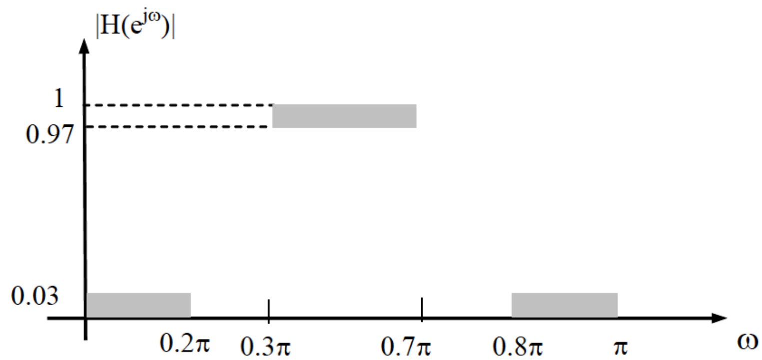
FIR bandpass filter design using window functions

1. Using Hamming, Blackman and Kaiser window functions, design a bandpass filter that has the following specifications,



a) Plot the resulting amplitude responses corresponding to the three designs.
b) Which windowing function leads to the filter with lowest order?

Answer :

a)

Important factors to design a windowing filter :

Window type	Window function (n)	Approximate width of main lobe
Hanning	$0.5 + 0.5\cos\left(\frac{\pi n}{m}\right)$	$\frac{8\pi}{m}$
Hamming	$0.54 + 0.46\cos\left(\frac{\pi n}{m}\right)$	$\frac{8\pi}{m}$
Blackman	$0.42 + 0.5\cos\left(\frac{\pi n}{m}\right) + 0.08\cos\left(\frac{2n\pi}{m}\right)$	$\frac{12\pi}{m}$

The window sequence should be an integer. In this problem, approximated 21 by the corresponding formula. The Passband order should be odd, here is 5.

$$\frac{w_{p_1} + w_{s_1}}{2} = \frac{25\pi}{100} \approx$$

In the Kaiser window, β beside window length should be chosen. So, we need two parameters, the cut-off frequency and the transition bandwidth (or roll-off). Also, the ripple error δ is the same for the passband and stopband by selecting the minimum term for both.

$$A = -20\log_{10}(\delta) \approx 36.4781$$

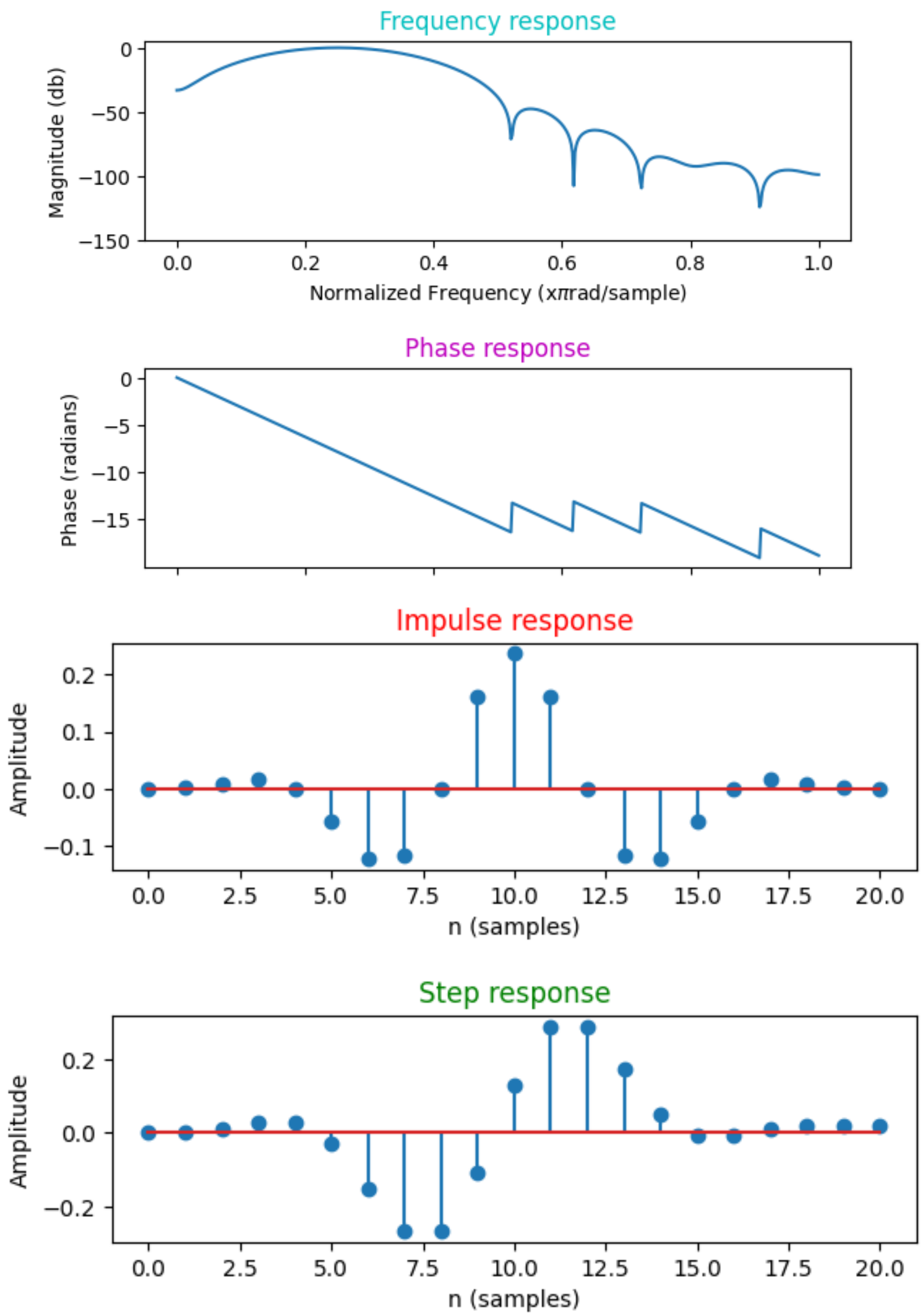
Large A results in a small ripple for the passband and β in the Kaiser window is determined based on below experimental formula:

$$\beta = \begin{cases} 0.1102\ (A - 8.7), & A > 50 \\ 0.5842\ (A-21)^{0.4} + 0.07886\ (A-21), & 21 \le A \le 50 \\ 0, & A \le 21 \end{cases}$$

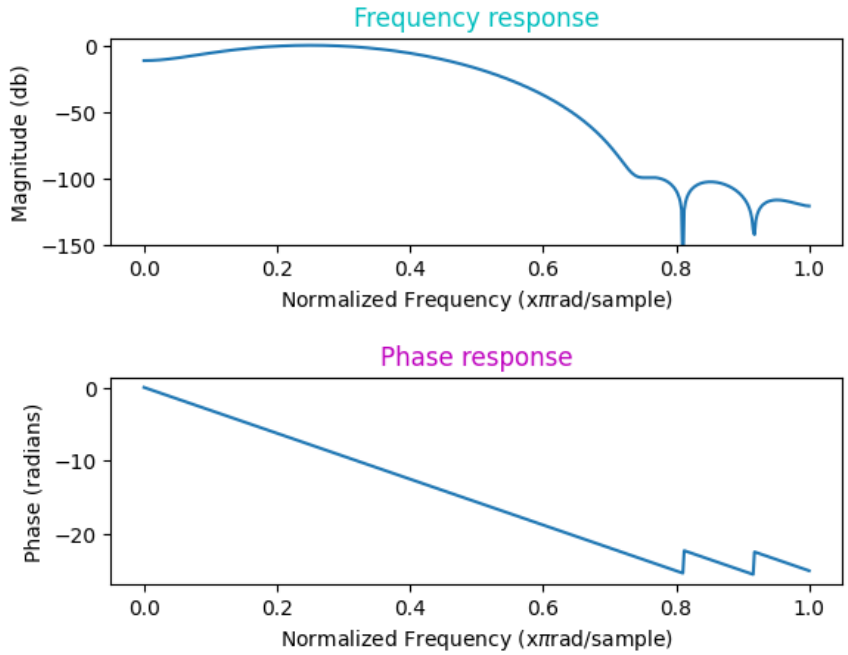
Therefore, our desired β is computed :

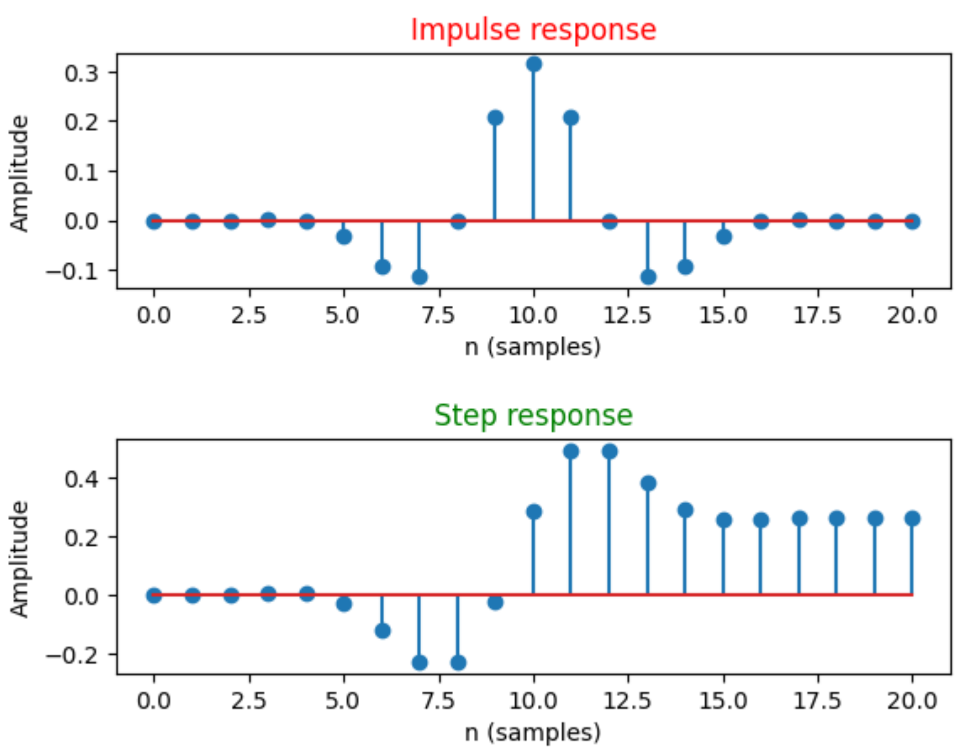
$$0.5842(36.47 - 21) + 0.07886(36.47 - 21) \approx 2.96$$

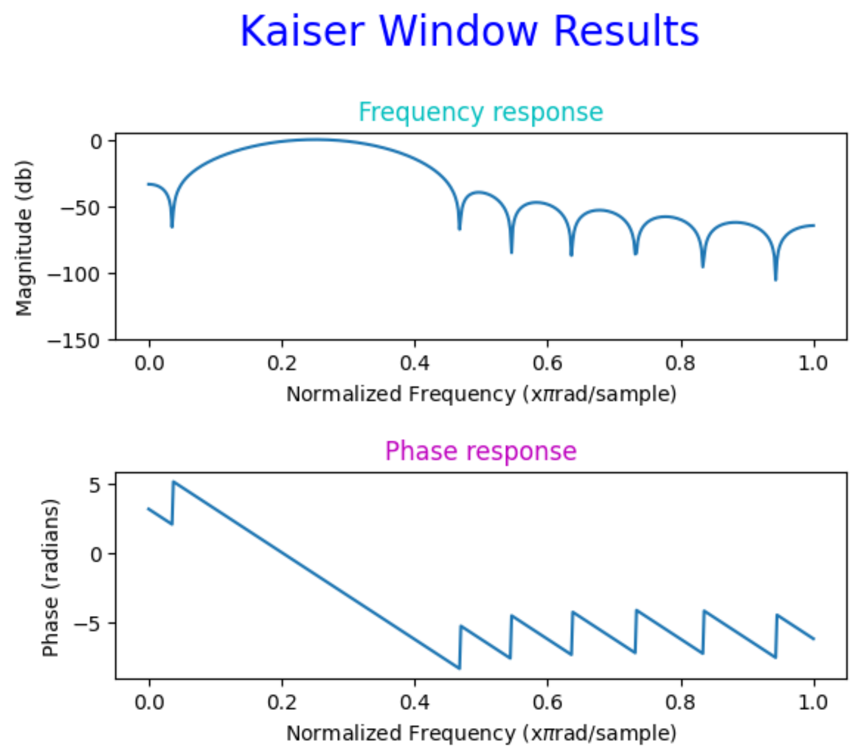
Hamming Window Results

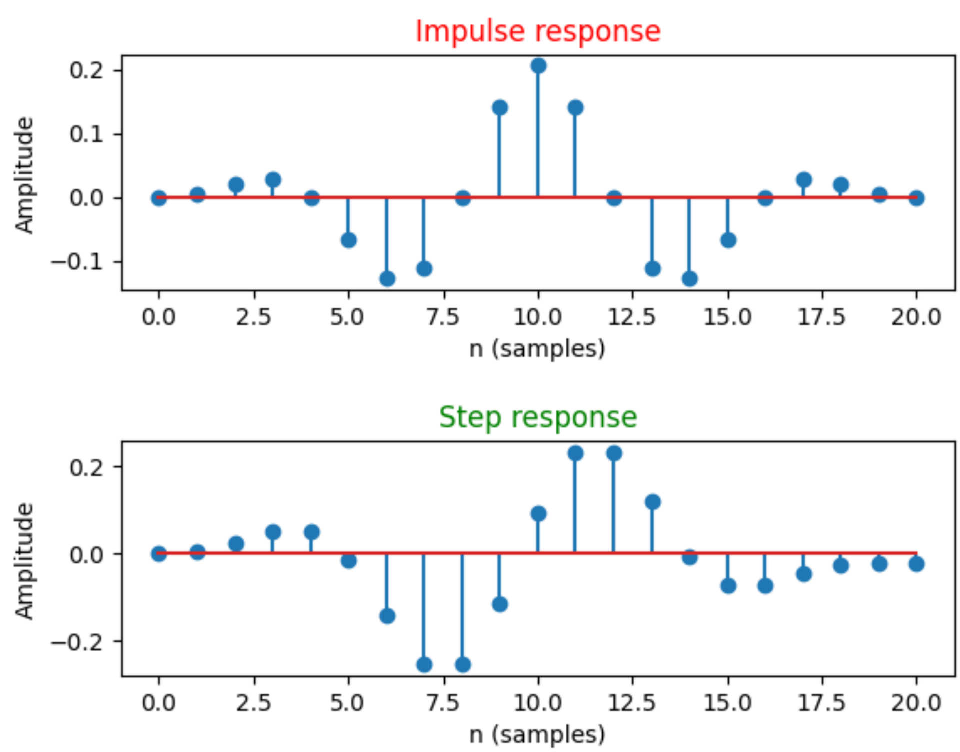


Blackman Window Results

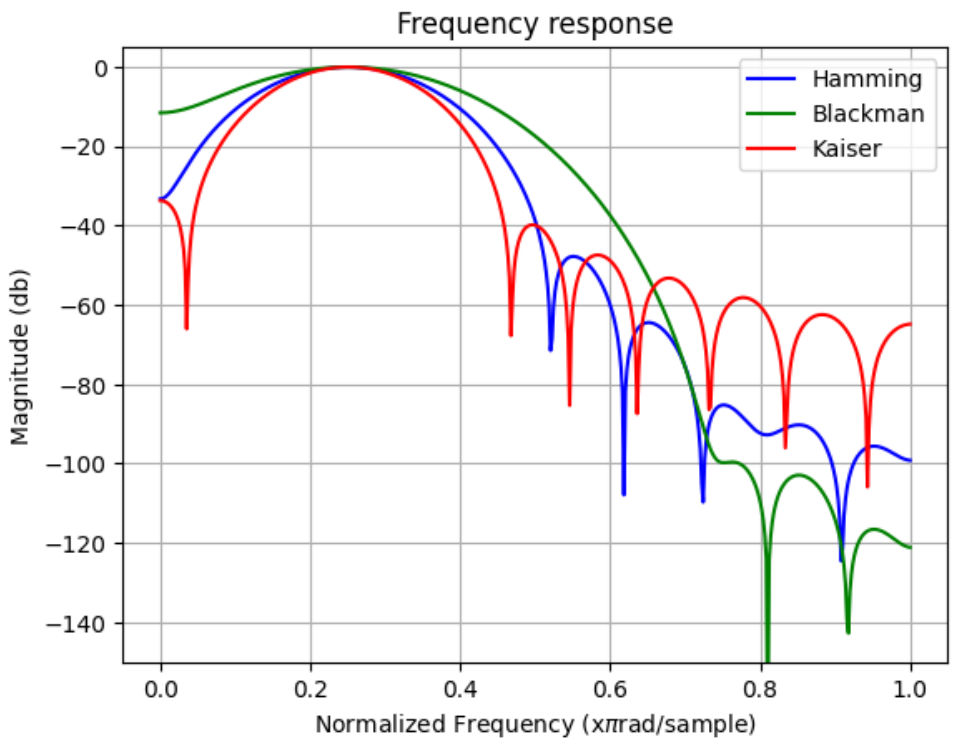





Kaiser Window Results
Frequency response
Magnitude (db)
0
−50
−100
−150
0.0
0.2
0.4
0.6
0.8
1.0
Normalized Frequency (xπrad/sample)
Phase response
Phase (radians)
5
0
−5
0.0
0.2
0.4
0.6
0.8
1.0
Normalized Frequency (xπrad/sample)


Impulse response
Amplitude
0.2
0.1
0.0
−0.1
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
n (samples)
Step response
Amplitude
0.2
0.0
−0.2
0.0
2.5
5.0
7.5
10.0
12.5
15.0
17.5
20.0
n (samples)

b)



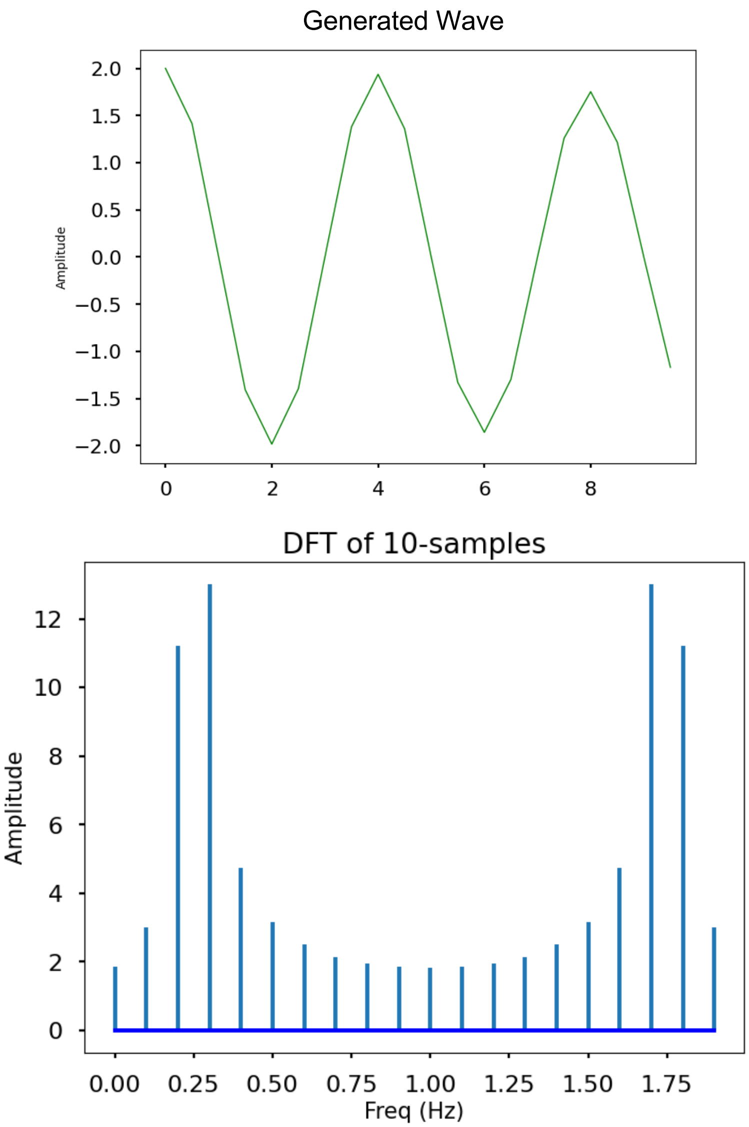
The Blackman window leads a filter with the lower order. Because of the smoother transition band (or roll of rate) compared to Hamming and Kaiser, as shown in the above plot. We know sharper transition band between the passband and the stopband generates a higher-order filter. The order of a filter indicates the minimum number of reactive components that the filter will be required. Generally, higher-order filters are used for narrow transition bands.

Identification of sinusoidal spectrums using DFT

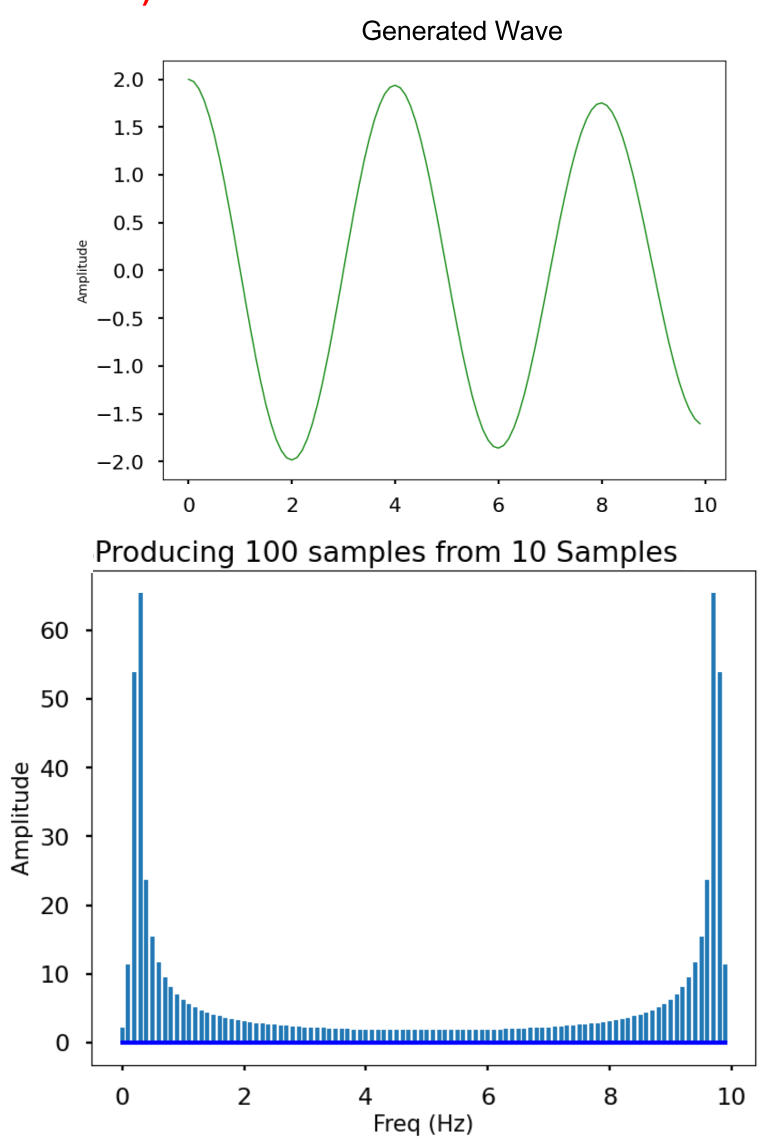
2. Consider the sequence $x(n)=\cos(0.48\pi n)+\cos(0.52\pi n)$. We want to determine its spectrum (amplitude of DFT values) based on the finite number of samples.
   a) Taking the first 10 samples of $x(n)$, $0\leq n\leq 9$, calculate and plot the 10-point DFT of $x(n)$.
   b) Using the 10 samples in part (a), calculate and plot the 100-point DFT.
   c) Taking the first 100 samples of $x(n)$, $0\leq n\leq 99$, plot the 100-point DFT of $x(n)$.
   d) Plot the 200-point DFT of $x(n)$ by using 200 samples.
   e) Discuss the results.

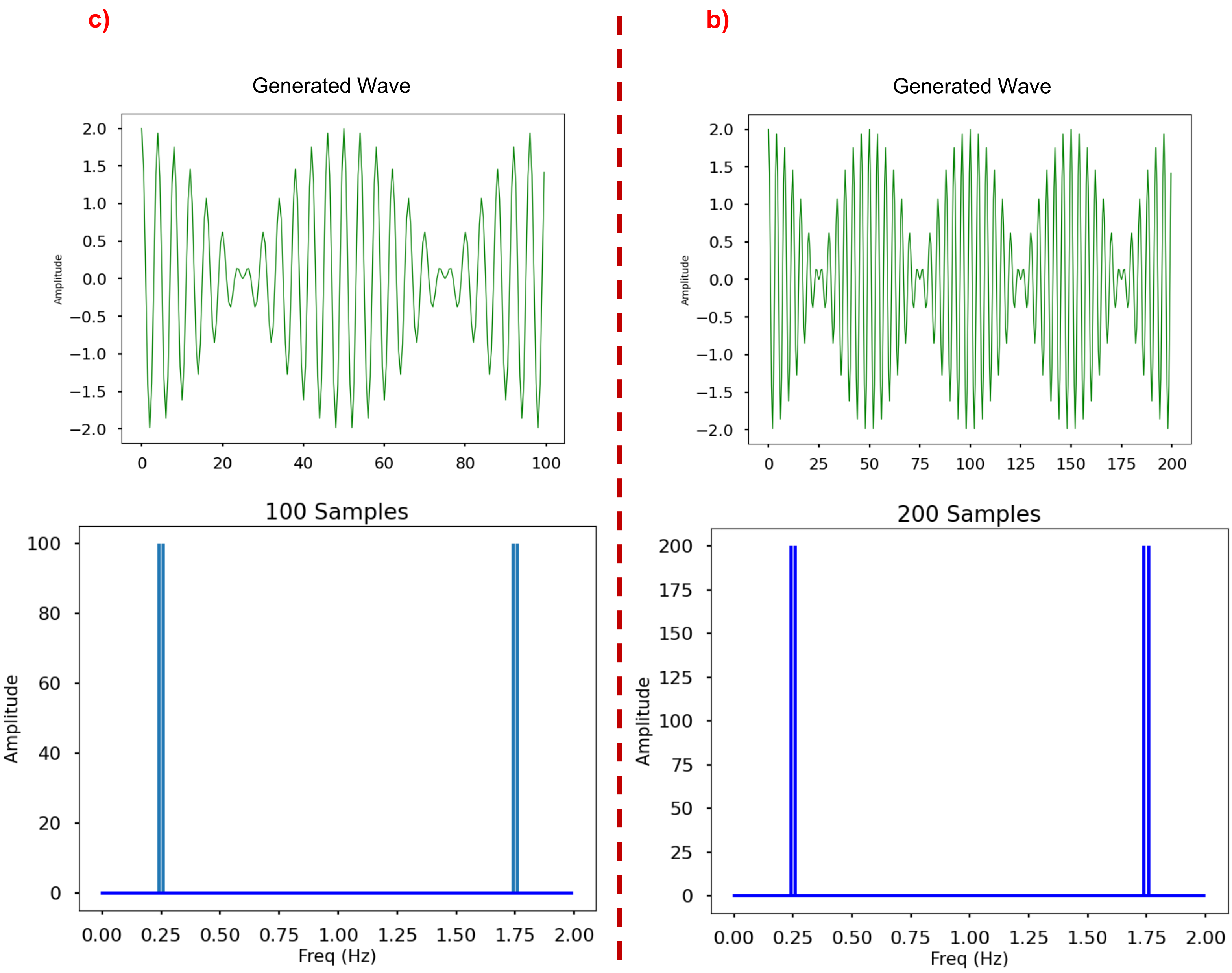
Answer)

a)



b)




c)
Generated Wave
Amplitude
100 Samples
Amplitude
Freq (Hz)
b)
Generated Wave
Amplitude
200 Samples
Amplitude
Freq (Hz)

e)

The input signal is: $x(n) = \cos\left(2\pi \times \left(\frac{24}{10}\right) n\right) + \cos(2\pi \left(\frac{26}{10}\right) n)$

We know it equals: $e^{j(2\pi\left(\frac{24}{10}\right)n)} + e^{-j(2\pi\left(\frac{24}{10}\right)n)} + e^{j(2\pi\left(\frac{26}{10}\right)n)} + e^{-j(2\pi\left(\frac{26}{10}\right)n)}$

Also, $x[n] = Ae^{j2\pi\frac{L}{N}n} = \sum_{k=0}^{N-1} \frac{1}{N} \underbrace{X[k]\, e^{2\pi\frac{k}{N}n}}_{\text{all } X \text{ equals to zero except k=L}} \Rightarrow A = \frac{1}{N} X[L]$

Here, sinusoidal frequencies of 0.24 and 0.26 are no longer an integer multiple of $\frac{L}{NT_s}$ in a $\left(L = 1, N = 10, T_s = {}^{1}\!/_{2}\right) \Longrightarrow \textbf{part a} \neq integer\ multiple\ of\ \frac{L}{NT_s}$. So, DFT is trying to express this sinusoid using the complex sinusoid, but the available frequencies don't correspond to the frequency of the sinusoid itself. Therefore, instead of having two non-zero coefficients, we can see many non-zero coefficients (spectral leakage). This matter is the same for the **part b** result, but DFT components are compressed because of the higher sampling rate of 10 samples per second to produce 100 samples from those 10 samples. DFT cannot create integer multiplication for those corresponding frequencies.

In **part c** $\frac{L}{NT_s} = \frac{1}{200} = 0.005$, DFT can generate integer multiplication coefficients for corresponding frequencies. In **part d** $\frac{L}{NT_s} = \frac{1}{400} = 0.0025$,same as part c, DFT can create multiply integer coefficients for frequencies 0.24 and 0.26 Hz. N-point DFT is referred to N-by-N square DFT matrix for input multiplication. So, for higher N (like part d ≈ 200 samples), more operation is needed and much more space, also, time consumption should be specified for an expansive operation compared to part c (part c ≈ 100 samples).