolio:

- **Portfolio 1**: Minimize the deviation from the target Δ.
- **Portfolio 2**: Minimize the deviation from the target Γ.
- **Protective Put**: Minimize the deviation from the target ν while reducing cost.

6.6 Optimization

The optimal strike is slightly OTM:

$$K^*_{\text{put}} \approx S_0 \times (1 - \epsilon)$$

Where ϵ is a small adjustment parameter, typically between 0.01 and 0.05.

7 Numerical Optimization Strategy

When analytical solutions are complex, numerical methods can be employed.

7.1 Objective Function

To minimize costs while maximizing sensitivities, solve:

$$\max_K \left[\Gamma(K) + \nu(K) - \lambda C(K)\right]$$

Where λ is a balancing parameter.

7.2 Numerical Methods

- **Gradient Descent**: Used to find the optimal strike.
- **Monte Carlo Simulations**: Test different strikes based on simulated market data.

8 Optimal Initial Data for the Strategy

8.1 General Portfolio Parameters

- **Initial Capital**:
  \$1,000,000 (or adjust based on investment capacity).
- **Target Volatility** (σ_{target}):
  15% annualized.
  Justification: This value aligns with the historical average volatility of the S&P 500.
- **Target Gamma** (Γ_{target}):
  Set initially to achieve a balanced exposure for large market moves. A value of $\Gamma_{\text{target}} = 0.5$ per \$1,000,000 capital is recommended.
- **Lookback Window** (N):
  20 trading days (approximately 1 month).
  Justification: Provides a short-term view of realized volatility to adapt quickly.
- **Rebalancing Frequency**:
  Daily rebalancing.
  Justification: Ensures timely adjustments, especially in volatile markets.

8.2 Market Data

- **Underlying Asset Prices** (S):
  Current price of the S&P 500 index (e.g., $S_0 = 4,500$).
- **Option Prices**:
  Use at-the-money (ATM) call and put options with 30 days to expiration.
- **Risk-Free Rate** (r):
  3% annualized (based on current U.S. Treasury yields).
- **Initial Implied Volatility** (σ_{implied}):
  18% annualized (reflects the VIX index as a proxy for S&P 500 implied volatility).

8.3 Initial Positions

- **Options Positions**:
  Start with a 70% allocation to Portfolio 1 (synthetic long positions) and 30% to Portfolio 2 (synthetic straddles).
  Justification: Ensures majority exposure to equity returns while reserving some capital for volatility hedging.

- **Initial Number of Contracts**:
  Assuming a contract size of 100:

$$n_{\text{Portfolio 1}} = \frac{0.7 \times \text{Initial Capital}}{S_0 \times 100}$$

$$n_{\text{Portfolio 2}} = \frac{0.3 \times \text{Initial Capital}}{\text{ATM Option Price} \times 100}$$

8.4 Technical Parameters

- **Contract Size**:
  100 units per contract for S&P 500 options.

- **Bid-Ask Spread**:
  Use a spread of 0.5% of the option price to account for transaction costs.

- **Constraints**:

  - Maximum Delta exposure: $|\Delta_{\text{Portfolio 1}}| < 0.9$.
  - Gamma exposure limit for Portfolio 2: $|\Gamma_{\text{Portfolio 2}}| < 1.5 \times \Gamma_{\text{target}}$.

8.5 Historical Data Logs

- **Historical Volatility Data**:
  Use the past 1-year S&P 500 data to calibrate the strategy.

- **Historical Option Prices and Greeks**:
  Include daily option data for both call and put positions.

- **Past Rebalancing Performance**:
  Simulate previous trades to validate and refine initial allocation.

9 Strategy parameters overview

9.1 General Portfolio Parameters

- **Initial Capital**:
  Total amount invested at the start.

- **Target Volatility** (σ_{target}):
  Desired level of portfolio volatility.

- **Target Gamma** (Γ_{target}):
  Desired gamma level for Portfolio 2 (straddles).

- **Lookback Window** (N):
  Time window used to calculate realized volatility (σ_{realized}).

- **Rebalancing Frequency**:
  Time interval for adjusting positions (daily, weekly, etc.).

9.2 Market Data

- **Underlying Asset Prices** (S):
  Historical prices of the underlying asset for computing returns and volatility.
- **Option Prices**:
  Prices of options (calls and puts) used for synthetic positions and straddles.
- **Risk-Free Rate** (r):
  Necessary for option pricing and Greeks calculation.
- **Initial Implied Volatility** (σ_{implied}):
  Implied volatility at the start of the strategy.

9.3 Initial Positions

- **Options Positions**:
  Number of contracts (calls/puts) to establish initial synthetic positions and straddles.
- **Initial Allocation Between Portfolios**:
  Initial distribution of capital between Portfolio 1 (synthetic long) and Portfolio 2 (straddles).

9.4 Technical Parameters

- **Contract Size**:
  Size of the options or futures contracts (e.g., 100 shares per contract).
- **Bid-Ask Spread**:
  Difference between buy and sell prices for options, to account for transaction costs.
- **Constraints**:
  Limits on positions (e.g., max Δ, Γ, or capital allocation).

9.5 Historical Data Logs

For testing and adjusting the strategy:

- **Historical Realized Volatility**.
- **Historical Prices and Greeks**.
- **Past Performance of Adjustments** (if prior simulations are available).