Statistical Arbitrage Strategy Rulebook

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March 18, 2025

Contents

1 Strategy Specification

1.1 Core Parameters

This subsection enumerates all the fundamental parameters involved in the statistical arbitrage strategy. These parameters are essential for model configuration, execution, and evaluation.

- **ETF Ticker** (T_{ETF}): The ticker symbol of the Exchange-Traded Fund used as the benchmark or hedging instrument.
- **Stock Ticker** (T_{Stock}): The ticker symbol of the individual stock being traded against the ETF.
- **Transaction Cost** ($C \in \mathbb{R}^+$): The total cost per transaction, including brokerage fees, slippage, and market impact.
- **Trading Days per Year** ($N_{yr} = 252$): The number of trading days in a year, used for annualizing performance metrics.
- **Position Size** (Q): The number of shares or units traded in each position.
- **Price Series** (P_t): The historical price data for both the ETF and the stock, including open, high, low, close, and volume.
- **Residuals** (ϵ_t): The difference between the observed stock price and the price predicted by the cointegration model.
- **Standardized Residuals** (z_t): The residuals normalized by their rolling mean and standard deviation.
- **Entry Threshold** (θ_{entry}): The number of standard deviations away from the mean residual required to trigger a trade.
- **Exit Threshold** (θ_{exit}): The number of standard deviations away from the mean residual required to close a trade.
- **Beta Window** (W_β): The rolling window size (in days) used to estimate the dynamic beta.
- **Residuals Window** (W_{res}): The rolling window size (in days) used to calculate the mean and standard deviation of residuals.
- **Maximum Holding Period** (T_{max}): The maximum number of days a position can remain open.
- **Rolling Mean** (μ_t): The mean of the residuals over the rolling window W_{res}.
- **Rolling Standard Deviation** (σ_t): The standard deviation of the residuals over the rolling window W_{res}.
- **Dynamic Beta** (β_t): The time-varying sensitivity of the stock's returns to the ETF's returns, estimated over the rolling window W_β.
- **Intercept** (α): The constant term in the cointegration model.
- **Profit and Loss** (PL): The profit or loss of a single trading transaction.
- **Opening Price** (P_{open}): The price at which a trading position is initiated.
- **Closing Price** (P_{close}): The price at which a trading position is closed.
- **Annualized Volatility** (σ_{annual}): The standard deviation of returns scaled to an annual basis.
- **Portfolio Capital** (C_{total}): The total capital allocated to the strategy.
- **Risk Fraction** (f_{risk}): The fraction of capital risked per trade.
- **Volatility Threshold** (θ_{vol}): The maximum allowed volatility for activating the strategy.
- **Slippage Coefficient** (a): The linear coefficient in the slippage model.
- **Slippage Exponent** (b): The exponent in the slippage model, capturing non-linear effects.
- **Average Daily Volume** (V_{avg}): The average daily trading volume of the stock, used in the slippage model.

1.2 Hyperparameters

This subsection lists all the hyperparameters involved in the statistical arbitrage strategy. These parameters are tuned to optimize the strategy's performance and risk management.

Symbol	Description	Typical Range
θ_{entry}	Entry threshold (in standard deviations)	[1.5, 2.5]
θ_{exit}	Exit threshold (in standard deviations)	[0.5, 1.0]
W_β	Rolling window size for beta estimation (days)	[60, 180]
W_{res}	Rolling window size for residuals calculation (days)	[90, 252]
T_{max}	Maximum holding period for a position (days)	[5, 21]
θ_{vol}	Volatility threshold for strategy activation	[0.1, 0.3]
f_{risk}	Risk fraction of capital allocated per trade	[0.01, 0.05]
θ_{profit}	Profit target threshold (as a multiple of)	[1.0, 2.0]
θ_{loss}	Stop-loss threshold (as a multiple of)	[1.0, 2.0]
θ_{trail}	Trailing stop threshold (as a percentage)	[0.01, 0.05]
$N_{lookback}$	Lookback period for historical data (days)	[252, 504]
θ_{conc}	Maximum position concentration (as a percentage)	[0.1, 0.3]
$\theta_{leverage}$	Maximum leverage ratio	[1.0, 3.0]
a	Slippage model linear coefficient	[0.01, 0.1]
b	Slippage model exponent	[1.0, 2.0]
θ_{corr}	Minimum correlation threshold for pair selection	[0.5, 0.8]
θ_{coint}	Cointegration test p-value threshold	[0.01, 0.05]

2 Model Framework

This section formalizes the mathematical models used in the statistical arbitrage strategy, along with the rationale for each modeling choice.

2.1 Cointegration Model

The cointegration model establishes a long-term equilibrium relationship between the stock and the ETF.

$$\ln P_t^{Stock} = \alpha + \beta \ln P_t^{ETF} + \epsilon_t \tag{1}$$

Rationale: Cointegration captures the long-term relationship between the stock and the ETF, allowing the strategy to identify mean-reverting deviations from equilibrium.

2.2 Residuals Calculation

The residuals represent deviations from the cointegration relationship.

$$\epsilon_t = \ln P_t^{Stock} - (\hat{\alpha} + \hat{\beta} \ln P_t^{ETF}) \tag{2}$$

Rationale: Residuals quantify the mispricing between the stock and the ETF, serving as the basis for trading signals.

2.3 Rolling Standardization

The residuals are standardized using a rolling window to account for time-varying volatility.

$$z_t = \frac{\epsilon_t - \mu_{[t-W_{res}:t]}}{\sigma_{[t-W_{res}:t]}} \tag{3}$$

Rationale: Standardization ensures that residuals are comparable over time, accounting for changes in market volatility and scaling deviations in terms of standard deviations.

2.4 Dynamic Beta Estimation

The sensitivity of the stock's returns to the ETF's returns is estimated dynamically using a rolling window.

$$\hat{\beta}_t = \frac{\sum_{i=t-W_\beta}^{t}(r_i^{ETF} - \bar{r}^{ETF})(r_i^{Stock} - \bar{r}^{Stock})}{\sum_{i=t-W_\beta}^{t}(r_i^{ETF} - \bar{r}^{ETF})^2} \tag{4}$$

Rationale: Dynamic beta captures time-varying market sensitivities, ensuring the model adapts to changing market conditions.

2.5 Volatility Modeling

A GARCH(1,1) model is used to estimate time-varying volatility.

$$\begin{aligned} r_t &= \mu + \epsilon_t \\ \sigma_t^2 &= \omega + \alpha\epsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \end{aligned}$$

Rationale: GARCH models provide accurate estimates of time-varying volatility, which is crucial for risk management and position sizing.

2.6 Residual Distribution

The standardized residuals are assumed to follow a normal distribution.

$$z_t \sim \mathcal{N}(0,1) \tag{5}$$

Rationale: Assuming normality simplifies the calculation of statistical thresholds (e.g., entry and exit signals) and facilitates hypothesis testing.

2.7 Stationarity Testing

The Augmented Dickey-Fuller (ADF) test is used to verify stationarity of the residuals.

$$\Delta\xi_t = \phi\xi_{t-1} + \sum_{k=1}^{p}\psi_k\Delta\xi_{t-k} + \varepsilon_t \tag{6}$$

Rationale: Stationarity ensures that the residuals exhibit mean-reverting behavior, a key assumption for statistical arbitrage strategies.

2.8 Correlation Analysis

The cross-correlation function measures the lead-lag relationship between the ETF and the stock.

$$\rho(\tau) = \frac{\mathbb{E}[(R_{A,t} - \mu_A)(R_{B,t+\tau} - \mu_B)]}{\sigma_A\sigma_B} \tag{7}$$

Rationale: Correlation analysis identifies potential lead-lag effects, which can improve trade timing and signal accuracy.

2.9 Slippage Model

Transaction costs are modeled as a function of trade size and market liquidity.

$$\text{Slippage} = a \cdot \left(\frac{Q}{V_{avg}}\right)^b \tag{8}$$

Rationale: Slippage modeling ensures realistic estimates of transaction costs, which are critical for accurate performance evaluation.

2.10 Performance Metrics

Key metrics for evaluating strategy performance are defined.

$$\text{Sharpe Ratio} = \frac{\mathbb{E}[r]}{\sigma_r}\sqrt{N_{yr}}$$
$$\text{Calmar Ratio} = \frac{\mathbb{E}[r]}{MDD}$$
$$\text{Hit Rate} = \frac{N_{win}}{N_{total}}$$
$$\text{Profit Factor} = \frac{\sum PL^+}{\sum PL^-}$$

Rationale: These metrics provide a comprehensive evaluation of risk-adjusted returns, drawdowns, and strategy consistency.

3 Trading Rules

This section defines the rules for entering and exiting trades, along with the rationale for each rule.

3.1 Entry Conditions

The strategy generates trading signals based on standardized residuals.

3.1.1 Long Signal

$$z_t < -\theta_{entry} \tag{9}$$

Rationale: A long signal is triggered when the stock is significantly undervalued relative to the ETF, indicating a potential buying opportunity.

3.1.2 Short Signal

$$z_t > \theta_{entry} \tag{10}$$

Rationale: A short signal is triggered when the stock is significantly overvalued relative to the ETF, indicating a potential selling opportunity.

3.2 Exit Conditions

The strategy defines both primary and alternative exit conditions to manage risk and lock in profits.

3.2.1 Primary Exit

$$|z_t| \leq \theta_{exit} \tag{11}$$

Rationale: The primary exit condition closes the position when the mispricing reverts to a predefined threshold, ensuring profits are captured when the mean-reversion occurs.

3.2.2 Alternative Exits

Alternative exit conditions are used to manage risk and prevent excessive losses.

- **Time-based Exit:**

  $$t_{hold} \geq T_{max} \tag{12}$$

  Rationale: Limits the duration of trades to prevent capital from being tied up in unprofitable positions for extended periods.

- **Stop-Loss Exit:**
$$PL \leq -C_{max} \tag{13}$$
Rationale: Protects against large losses by closing positions when the unrealized loss exceeds a predefined threshold.

- **Profit Target Exit:**
$$PL \geq C_{target} \tag{14}$$
Rationale: Locks in profits by closing positions when the unrealized gain reaches a predefined target.

3.3 Position Sizing

The strategy determines the size of each trade based on risk management principles.

$$Q_t = \frac{f_{risk} \cdot C_{total}}{\sigma_t \sqrt{T_{max}}} \tag{15}$$

Rationale: Position sizing ensures that each trade's risk is proportional to the portfolio's total capital and the asset's volatility, maintaining consistent risk exposure.

3.4 Trade Execution

Trades are executed using limit orders to minimize slippage.

$$P_{order} = P_{mid} \pm \delta \tag{16}$$

Rationale: Limit orders reduce transaction costs by avoiding market orders, which can be adversely affected by bid-ask spreads and market impact.

4 Risk Management

This section outlines the rules and methodologies used to manage risk in the statistical arbitrage strategy.

4.1 Position Sizing

Position sizing ensures that each trade's risk is proportional to the portfolio's total capital and the asset's volatility.

$$Q_t = \frac{K \cdot C_{total}}{\sigma_{annual} \sqrt{T_{max}/N_{yr}}} \tag{17}$$

Rationale: Volatility-adjusted sizing balances risk and reward by allocating more capital to less volatile assets and less capital to more volatile assets, ensuring consistent risk exposure across trades.

4.2 Portfolio Constraints

Portfolio-level constraints are enforced to limit exposure and diversify risk.

4.2.1 Maximum Exposure

$$\sum_i Q_i P_i \leq 0.3 C_{total} \tag{18}$$

Rationale: Limits the total capital allocated to open positions, reducing the impact of adverse market movements and preventing over-concentration in a single strategy.

4.2.2 Sector Limit

$$\sum_{j \in S} Q_j P_j \leq 0.15 C_{total} \tag{19}$$

Rationale: Diversifies risk by limiting exposure to any single sector, reducing the portfolio's vulnerability to sector-specific shocks.

4.2.3 Leverage Constraint

$$\sum_i Q_i P_i \leq \theta_{leverage} \cdot C_{total} \tag{20}$$

Rationale: Prevents excessive leverage, which can amplify losses during market downturns and ensure the portfolio remains within acceptable risk limits.

4.3 Stop-Loss Rules

Stop-loss rules are implemented to limit losses on individual trades.

$$PL \leq -C_{max} \tag{21}$$

Rationale: Protects against large losses by automatically closing positions when the unrealized loss exceeds a predefined threshold.

4.4 Volatility Filtering

The strategy is deactivated during periods of excessively high volatility.

$$\sigma_t \leq \theta_{vol} \cdot \bar{\sigma}_{1yr} \tag{22}$$

Rationale: High volatility increases the risk of large price swings and reduces the reliability of mean-reversion signals, making it prudent to avoid trading during such periods.

4.5 Drawdown Control

A maximum drawdown limit is enforced at the portfolio level.

$$MDD \leq \theta_{drawdown} \cdot C_{total} \tag{23}$$

Rationale: Ensures that the portfolio's losses do not exceed a predefined percentage of total capital, preserving capital for future opportunities.

4.6 Correlation Monitoring

The strategy monitors the correlation between the ETF and the stock to ensure the relationship remains stable.

$$\rho_t \geq \theta_{corr} \tag{24}$$

Rationale: A stable correlation is essential for the cointegration model to remain valid. If the correlation falls below a threshold, the strategy is paused to avoid trading on unreliable signals.

4.7 Liquidity Constraints

Trades are only executed in liquid assets to minimize slippage and market impact.

$$V_{avg} \geq \theta_{liquidity} \tag{25}$$

Rationale: Ensures that trades can be executed efficiently without significantly impacting the market price, reducing transaction costs.

5 Performance Metrics

This section defines the key metrics used to evaluate the performance of the statistical arbitrage strategy, along with the rationale for each metric.

5.1 Risk-Adjusted Returns

Risk-adjusted returns measure the strategy's performance relative to the risk taken.

5.1.1 Sharpe Ratio

$$SR = \frac{\mathbb{E}[r]}{\sigma_r}\sqrt{N_{yr}} \tag{26}$$

Rationale: The Sharpe Ratio quantifies the excess return per unit of risk (volatility), providing a standardized measure of risk-adjusted performance that is comparable across different strategies and time periods.

5.1.2 Calmar Ratio

$$CR = \frac{\mathbb{E}[r]}{MDD} \tag{27}$$

Rationale: The Calmar Ratio measures the return relative to the maximum drawdown, highlighting the strategy's ability to generate returns while managing downside risk.

5.2 Strategy Statistics

These statistics provide insights into the strategy's consistency, profitability, and trading activity.

5.2.1 Hit Rate

$$HR = \frac{N_{win}}{N_{total}} \tag{28}$$

Rationale: The Hit Rate measures the proportion of winning trades, indicating the strategy's consistency in generating profitable signals.

5.2.2 Profit Factor

$$PF = \frac{\sum PL^{+}}{\sum PL^{-}} \tag{29}$$

Rationale: The Profit Factor compares the total profits from winning trades to the total losses from losing trades, providing a measure of the strategy's overall profitability.

5.2.3 Turnover

$$TO = \frac{\sum |Q_t|}{C_{total}} \tag{30}$$

Rationale: Turnover measures the trading activity relative to the portfolio's total capital, helping to assess the strategy's transaction costs and liquidity requirements.

5.3 Additional Metrics

Additional metrics are used to provide a comprehensive evaluation of the strategy's performance.

5.3.1 Annualized Return

$$R_{annual} = \left(\prod_{t=1}^{N_{yr}}(1 + r_t)\right)^{\frac{N_{yr}}{T}} - 1 \tag{31}$$

Rationale: Annualized Return provides a standardized measure of the strategy's performance over time, facilitating comparison with benchmarks and other investments.

5.3.2 Sortino Ratio

$$Sortino = \frac{\mathbb{E}[r]}{\sigma_{down}} \tag{32}$$

Rationale: The Sortino Ratio focuses on downside volatility, providing a more targeted measure of risk-adjusted performance for strategies where minimizing losses is a priority.

5.3.3 Win/Loss Ratio

$$WLR = \frac{\mathbb{E}[PL^+]}{\mathbb{E}[PL^-]} \tag{33}$$

Rationale: The Win/Loss Ratio compares the average profit of winning trades to the average loss of losing trades, offering insights into the strategy's risk-reward profile.

5.3.4 Maximum Drawdown Duration

$$MDD_{duration} = \max(t_{end} - t_{start}) \tag{34}$$

Rationale: The Maximum Drawdown Duration measures the longest period of time required to recover from a peak to a new high, providing insights into the strategy's recovery capability.

6 Backtesting Protocol

This section outlines the methodology for backtesting the statistical arbitrage strategy to evaluate its historical performance.

6.1 Walk-Forward Testing

Walk-forward testing is used to validate the strategy's robustness over time.

$$T_{train} : T_{test} = 3 : 1 \quad \text{(rolling window)} \tag{35}$$

Rationale: Walk-forward testing splits the data into training and testing periods, ensuring the strategy is validated on out-of-sample data. A 3:1 ratio balances the need for sufficient training data and meaningful testing periods.

6.2 Slippage Model

Transaction costs are modeled to account for market impact and liquidity constraints.

$$\text{Slippage} = a \cdot \left(\frac{Q}{V_{avg}}\right)^b \tag{36}$$

where V_{avg} is the average daily trading volume.

Rationale: The slippage model estimates the cost of executing trades, ensuring that backtest results reflect realistic trading conditions.

7 Implementation

This section describes the execution logic and monitoring tools used to implement the statistical arbitrage strategy.

7.1 Execution Logic

The trading strategy is implemented using the following algorithm:

[1] Calculate standardized residuals z_t Check entry conditions long signal Calculate position size Q_t Place limit order: $P_{entry} = P_{mid} - \delta$ short signal Calculate position size Q_t Place limit order: $P_{entry} = P_{mid} + \delta$ Monitor exit conditions Log all transactions

Rationale: The execution logic ensures that trades are executed efficiently using limit orders, minimizing slippage and transaction costs.

7.2 Monitoring Dashboard

The strategy's performance is monitored using a dashboard with the following components:

- **Residuals Heatmap by Sector:** Visualizes the distribution of residuals across different sectors.
- **Cumulative Performance Curve:** Tracks the strategy's cumulative profit and loss over time.
- **P&L Distribution:** Displays the distribution of individual trade profits and losses.
- **Rolling Sharpe Ratio:** Monitors the strategy's risk-adjusted performance over time.

Rationale: The monitoring dashboard provides real-time insights into the strategy's performance, enabling timely adjustments and risk management.