

Machine Learning

HW 1

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Ex. 1.

x_1	x_2	x_3	y
5	8	6	1
6	4	3	1
8	5	9	0
7	9	1	0

$$\mathcal{L} = 0.1$$

$$w_1 = 0.08$$

$$w_2 = 0.06$$

$$w_3 = 0.03$$

$$\begin{cases} 1; z \geq 0 \\ 0; \text{otherwise} \end{cases}$$

$$b = 0$$

Epoch 1.

$$1. \ z = w_1 x_1 + w_2 x_2 + w_3 x_3 + b$$

$$z = 0.08 \cdot 5 + 0.06 \cdot 8 + 0.03 \cdot 6 = 1.06$$

$$G(z) = G(1.06) = 1 \quad (\text{correct, no update on weights})$$

($y = \hat{y}) \Rightarrow \Delta \text{ values will be } 0$

$$2. \ z = 0.08 \cdot 6 + 0.06 \cdot 4 + 0.03 \cdot 3 + 0 = 0.81$$

$$G(0.81) = 1 \quad (y = \hat{y} \Rightarrow \text{no update on weights})$$

$$3. \ z = 0.08 \cdot 8 + 0.06 \cdot 5 + 0.03 \cdot 9 + 0 = 1.21$$

$$G(1.21) = 1 \quad y \neq \hat{y}, \text{ so we update}$$

$$\Delta w_1 = \mathcal{L} (y^i - \hat{y}^i) x_j^i = 0.1 (0 - 1) \cdot 8 = -0.8$$

$$\Delta w_2 = 0.1 (0 - 1) \cdot 5 = -0.5$$

$$\Delta w_3 = 0.1 (0 - 1) \cdot 9 = -0.9$$

$$\Delta b = 0.1 (0 - 1) = -0.1$$

$$w_1 = 0.08 - 0.8 = -0.72$$

$$w_2 = 0.06 - 0.5 = -0.44$$

$$w_3 = 0.03 - 0.9 = -0.87$$

$$b = 0 - 0.1 = -0.1$$

$$\text{errors} = 1$$

$$4. z = -0.72 \cdot 7 - 0.44 \cdot 9 - 0.87 \cdot 1 - 0.1 = -9.97$$

$$G(-9.97) = 0 \quad (y = \hat{y} \Rightarrow \text{no update needed})$$

$$\text{errors} = [1]$$

Epoch 2.

$$1. z = -0.72 \cdot 5 - 0.44 \cdot 8 - 0.87 \cdot 6 - 0.1 = -12.44$$

$$G(-12.44) = 0 \quad (y \neq \hat{y} \Rightarrow \text{we update})$$

$$\Delta w_1 = 0.1 (1 - 0) \cdot 5 = 0.5$$

$$\Delta w_2 = 0.1 (1 - 0) \cdot 8 = 0.8$$

$$\Delta w_3 = 0.1 (1 - 0) \cdot 6 = 0.6$$

$$\Delta b = 0.1 (1 - 0) = 0.1$$

$$w_1 = -0.72 + 0.5 = -0.22$$

$$w_2 = -0.44 + 0.8 = 0.36$$

$$w_3 = -0.87 + 0.6 = -0.27$$

$$b = -0.1 + 0.1 = 0$$

$$\text{errors} += 1$$

$$2. z = -0.22 \cdot 6 + 0.36 \cdot 4 - 0.27 \cdot 3 + 0 = -0.69$$

$$G(-0.69) = 0 \quad (y \neq \hat{y} \Rightarrow \text{we update})$$

$$\Delta w_1 = 0.1 (1 - 0) \cdot 6 = 0.6$$

$$\Delta w_2 = 0.1 (1 - 0) \cdot 4 = 0.4$$

$$\Delta w_3 = 0.1 (1 - 0) \cdot 3 = 0.3$$

$$\Delta b = 0.1 (1 - 0) = 0.1$$

$$w_1 = -0.22 + 0.6 = 0.38$$

$$w_2 = 0.36 + 0.4 = 0.76$$

$$w_3 = -0.27 + 0.3 = 0.03$$

$$b = 0 + 0.1 = 0.1$$

$$\text{errors} += 1$$

$$3. z = 0.38 \cdot 8 + 0.76 \cdot 5 + 0.03 \cdot 9 + 0.1 = 7.21$$

$$G(z) = G(7.21) = 1 \quad (y \neq \hat{y} \Rightarrow \text{we update})$$

$$\Delta w_1 = 0.1 (0 - 1) \cdot 8 = -0.8$$

$$\Delta w_2 = 0.1 (0 - 1) \cdot 5 = -0.5$$

$$\Delta w_3 = 0.1 (0 - 1) \cdot 9 = -0.9$$

$$\Delta b = 0.1 (0 - 1) = -0.1$$

$$w_1 = 0.38 - 0.8 = -0.42$$

$$w_2 = 0.76 - 0.5 = 0.26$$

$$w_3 = 0.03 - 0.9 = -0.87$$

$$b = 0.1 - 0.1 = 0$$

$$\text{errors} += 1$$

$$4. z = -0.42 \cdot 7 + 0.26 \cdot 9 - 0.87 \cdot 1 + 0 = -1.47$$

$$G(-1.47) = 0 \quad (y = \hat{y} \Rightarrow \text{no update})$$

$$\text{errors} = [1, 3]$$

Epoch 3

$$1. z = -0.42 \cdot 5 + 0.26 \cdot 8 - 0.87 \cdot 6 + 0 = -5.24$$

$$G(-5.24) = 0 \quad (y \neq \hat{y} \Rightarrow \text{we update})$$

$$\Delta w_1 = 0.1 (1 - 0) \cdot 5 = 0.5$$

$$\Delta w_2 = 0.1 (1 - 0) \cdot 8 = 0.8$$

$$\Delta w_3 = 0.1 (1 - 0) \cdot 6 = 0.6$$

$$\Delta b = 0.1 (1 - 0) = 0.1$$

$$w_1 = -0.42 + 0.5 = 0.08$$

$$w_2 = 0.26 + 0.8 = 1.06$$

$$w_3 = -0.87 + 0.6 = -0.27$$

$$b = 0 + 0.1 = 0.1$$

$$\text{errors} += 1$$

2. $z = 0.08 \cdot 6 + 1.06 \cdot 4 - 0.27 \cdot 3 + 0.1 = 4.01$
 $G(4.01) = 1$ ($y = \hat{y} \Rightarrow$ no update)

3. $z = 0.08 \cdot 8 + 1.06 \cdot 5 - 0.27 \cdot 9 + 0.1 = 3.51$
 $G(z) = G(3.51) = 1$ ($y \neq \hat{y} \Rightarrow$ we update)

$$\Delta w_1 = 0.1(0-1) \cdot 8 = -0.8$$

$$\Delta w_2 = 0.1(0-1) \cdot 5 = -0.5$$

$$\Delta w_3 = 0.1(0-1) \cdot 9 = -0.9$$

$$\Delta b = 0.1(0-1) = -0.1$$

$$w_1 = 0.08 - 0.8 = -0.72$$

$$w_2 = 1.06 - 0.5 = 0.56$$

$$w_3 = -0.27 - 0.9 = -1.17$$

$$b = +0.1 - 0.1 = 0$$

errors += 1

4. $z = -0.72 \cdot 7 + 0.56 \cdot 9 - 1.17 \cdot 1 + 0 = -1.17$
 $G(-1.17) = 0$ ($y = \hat{y} \Rightarrow$ no update)

errors = [1; 3; 2]

Epoch 4)

1. $z = -0.72 \cdot 5 + 0.56 \cdot 8 - 1.17 \cdot 6 + 0 = -6.14$
 $G(-6.14) = 0$ ($y \neq \hat{y} \Rightarrow$ update weights)

$$\Delta w_1 = 0.1(1-0) \cdot 5 = 0.5$$

$$\Delta w_2 = 0.1(1-0) \cdot 8 = 0.8$$

$$\Delta w_3 = 0.1(1-0) \cdot 6 = 0.6$$

$$\Delta b = 0.1(1-0) = 0.1$$

$$w_1 = -0.72 + 0.5 = -0.22$$

$$w_2 = 0.56 + 0.8 = 1.36$$

$$w_3 = -1.17 + 0.6 = -0.57$$

$$b = 0 + 0.1 = 0.1$$

errors += 1

$$2. z = -0.22 \cdot 6 + 1.36 \cdot 4 - 0.57 \cdot 3 + 0.1 = 2.51$$

$$G(2.51) = 1 \quad (y = \hat{y} \Rightarrow \text{no update})$$

$$3. z = -0.22 \cdot 8 + 1.36 \cdot 5 - 0.57 \cdot 9 + 0.1 = 0.01$$



So closeooooo

AAAAA...

$$G(0.01) = 1 \quad (y \neq \hat{y} = \text{update weights!!!})$$

$$\Delta w_1 = 0.1(0-1) \cdot 8 = -0.8$$

$$\Delta w_2 = 0.1(0-1) \cdot 5 = -0.5$$

$$\Delta w_3 = 0.1(0-1) \cdot 9 = -0.9$$

$$\Delta b = 0.1(0-1) = -0.1$$

$$w_1 = -0.22 - 0.8 = -1.02$$

$$w_2 = 1.36 - 0.5 = 0.86$$

$$w_3 = -0.57 - 0.9 = -1.47$$

$$b = 0.1 - 0.1 = 0$$

errors += 1

$$4. z = -1.02 \cdot 7 + 0.86 \cdot 9 - 1.47 \cdot 1 + 0 = -0.87$$

$$G(-0.87) = 0 \quad (y \neq \hat{y} \Rightarrow \text{no update})$$

$$\text{errors} = [1; 3; 2; 2]$$

Epoch 5

$$1. z = -1.02 \cdot 5 + 0.86 \cdot 8 - 1.47 \cdot 6 + 0 = -7.04$$

$$G(-7.04) = 0 \quad (y \neq \hat{y} \Rightarrow \text{update weights})$$

$$\Delta w_1 = 0.1(1-0) \cdot 5 = 0.5$$

$$w_1 = -1.02 + 0.5 = -0.52$$

$$\Delta w_2 = 0.1(1-0) \cdot 8 = 0.8$$

$$w_2 = 0.86 + 0.8 = 1.66$$

$$\Delta w_3 = 0.1(1-0) \cdot 6 = 0.6$$

$$w_3 = -1.47 + 0.6 = -0.87$$

$$\Delta b = 0.1(1-0) = 0.1$$

$$b = 0 + 0.1 = 0.1$$

errors += 1

$$2. z = -0.52 \cdot 6 + 1.66 \cdot 4 - 0.87 \cdot 3 + 0.1 = 1.01$$

$$G(1.01) = 1 \quad (y = \hat{y} \Rightarrow \text{no update})$$

$$3. z = -0.52 \cdot 8 + 1.66 \cdot 5 - 0.87 \cdot 9 + 0.1 = -3.59$$

$$G(-3.59) = 0 \quad (y = \hat{y} \Rightarrow \text{no update})$$

$$4. z = -0.52 \cdot 7 + 1.66 \cdot 9 - 0.87 \cdot 1 + 0.1 = 10.53$$

$$G(10.53) = 1 \quad (y \neq \hat{y} \Rightarrow \text{we need update})$$

$$\Delta w_1 = 0.1(0-1) \cdot 7 = -0.7$$

$$w_1 = -0.52 - 0.7 = -1.22$$

$$\Delta w_2 = 0.1(0-1) \cdot 9 = -0.9$$

$$w_2 = 1.66 - 0.9 = 0.76$$

$$\Delta w_3 = 0.1(0-1) \cdot 1 = -0.1$$

$$w_3 = -0.87 - 0.1 = -0.97$$

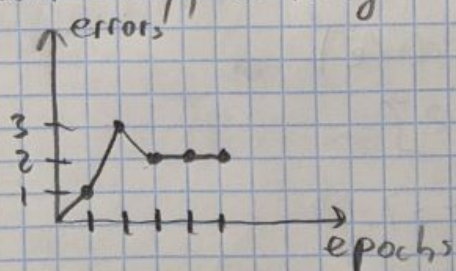
$$\Delta b = 0.1(0-1) = -0.1$$

$$b = 0.1 - 0.1 = 0$$

errors += 1

$$\text{errors} = [1; 3; 2; 2; 2]$$

The algorithm did not converge after 5 epochs. Currently, looking at our errors, it can seem like



the algorithm is in cyclic or plateau phase. We can suspect that learning rate, for this scenario might be too large & cause such

behaviour (too large weight updates)