Impulse and Frequency Response

1. Assume a causal LTI system with transfer function

$$H(z) = \frac{1 - z^{-1}}{1 - 0.75z^{-1} + 0.12z^{-2}}$$

a) Plot the impulse response
b) Plot the amplitude and phase responses
c) What type of filter does it represent?
d) Plot the poles and zeros of the system

Answer :

a)

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Piecewise((-(3/4)**n/3 + 4*UnitImpulse(n)/3 + 3*UnitImpulse(n - 2)/25, n >= 0))

impulse response = (-3/4) power of (n/3)) + (4/3 * u[n]) + (3/25 *u[n-2])
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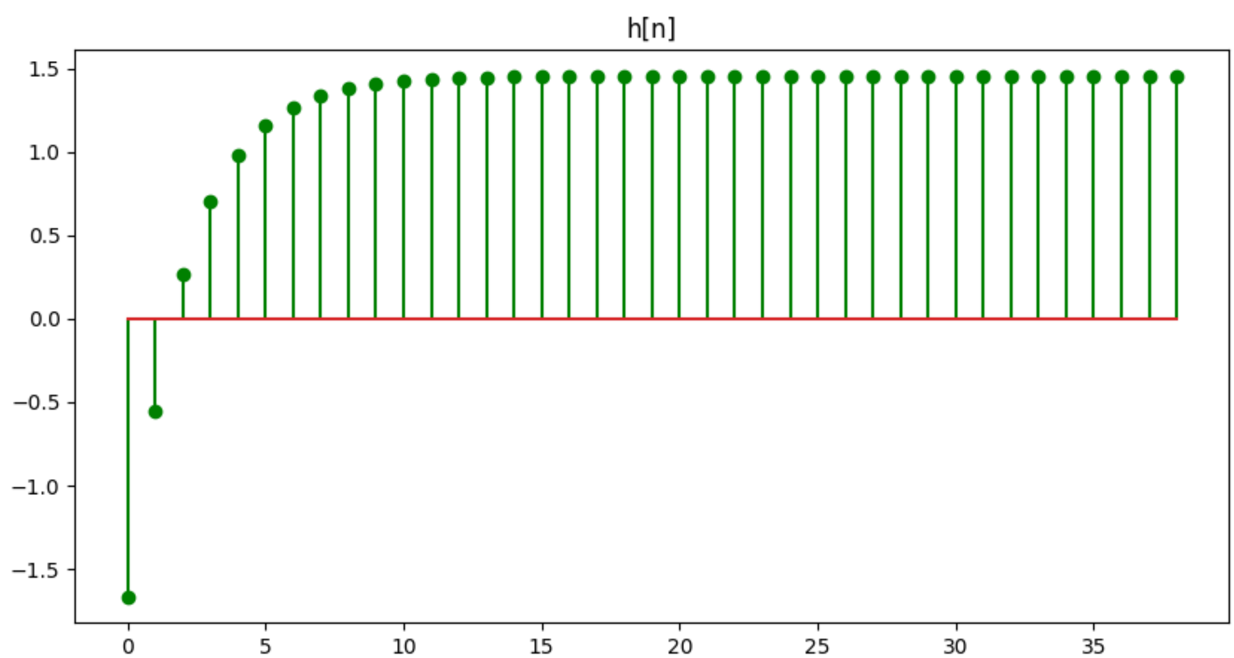
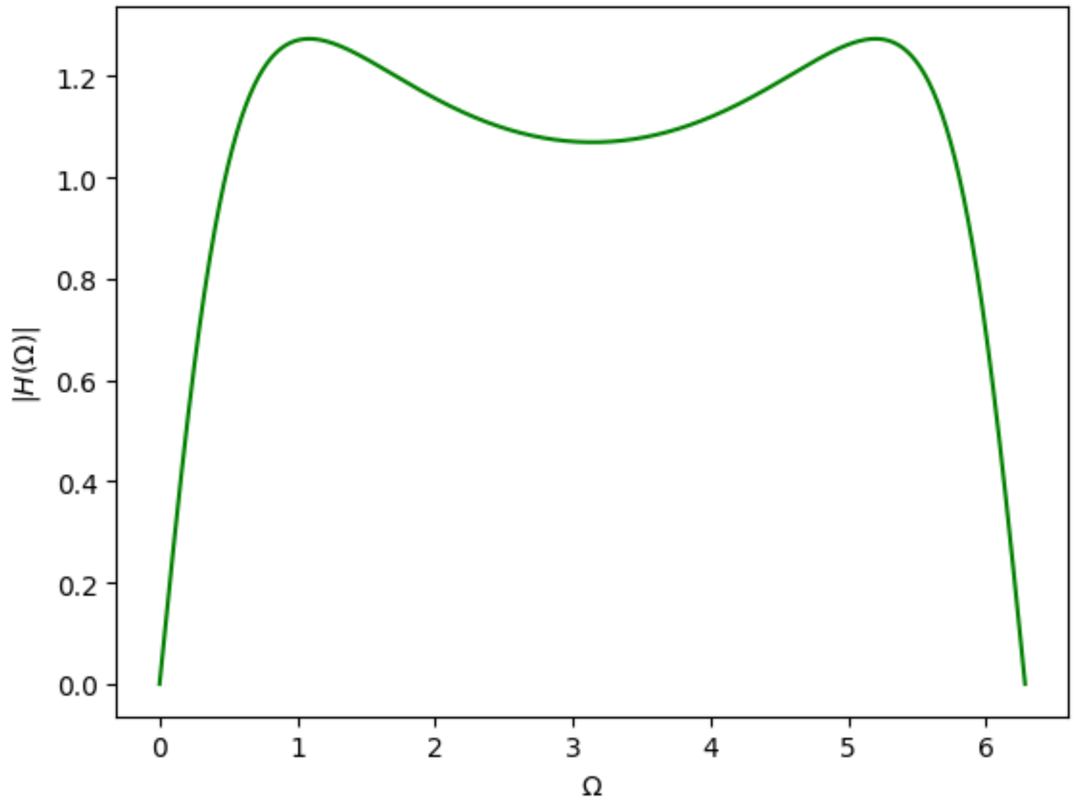
$$\Downarrow$$

$$h[n] = -({}^{3}/_{4})^{\frac{n}{3}} + ({}^{4}/_{3} \times u[n]) + ({}^{3}/_{25} \times u[n-2])$$



b)



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c) This filter includes succession narrow band filters including real axis. Therefore, we reach a band-pass filter.

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d)

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Zeros = [1.]
Poles = [0.51861407 0.23138593]
Gain = 1.0
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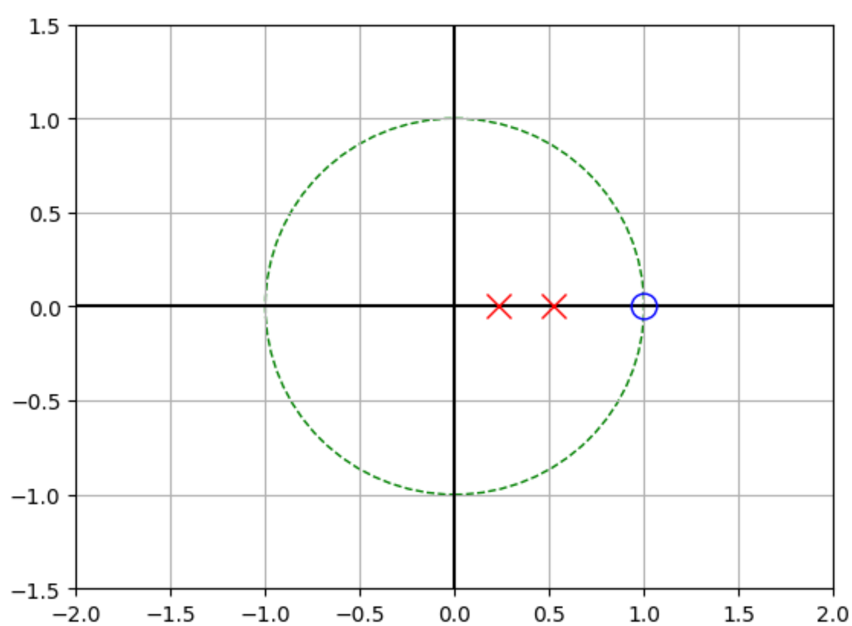


Response of LTI Systems to Sinusoidal Signals

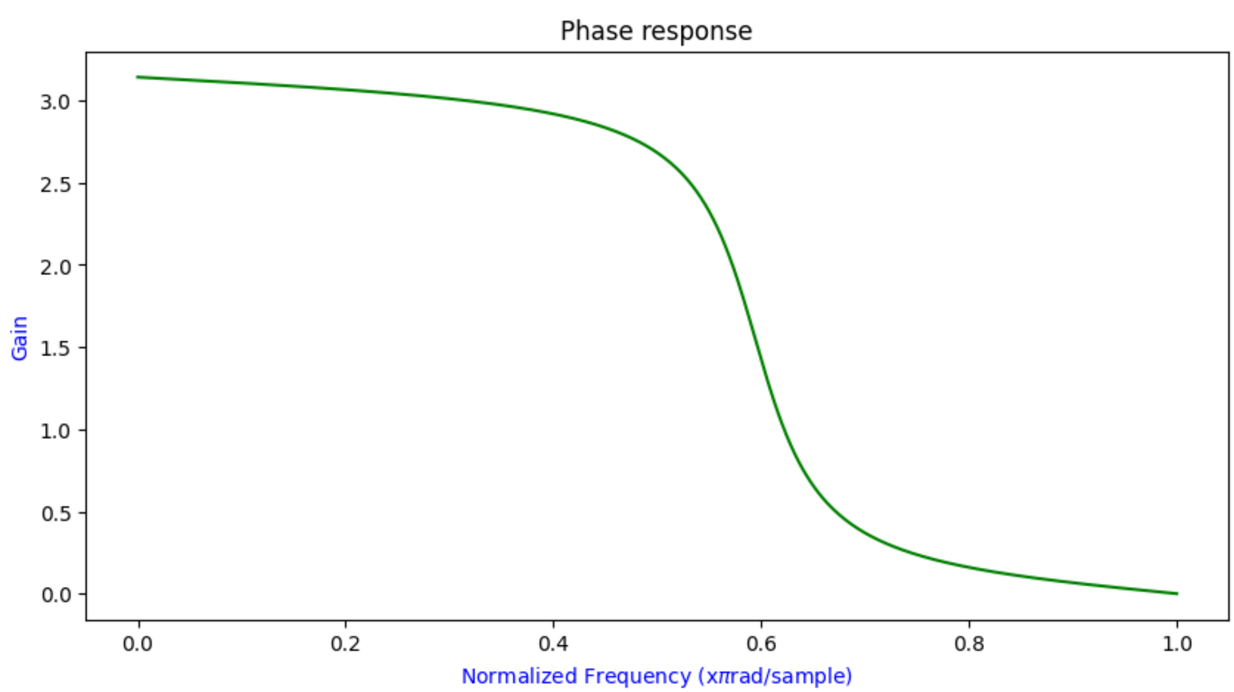
2. Assume a causal LTI system with transfer function

$$H(z) = \frac{0.25 - 0.54z^{-1} + 0.25z^{-2}}{1 + 0.5z^{-1} + 0.75z^{-2}}$$

a) Compute the frequency response.
b) Using the frequency response, find the output of the system for an input $x_1(n)=2\cos(\pi n/10)$ and $x_2(n)=\cos(3\pi n/5)$, for $0 \le n \le 200$. Then use additive property to get the output sequence for $x(n)= x_1(n)+x_2(n)$. Discuss the results.
c) Find the difference equation that describes $H(z)$.
d) Using the difference equation, find the output of the system for the input x(n) in b). Hint: Treat the system and the input as causal.
e) Do the methods in b) and d) give the same result? Can you tell the difference between the two systems?

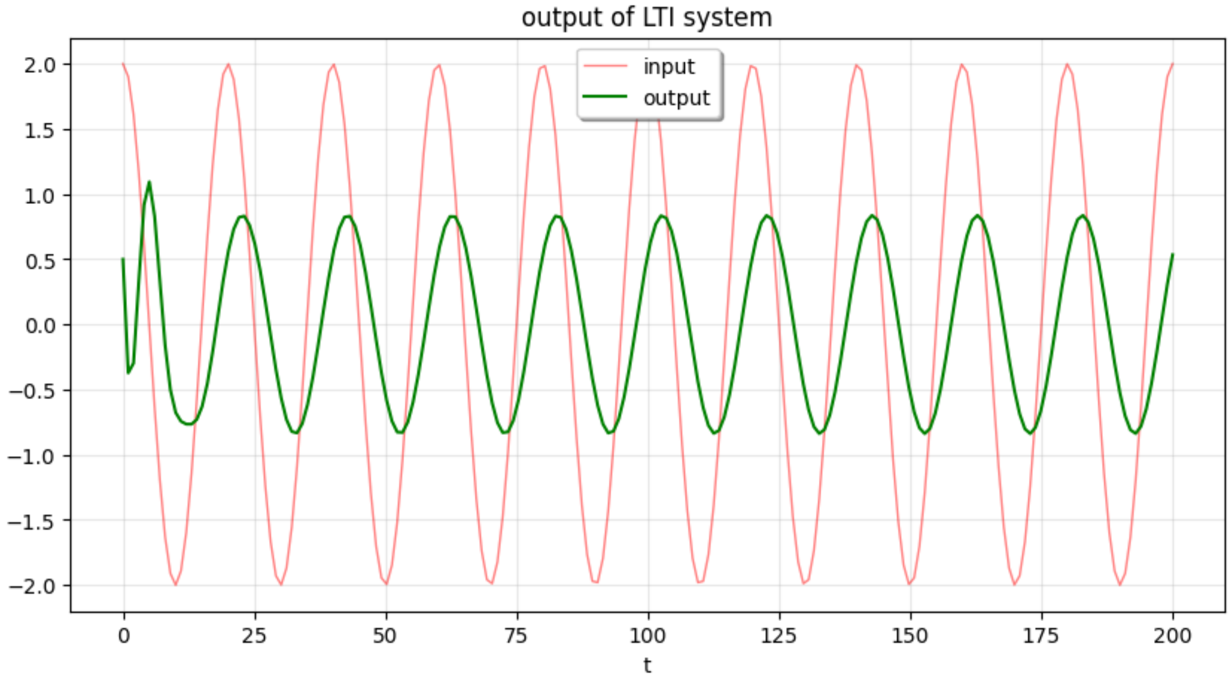
Answer :

a)

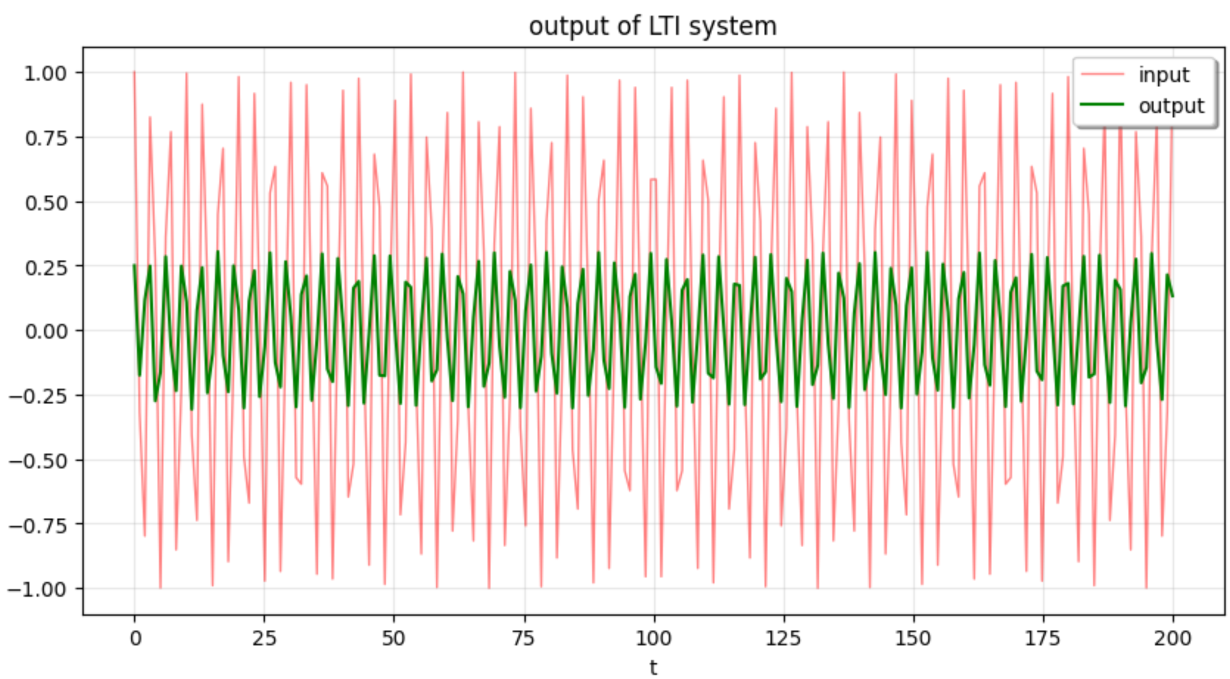


b)

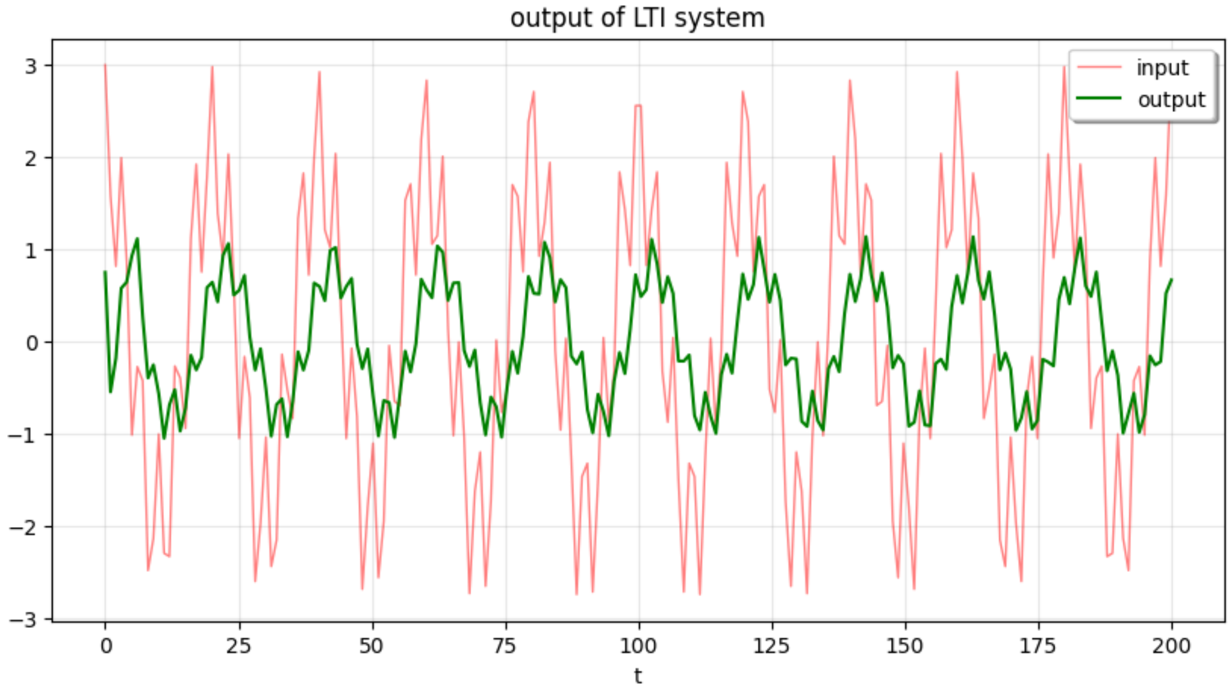
$x_1(n)=2\cos(\pi n/10)$



$x_2(n)=\cos(3\pi n/5)$



$x(n) = x_1(n) + x_2(n)$



The additive property says that if the response of a system to input x_1 is y_1 and the response x_2 is y_2, then the response to input $[x_1 + x_2]$ is equal $[y_1 + y_2]$. This intuition comes from three properties of an LTI system which include scaling, additivity, and time invariance. Generally, additivity and scaling properties together are named superposition property.

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c)

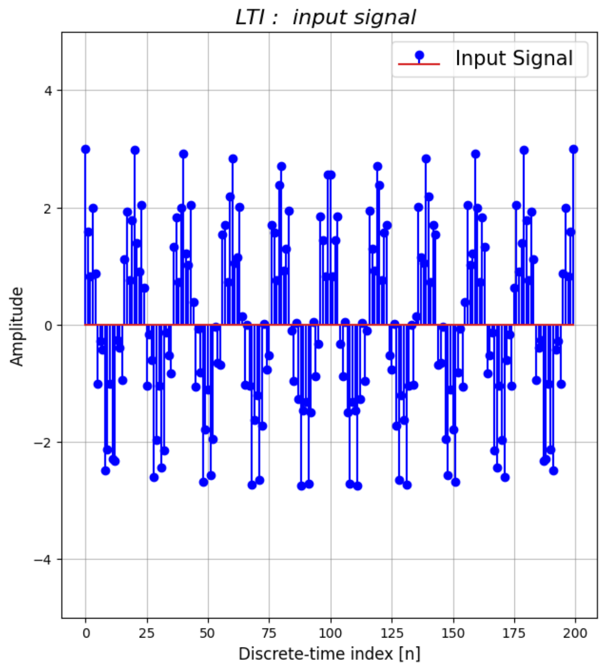
$H(z)$ can be computed by given input $x[n]$ and output $y[n]$ with difference equation :

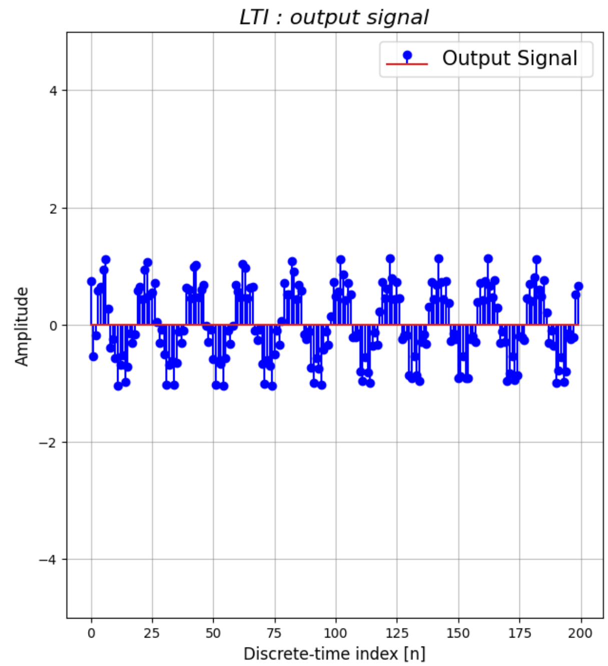
$$\sum_{k=0}^{N} a_k y[n-h] = \sum_{k=0}^{M} b_k [n-k] \implies H(z) = \frac{N(z)}{D(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{\sum_{k=0}^{N} a_k z^{-k}}$$

Difference equation method follows three steps:

1) Calculate $X(z)$ and $Y(z)$
2) Finding coefficients of b_k and a_k
3) Calculating $H(z)$

Point: This estimation method is not the best.



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==== Difference Equation that Describes H(z) ====

         0.75 + 0.43z^(-1)  -0.001z^(-2)  -0.006z^(-3)  -0.693z^(-4)  -0.793z^(-5)  -0.101z^(-6)  -0.045z^(-7)  + 0.294z^(-8)  + 0.917z^(-9)
H(z) = ------------------------------------------------------------------------------------------------------------------------------------
         3.0 + 1.032z^(-1)  -0.153z^(-2)  0.0z^(-3)  -2.379z^(-4)  -2.379z^(-5)  -0.0z^(-6)  -0.153z^(-7)  + 1.032z^(-8)  + 3.0z^(-9)
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Coefficients of H(z) within 10 first orders

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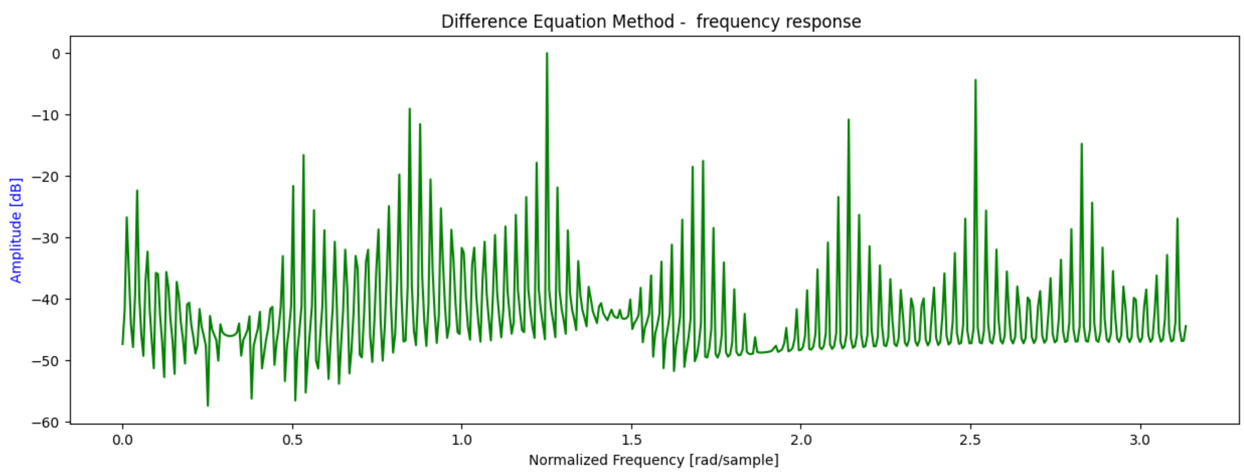
d)



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e)

There is a damaging term since the system parameters tend to be estimated from discrete-time data in the difference equation method. The output of a linear time-invariant (LTI) system is calculated from convolving impulse response $h[n]$ and input $x[n]$. A popular different equation method for the discrete-time LTI system is by defining constant coefficients. A system can be explained by a linear difference equation when it's linear and time-invariant. But it's not possible for finite discrete-time LTI systems. We generally can't describe an arbitrary discrete-time LTI system by a difference equation.

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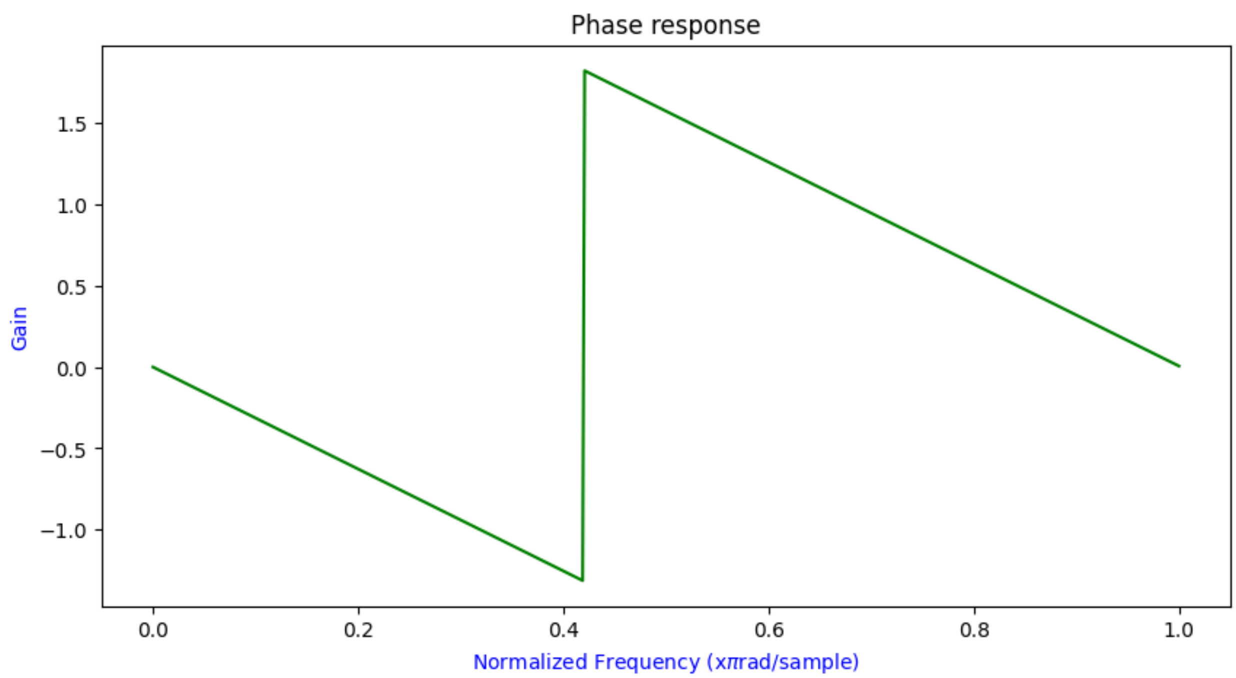
3. Assume a causal LTI system with transfer function

$$H(z) = 1 - 0.5z^{-1} + z^{-2}$$

Repeat Q.2. a)-d) and compare the results.

Answer :

a)



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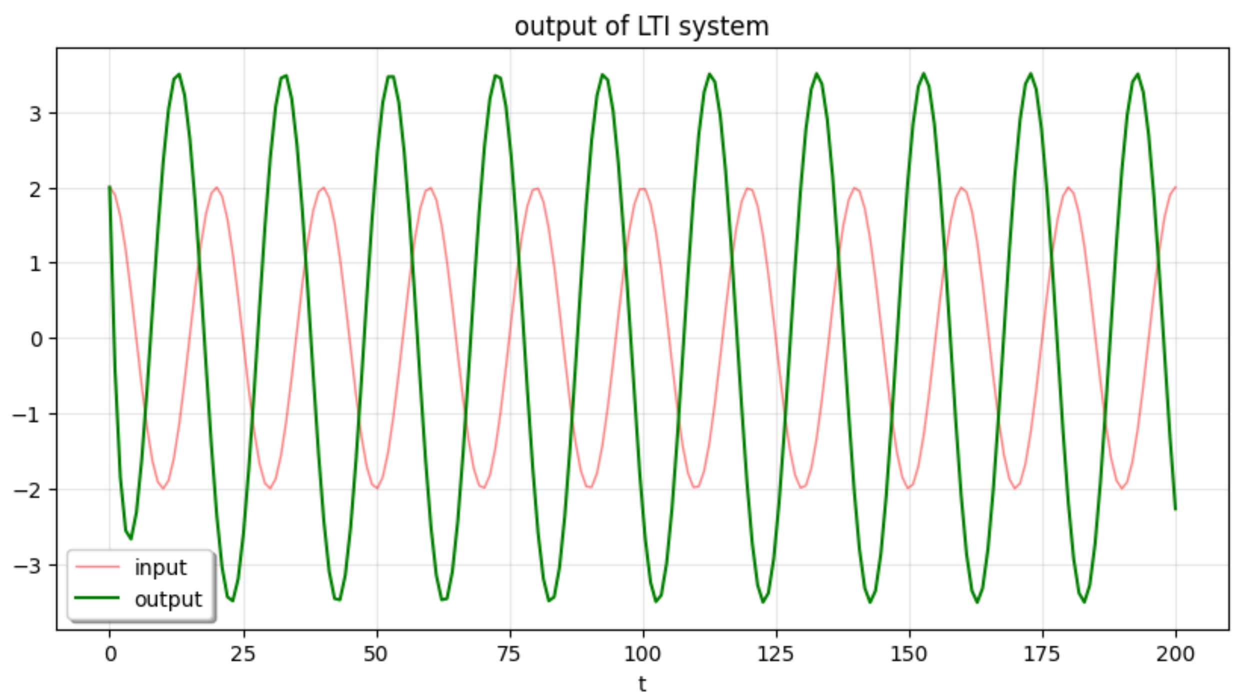
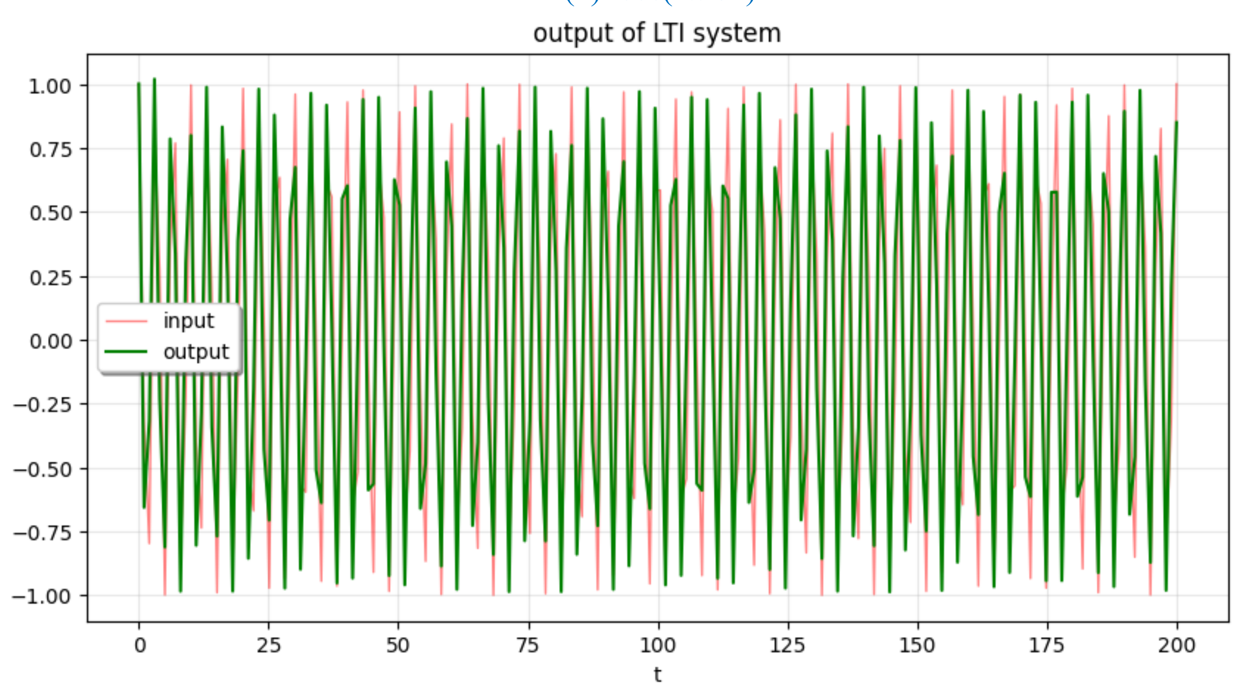
b)

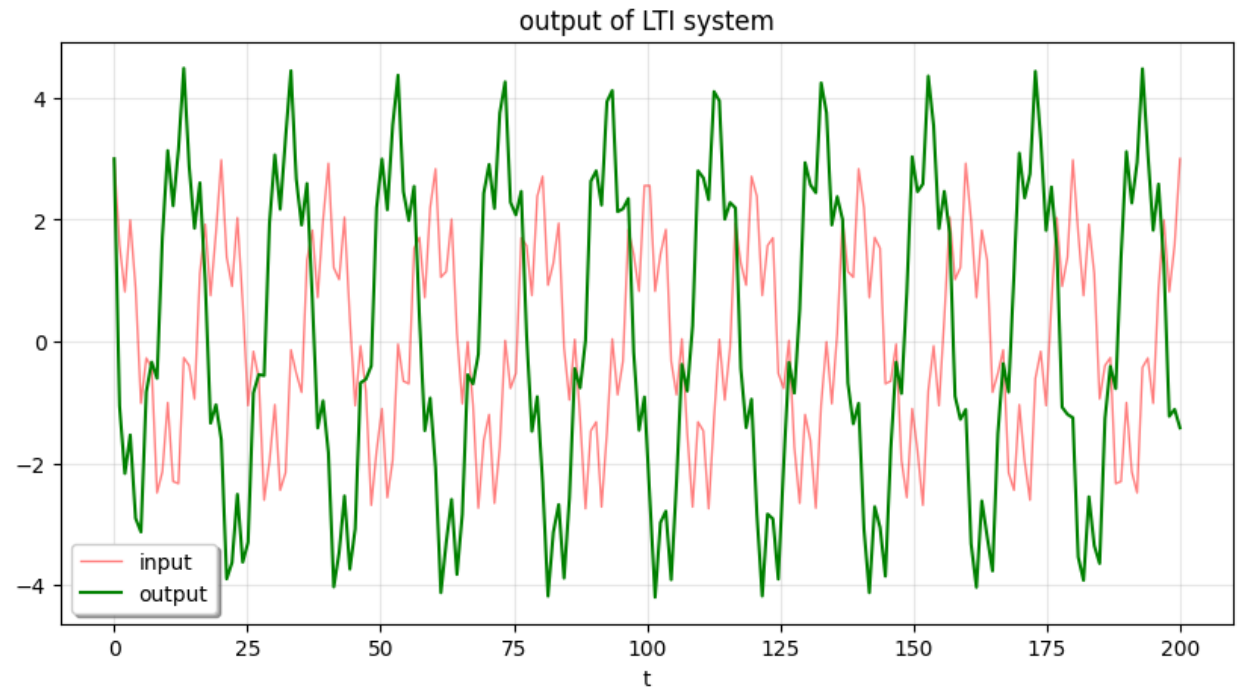
$x_1(n)=2\cos(\pi n/10)$



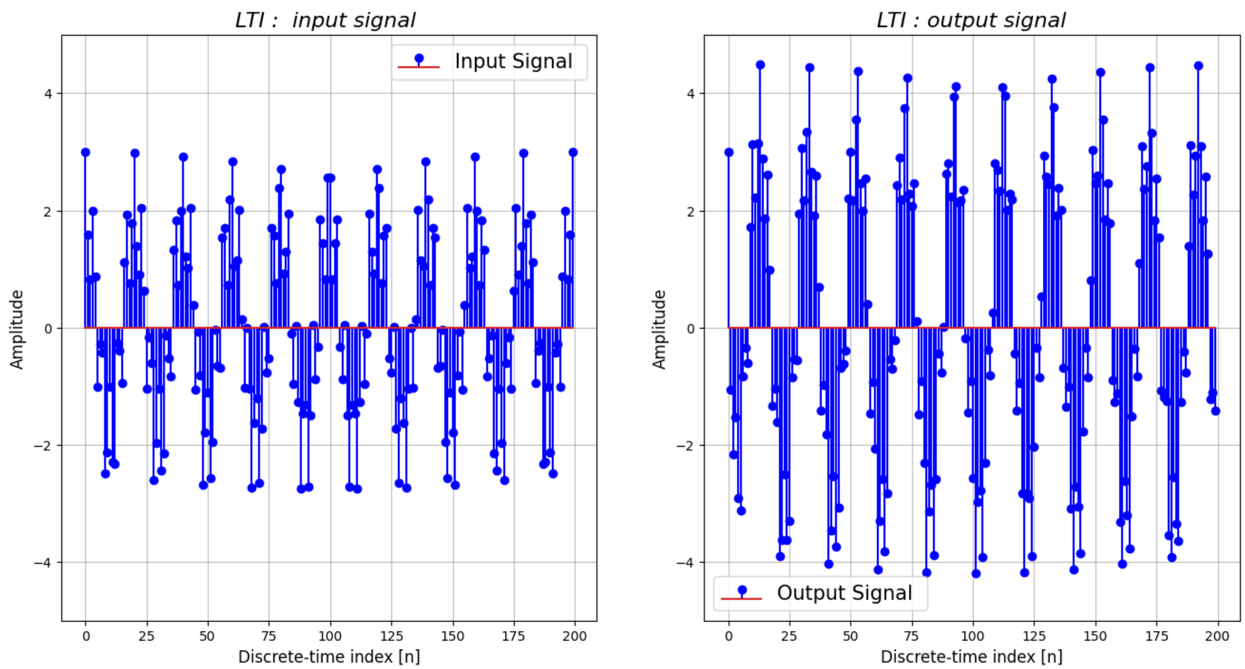
$x_2(n)=\cos(3\pi n/5)$



$x(n) = x_1(n) + x_2(n)$



c)

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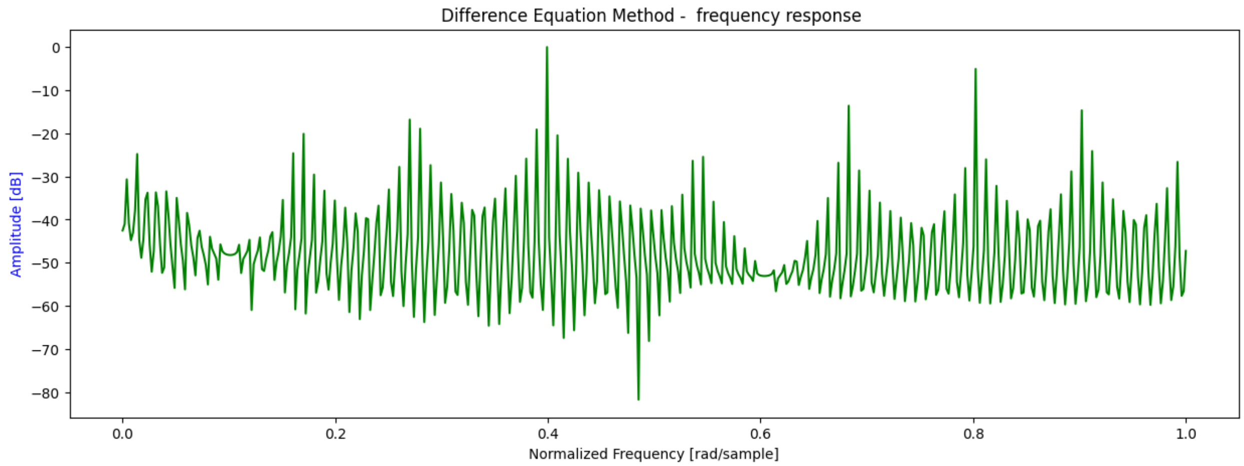
==== Difference Equation that Describes H(z) ====

         3.0 -2.595z^(-1)  -0.014z^(-2)  + 0.041z^(-3)  + 4.116z^(-4)  + 4.759z^(-5)  + 0.642z^(-6)  + 0.264z^(-7)  -1.744z^(-8)  -5.469z^(-9)
H(z) = -----------------------------------------------------------------------------------------------------------------------------------
         3.0 + 1.032z^(-1)  -0.153z^(-2)  0.0z^(-3)  -2.379z^(-4)  -2.379z^(-5)  -0.0z^(-6)  -0.153z^(-7)  + 1.032z^(-8)  + 3.0z^(-9)
```

Coefficients of H(z) within 10 first orders d)



Comparison with problem 2 :

- For sinusoidal inputs, we have sinusoidal outputs. The first filter (Problem 2) is in higher order, so the gain is bigger. But two filters have the same frequency behaviour.
- We have phase shifting ∅ and magnitude variations (amplitude variation $|G(jw)|$).
- There is the same peak response and attenuating signal below -40DB (cross-ever frequency) for filters. Also, a magnitude cross below -60DB is called bandwidth frequency. It means attenuation below the RMS value of the input in filter problem 3. So, this filter's upper limit is higher than filter problem 2, which is better to control by a dynamic system.
- Filter problem 3 has a larger range of passing frequencies.