Volatility-Driven Multi-Asset Arbitrage Rulebook

Contents

1 Introduction

The **Volatility-Driven Multi-Asset Arbitrage** strategy exploits discrepancies between implied and realized volatility across multiple asset classes, as well as misalignments in correlations. This rulebook outlines the guiding principles for designing, implementing, and monitoring the strategy.

2 Strategy Objectives

- **Capture implied volatility premiums** when overvalued relative to realized volatility.
- **Arbitrage correlation discrepancies** between asset classes.
- **Multi-asset diversification** to reduce systemic risk.

3 Initial Configuration

3.1 Asset Classes

- **Equities**: Indices (e.g., S&P 500, NASDAQ) — 25/100 of the capital.
- **Bonds**: Futures on T-Notes, Bunds —25/100 of the capital.
- **FX**: EUR/USD, USD/JPY —25/100 of the capital.
- **Commodities**: WTI, Gold —25/100 of the capital.

3.2 Key Parameters

- **Lookback Window**: 20 days for realized volatility.
- **Rebalancing Frequency**: weekly.
- **Option Maturity**: 30 days (short-term options).
- **Initial Allocation**:
  - 60%: Volatility Arbitrage.
  - 40%: Correlation Arbitrage.

4 Quantitative Methodology

4.1 Volatility Calculations

4.1.1 Realized Volatility (σ_{real})

$$\sigma_{\text{real}} = \sqrt{\frac{252}{N} \sum_{t=1}^{N} (R_t - \bar{R})^2}$$

where N is the lookback window.

4.1.2 calculates implied volatility datas

Use yahoo finance to retrieve options datas and calculate implied volatilities using reversed Black and Scholes equation.

Collect option datas:

To calculate implied volatilities and collect option data, refer to the Python scripts hosted in the GitHub repository:

- Collect Option Data.
- Calculating Implied Volatilities with BSM.

4.1.3 Weighting Implied Volatilities

To calculate the implied volatility of a portfolio composed of multiple assets, a weighting method based on the covariance matrix is used. The general formula for the portfolio volatility is given by:

$$\sigma_{\text{portfolio}} = \sqrt{\mathbf{w}^\top \Sigma \mathbf{w}}$$

where:

- $\mathbf{w}$ is the vector of weights of the assets in the portfolio;
- Σ is the covariance matrix of the assets, derived from the correlations between the assets and their individual implied volatilities (σ_{impl}).

Example. Consider a portfolio composed of 50% Nasdaq ($\sigma_{\text{impl, Nasdaq}} = 20\%$) and 50% S&P 500 ($\sigma_{\text{impl, SP}} = 15\%$), with a correlation between the two indices of $\rho = 0.8$. The covariance matrix is calculated as follows:

$$\Sigma = \begin{bmatrix} \sigma^2_{\text{impl, Nasdaq}} & \rho \cdot \sigma_{\text{impl, Nasdaq}} \cdot \sigma_{\text{impl, SP}} \\ \rho \cdot \sigma_{\text{impl, Nasdaq}} \cdot \sigma_{\text{impl, SP}} & \sigma^2_{\text{impl, SP}} \end{bmatrix}.$$

The portfolio volatility is then:

$$\sigma_{\text{portfolio}} = \sqrt{(0.5)^2(0.2)^2 + (0.5)^2(0.15)^2 + 2(0.5)(0.5)(0.8)(0.2)(0.15)}.$$

Diversification and Volatility.

- If the portfolio assets are highly correlated (ρ close to 1), diversification will have a limited effect on reducing the overall portfolio volatility.
- Conversely, if the correlations between assets are low (ρ close to 0 or negative), the portfolio volatility will be lower than the weighted sum of individual volatilities, thereby reducing overall risk.

4.2 Arbitrage Opportunities

4.2.1 General Formula

The number of options to trade can be calculated based on the capital you want to allocate and the cost per option:

For Buying Options

$$\text{Number of Options to Buy} = \frac{\text{Capital Allocated}}{\text{Cost per Option}}$$

For Selling Options

$$\text{Number of Options to Sell} = \frac{\text{Capital Allocated}}{\text{Margin Requirement per Option}}$$

4.2.2 Key Factors to Consider

Size of the Arbitrage Opportunity

$$\sigma_{\text{real}} - \sigma_{\text{impl}}$$

Larger discrepancies between σ_{real} and σ_{impl} indicate a stronger arbitrage opportunity. Allocate more capital to larger opportunities but maintain diversification to manage risks.

4.2.3 Adjusting for Volatility Mean Reversion

To optimize the portfolio and maximize returns, the strategy aims to achieve a ratio of:

$$\text{Target Ratio} = \frac{\sigma_{\text{impl}}}{\sigma_{\text{real}}} = 1$$

This target indicates that the implied volatility (σ_{impl}) and realized volatility (σ_{real}) converge, removing any arbitrage opportunity. Adjustments are required to exploit this convergence dynamically.

1. Key Formula for Adjustments

To achieve this, the number of options to trade (n_i) can be determined by calculating the required exposure to implied volatility changes to push the ratio towards 1:

$$n_i = \frac{\text{Target Ratio} - \text{Current Ratio}}{\text{Expected Impact per Option}}$$

where:

- Target Ratio $= 1$,
- Current Ratio $= \frac{\sigma_{\text{impl}}}{\sigma_{\text{real}}}$,
- Expected Impact per Option: The expected change in the ratio per option traded, based on Vega sensitivity and the implied volatility of the straddle or strangle.

2. Adjusting for Overvalued Implied Volatility ($\sigma_{\text{impl}} > \sigma_{\text{real}}$)

If Current Ratio > 1:

- The implied volatility is overestimated.
- Adjustments:

- Sell options (e.g., straddles or strangles) to capture the premium from the overvalued implied volatility.
- The number of options (n_i) to sell is proportional to the gap between the current ratio and the target ratio:

$$n_i = \frac{\sigma_{\text{impl}} - \sigma_{\text{real}}}{\text{Expected Impact per Option}}$$

3. Adjusting for Undervalued Implied Volatility ($\sigma_{\text{impl}} < \sigma_{\text{real}}$)

If Current Ratio < 1:

- The implied volatility is underestimated.
- Adjustments:
  - Buy options (e.g., straddles or strangles) to benefit from the correction of implied volatility upwards.
  - The number of options (n_i) to buy is calculated similarly:

$$n_i = \frac{\sigma_{\text{real}} - \sigma_{\text{impl}}}{\text{Expected Impact per Option}}$$

4. Dynamic Rebalancing

As market conditions evolve, regularly monitor the ratio $\frac{\sigma_{\text{impl}}}{\sigma_{\text{real}}}$ and rebalance the portfolio dynamically:

1. **Monitor the Ratio**: Compute the current ratio at regular intervals.
2. **Adjust Positions**: Buy or sell straddles/strangles based on the gap from the target ratio.
3. **Convergence to Target**: Maintain positions until the ratio approaches 1, at which point you can unwind positions to lock in profits.

5. Example Calculation

- Current implied volatility: $\sigma_{\text{impl}} = 25\%$,
- Current realized volatility: $\sigma_{\text{real}} = 20\%$,
- Current ratio: $\frac{\sigma_{\text{impl}}}{\sigma_{\text{real}}} = 1.25$,
- Target ratio: 1,
- Expected impact per straddle: 0.01 per unit.

The number of straddles to sell:

$$n_i = \frac{1.25 - 1}{0.01} = 25$$

Sell 25 straddles to adjust the portfolio and profit as the ratio converges to 1.

6. Risk Management

While adjusting for the target ratio:

- Monitor changes in both implied and realized volatility.
- Avoid excessive leverage or overexposure to a single asset.
- Reassess the target if market conditions change significantly.

4.3 Historical Correlation (ρ_{xy}) Formula

The historical correlation between two assets X and Y is defined as:

$$\rho_{xy} = \frac{\mathrm{Cov}(X, Y)}{\sigma_X \sigma_Y}$$

where:

- $\mathrm{Cov}(X, Y)$ is the covariance between the returns of X and Y,
- σ_X and σ_Y are the standard deviations of the returns of X and Y, respectively.

4.4 Identifying Misalignments

We exploit the correlation to identify pairs of assets that deviate from their expected behavior:

4.4.1 Highly Correlated Pairs ($\rho_{xy} \approx 1$)

If two assets are typically highly correlated but show a temporary drop in correlation, it may indicate an arbitrage opportunity.
Action: Go long on both assets (mean reversion) as their correlation may revert to historical norms.

4.4.2 Decorrelation ($\rho_{xy} \to 0$)

If two assets that are usually decorrelated suddenly show high correlation, it may indicate temporary co-movement due to market inefficiencies.
Action: Go short on both assets, as their decorrelation is expected to normalize.

4.5 Strategy Execution

4.5.1 Long on Highly Correlated Pairs

When the correlation of a pair drops significantly from its historical average, act as follows:

1. Calculate the historical average correlation ($\bar{\rho}_{xy}$).
2. Identify pairs where:
$$\rho_{xy} < \bar{\rho}_{xy} - \text{Threshold}$$
where Threshold is a predefined deviation (e.g., 1 standard deviation).

3. **Positioning:** Go long on both X and Y (e.g., buy equities, ETFs, or futures contracts). Expect prices of X and Y to revert to their correlated behavior.

4.5.2 Short on Recently Decorrelated Pairs

When the correlation of a pair increases significantly from its historical average:

1. Identify pairs where:
$$\rho_{xy} > \bar{\rho}_{xy} + \text{Threshold}$$
2. **Positioning:** Short both X and Y (e.g., sell equities, ETFs, or futures contracts). Expect their prices to diverge as the correlation normalizes.

4.6 Risk Management

Correlation strategies can be risky due to unexpected market conditions. Mitigate risks as follows:

- **Stop-Loss Orders:** Define a maximum loss limit for each position and automate exits if the threshold is breached.
- **Diversification:** Avoid overexposure to a single pair by diversifying across multiple correlated pairs.
- **Position Sizing:** Allocate capital proportionally to the deviation from historical correlation ($\bar{\rho}_{xy}$):
$$\text{Position Size} \propto |\rho_{xy} - \bar{\rho}_{xy}|$$

4.7 Example: Trade Based on Correlation

Scenario:

- Historical Correlation ($\bar{\rho}_{xy}$): 0.8
- Current Correlation (ρ_{xy}): 0.5
- Threshold: 0.2

Action: Since:
$$\rho_{xy} < \bar{\rho}_{xy} - \text{Threshold}$$
Go long on both assets X and Y.

Position sizing: Allocate more capital if:

$$|\rho_{xy} - \bar{\rho}_{xy}|$$

is large, as the misalignment is more significant.

4.8 Mathematical Framework for Optimization

To optimize this strategy:

1. **Define an Objective Function:** Maximize the expected return based on correlation misalignments:

$$\max \sum_i \alpha_i (\rho_{xy,i} - \bar{\rho}_{xy,i})^2$$

where:

- α_i is the weight for each pair i,
- $(\rho_{xy,i} - \bar{\rho}_{xy,i})$ is the deviation.

2. **Solve Using Constraints:**

- **Budget:** Total capital allocated cannot exceed the available budget.
- **Risk:** Limit exposure to highly volatile pairs.

4.8.1 Calculation of the Threshold Using Historical Data

To compute the threshold based on historical data, we identify significant deviations from the historical average correlation. Let the historical correlation values be denoted as:

$$\rho_1, \rho_2, \ldots, \rho_N$$

where N represents the total number of historical observations. The threshold is defined as a multiple of the standard deviation of these values.

Step 1: Compute the Historical Average Correlation The historical average correlation, $\bar{\rho}$, is given by:

$$\bar{\rho} = \frac{1}{N} \sum_{i=1}^{N} \rho_i$$

Step 2: Compute the Standard Deviation of Correlation Values The standard deviation of the correlation values, σ_ρ, is computed as:

$$\sigma_\rho = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (\rho_i - \bar{\rho})^2}$$

Step 3: Define the Threshold The threshold is defined as:

$$\text{Threshold} = k \cdot \sigma_\rho$$

where k is a constant multiplier that determines the sensitivity of the threshold. Common values for k are:

- $k = 1$: Captures 68% of deviations.
- $k = 2$: Captures 95% of deviations.
- $k = 3$: Captures 99.7% of deviations.

Step 4: Apply the Threshold to Identify Opportunities A significant deviation occurs if the current correlation, ρ_{current}, satisfies:

$$|\rho_{\text{current}} - \bar{\rho}| > \text{Threshold}$$

Example:

- Historical correlation values: $\rho_1 = 0.75, \rho_2 = 0.8, \rho_3 = 0.85$.
- Compute $\bar{\rho}$:
$$\bar{\rho} = \frac{0.75 + 0.8 + 0.85}{3} = 0.8$$
- Compute σ_ρ:
$$\sigma_\rho = \sqrt{\frac{1}{3}\left[(0.75 - 0.8)^2 + (0.8 - 0.8)^2 + (0.85 - 0.8)^2\right]} = 0.05$$
- Define the threshold with $k = 2$:
$$\text{Threshold} = 2 \cdot 0.05 = 0.1$$
- Identify opportunities:
$$\text{If } |\rho_{\text{current}} - 0.8| > 0.1, \text{ then act.}$$

5 Risk Management

5.1 Calculation of Total Transaction Costs

To calculate the total transaction costs ($C_{\text{transaction}}$) associated with a portfolio of N strangles, we consider the cost of each individual strangle. The formula is given by:

$$C_{\text{transaction}} = \sum_{i=1}^{N} C_{\text{option},i} \cdot n_i$$

where:

- $C_{\text{option},i}$: The cost of the i-th strangle (call premium + put premium).
- n_i: The number of strangles purchased or sold for the i-th type.

Breakdown of Costs for Each Strangle: Each strangle consists of a call option and a put option. The cost for the i-th strangle is calculated as:

$$C_{\text{option},i} = \text{Premium}_{\text{call},i} + \text{Premium}_{\text{put},i} + C_{\text{fees},i}$$

where:

- $\text{Premium}_{\text{call},i}$: Premium for the call option in the i-th strangle.
- $\text{Premium}_{\text{put},i}$: Premium for the put option in the i-th strangle.
- $C_{\text{fees},i}$: Additional fees, including bid-ask spread, commissions, or exchange fees.

Final Total Transaction Costs: Thus, combining the above, the total transaction costs for all strangles in the portfolio is:

$$C_{\text{transaction}} = \sum_{i=1}^{N} \left((\text{Premium}_{\text{call},i} + \text{Premium}_{\text{put},i} + C_{\text{fees},i}) \cdot n_i \right)$$

Example Calculation:

- Number of strangles: $N = 3$.
- For the first strangle:

$$\text{Premium}_{\text{call},1} = 5, \quad \text{Premium}_{\text{put},1} = 4, \quad C_{\text{fees},1} = 1$$

$$C_{\text{option},1} = 5 + 4 + 1 = 10$$

- Number of strangles purchased: $n_1 = 20$.
- Repeat for n_2 and n_3.

  Total transaction costs:

$$C_{\text{transaction}} = \sum_{i=1}^{3} C_{\text{option},i} \cdot n_i$$

5.2 Maximize profits with max Gamma and min Cost

5.2.1 Gamma (Γ)

The gamma measures the sensitivity of the delta of an option to the movement in the price of the underlying asset. A high gamma indicates that the options will respond strongly to price fluctuations, which is desirable for a strategy based on significant movements.

5.2.2 Transaction Costs ($C_{\text{transaction}}$)

Transaction costs include the purchase and sale costs of the options (C_{option}) and other fees (bid/ask spread, commissions, etc.). Minimizing these costs improves net profitability.

5.2.3 Optimization Objective

The optimization goal is to maximize an objective function that combines:

- Maximization of the portfolio gamma (Γ_{total}).
- Minimization of transaction costs ($C_{\text{transaction}}$).

  The objective function is defined as:

$$\text{Maximize } \mathcal{L} = w_1 \cdot \Gamma_{\text{total}} - w_2 \cdot C_{\text{transaction}}$$

  where:

- w_1 and w_2 are weights to balance the relative importance of gamma and costs,
- $\Gamma_{\text{total}} = \sum_{i=1}^{N} \Gamma_i$, the sum of the gammas of the N strangles in the portfolio,
- $C_{\text{transaction}} = \sum_{i=1}^{N} C_{\text{option},i}$, the total cost of the options.

5.2.4 Constraints

The optimization must respect the following constraints:

5.2.5 Budget Constraint

$$\sum_{i=1}^{N} n_i \cdot P_i \leq \text{Available Capital}$$

where P_i is the price of the i-th strangle.

5.2.6 Implied Volatility

Prioritize options with favorable implied volatilities (e.g., $\sigma_{\text{impl}} < \sigma_{\text{real}}$). (TRY : let's take the 80/100 highest implied vol)

5.2.7 Excluding Options with Large Bid/Ask Spreads (or Liquidity)

To reduce transaction costs, it is essential to exclude options with excessively large bid/ask spreads. The spread reflects the difference between the price at which you can buy (ask price) and sell (bid price) an option. A large spread increases costs and reduces profitability, especially in volatility arbitrage strategies.

1. General Rule of Thumb For most liquid options, the bid/ask spread should ideally be less than 10% of the mid-price. The bid/ask spread percentage can be calculated as:

$$\text{Bid/Ask Spread Percentage} = \frac{\text{Ask Price} - \text{Bid Price}}{\text{Mid Price}} \times 100$$

where:

$$\text{Mid Price} = \frac{\text{Ask Price} + \text{Bid Price}}{2}$$

2. Typical Acceptable Ranges

- **Highly Liquid Options (e.g., S&P 500, NASDAQ options):**
  - Spreads are typically $1 - 5\%$ of the mid-price.
  - Avoid options where the spread exceeds 5% of the mid-price.
- **Moderately Liquid Options (less popular equities, ETFs):**
  - Spreads generally fall between $5 - 10\%$ of the mid-price.
  - Avoid options where the spread exceeds 10%.
- **Illiquid Options (exotic or deep OTM options):**
  - Spreads can exceed 20% of the mid-price.
  - Avoid options where the spread exceeds $15 - 20\%$ unless there is a compelling reason to trade.

3. Why Tight Spreads Are Critical

- **Impact on Profitability:** Large bid/ask spreads increase transaction costs, reducing profit margins.
- **Indicator of Liquidity:** A wide spread often signals low liquidity, making it harder to enter or exit positions efficiently.
- **Risk of Adverse Market Moves:** In volatile markets, executing trades with wide spreads increases the risk of unfavorable price execution.

4. Example of Acceptable and Unacceptable Spreads

- **Case 1: Tight Spread (Good Trade):**
  - Bid Price: \$2.00, Ask Price: \$2.10.
  - Mid Price: $\frac{2.00+2.10}{2} = 2.05$.
  - Spread Percentage: $\frac{2.10-2.00}{2.05} \times 100 = 4.88\%$.
  - This is acceptable for a liquid option.
- **Case 2: Wide Spread (Avoid Trade):**
  - Bid Price: \$2.00, Ask Price: \$2.50.
  - Mid Price: $\frac{2.00+2.50}{2} = 2.25$.
  - Spread Percentage: $\frac{2.50-2.00}{2.25} \times 100 = 22.22\%$.
  - This spread is too wide and should be avoided.

5. Practical Recommendations

- Define a maximum acceptable spread percentage ($e.g., 5\% - 10\%$) in your trading rules.
- Focus on highly liquid options to naturally avoid wide spreads.
- Avoid illiquid options with excessively large spreads unless the potential payoff justifies the trade.

6. Implementation in Automated Strategies In systematic trading, exclude options with large spreads by applying a filter:

$$\frac{\text{Ask Price} - \text{Bid Price}}{\text{Mid Price}} > \text{Threshold Percentage}$$

This ensures that only options with acceptable spreads are included in the trading strategy.

5.3 Mathematical Formulation

The optimization can be formulated as a linear or non-linear programming problem:

$$\max_{\{n_i\}} \mathcal{L} = w_1 \cdot \sum_{i=1}^{N} n_i \cdot \Gamma_i - w_2 \cdot \sum_{i=1}^{N} n_i \cdot C_{\text{option},i}$$

subject to:

- $\sum_{i=1}^{N} n_i \cdot P_i \leq \text{Available Capital}$,
- $\sum_{i=1}^{N} n_i \cdot \Delta_i = 0 \quad \text{(optional)}$,
- $n_i \in Z^+ \quad \text{(positive integer number of options)}$.

where n_i is the number of strangles of type i.

5.4 Optimization Algorithm

5.4.1 Initialization

- Collect data on available options $(\Gamma_i, C_{\text{option},i}, P_i, \Delta_i)$.
- Set the parameters w_1 and w_2.

5.4.2 Greedy Heuristic (Simplified Approach)

- Sort the strangles by their ratio $\frac{\Gamma_i}{C_{\text{option},i}}$.
- Select the strangles with the best ratio until the available capital is exhausted.

5.4.3 Numerical Optimization

Use an optimization method such as quadratic programming or a genetic algorithm to solve the global problem.

5.4.4 Numerical Example

Data

- $N = 3$ available strangles,
- $\Gamma = [0.1, 0.08, 0.12]$,
- $C_{\text{option}} = [5, 4, 6]$,
- $P = [100, 80, 120]$,
- Available capital $= 300$.

Greedy Approach

- Compute the ratio $\frac{\Gamma_i}{C_{\text{option},i}}$:

$$\frac{\Gamma}{C_{\text{option}}} = [0.02, 0.02, 0.02]$$

  All ratios are equal here, so the selection is based on total cost.

- Selection: Choose the first and second strangles since their costs $(100 + 80 = 180)$ are below the available capital.

5.5 Visualization

To visualize the optimization results, a Python script can be used to compare the gammas and costs of the selected strangles. The script generates a bar chart or scatter plot to illustrate the relationship between gamma, transaction costs, and the selected strangles.

5.5.1 Example Python Code

Below is a high-level description of the Python code:

1. Collect data on gamma (Γ), transaction costs (C_{option}), and selected strangles.
2. Use libraries such as `matplotlib` and `pandas` to generate visualizations.
3. Plot:
   - A bar chart showing the gamma values of selected strangles.
   - A scatter plot comparing the gamma-to-cost ratio $(\frac{\Gamma}{C_{\text{option}}})$ for all available strangles.

5.5.2 Python Code and Repository

The Python script implementing this visualization can be found on GitHub: `GitHub Repository: Plot Max Gamma and Min Cost FunctionPython Visualization for Strangle Optimization`.

This script can be customized to include additional data points or constraints, depending on your specific requirements.

5.5.3 Example Visualization

Once the script is executed, the output may look as follows:

- A bar chart displaying the gamma of the selected strangles alongside their transaction costs.
- A scatter plot highlighting the strangles with the highest gamma-to-cost ratio.

The visualizations provide an intuitive way to assess the trade-offs between gamma maximization and transaction cost minimization.

5.6 Delta Neutrality

To maintain a delta-neutral portfolio, the total portfolio delta ($\Delta_{\text{portfolio}}$) must satisfy:

$$\Delta_{\text{portfolio}} = 0$$

This means that the portfolio's overall sensitivity to changes in the price of the underlying asset is eliminated, ensuring the portfolio is hedged against small price movements. Below are the key steps to achieve delta neutrality:

1. Calculate the Portfolio Delta The portfolio delta is the sum of the deltas of all individual positions:

$$\Delta_{\text{portfolio}} = \sum_{i=1}^{N} n_i \cdot \Delta_i$$

where:

- N: Number of options in the portfolio.
- n_i: Number of units (contracts) of the i-th option.
- Δ_i: Delta of the i-th option.

2. Adjusting Positions to Neutralize Delta To neutralize the portfolio delta, adjust the number of units of a hedge instrument (such as the underlying asset or another option). The adjustment is given by:

$$n_{\text{hedge}} = -\frac{\Delta_{\text{portfolio}}}{\Delta_{\text{hedge}}}$$

where:

- n_{hedge}: Number of units of the hedge instrument to buy or sell.
- Δ_{hedge}: Delta of the hedge instrument (e.g., delta of the underlying asset is typically 1).

3. Using the Underlying Asset for Hedging If the hedge instrument is the underlying asset:

$$n_{\text{hedge}} = -\Delta_{\text{portfolio}}$$

This involves buying ($n_{\text{hedge}} > 0$) or selling ($n_{\text{hedge}} < 0$) units of the underlying asset to offset the portfolio delta.

4. Using Options for Hedging Alternatively, another option can be used for hedging. In this case, solve:

$$n_{\text{hedge}} = -\frac{\Delta_{\text{portfolio}}}{\Delta_{\text{option}}}$$

where Δ_{option} is the delta of the chosen hedging option.

5. Rebalancing for Delta Neutrality Market conditions change over time, causing deltas to fluctuate. Regularly rebalance the portfolio to maintain delta neutrality:

- Monitor changes in $\Delta_{\text{portfolio}}$.
- Adjust the hedge position as needed.

Example:

- Portfolio contains 10 call options with $\Delta = 0.5$.
- Portfolio delta: $\Delta_{\text{portfolio}} = 10 \cdot 0.5 = 5$.
- To neutralize delta using the underlying asset:

$$n_{\text{hedge}} = -\Delta_{\text{portfolio}} = -5$$

  This means selling 5 units of the underlying asset.

5.7 Vega Exposure

Vega measures the sensitivity of an option's price to changes in implied volatility. To control Vega and limit losses from sudden changes in implied volatility, the following steps can be implemented:

1. Understanding Portfolio Vega The portfolio Vega ($\text{Vega}_{\text{portfolio}}$) is the sum of the Vegas of all individual options in the portfolio:

$$\text{Vega}_{\text{portfolio}} = \sum_{i=1}^{N} n_i \cdot \text{Vega}_i$$

where:

- N: Number of options in the portfolio,
- n_i: Number of units (contracts) of the i-th option,
- Vega_i: Vega of the i-th option.

2. Diversify Option Types To reduce concentrated Vega exposure:

- Combine options with positive and negative Vegas.
- Use options across multiple strikes, maturities, and underlying assets to diversify volatility exposure.

3. Hedging Vega Vega exposure can be hedged using other options or volatility instruments:

- Buy options with negative Vega to offset positive Vega in the portfolio:

$$\text{Vega}_{\text{hedge}} = -\text{Vega}_{\text{portfolio}}$$

- Trade volatility-based instruments, such as variance swaps or VIX futures, to neutralize implied volatility risks.

4. Adjusting Portfolio Vega Dynamically Volatility changes over time, impacting the portfolio Vega. To control exposure:

- Monitor implied volatility levels regularly.
- Rebalance the portfolio by adjusting positions in options or volatility instruments.
- Reduce exposure during periods of extreme volatility or uncertainty.

5. Example:

- Portfolio contains 10 call options with $\text{Vega}_i = 0.25$.
- Portfolio Vega:
$$\text{Vega}_{\text{portfolio}} = 10 \cdot 0.25 = 2.5$$
- To neutralize Vega, sell options with a total Vega of 2.5, or trade volatility instruments to offset the risk.

6. Managing Vega Sensitivity For options near expiration, Vega is more sensitive to changes in implied volatility. Limit the exposure of short-dated options during volatile periods to avoid large losses. Additionally:

- Use longer-dated options with lower Vega sensitivity when hedging volatility.(TRY : the 80/100 longest-dated options)
- Monitor implied volatility skew across different strikes and maturities to identify attractive hedging opportunities.

5.8 Correlation Risk

Apply a stop-loss if correlation does not revert within a specified time.
(TRY : for each underlying class, stop loss 25/100 of the invested amount)

5.9 Exposure Limits

- Maximum exposure per asset class: 25%.
- Gamma limit for our options strategy exposition: $1.5 \times \Gamma_{\text{target}}$.

6 Performance Monitoring

6.1 Key Metrics

- **Sharpe Ratio**: Monitor risk-adjusted returns.
- **Value at Risk (VaR)**: Assess potential losses.
- Profit and loss (P&L) breakdown: Volatility vs. correlation components.

7 Exploration of differents methods of optimization

7.1 Optimization of Algorithms

- **Gradient Descent**: Adjust optimal strikes (ϵ).
- **Machine Learning**: Predictive models for volatility and correlation dislocations.

8 Conclusion

The **Volatility-Driven Multi-Asset Arbitrage** strategy captures temporary volatility and correlation dislocations across various assets. With dynamic risk management and multi-asset diversification, it offers a strong risk-adjusted return potential while maintaining low directional exposure.