Pairs Trading Strategy: Mathematical Rulebook

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1 Objective

The objective of this strategy is to systematically capitalize on transient price divergences between two historically co-integrated assets, $S_1(t)$ and $S_2(t)$, via statistical arbitrage. By identifying a mean-reverting spread and validating the regime with the Hurst exponent, the strategy aims to open long-short positions whenever deviations from the long-term equilibrium become statistically significant, thus maximizing risk-adjusted returns.

2 Pairs Formation

2.1 Normalized Prices

Normalize price series for assets over a **formation period** $T_{\text{formation}}$:

$$P_i(t) = \frac{S_i(t)}{S_i(0)}, \quad i = 1, 2,\ t \in T_{\text{formation}}.$$

2.2 Distance Measure

Define the distance $d_{1,2}$ between two assets as:

$$d_{1,2} = \sqrt{\sum_{t \in T_{\text{formation}}} \left(P_1(t) - P_2(t)\right)^2}.$$

Select pairs with the smallest $d_{1,2}$.

2.3 Co-Integration

Co-integration is a statistical method used to determine whether two non-stationary time series maintain a stable long-term relationship. This is essential in pairs trading to ensure that the price deviations between two assets converge over time.

2.3.1 Definition

Two time series $P_1(t)$ and $P_2(t)$ are said to be **co-integrated** if there exists a linear combination of these series that is stationary, even if $P_1(t)$ and $P_2(t)$ themselves are non-stationary.

Formally, this means there exists a parameter β such that the residuals $\epsilon(t)$ defined by:

$$\epsilon(t) = P_1(t) - \beta P_2(t),$$

are **stationary**, meaning $\epsilon(t)$ has a constant mean, finite variance, and time-independent properties.

2.3.2 Estimating the Parameter β

The coefficient β is estimated through ordinary least squares (OLS) regression, which models the linear relationship between two time series $P_1(t)$ and $P_2(t)$. Mathematically, β is given by:

$$\beta = \frac{\text{Cov}(P_1, P_2)}{\text{Var}(P_2)},$$

where:

- $\text{Cov}(P_1, P_2)$ is the covariance between the series $P_1(t)$ and $P_2(t)$,
- $\text{Var}(P_2)$ is the variance of $P_2(t)$.

Formula for Covariance The covariance between $P_1(t)$ and $P_2(t)$ over a given time period $T_{\text{formation}}$ is calculated as:

$$\text{Cov}(P_1, P_2) = \frac{1}{N} \sum_{t \in T_{\text{formation}}} (P_1(t) - \mu_1)(P_2(t) - \mu_2),$$

where:

- N is the number of observations in $T_{\text{formation}}$,
- $\mu_1 = \frac{1}{N} \sum_{t \in T_{\text{formation}}} P_1(t)$ is the mean of $P_1(t)$,
- $\mu_2 = \frac{1}{N} \sum_{t \in T_{\text{formation}}} P_2(t)$ is the mean of $P_2(t)$.

Formula for Variance The variance of $P_2(t)$ is calculated as:

$$\text{Var}(P_2) = \frac{1}{N} \sum_{t \in T_{\text{formation}}} (P_2(t) - \mu_2)^2 .$$

Linear Relationship Between $P_1(t)$ and $P_2(t)$ Using the estimated β, the linear relationship between $P_1(t)$ and $P_2(t)$ is expressed as:

$$P_1(t) = \beta P_2(t) + \epsilon(t),$$

where:

- $\epsilon(t)$ represents the residuals, which capture the deviations of $P_1(t)$ from the estimated relationship.

2.3.3 Testing Residuals for Stationarity

To verify whether the residuals $\epsilon(t)$ are stationary, unit root tests such as the **Augmented Dickey-Fuller (ADF)** test are used.

ADF Test Hypotheses

- H_0 (null hypothesis): The residuals $\epsilon(t)$ are not stationary.
- H_1 (alternative hypothesis): The residuals $\epsilon(t)$ are stationary.

The ADF test examines the following equation:

$$\Delta\epsilon(t) = \phi\epsilon(t-1) + \sum_{i=1}^{k} \gamma_i \Delta\epsilon(t-i) + \nu(t),$$

where:

- $\Delta\epsilon(t) = \epsilon(t) - \epsilon(t-1)$ is the first difference of the residuals,
- ϕ is the stationarity parameter,
- γ_i are the coefficients of lagged terms,
- $\nu(t)$ is a white noise error term.

Decision Criteria - If $\phi = 0$, the residuals $\epsilon(t)$ follow a non-stationary process. - If $\phi < 0$, the residuals $\epsilon(t)$ are stationary.

The t-statistic associated with ϕ is compared against critical values from statistical tables to determine if H_0 can be rejected.

2.3.4 Calculation via R/S Analysis

To estimate the Hurst exponent H of the residuals or spread $\epsilon(t)$, we use the **R/S (rescaled range) analysis**. The procedure can be summarized as follows:

1. **Partition the Time Series:**
   Consider a time series of length N, which in our context is the spread $\epsilon(t)$. You can choose multiple segments (or sub-periods) of length n (e.g., $n = N/2, N/4, \ldots$) to analyze the scaling behavior.

2. **Compute the Mean:**
   For a sub-period of length n, let the mean of $\epsilon(t)$ be:
$$\bar{\epsilon} = \frac{1}{n}\sum_{t=1}^{n}\epsilon(t).$$

3. **Form the Cumulative Deviation Series:**
   Define the cumulative deviation from the mean at each time point k (with $1 \le k \le n$):
$$Y(k) = \sum_{t=1}^{k}\big[\epsilon(t) - \bar{\epsilon}\big].$$

4. **Calculate the Range $R(n)$:**
   The *range* $R(n)$ is the difference between the maximum and minimum of $Y(k)$ over $1 \le k \le n$:
$$R(n) = \max_{1\le k\le n} Y(k) \; - \; \min_{1\le k\le n} Y(k).$$

5. **Calculate the Standard Deviation $S(n)$:**
   The standard deviation of the original sub-period $\epsilon(t)$ (of length n) is:
$$S(n) = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\Big(\epsilon(t) - \bar{\epsilon}\Big)^2}.$$

6. **Compute the Rescaled Range $\frac{R(n)}{S(n)}$:**
   For each sub-period of length n, compute:
$$\frac{R(n)}{S(n)}.$$
   Repeat this process for different values of n (e.g., multiple scales).

7. **Log-Log Regression:**
   Plot $\log\big(R(n)/S(n)\big)$ against $\log(n)$ across the different segment lengths n. The slope of the best-fit line in this log-log plot gives an estimate of the Hurst exponent H:
$$\log\big(R(n)/S(n)\big) \;\approx\; H\,\log(n) \;+\; \text{constant}.$$

In this rulebook, if we observe:

$$H < 0.5 \quad \longrightarrow \quad \text{Anti-persistence (mean-reverting)},$$

$$H \approx 0.5 \quad \longrightarrow \quad \text{Random walk},$$

$$H > 0.5 \quad \longrightarrow \quad \text{Persistence (trending)}.$$

By periodically estimating H for $\epsilon(t)$, we can determine whether the spread is likely to revert to its mean or continue in a trend.

2.3.5 Application to Pairs Trading

If the residuals $\epsilon(t)$ are stationary:

- The two assets $P_1(t)$ and $P_2(t)$ are co-integrated.
- A stable long-term relationship exists between the two assets.
- Price deviations ($\epsilon(t)$) can be exploited in pairs trading, as they tend to revert to their mean.

3 Entry Rule

3.1 Spread Definition

In a co-integration framework, let $P_1(t)$ and $P_2(t)$ be the (normalized) prices of two assets over time, and let β be the coefficient estimated via Ordinary Least Squares (OLS) such that $\epsilon(t) = P_1(t) - \beta\, P_2(t)$ is (approximately) stationary. We define our trading spread $\Delta(t)$ as:

$$\Delta(t) = P_1(t) - \beta\, P_2(t).$$

When $\Delta(t)$ significantly deviates from its historical mean, we anticipate mean reversion.

3.2 Mean and Volatility of $\Delta(t)$

3.2.1 Mean (μ_Δ)

Over the formation period $T_{\text{formation}}$, let the number of observations be N. We define the sample mean of the spread:

$$\mu_\Delta = \frac{1}{N} \sum_{t \in T_{\text{formation}}} \Delta(t).$$

3.2.2 Variance and Standard Deviation

The sample variance of $\Delta(t)$ over $T_{\text{formation}}$ is:

$$\text{Var}[\Delta(t)] = \frac{1}{N} \sum_{t \in T_{\text{formation}}} \Big(\Delta(t) - \mu_\Delta\Big)^2.$$

Hence, the standard deviation is:

$$\sigma_\Delta = \sqrt{\text{Var}[\Delta(t)]}.$$

3.2.3 Thresholds

Using μ_Δ and σ_Δ, we define thresholds for potential mean-reversion signals:

$$\text{Long threshold (lower bound): } \mu_\Delta - k\,\sigma_\Delta,$$
$$\text{Short threshold (upper bound): } \mu_\Delta + k\,\sigma_\Delta,$$

where k is a predefined multiplier (commonly $k = 2$).

3.3 Trigger Threshold

A trading signal is generated when the spread diverges from its mean by at least k standard deviations:

$$\big|\Delta(t) - \mu_\Delta\big| \quad \geq \quad k\,\sigma_\Delta.$$

3.4 Positioning

Assume S_1 corresponds to P_1 and S_2 to P_2:

- **Long Position on the Spread:** If $\Delta(t) > \mu_\Delta + k\,\sigma_\Delta$, we expect $\Delta(t)$ to *decrease* (revert), so we **short** S_1 and **long** S_2.
- **Short Position on the Spread:** If $\Delta(t) < \mu_\Delta - k\,\sigma_\Delta$, we expect $\Delta(t)$ to *increase*, so we **long** S_1 and **short** S_2.

4 Exit Rule

4.1 Mean Reversion Exit

Close positions when the spread $\Delta(t)$ returns near its mean:

$$\Delta(t) \approx \mu_\Delta.$$

In practice, one can set an exit threshold (e.g., $\pm 0.2\,\sigma_\Delta$ around μ_Δ) to account for noise.

4.2 Time-Based (Forced) Closure

Close all open positions at the end of the *trading period* T_{trading} to avoid excessive overnight or regime-change risks.

5 Return Calculation

5.1 Daily Returns

Let $w_1(t)$ and $w_2(t)$ be the (possibly time-varying) position sizes in S_1 and S_2. The portfolio return at time t is defined by:

$$r(t) = w_1(t)\,\frac{S_1(t+1) - S_1(t)}{S_1(t)} + w_2(t)\,\frac{S_2(t+1) - S_2(t)}{S_2(t)}.$$

Depending on margin requirements or portfolio constraints, $w_1(t)$ and $w_2(t)$ can be leveraged.

5.2 Cumulative Return

We define the cumulative return over the trading horizon T_{trading} as the product of daily gross returns, minus 1:

$$R = \prod_{t \in T_{\text{trading}}} \big(1 + r(t)\big) \; - \; 1.$$

This formula captures compounding effects.

5.3 Risk-Adjusted Return

We measure strategy performance using the Sharpe Ratio:

$$\text{Sharpe Ratio} = \frac{\mathbb{E}[r] - r_f}{\sigma_r},$$

where:

- $\mathbb{E}[r]$ is the expected (mean) *daily* return of the strategy,
- r_f is the risk-free rate (annualized or daily, consistently with $\mathbb{E}[r]$),
- σ_r is the standard deviation of the *daily* returns.

In practice, to annualize the Sharpe Ratio, we scale it by $\sqrt{D}$, where D is the number of trading days in a year (commonly $D \approx 252$). The target is typically to maintain a minimum Sharpe Ratio of:

$$\text{Sharpe Ratio} \;\geq\; 1.$$

6 Remarks on Implementation and Risk

- **Stationarity / Co-Integration:** Ensure $\Delta(t)$ is truly mean-reverting by testing stationarity (e.g. Augmented Dickey-Fuller). If the residuals lose stationarity, re-estimate β or switch pairs.
- **Transaction Costs:** Since pairs trading involves simultaneous long and short, factor in fees, bid-ask spreads, and slippage.
- **Position Sizing:** Calibrate $w_1(t)$ and $w_2(t)$ based on volatility to control drawdowns.
- **Regime Changes:** Monitor market conditions (e.g. via Hurst exponent or volatility shifts). A strongly trending market can invalidate mean-reversion assumptions.

7 Return Calculation

7.1 Daily Returns

Let $w_1(t)$ and $w_2(t)$ denote positions in S_1 and S_2. The portfolio return at time t is:

$$r(t) = w_1(t) \cdot \frac{S_1(t+1) - S_1(t)}{S_1(t)} + w_2(t) \cdot \frac{S_2(t+1) - S_2(t)}{S_2(t)}.$$

7.2 Cumulative Return

The cumulative return over the trading period is:

$$R = \prod_{t \in T_{\text{trading}}} (1 + r(t)) - 1.$$

7.3 Risk-Adjusted Return

Use Sharpe Ratio to measure performance:

$$\text{Sharpe Ratio} = \frac{\mathbb{E}[r] - rf}{\sigma_r},$$

where $\mathbb{E}[r]$ is the mean return, σ_r is the standard deviation of returns and rf the risk-free rate. objective : reach a minimum of 1.

8 Transaction Costs

8.1 Calculation of Transaction Costs

Transaction costs are a crucial factor in evaluating the profitability of a trading strategy. They typically include:

- **Bid-Ask Spread:** The difference between the buy (ask) and sell (bid) prices.
- **Fixed Fees:** Brokerage or exchange fees incurred per trade.
- **Slippage:** The difference between the expected and actual execution price due to market conditions.

8.1.1 Incorporating Transaction Costs into Returns

The net return after accounting for transaction costs at time t is given by:

$$r_{\text{net}}(t) = r(t) - c_{\text{transaction}}(t),$$

where:

- $r(t)$ is the gross return at time t,
- $c_{\text{transaction}}(t)$ is the total transaction cost incurred at time t.

Bid-Ask Spread The cost due to the bid-ask spread can be calculated as:

$$c_{\text{bid-ask}} = \frac{\text{Spread}}{\text{Mid Price}},$$

where:

- Spread = Ask Price − Bid Price,
- Mid Price = $\frac{\text{Ask Price}+\text{Bid Price}}{2}$.

Fixed Fees Fixed fees are added to transaction costs as:

$$c_{\text{fixed}} = f_{\text{broker}} \cdot N_{\text{trades}},$$

where:

- f_{broker} is the fee per trade (e.g., \$10 per trade),
- N_{trades} is the number of trades executed.

Slippage Slippage is estimated as:

$$c_{\text{slippage}} = \frac{|P_{\text{execution}} - P_{\text{expected}}|}{P_{\text{expected}}},$$

where:

- $P_{\text{execution}}$ is the actual execution price,
- P_{expected} is the expected execution price.

The expected execution price, P_{expected}, could be the theoretical price of the asset at the time of placing the order, calculated using:

The bid-ask midpoint:

$$P_{\text{expected}} = \frac{\text{Bid Price} + \text{Ask Price}}{2}$$

This is commonly used in backtests or high-frequency trading to represent a "fair" execution price.

8.1.2 Total Transaction Costs

The total transaction cost is the sum of the individual components:

$$c_{\text{transaction}}(t) = c_{\text{bid-ask}}(t) + c_{\text{fixed}}(t) + c_{\text{slippage}}(t).$$

8.1.3 Impact on Cumulative Return

The cumulative net return over the trading period T_{trading} is adjusted for transaction costs:

$$R_{\text{net}} = \prod_{t \in T_{\text{trading}}} (1 + r_{\text{net}}(t)) - 1.$$

Substituting $r_{\text{net}}(t) = r(t) - c_{\text{transaction}}(t)$:

$$R_{\text{net}} = \prod_{t \in T_{\text{trading}}} (1 + r(t) - c_{\text{transaction}}(t)) - 1.$$

8.1.4 Usage in Strategy Evaluation

Transaction costs are used to:

- Evaluate the true profitability of the strategy after accounting for market frictions.
- Optimize trade frequency to minimize costs while maximizing returns.
- Compare strategies with similar gross returns but differing transaction costs.

8.1.5 Example

Assume:

- Bid price = \$100, Ask price = \$102.
- Fixed fee = \$10 per trade.
- Execution price = \$101.5, Expected price = \$101.

Step 1: Calculate Bid-Ask Spread

$$c_{\text{bid-ask}} = \frac{\text{Ask Price} - \text{Bid Price}}{\text{Mid Price}} = \frac{102 - 100}{\frac{102+100}{2}} = \frac{2}{101} \approx 1.98\%.$$

Step 2: Calculate Fixed Fees If $N_{\text{trades}} = 5$:

$$c_{\text{fixed}} = 10 \cdot 5 = \$50.$$

Step 3: Calculate Slippage

$$c_{\text{slippage}} = \frac{|101.5 - 101|}{101} = \frac{0.5}{101} \approx 0.50\%.$$

Step 4: Total Transaction Cost

$$c_{\text{transaction}} = c_{\text{bid-ask}} + c_{\text{fixed}} + c_{\text{slippage}}.$$

$$c_{\text{transaction}} = 1.98\% + 50 + 0.50\%.$$

8.2 Adjusted Profitability

Calculate adjusted excess return:

$$R_{\text{net}} = \prod_{t \in T_{\text{trading}}} (1 + r_{\text{net}}(t)) - 1.$$

9 Risk Management

9.1 Leverage

Definition Leverage refers to the use of borrowed capital to increase the potential return on an investment. While leverage amplifies profits, it also significantly increases risk. Managing leverage effectively is crucial to controlling downside risk in a trading strategy.

To limit the risk of excessive leverage, **Value-at-Risk (VaR)** is often used as a guiding metric. The **VaR** at a confidence level α is defined as:

$$\text{VaR}_\alpha = \inf\{x : P(R < -x) \leq \alpha\},$$

where:

- R is the portfolio return over a given time horizon,
- α is the confidence level (e.g., $\alpha = 0.05$ for a 95% confidence interval),
- $-x$ is the loss threshold corresponding to the specified confidence level.

Interpretation The VaR_α represents the maximum expected loss of the portfolio at the confidence level α over a specified time horizon. For example:

- If $\text{VaR}_{0.05} = \$10,000$, there is a 5% chance that the portfolio will lose more than \$10,000 in the given time period.
- The remaining 95% of the time, losses are expected to be less than \$10,000.

9.1.1 Implementation in Leverage Management

Step 1: Calculate Portfolio VaR

1. **Determine Portfolio Returns:** Compute the daily returns $R(t)$ of the portfolio.
2. **Estimate the Distribution of Returns:** Assume $R(t)$ follows a normal distribution, or use historical simulation methods to estimate the empirical distribution.
3. **Compute VaR:** For a confidence level α, calculate:

$$\mathrm{VaR}_\alpha = -\Phi^{-1}(\alpha) \cdot \sigma_R \cdot \sqrt{T},$$

   where:

   - $\Phi^{-1}(\alpha)$ is the inverse cumulative distribution function (z-score) for the normal distribution,
   - σ_R is the standard deviation of portfolio returns,
   - T is the time horizon (e.g., one day, one week).

Step 2: Set Leverage Limits

- Define a maximum allowable VaR as a percentage of the portfolio value:

$$\text{Leverage Limit} = \frac{\mathrm{VaR}_\alpha}{\text{Maximum Allowable Loss}}.$$

- Adjust portfolio exposure such that the leverage does not exceed this limit:

$$\text{Portfolio Leverage} = \frac{\text{Gross Portfolio Value}}{\text{Equity}} \leq \text{Leverage Limit}.$$

$$\text{Equity} = \text{Gross Portfolio Value} - \text{Liabilities}$$

$$\text{Liabilities} = \text{Margin Loans} + \text{Short Positions} + \text{Unsettled Trades} + \text{Accrued Expenses} + \text{Derivatives Losses}$$

9.1.2 1. Monitoring Risk

The goal is to evaluate risk relative to the VaR and reduce exposure if the risk exceeds an acceptable threshold.

Mathematical Formula:

$$\mathrm{VaR}_\alpha = -\Phi^{-1}(\alpha) \cdot \sigma_R \cdot \sqrt{T}$$

Where:

- $\Phi^{-1}(\alpha)$: Z-score corresponding to the confidence level α,
- σ_R: Standard deviation of portfolio returns,
- T: Time horizon.

Rule:

If the actual or expected loss exceeds the VaR, reduce exposure proportionally:

$$\text{Adjusted Exposure} = \frac{\text{Maximum Allowable Loss}}{\text{Observed VaR}} \cdot \text{Current Exposure}$$

This ensures an adjustment proportional to the acceptable VaR.
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9.1.3 2. Setting Stop-Loss Thresholds

Stop-loss thresholds combine VaR and leverage limits to prevent significant drawdowns.

Stop-Loss Based on VaR:

$$\text{Stop-Loss} = \text{Portfolio Value} \cdot \text{VaR}_\alpha$$

If the loss exceeds this threshold, the position is automatically closed to protect the portfolio.

Stop-Loss Based on Leverage:

The maximum allowable leverage is defined as:

$$\text{Leverage Limit} = \frac{\text{VaR}_\alpha}{\text{Maximum Allowable Loss}}$$

The portfolio should always satisfy the constraint:

$$\text{Gross Portfolio Value} \leq \text{Leverage Limit} \cdot \text{Equity}$$

9.2 Drawdown

Definition The **Maximum Drawdown (MDD)** is a measure of downside risk that quantifies the largest peak-to-trough decline in the value of a portfolio or strategy over a given time period. It is mathematically defined as:

$$\text{Max Drawdown} = \max_{t \in T_{\text{trading}}} \left(1 - \frac{R(t)}{\max_{s \leq t} R(s)}\right),$$

where:

- $R(t)$ is the cumulative return of the portfolio at time t,
- $\max_{s \leq t} R(s)$ is the maximum cumulative return observed up to time t,
- T_{trading} is the trading period under consideration.

Drawdown at Time t At any time t, the drawdown ($DD(t)$) is the percentage decline from the historical peak return:

$$DD(t) = 1 - \frac{R(t)}{\max_{s \leq t} R(s)}.$$

Maximum Drawdown The **Maximum Drawdown** is the largest drawdown observed over the trading period T_{trading}:

$$\text{Max Drawdown} = \max_{t \in T_{\text{trading}}} DD(t).$$

9.2.1 Usage in Risk Management

Maximum Drawdown is a key metric in assessing the performance and risk of a trading strategy. It can be used as follows:

- **Risk Monitoring:** - Continuously monitor the drawdown to identify periods of significant losses. - Set a maximum allowable drawdown threshold (e.g., 20%) to trigger risk mitigation actions.
- **Strategy Evaluation:** - Compare the MDD of different strategies to assess their relative risk. - A strategy with a lower MDD is considered less risky, even if returns are similar.
- **Stop-Loss Rules:** - Use MDD as a criterion for stopping trading when losses exceed a tolerable limit. - For example, terminate trading if Max Drawdown > 25%.
- **Risk-Adjusted Metrics:** - Combine MDD with return metrics to compute risk-adjusted performance measures, such as the **Calmar Ratio**:

$$\text{Calmar Ratio} = \frac{\text{Annualized Return}}{\text{Maximum Drawdown}}.$$

9.2.2 Example in Practice

Assume the following cumulative returns $R(t)$ over a trading period:

t	$R(t)$	$\max_{s \leq t} R(s)$	$DD(t)$
1	5%	5%	0%
2	10%	10%	0%
3	8%	10%	20%
4	12%	12%	0%
5	9%	12%	25%

In this example:

$$\text{Max Drawdown} = 25\%.$$

9.3 Stop-Loss

Implement stop-loss rules if $|\Delta(t)| >$ Max Divergence.

9.3.1 How Max Divergence is Determined

The threshold for **Max Divergence** can be set using one of the following methods:

Empirical Analysis Based on historical data, identify extreme values of the spread $\Delta(t)$ during the formation period $T_{\text{formation}}$. For example:

- Calculate the 99th percentile of $|\Delta(t)|$, which captures the most extreme deviations observed historically.
- Use this percentile as the threshold for Max Divergence.

10 Conclusion

The Pairs Trading Strategy detailed in this rulebook provides a rigorous framework for exploiting price deviations between co-integrated assets through statistical arbitrage. Each component is mathematically defined to ensure precision and adaptability. Key elements of the strategy include:

- **Robust Pair Selection:** The use of normalized prices, distance measures, and co-integration tests ensures that selected asset pairs exhibit a stable long-term relationship, which is essential for mean reversion trading.
- **Defined Entry and Exit Rules:** Entry thresholds based on historical spread mean (μ_Δ) and volatility (σ_Δ) identify trading opportunities with systematic rules for opening and closing long and short positions.
- **Transaction Costs Integration:** Transaction costs such as bid-ask spreads, fixed fees, and slippage are explicitly modeled, ensuring realistic assessments of net profitability while avoiding overestimation of returns.
- **Risk Management:** The strategy incorporates robust risk management measures, including leverage control via Value-at-Risk (VaR), Maximum Drawdown (MDD) monitoring, and stop-loss mechanisms, to safeguard against excessive losses.
- **Practical Application:** The inclusion of realistic metrics and thresholds, such as Max Divergence and empirical calibration of trading parameters, ensures the strategy can be effectively implemented in real-world trading environments.

Incorporating the Hurst exponent ($\hat{H} < 0.5$) as a filter ensures that the spread remains anti-persistent and more likely to revert to its mean. This two-step validation (co-integration + Hurst filter) significantly strengthens the robustness of the Pairs Trading Strategy.