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A Proportional Plus Damping Injection Controller for Teleoperators with Joint Flexibility and Time–Delays

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***Abstract*— The problem of controlling a rigid bilateral teleoperator with time-delays has been effectively addressed since the late 80's. However, the control of flexible joint manipulators in a bilateral teleoperation scenario is still an open problem. In the present paper we report two versions of a proportional plus damping injection controller that are capable of globally stabilizing a nonlinear bilateral teleoperator with joint flexibility and variable time-delays. The first version controls a teleoperator composed by a rigid local manipulator and a flexible joint remote manipulator and the second version deals with local and remote manipulators with joint flexibility. For both schemes, it is proved that the joint and motor velocities and the local and remote position error are bounded. Moreover, if the human operator and remote environment forces are zero then velocities asymptotically converge to zero and position tracking is established. Simulations are presented to show the performance of the proposed controllers.**

I. INTRODUCTION

A teleoperator is composed of a *human operator*, a *local manipulator*, a *communication channel*, a *remote manipulator* and a remote *environment*. The main objective of such a scheme is to extend the human manipulation capabilities to a remote environment. To this end, the local and remote manipulators exchange control signals through the communication channel and the remote force interaction is reflected back to the operator. Controlling these systems has become a highly active research field. For a recent historical survey on this research line the reader may refer to [1] and, for a tutorial on teleoperators control, to [2].

Since its introduction, in the late 80's by Anderson and Spong [3], the scattering transformation has dominated the field of teleoperators control. However, one of the issues with most of the scattering-based schemes is position drift (an exception is [4]). Recently, without the use of the scattering transformation, Chopra et al. [5] have proposed the use of adaptive schemes to overcome the effects of position drift. Along the same line Nuño et al. [6] report a different adaptive scheme that is capable of synchronizing the local and remote positions despite constant time-delays. In [7] the adaptive controller of [6], is extended to the synchronization of multiple robots networks with delays.

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Lee and Spong [8] suggest the use of a proportional-derivative plus damping controller to control a teleoperator with constant time-delays. Later, in [9] for the constant time-delay case and and in [10] for the variable time-delay case, Nuño et al. show that simple proportional plus damping controllers are capable of providing position tracking in delayed teleoperators. All these previous schemes analyzed the case when the local and remote manipulators are composed by rigid links. Moreover, it should be underscored that in diverse applications, including space and surgical telerobotics, the use of thin, lightweight and cable-driven manipulators is increasing. It was shown in [11] that the lumped (linear) dynamics of a flexible link are identical to the (linear) dynamics of a flexible joint. However, to the authors knowledge, most of the teleoperator controllers reported until today deal with rigid manipulators. Few exceptions, for linearized teleoperators and without time-delays, are: [11], [12], [13], [14] and [15].

The present work, reports some first results on the control of nonlinear teleoperators with joint flexibility and time-delays. Joint flexibility is a major source of oscillatory behaviors in robot manipulators, it can be caused by transmission elements such as harmonic drives, belts, or long shafts. Joint flexibility is modeled using motor rotor and link positions and velocities, hence, the order of the model is twice that of rigid joints. By using a proportional plus damping controller, and assuming that the human operator and the environment behave as passive systems, it is proved that all velocity and position error signals are bounded. Moreover, if the human operator does not inject forces on the local manipulator then the velocities and the position error are shown to be asymptotically convergent to zero. The proposed scheme is proved to be robust to variable time-delays. The proofs of such properties employ formal arguments based on Barbalăt's Lemma. First, the case when only the remote manipulator exhibits joint flexibility is studied and then such results are extended to the more challenging case when both local and remote manipulators exhibit joint flexibility.

To streamline the presentation, the following notation is introduced. Lower case letters denote scalar functions, *e*.g. t, bold lower case letters denote vector functions, *e*.g. $\mathbf{x}$, and bold upper case letters denote matrices, *e*.g. $\mathbf{A}$. Additionally, $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$. $\lambda_m\{\mathbf{A}\}$ and $\lambda_M\{\mathbf{A}\}$ represent the minimum and maximum eigenvalue of matrix $\mathbf{A}$, respectively while $||\mathbf{A}||$ denotes the matrix–induced 2-norm. $|\mathbf{x}|$ stands for the standard Euclidean norm of vector $\mathbf{x}$. For any function $\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-

norm is defined as $\|\mathbf{f}\|_\infty := \sup_{t\geq 0}|\mathbf{f}(t)|$, and the square of the $\mathcal{L}_2$-norm as $\|\mathbf{f}\|_2^2 := \int_0^\infty |\mathbf{f}(t)|^2 dt$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_\infty < \infty\}$ and $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_2 < \infty\}$, respectively.

II. Modeling the Teleoperator with Remote Flexibility

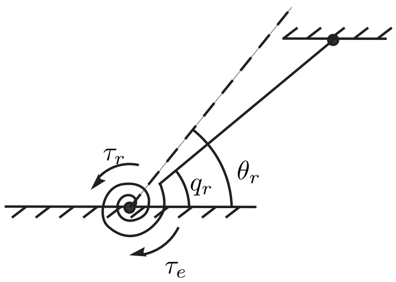

Fig. 1. One-DOF remote manipulator with a flexible joint.

In the first case, the local manipulator is modeled as a rigid n–degree of freedom (DOF) manipulator composed by revolute joints. Its nonlinear dynamic behavior is given by

$$\mathbf{M}_l(\mathbf{q}_l)\ddot{\mathbf{q}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\dot{\mathbf{q}}_l = \boldsymbol{\tau}_h - \boldsymbol{\tau}_l. \tag{1}$$

The remote manipulator is assumed to be a n–DOF manipulator with revolute flexible joints (Fig. 1 shows, schematically, a one-DOF manipulator with joint flexibility), whose dynamical behavior is governed by

$$\mathbf{M}_r(\mathbf{q}_r)\ddot{\mathbf{q}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r + \mathbf{S}_r[\mathbf{q}_r - \boldsymbol{\theta}_r] = -\boldsymbol{\tau}_e \tag{2a}$$
$$\mathbf{J}_r\ddot{\boldsymbol{\theta}}_r + \mathbf{S}_r[\boldsymbol{\theta}_r - \mathbf{q}_r] = \boldsymbol{\tau}_r \tag{2b}$$

where, using the subindex $i \in \{l, r\}$ for local and remote manipulators, respectively, $\mathbf{q}_i \in \mathbb{R}^n$ is the link position and $\boldsymbol{\theta}_r \in \mathbb{R}^n$ is the remote joint (motor) position. $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ is the coriolis and centrifugal effects matrix, defined via the Christoffel symbols of the first kind, $\mathbf{J}_r \in \mathbb{R}^{n\times n}$ is a diagonal matrix corresponding to the remote actuator inertia, $\mathbf{S}_r \in \mathbb{R}^{n\times n}$ is a diagonal and positive definite matrix that contains the remote joint stiffness, $\boldsymbol{\tau}_i \in \mathbb{R}^n$ is the control signal and $\boldsymbol{\tau}_h \in \mathbb{R}^n$, $\boldsymbol{\tau}_e \in \mathbb{R}^n$ are the forces exerted by the human operator and the environment interaction, respectively. It is assumed that gravity is compensated in both, the local and remote manipulators.

Throughout the paper, the following standard assumption is made: $\mathbf{M}_i(\mathbf{q}_i)$ is symmetric positive definite and bounded for all $\mathbf{q}_i$. Further, it is well-known that dynamics (1) and (2a) enjoy the following properties [16], [17], [10]:

P1. For all $\mathbf{x} \in \mathbb{R}^n$, $\mathbf{x}^\top[\dot{\mathbf{M}}_i(\mathbf{q}_i) - 2\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)]\mathbf{x} = 0$.

P2. For all $\mathbf{q}_i, \dot{\mathbf{q}}_i \in \mathbb{R}^n$, $\exists c_i \in \mathbb{R}_{>0}$ such that $|\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i| \leq c_i|\dot{\mathbf{q}}_i|^2$.

P3. If $\dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$ then $\frac{d}{dt}\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ is a bounded operator.

Remark 1: It is well-known that (1) satisfies the power-balance equation $\dot{V}_l = \dot{\mathbf{q}}_l^\top(\boldsymbol{\tau}_h - \boldsymbol{\tau}_l)$, where $V_l : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is given by $V_l(\mathbf{q}_l, \dot{\mathbf{q}}_l) = \frac{1}{2}\dot{\mathbf{q}}_l^\top \mathbf{M}_l(\mathbf{q}_l)\dot{\mathbf{q}}_l$.

On the other hand (2) satisfies $\dot{V}_r = -\dot{\mathbf{q}}_l^\top\boldsymbol{\tau}_e + \dot{\boldsymbol{\theta}}_r^\top\boldsymbol{\tau}_r$ where $V_r : \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ is given by

$$V_r(\mathbf{q}_r, \dot{\mathbf{q}}_r, \boldsymbol{\theta}_r, \dot{\boldsymbol{\theta}}_r) = \frac{1}{2}\dot{\mathbf{q}}_r^\top \mathbf{M}_r(\mathbf{q}_r)\dot{\mathbf{q}}_r + \frac{1}{2}\dot{\boldsymbol{\theta}}_r^\top \mathbf{J}_r\dot{\boldsymbol{\theta}}_r + \frac{1}{2}(\mathbf{q}_r - \boldsymbol{\theta}_r)^\top \mathbf{S}_r(\mathbf{q}_r - \boldsymbol{\theta}_r). \tag{3}$$

Regarding the human operator, the environment and the time-delays in the communication channel, this paper employs the following standard assumptions:

A1. The variable time-delays $T_i(t)$ have known upper bounds ${}^*T_i$, i.e., $0 \leq T_i(t) \leq {}^*T_i < \infty$.

A2. The human operator and the environment define passive (force to velocity) maps, that is, there exists $\kappa_i \in \mathbb{R}_{\geq 0}$ such that, for all $t \geq 0$,

$$E_h := -\int_0^t \dot{\mathbf{q}}_l^\top(\sigma)\boldsymbol{\tau}_h(\sigma)d\sigma + \kappa_l \geq 0, \tag{4a}$$
$$E_e := \int_0^t \dot{\mathbf{q}}_r^\top(\sigma)\boldsymbol{\tau}_e(\sigma)d\sigma + \kappa_r \geq 0. \tag{4b}$$

III. Proportional Position Error Plus Damping Injection Controller

Suppose that the forces applied on the local and remote manipulators are proportional to their position errors plus a damping injection term. Assume, that on the remote side, only the position and velocity of the joint (motor) are available for measurement. In this scenario the control laws are given by[1]

$$\boldsymbol{\tau}_l = K_l[\mathbf{q}_l - \boldsymbol{\theta}_r(t - T_r(t))] + B_l\dot{\mathbf{q}}_l \tag{5a}$$
$$\boldsymbol{\tau}_r = K_r[\mathbf{q}_l(t - T_l(t)) - \boldsymbol{\theta}_r] - B_r\dot{\boldsymbol{\theta}}_r \tag{5b}$$

where K_i and B_i are positive constants.[2]

Before going through the stability properties, a lemma that is instrumental in the analysis is presented for completeness. Its proof can be found in [10].

Lemma 1: For any vector signals $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, any variable time-delay $0 \leq T(t) \leq {}^*T < \infty$ and any constant $\alpha > 0$, the following inequality holds

$$-\int_0^t \mathbf{x}^\top(\sigma)\int_{-T(\sigma)}^0 \mathbf{y}(\sigma + \theta)d\theta d\sigma \leq \frac{\alpha}{2}\|\mathbf{x}\|_2^2 + \frac{{}^*T^2}{2\alpha}\|\mathbf{y}\|_2^2.$$

■

Proposition 1: Consider the teleoperator (1)–(2), controlled by (5) with $\boldsymbol{\tau}_h, \boldsymbol{\tau}_e$ verifying (4). Set the control gains such that

$$4B_lB_r > ({}^*T_l + {}^*T_r)^2 K_lK_r \tag{6}$$

Then:

I. Velocities and position errors are bounded, i.e., $\dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_r, \mathbf{q}_r - \boldsymbol{\theta}_r, \mathbf{q}_l - \boldsymbol{\theta}_r \in \mathcal{L}_\infty$. Moreover, $\dot{\mathbf{q}}_l, \dot{\boldsymbol{\theta}}_r \in \mathcal{L}_2$ and $|\ddot{\boldsymbol{\theta}}_r|, |\dot{\boldsymbol{\theta}}_r| \to 0$ as $t \to \infty$.

[1]To avoid cluttering the notation, the argument of all time signals is omitted except for the case when it appears delayed.

[2]The assumption of scalar gains is made for simplicity, the case when these gains are positive definite diagonal matrices can be treated with slight modifications to the proof.

II. In the case that the human and environment forces are bounded, i.e., $\boldsymbol{\tau}_h, \boldsymbol{\tau}_e \in \mathcal{L}_\infty$, link velocities asymptotically converge to zero, i.e., $|\dot{\mathbf{q}}_i| \to 0$ as $t \to \infty$.

III. If additionally, the human operator does not inject any forces on the local manipulator, i.e. $\boldsymbol{\tau}_h = \mathbf{0}$, the local and remote joint position error and remote joint and motor position error asymptotically converge to zero, i.e., $\lim_{t\to\infty} |\mathbf{q}_l(t) - \mathbf{q}_r(t)| = \lim_{t\to\infty} |\mathbf{q}_r(t) - \boldsymbol{\theta}_r(t)| = 0$.

Proof: Consider the following function $W(\mathbf{q}_i, \dot{\mathbf{q}}_i, \boldsymbol{\theta}_r, \dot{\boldsymbol{\theta}}_r, t)$ given by

$$W = V_l + \frac{K_l}{K_r} V_r + E_h + \frac{K_l}{K_r} E_e + \frac{K_l}{2} |\mathbf{q}_l - \boldsymbol{\theta}_r|^2$$

where V_i are defined in Remark 1 and E_h and E_e in (4a) and (4b), respectively. Note that W is positive semi-definite and radially unbounded with regards to $\dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_r, |\mathbf{q}_r - \boldsymbol{\theta}_r|$ and $|\mathbf{q}_l - \boldsymbol{\theta}_r|$. Calculating its time-derivative along (1), (2) and using Property P1, yields

$$\dot{W} = -\dot{\mathbf{q}}_l^\top \boldsymbol{\tau}_l + \frac{K_l}{K_r} \dot{\boldsymbol{\theta}}_r^\top \boldsymbol{\tau}_r + K_l (\mathbf{q}_l - \boldsymbol{\theta}_r)^\top (\dot{\mathbf{q}}_l - \dot{\boldsymbol{\theta}}_r).$$

Substituting the controllers (5), we get

$$\begin{aligned} \dot{W} = & - B_l |\dot{\mathbf{q}}_l|^2 - K_l \dot{\mathbf{q}}_l^\top [\boldsymbol{\theta}_r - \boldsymbol{\theta}_r(t - T_r(t))] - \\ & - \frac{K_l B_r}{K_r} |\dot{\boldsymbol{\theta}}_r|^2 - K_l \dot{\boldsymbol{\theta}}_r^\top [\mathbf{q}_l - \mathbf{q}_l(t - T_l(t))], \end{aligned}$$

Noting that, for any $\mathbf{x} \in \mathbb{R}^n$,

$$\mathbf{x} - \mathbf{x}(t - T(t)) = \int_{-T(t)}^{0} \dot{\mathbf{x}}(t + \sigma) d\sigma, \qquad (7)$$

$\dot{W}$ can be written as

$$\begin{aligned} \dot{W} &= -B_l |\dot{\mathbf{q}}_l|^2 - K_l \dot{\mathbf{q}}_l^\top \int_{-T_r(t)}^{0} \dot{\boldsymbol{\theta}}_r(t + \sigma) d\sigma - \\ &- \frac{K_l B_r}{K_r} |\dot{\boldsymbol{\theta}}_r|^2 - K_l \dot{\boldsymbol{\theta}}_r^\top \int_{-T_l(t)}^{0} \dot{\mathbf{q}}_l(t + \sigma) d\sigma. \end{aligned}$$

Integrating from 0 to t, and invoking Lemma 1 on the above integral terms, with α_l and α_r, respectively, yields

$$\begin{aligned} W(t) &- W(0) \le - \left[B_l - \frac{K_l}{2} \left(\alpha_l + \frac{{}^*T_l^2}{\alpha_r} \right) \right] \|\dot{\mathbf{q}}_l\|_2^2 - \\ &- \left[\frac{K_l B_r}{K_r} - \frac{K_l}{2} \left(\alpha_r + \frac{{}^*T_r^2}{\alpha_l} \right) \right] \|\dot{\boldsymbol{\theta}}_r\|_2^2. \end{aligned}$$

Let us define $\lambda_i \in \mathbb{R}$ as $\lambda_l := B_l - \frac{K_l}{2}\left(\alpha_l + \frac{{}^*T_l^2}{\alpha_r}\right)$ and $\lambda_r := B_r - \frac{K_r}{2}\left(\alpha_r + \frac{{}^*T_r^2}{\alpha_l}\right)$. Note that if there exist $\lambda_i > 0$ then $\lambda_l \|\dot{\mathbf{q}}_l\|_2^2 + \lambda_r \|\dot{\boldsymbol{\theta}}_r\|_2^2 \le W(0)$, thus $\dot{\mathbf{q}}_l, \dot{\boldsymbol{\theta}}_r \in \mathcal{L}_2$.

Solving simultaneously for $\lambda_i > 0$ and $\alpha_i > 0$, there will exist a solution if $4 B_l B_r > ({}^*T_l + {}^*T_r)^2 K_l K_r$. Thus, setting the control gains fulfilling this last inequality, ensures that $\exists \alpha_i > 0$ such that $\lambda_i > 0$ and $\dot{\mathbf{q}}_l, \dot{\boldsymbol{\theta}}_r \in \mathcal{L}_2$, which in turn implies that $W(t) \le W(0)$. Hence $\dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_r, |\mathbf{q}_r - \boldsymbol{\theta}_r|, |\mathbf{q}_l - \boldsymbol{\theta}_r| \in \mathcal{L}_\infty$.

In order to prove that $\lim_{t\to\infty} |\dot{\boldsymbol{\theta}}_r(t)| = 0$, let us rewrite $\boldsymbol{\tau}_r$ as

$$\boldsymbol{\tau}_r = K_r (\mathbf{q}_l - \boldsymbol{\theta}_r) - B_r \dot{\boldsymbol{\theta}}_r - K_r \int_{t - T_l(t)}^{t} \dot{\mathbf{q}}_l(\sigma) d\sigma.$$

Note that, since $\dot{\mathbf{q}}_l \in \mathcal{L}_2$, $\int_{t-T_l(t)}^{t} \dot{\mathbf{q}}_l(\sigma) d\sigma \le {}^*T_r^{\frac{1}{2}} \|\dot{\mathbf{q}}_l\|_2 < \infty$ (using Schwartz's inequality), hence, from (2b), it is proved that $\ddot{\boldsymbol{\theta}}_r \in \mathcal{L}_\infty$. This last, together with the fact that $\dot{\boldsymbol{\theta}}_r \in \mathcal{L}_\infty \cap \mathcal{L}_2$ ensures that $\lim_{t\to\infty} |\dot{\boldsymbol{\theta}}_r(t)| = 0$. Moreover, since $\dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_r, \ddot{\boldsymbol{\theta}}_r \in \mathcal{L}_\infty$, it can be established that $\frac{d}{dt}\ddot{\boldsymbol{\theta}}_r \in \mathcal{L}_\infty$, which implies that $\ddot{\boldsymbol{\theta}}_r$ is uniformly continuous. This, and the boundedness and existence of the following limit

$$\lim_{t\to\infty} \int_0^t \ddot{\boldsymbol{\theta}}_r(\sigma) d\sigma = -\dot{\boldsymbol{\theta}}_r(0) < \infty$$

proves the claim that $|\ddot{\boldsymbol{\theta}}_r| \to 0$. This concludes the proof of Part I of Proposition 1.

The proof that $|\dot{\mathbf{q}}_l| \to 0$ can be easily established noting that boundedness of $\boldsymbol{\tau}_h, \dot{\mathbf{q}}_l$ and $|\mathbf{q}_l - \boldsymbol{\theta}_r|$ imply, from (1), that $\ddot{\mathbf{q}}_l \in \mathcal{L}_\infty$. Since $\dot{\mathbf{q}}_l \in \mathcal{L}_\infty \cap \mathcal{L}_2$, $|\dot{\mathbf{q}}_l| \to 0$.

On the other hand, boundedness of $\boldsymbol{\tau}_e, \dot{\mathbf{q}}_r$ and $|\mathbf{q}_r - \boldsymbol{\theta}_r|$ imply, from (2a), that $\ddot{\mathbf{q}}_r \in \mathcal{L}_\infty$. Differentiating (2b) yields

$$\frac{d}{dt}\ddot{\boldsymbol{\theta}}_r = \mathbf{J}_r^{-1} \left[\mathbf{S}_r [\dot{\mathbf{q}}_r - \dot{\boldsymbol{\theta}}_r] + K_r [\dot{\mathbf{q}}_l(t - T_l(t)) - \dot{\boldsymbol{\theta}}_r] - B_r \ddot{\boldsymbol{\theta}}_r \right].$$

Since all the signals inside the brackets converge to zero, except $\dot{\mathbf{q}}_r$, the proof that $|\dot{\mathbf{q}}_r| \to 0$ is established if it can be proved that $|\frac{d}{dt}\ddot{\boldsymbol{\theta}}_r| \to 0$. For, it suffices to prove that $\frac{d^2}{dt^2}\ddot{\boldsymbol{\theta}}_r \in \mathcal{L}_\infty$. Indeed, the fact that $\ddot{\mathbf{q}}_i, \frac{d}{dt}\ddot{\boldsymbol{\theta}}_r, \ddot{\boldsymbol{\theta}}_r \in \mathcal{L}_\infty$ ensure that $\frac{d^2}{dt^2}\ddot{\boldsymbol{\theta}}_r \in \mathcal{L}_\infty$ as needed, establishing the proof of Part II.

When $\boldsymbol{\tau}_h = \mathbf{0}$, the closed-loop system (1) and (5a) is

$$\ddot{\mathbf{q}}_l = -\mathbf{M}_l^{-1} \left([\mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l) + B_l] \dot{\mathbf{q}}_l - K_l [\mathbf{q}_l - \boldsymbol{\theta}_r(t - T_r(t))] \right).$$

The fact that $\lim_{t\to\infty} |\dot{\mathbf{q}}_l(t)| = 0$ allows to show that, if it is proved that $|\ddot{\mathbf{q}}_l| \to 0$, the term $|\mathbf{q}_l - \boldsymbol{\theta}_r(t - T_r(t))| \to 0$. For, recall that with Properties P1, P2, P3 and boundedness of $\ddot{\mathbf{q}}_l, \dot{\mathbf{q}}_l, \dot{\boldsymbol{\theta}}_r$ and $|\mathbf{q}_l - \boldsymbol{\theta}_r|$ it can be shown that $\frac{d}{dt}\ddot{\mathbf{q}}_l \in \mathcal{L}_\infty$. On the other hand, $|\dot{\mathbf{q}}_l| \to 0$ implies that $\lim_{t\to\infty} \int_0^t \ddot{\mathbf{q}}_l(\sigma) d\sigma$ exists and it is finite. Hence, $|\ddot{\mathbf{q}}_l| \to 0$ as required.

Now, using

$$\mathbf{q}_l - \boldsymbol{\theta}_r(t - T_r(t)) = \mathbf{q}_l - \boldsymbol{\theta}_r + \int_{t - T_r(t)}^{t} \dot{\boldsymbol{\theta}}_r(\sigma) d\sigma$$

together with convergence to zero of $\mathbf{q}_l - \boldsymbol{\theta}_r(t - T_r(t))$ and $\dot{\boldsymbol{\theta}}_r$, it is proved that $\lim_{t\to\infty} |\mathbf{q}_l - \boldsymbol{\theta}_r| = 0$. Now, rewriting (2b) in closed-loop with (5b), yields

$$\mathbf{J}_r \ddot{\boldsymbol{\theta}}_r = \mathbf{S}_r (\mathbf{q}_r - \boldsymbol{\theta}_r) + K_r (\mathbf{q}_l - \boldsymbol{\theta}_r) - K_r \int_{t - T_l(t)}^{t} \dot{\mathbf{q}}_l(\sigma) d\sigma - B_r \dot{\boldsymbol{\theta}}_r,$$

note that convergence to zero of $\ddot{\boldsymbol{\theta}}_r, \dot{\boldsymbol{\theta}}_r, \dot{\mathbf{q}}_l$ and $|\mathbf{q}_l - \boldsymbol{\theta}_r|$ supports the claim that $\lim_{t\to\infty} |\mathbf{q}_r - \boldsymbol{\theta}_r| = 0$. Finally, note that

$$\lim_{t\to\infty} |\mathbf{q}_l - \boldsymbol{\theta}_r| = \lim_{t\to\infty} |\mathbf{q}_l - \mathbf{q}_r + \mathbf{q}_r - \boldsymbol{\theta}_r| = 0$$

and $\lim_{t\to\infty} |\mathbf{q}_r - \boldsymbol{\theta}_r| = 0$ thus $\lim_{t\to\infty} |\mathbf{q}_l - \mathbf{q}_r| = 0$. This completes the proof. ■

IV. THE CASE WITH LOCAL AND REMOTE FLEXIBILITY

When the local manipulator exhibits joint flexibility, its dynamical behavior (1) transforms to

$$\mathbf{M}_l(\mathbf{q}_l)\ddot{\mathbf{q}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\dot{\mathbf{q}}_l + \mathbf{S}_l[\mathbf{q}_l - \boldsymbol{\theta}_l] = \boldsymbol{\tau}_h \quad (8a)$$
$$\mathbf{J}_l\ddot{\boldsymbol{\theta}}_l + \mathbf{S}_l[\boldsymbol{\theta}_l - \mathbf{q}_l] = -\boldsymbol{\tau}_l \quad (8b)$$

where $\mathbf{J}_l \in \mathbb{R}^{n\times n}$ is a diagonal matrix corresponding to the local actuator inertia and $\mathbf{S}_l \in \mathbb{R}^{n\times n}$ is a diagonal and positive definite matrix that contains the local joint stiffness.

Proposition 2: Consider the teleoperator (8) and (2), controlled by

$$\boldsymbol{\tau}_l = K_l[\boldsymbol{\theta}_l - \boldsymbol{\theta}_r(t - T_r(t))] + B_l\dot{\boldsymbol{\theta}}_l \quad (9a)$$
$$\boldsymbol{\tau}_r = K_r[\boldsymbol{\theta}_l(t - T_l(t)) - \boldsymbol{\theta}_r] - B_r\dot{\boldsymbol{\theta}}_r \quad (9b)$$

with $\boldsymbol{\tau}_h, \boldsymbol{\tau}_e$ verifying (4). Set the control gains such that (6) holds. Then:

I. All velocity signals and position errors are bounded, i.e., $\dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_i, \mathbf{q}_i - \boldsymbol{\theta}_i, \boldsymbol{\theta}_l - \boldsymbol{\theta}_r \in \mathcal{L}_\infty$. Moreover, $\dot{\boldsymbol{\theta}}_i \in \mathcal{L}_2$ and $|\ddot{\boldsymbol{\theta}}_i|, |\dot{\boldsymbol{\theta}}_i| \to 0$ as $t \to \infty$.

II. In the case that the human and environment forces are bounded, i.e., $\boldsymbol{\tau}_h, \boldsymbol{\tau}_e \in \mathcal{L}_\infty$, $|\dot{\mathbf{q}}_i| \to 0$ as $t \to \infty$.

III. If additionally, the human operator and environment do not inject any forces on the local and remote manipulators, respectively, i.e. $\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$, the local and remote joint position error and remote joint and motor position error asymptotically converge to zero, i.e., $\lim_{t\to\infty} |\mathbf{q}_l(t) - \mathbf{q}_r(t)| = \lim_{t\to\infty} |\mathbf{q}_i(t) - \boldsymbol{\theta}_i(t)| = 0$.

Proof: The proof follows the same arguments as the proof of Proposition 1. Hence, only the main steps are presented here. Consider the function

$$U = V_l + \frac{K_l}{K_r}V_r + E_h + \frac{K_l}{K_r}E_e + \frac{K_l}{2}|\boldsymbol{\theta}_l - \boldsymbol{\theta}_r|^2$$

where V_l is similarly defined as V_r in (3), substituting the subscript r for l. E_h and E_e are defined in (4a) and (4b), respectively. Note that U is positive semi-definite and radially unbounded with regards to $\dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_i, |\mathbf{q}_i - \boldsymbol{\theta}_i|$ and $|\boldsymbol{\theta}_l - \boldsymbol{\theta}_r|$. Its time-derivative along (2), (8), (9), using Property P1 and (7), yields

$$\dot{U} = -B_l|\dot{\boldsymbol{\theta}}_l|^2 - K_l\dot{\boldsymbol{\theta}}_l^\top \int_{-T_r(t)}^{0} \dot{\boldsymbol{\theta}}_r(t+\sigma)d\sigma - $$
$$- \frac{K_l B_r}{K_r}|\dot{\boldsymbol{\theta}}_r|^2 - K_l\dot{\boldsymbol{\theta}}_r^\top \int_{-T_l(t)}^{0} \dot{\boldsymbol{\theta}}_l(t+\sigma)d\sigma.$$

Integrating from 0 to t, invoking Lemma 1 on the above integral terms, with α_l and α_r, respectively, yields

$$\lambda_l\|\dot{\boldsymbol{\theta}}_l\|_2^2 + \lambda_r\|\dot{\boldsymbol{\theta}}_r\|_2^2 \le U(0),$$

where λ_i are given, as in the previous proof, by $\lambda_l := B_l - \frac{K_l}{2}\left(\alpha_l + \frac{{}^*T_l^2}{\alpha_r}\right)$ and $\lambda_r := B_r - \frac{K_r}{2}\left(\alpha_r + \frac{{}^*T_r^2}{\alpha_l}\right)$. Hence, setting $4B_lB_r > ({}^*T_l + {}^*T_r)^2K_lK_r$ ensures that $\exists\alpha_i > 0$ such that $\lambda_i > 0$ and $\dot{\boldsymbol{\theta}}_i \in \mathcal{L}_2$, which in turn implies that $\dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_i, |\mathbf{q}_i - \boldsymbol{\theta}_i|, |\mathbf{q}_l - \boldsymbol{\theta}_r| \in \mathcal{L}_\infty$. Using these bounded signals, together with the expressions in (2), (8) and (9) it can be easily proved that $\lim_{t\to\infty} |\ddot{\boldsymbol{\theta}}_i(t)| = 0$ and $\lim_{t\to\infty} |\dot{\boldsymbol{\theta}}_i(t)| = 0$. This completes Part I of the proof.

Boundedness of $\boldsymbol{\tau}_h, \boldsymbol{\tau}_e, \dot{\mathbf{q}}_i$ and $|\mathbf{q}_i - \boldsymbol{\theta}_i|$ imply, from (2a) and (8a), that $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$. Moreover, after differentiation of (2b) and (8b), it can be proved that $|\frac{d}{dt}\ddot{\boldsymbol{\theta}}_i| \to 0$. This last and convergence to zero of $\ddot{\boldsymbol{\theta}}_i$ and $\dot{\boldsymbol{\theta}}_i$ supports the asymptotical convergence of $\dot{\mathbf{q}}_i$ to zero, establishing the proof of Part II.

Note that, for the proof of Part III, it can be written (for the local and remote manipulators)

$$\ddot{\mathbf{q}}_i = -\mathbf{M}_i^{-1}\left(\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i - \mathbf{S}_i[\mathbf{q}_i - \boldsymbol{\theta}_i]\right)$$

and, since $|\dot{\mathbf{q}}_i| \to 0$, convergence to zero of $\ddot{\mathbf{q}}_i$ implies that $|\mathbf{q}_i - \boldsymbol{\theta}_i| \to 0$ which in turn implies, from (8b), that $|\boldsymbol{\theta}_l(t) - \boldsymbol{\theta}_r(t - T_r(t))| \to 0$. Now, $|\dot{\mathbf{q}}_i| \to 0$ implies that $\lim_{t\to\infty}\int_0^t \ddot{\mathbf{q}}_i(\sigma)d\sigma$ exists and it is finite. Moreover, boundedness of $\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \dot{\boldsymbol{\theta}}_i$, together with P3, ensures that $\ddot{\mathbf{q}}_i$ is uniformly continuous. Hence $|\ddot{\mathbf{q}}_i| \to 0$ as required.

Finally,

$$\mathbf{q}_l - \mathbf{q}_r = \mathbf{q}_l - \boldsymbol{\theta}_l + \boldsymbol{\theta}_l - \boldsymbol{\theta}_r(t - T_r(t)) - \mathbf{q}_r + \boldsymbol{\theta}_r - \int_{t-T_r(t)}^{t} \dot{\boldsymbol{\theta}}_r(\sigma)d\sigma,$$

convergence to zero of $|\mathbf{q}_i - \boldsymbol{\theta}_i|, |\boldsymbol{\theta}_l - \boldsymbol{\theta}_r(t - T_r(t))|$ and $|\dot{\boldsymbol{\theta}}_r|$ supports the claim that $\lim_{t\to\infty} |\mathbf{q}_l - \mathbf{q}_r| = 0$. This completes the proof. ■

V. SIMULATIONS

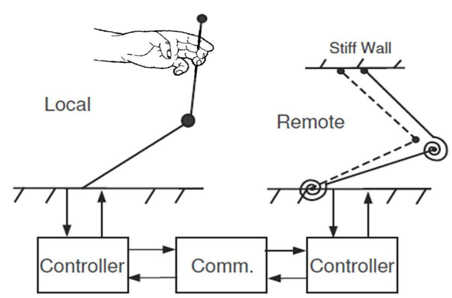

Fig. 2. Simulations test-bed.

To show the effectiveness of the proposed control schemes, some simulations, in which the local and remote manipulators are modeled as a pair of 2 DOF serial links with revolute joints (see Fig. 2), have been performed. Due to space limitations, only the simulations of the teleoperator composed by a rigid local manipulator and a remote manipulator with flexible joints are presented. The corresponding nonlinear dynamics are modeled by (1) and (2). In what follows $\alpha_i := l_{2_i}^2 m_{2_i} + l_{1_i}^2(m_{1_i} + m_{2_i})$, $\beta_i := l_{1_i}l_{2_i}m_{2_i}$ and $\delta_i := l_{2_i}^2 m_{2_i}$. The inertia matrices $\mathbf{M}_i(\mathbf{q}_i) := [M_{mn}]$ are given by: $M_{11} = \alpha_i + 2\beta_i c_{2_i}$, $M_{12} = M_{21} = \delta_i + \beta_i c_{2_i}$ and $M_{22} = \delta_i$. c_{2_i} is the short notation for $\cos(q_{2_i})$. q_{k_i} is the position of link k of manipulator i, with $k \in \{1,2\}$. The Coriolis and centrifugal effects are modeled by $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i)=[C_{mn}]$ and are given by $C_{11}=-2\beta_i \mathrm{s}_{2_i}\dot{q}_{2_i}$, $C_{12}=-\beta_i \mathrm{s}_{2_i}\dot{q}_{2_i}$, $C_{21}=\beta_i \mathrm{s}_{2_i}\dot{q}_{1_i}$ and $C_{22}=0$. s_{2_i} is the short notation for $\sin(q_{2_i})$. $\dot{q}_{1_i}$ and $\dot{q}_{2_i}$ are the respective revolute velocities of the two links. l_{k_i} and m_{k_i} are the respective lengths and masses of each link.

The remote motor inertia is given by $\mathbf{J}_r = 0.2\mathbf{I}$Kgm $\in \mathbb{R}^{2\times 2}$, the stiffness of the remote flexible joints is $\mathbf{S}_r = 500\mathbf{I}$Nm $\in \mathbb{R}^{2\times 2}$. The physical parameters for the manipulators are: the length of links $l_{1_i} = l_{2_i} = 0.38$m; the masses of the links are $m_{1_l} = 3.5$Kg, $m_{2_l} = 0.5$Kg, $m_{1_r} = 0.5$Kg and $m_{2_r} = 0.35$Kg. The initial conditions are $\ddot{\mathbf{q}}_i(0) = \dot{\mathbf{q}}_i(0) = \ddot{\boldsymbol{\theta}}_r(0) = \dot{\boldsymbol{\theta}}_r(0)\mathbf{0}$, $\mathbf{q}_l^{\top}(0) = [-1/8\pi, 1/8\pi]^{\top}$ and $\mathbf{q}_r^{\top}(0) = \boldsymbol{\theta}_r(0) = [1/5\pi, -1/2\pi]^{\top}$. The human operator is modeled as a spring-damper system with gains $K_s = 10$Nm and $K_d = 2$Nms, respectively. Fig. 3 shows the desired trajectory for the human operator spring-damper model. The local and remote controllers gains are set to $B_l = 3$Nms, $K_l = 5$Nm, $B_r = 8$Nms and $K_r = 15$Nm. The time-delays upper bound is ${}^*T_i = 0.55$s (Fig. 4).

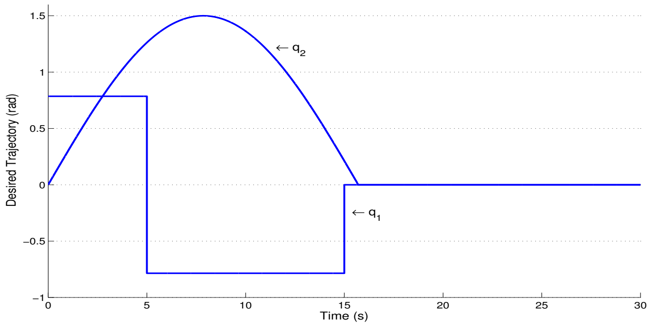

Fig. 3. Desired trajectory of the human operator.

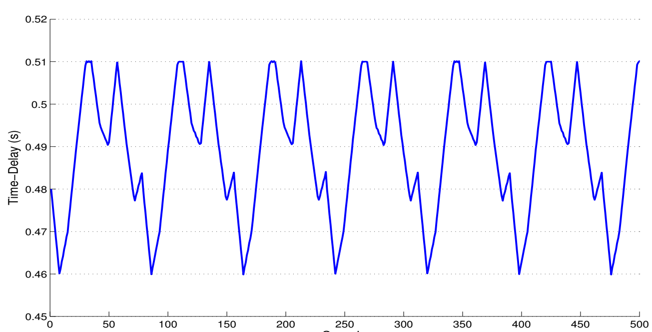

Fig. 4. Variable time-delay.

The first set of simulations presents the results when both local and remote manipulators move freely in the space, i.e., $\boldsymbol{\tau}_e = \mathbf{0}$. Fig. 5 shows the local and remote joint positions and remote joint-motor position error together with the local and remote position error. The asymptotic behavior of both local and remote position error and remote joint-motor position error can be clearly observed, i.e., convergence to zero of $|\mathbf{q}_l - \boldsymbol{\theta}_r|$ and $\boldsymbol{\theta}_r - \mathbf{q}_r$. Fig. 6 depicts the tracking results in Cartesian space. Both joint and Cartesian position errors asymptotically converge to zero despite time-delays.

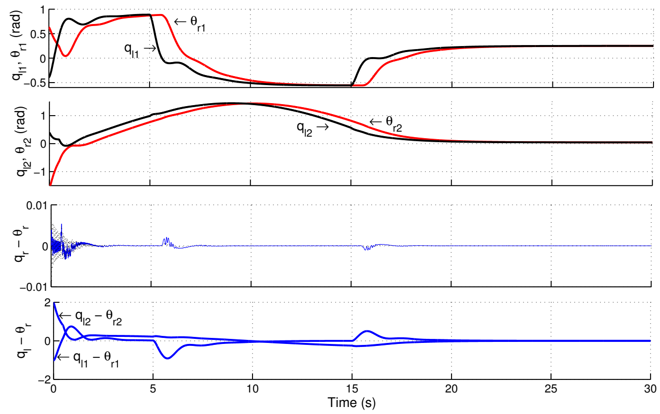

Fig. 5. Local and remote joint positions and position error.

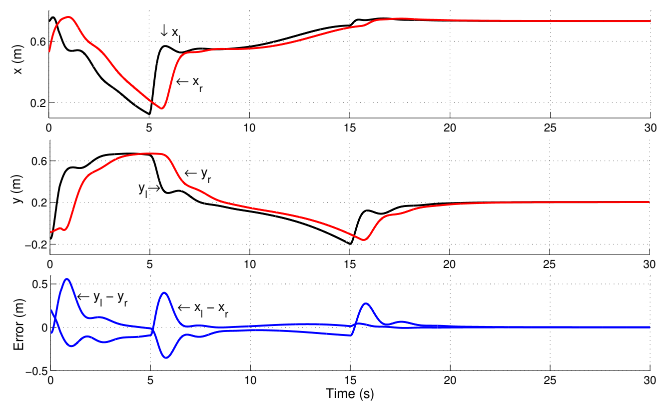

Fig. 6. Cartesian local and remote positions and Cartesian position error.

In the second set of simulations the remote environment is modeled as a stiff virtual wall, using a virtual spring-damper system with stiffness equal to 20000 Nm and damping equal to 200 Nms, located at 0.5m in the $y-$coordinate.

The local and remote joint positions and the remote joint-motor position error together with the local and remote position error can be seen in Fig. 7 when the remote manipulator interacts with a stiff virtual wall. Again, the asymptotic behavior of both local and remote position error and remote joint-motor position error can be clearly observed. Fig. 8 depicts the tracking results in Cartesian space. Note that, even though the remote manipulator cannot go beyond 0.5m in the $y-$coordinate and even in the presence of variable time-delays, position tracking without contact is established. The resulting controller torques are shown in Fig. 9.

VI. Conclusions

This paper proposes a proportional plus damping controller that can achieve asymptotic convergence to zero of the local and remote position errors for nonlinear teleoperators with flexible joint manipulators. Without using strict

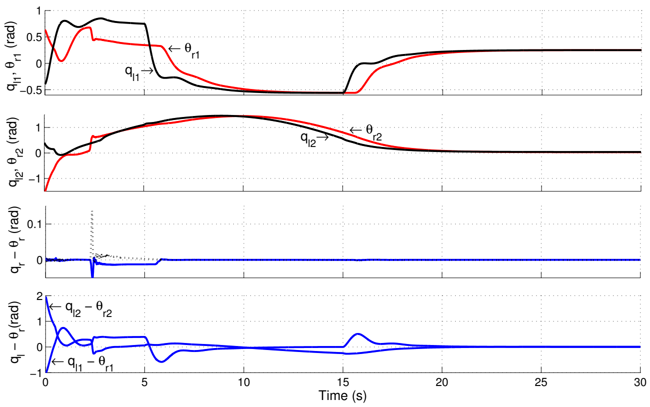

Fig. 7. Local and remote joint positions and position error when interacting with a stiff wall.

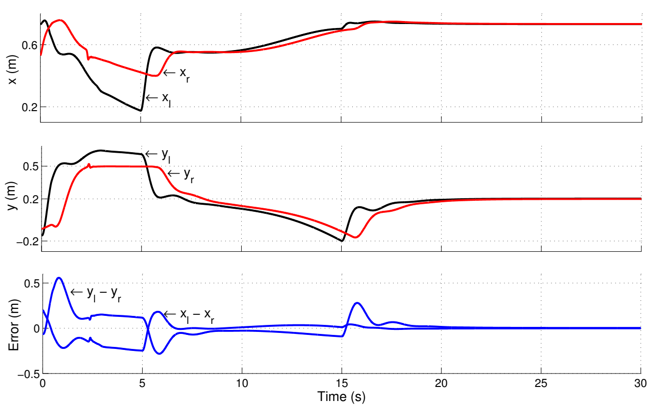

Fig. 8. Cartesian local and remote positions and Cartesian position error when interacting with a stiff wall.

Lyapunov functions but with the application of Barbalăt's Lemma and some signal bounding, convergence of position error and velocities to zero is established even in the presence of joint flexibility and variable time-delays. The theoretical results are supported by simulations with a rigid local manipulator and a flexible joint remote manipulator.

Future research includes the design of velocity observers together with position tracking controllers to give a solution to the teleoperators control when joint velocities are not measurable and will also include an evaluation of performance of the proposed schemes, see [18] for a solution to the observer-controller case for rigid manipulators.

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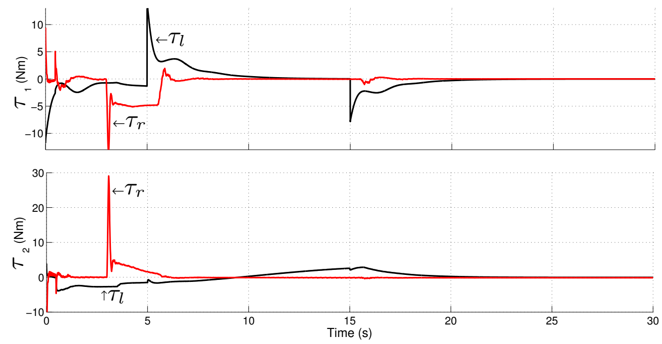

Fig. 9. Local and remote controller torques when interacting with a stiff wall.

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