Multi-Factor Adaptive Market-Making (MFAMM) Strategy Rulebook

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1 Strategy Overview

Stratégie hybride haute/moyenne fréquence combinant :

- Market-making prédictif avec contrôle optimal stochastique
- Arbitrage cross-asset par cointégration fractale
- Allocation méta-dynamique via apprentissage par renforcement

Notation Glossary

- $\delta_t^{\pm}$: Bid/Ask spreads (control variables)
- $\mathcal{A}$: Admissible control space (spreads $\in [\delta_{\min}, \delta_{\max}]$)
- $N_t^{\pm}$: Counting processes for filled orders (bid/ask)
- q_t: Inventory position (shares held)
- γ: Risk aversion coefficient (PnL vs. inventory risk tradeoff)
- σ_t: Volatility process (filtered realized volatility)
- $\lambda_t^{\pm}$: Order arrival intensities (probabilistic execution rates)
- Θ_t^{L3}: Order book state features matrix
- VPIN_t: Volume-synchronized informed trading probability

- DOI_t: Deep Order Book Imbalance (CNN output $\in [-1, 1]$)
- κ: VPIN smoothing factor (time-volume normalization)
- V: Number of volume buckets for VPIN calculation
- $\mathcal{B}_i/\mathcal{S}_i$: Buy/Sell volumes in i^{th} VPIN bucket
- p_k: Price at k^{th} order book level
- p_{mid}: Mid-price $(p_{\text{best bid}} + p_{\text{best ask}})/2$
- $v_k^{\text{bid/ask}}$: Volume at k^{th} bid/ask level
- $\mathcal{H}(q, t)$: Value function (Hamilton-Jacobi-Bellman PDE solution)
- α: Terminal inventory penalty coefficient
- $\beta_{0:3}$: Intensity model coefficients (XGBoost survival analysis)
- $\phi(\cdot)$: CNN embedding function for DOI calculation
- $a/p/n$: Anchor/positive/negative samples in triplet loss
- ϵ: Triplet loss margin (typically 0.2-0.5)
- $\mathcal{F}_t$: Market filtration (adapted σ-algebra)

Key Relationships

- Spread-Intensity Relationship: $\delta^{\pm} \propto 1/\lambda^{\pm}$ (nonlinear via XGBoost)
- Inventory Risk: $\gamma\sigma^2 q^2$ penalizes directional exposure
- VPIN Thresholding: VPIN > 0.7 triggers reduced quoting activity
- DOI Interpretation: DOI > 0.3 = buying pressure, DOI < -0.3 = selling pressure

2 Core Models

2.1 Predictive Market-Making

The model combines stochastic optimal control with machine learning-based order flow prediction to maximize expected profits while controlling inventory risk.

$$\sup_{\delta_t^\pm \in \mathcal{A}} \mathbb{E}\left[\int_0^T \left(\delta_t^+ dN_t^+ + \delta_t^- dN_t^-\right) - \gamma\sigma_t^2 q_t^2 dt \;\middle|\; \mathcal{F}_t\right] \tag{1}$$

System Dynamics

$$\begin{aligned} dq_t &= dN_t^- - dN_t^+ \quad \text{(Inventory evolution)} && (2) \\ \lambda_t^\pm &= \exp\left(\beta_0 + \beta_1\Theta_t^{\mathrm{L3}} + \beta_2\mathrm{VPIN}_t + \beta_3\mathrm{DOI}_t\right) && (3) \\ \mathbb{P}(dN_t^\pm = 1) &= \lambda_t^\pm dt + o(dt) \quad \text{(Counting process)} && (4) \end{aligned}$$

Feature Engineering

- **VPIN (Volume-Synchronized Probability of Informed Trading)**:

$$\mathrm{VPIN}_t = \frac{1}{\kappa V}\sum_{i=1}^{V} |\mathcal{B}_i - \mathcal{S}_i| \tag{5}$$

$$\begin{aligned} \text{where } \mathcal{B}_i &= \text{Buy volume in } i^{th} \text{ volume bucket} \\ \mathcal{S}_i &= \text{Sell volume in } i^{th} \text{ volume bucket} \\ \kappa &= \text{Smoothing parameter (typically 0.1 to 0.5)} \end{aligned}$$

- **DOI (Deep Order Book Imbalance)**:

$$\mathrm{DOI}_t = \mathrm{CNN}\left(\left\{\frac{p_k - p_{\mathrm{mid}}}{p_{\mathrm{mid}}}, \frac{v_k^{\mathrm{bid}} - v_k^{\mathrm{ask}}}{\max(v_k^{\mathrm{bid}}, v_k^{\mathrm{ask}})}\right\}_{k=1}^{K}\right) \tag{6}$$

  where K order book levels are encoded as a normalized 10×10 grid.

- **L3 Features** $\Theta_t^{\mathbf{L3}}$:

$$\Theta_t^{\mathrm{L3}} = \begin{bmatrix} \sum_{k=1}^{5} v_k^{\mathrm{bid}} & \sum_{k=1}^{5} v_k^{\mathrm{ask}} \\ \frac{\partial^2 p}{\partial v^2}\Big|_{\text{best ask}} & \text{Mean book slope} \\ \mathbb{I}_{\{\text{Fat finger}\}} & \text{Volume entropy} \end{bmatrix} \tag{7}$$

Numerical Optimization

Solution via adaptive Han-Wong algorithm:

$$\begin{aligned} \mathcal{H}(q,t) = \sup_{\delta^{\pm}} \Big\{ & \lambda^+(\delta^+)\delta^+ + \lambda^-(\delta^-)\delta^- - \gamma\sigma^2 q^2 \\ & + \frac{\partial \mathcal{H}}{\partial t} + (\lambda^- - \lambda^+)\frac{\partial \mathcal{H}}{\partial q} \Big\} \end{aligned} \tag{8}$$

Finite difference approximation with boundary conditions:

$$\mathcal{H}(q,T) = -\alpha q^2 \quad \text{(Terminal penalty)} \tag{9}$$

$$\frac{\partial \mathcal{H}}{\partial \delta^{\pm}}\bigg|_{\delta^{\pm}=\delta^{\pm}_{\mathrm{opt}}} = 0 \tag{10}$$

Parameter Calibration

- β estimation in (3) via censored maximum likelihood:

$$\mathcal{L}(\beta) = \prod_{i=1}^{n} \left(\lambda(t_i)^{\Delta N_i} \exp\left(-\int_{t_{i-1}}^{t_i} \lambda(s)ds \right) \right) \tag{11}$$

- CNN training for DOI using triplet loss:

$$\mathcal{L}_{\mathrm{triplet}} = \max\left(0, \|\phi(a) - \phi(p)\|_2 - \|\phi(a) - \phi(n)\|_2 + \epsilon\right) \tag{12}$$

  where a=anchor, p=positive, n=negative examples.

Key Parameters

$$\begin{aligned} \gamma &= 0.1 \text{ (risk aversion)} \\ \sigma_t &= \text{Rolling volatility (5min, 99\% corse filter)} \\ \kappa &= 0.25 \text{ (VPIN smoothing)} \\ \alpha &= 10^{-4} \text{ (Terminal penalty)} \end{aligned}$$

2.2 Cross-Asset Arbitrage

2.2.1 Cointegration Fractale

Décomposition en ondelettes des spreads :

$$W_\psi[s](a,b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} s(t)\psi\left(\frac{t-b}{a}\right) dt \tag{13}$$

Relation de cointégration multi-échelle :

$$\exists \beta_{a_1,\ldots,a_k} : \sum_{k=1}^{K} \|W_\psi[\sum_{i=1}^{n} \beta_i^{(k)} X_i](a_k, b)\|_2^2 < \epsilon \tag{14}$$

2.2.2 Exécution Optimale

Problème d'optimal transport :

$$\min_{\pi \in \Pi(\mu,\nu)} \sum_{i,j} C_{ij}\pi_{ij} + \lambda \mathrm{KL}(\pi || \mu \otimes \nu) \tag{15}$$

Où C_{ij} encode l'impact croisé entre actifs.

2.3 Meta-Allocation

Réseau de neurones hyperboliques :

$$h_i^{(l+1)} = \exp_{\mathbf{0}}^{\mathbb{D}^d}\left(\sum_{j \in \mathcal{N}(i)} \alpha_{ij} \log_{\mathbf{0}}^{\mathbb{D}^d}(h_j^{(l)})\right) \tag{16}$$

Fonction de reward RL :

$$r_t = \underbrace{\sum_{k=1}^{K} \pi_t^k}_{\text{PnL}} - \lambda\sqrt{\mathrm{tr}(\Omega\Sigma_t\Omega^\top)} + \mu\mathcal{D}_{\mathrm{JS}}(p_t || p_{t-1}) \tag{17}$$

3 Risk Management

3.1 Dynamic CVaR

Estimation par normalizing flows :

$$\mathrm{CVaR}_\alpha = \mathbb{E}[L|L > F^{-1}(\alpha)] \approx \frac{1}{N(1-\alpha)} \sum_{i=1}^{N} l_i \prod_{k=1}^{K} \phi_k^{-1}(z_{i,k}) \tag{18}$$

Contrainte de drawdown :

$$\mathbb{P}\left(\exists t \in [0,T] : \mathrm{PnL}_t < -\eta \mathrm{VaR}_t\right) \leq \epsilon \tag{19}$$

4 Execution Policy

4.1 Market Impact Model

Noyau non-linéaire :

$$\Delta S_t = \int_{-\infty}^{t} f(t-s) g(\theta_s) ds + \mathrm{HFT}_{\text{adversaire}}(X_t) \tag{20}$$

Solution via réseaux de neurones à retard :

$$\hat{g}(\theta) = \mathrm{LiNet}(\{\theta_{t-k\delta}\}_{k=0}^{K}) \tag{21}$$

4.2 Quantum-Inspired Routing

Formulation QAOA :

$$H_C = \sum_{(i,j)\in E} w_{ij} Z_i Z_j \tag{22}$$

$$H_B = \sum_i X_i \tag{23}$$

$$\min_{\beta,\gamma} \langle \psi(\beta,\gamma) | H_C | \psi(\beta,\gamma) \rangle \tag{24}$$

Appendix

- Infrastructure requirements (FPGA, colocation)
- Calibration procedure (CMA-ES algorithm)
- Adversarial testing framework