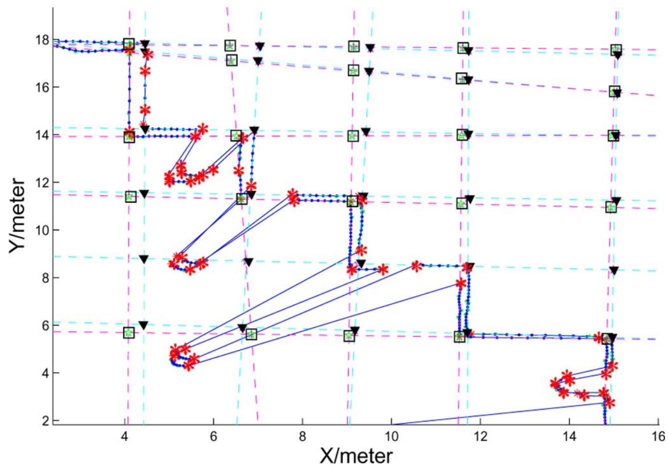

Fig. 11. Two successive outdoor range scans, smoothed and segmented by the line mask. The line intersection points (□) in the first scan, and inverted triangles (▼) in the second, can be used as occlusion-invariant, point features.

VII. Conclusion

Unlike RANSAC, the presented algorithm smoothes range data, to produce a local description of features, which, in some circumstances, can be more beneficial than global descriptions for robot navigation. Also, its computational complexity is independent of the number of model outliers, and is less affected by the use of higher order geometric models.

The smoothing and segmentation of range data is fundamentally different from that of image data. Structure preserving, and noise reduction algorithms in vision, use the local intensity gradient as a measure of noise. Range values are completely environment dependent, and not constant between features. Therefore, in this algorithm, the Mahalanobis distance between observed range values and their geometric-model-based predictions is used as the "measure of noise." A mask weighting function of the Mahalanobis distance was derived, which behaves as the diffusion coefficient in the anisotropic diffusion equation, often applied in vision, which guarantees that no new features are introduced with increase of scale. This mask can be applied iteratively, providing smoothing at different scales. The results demonstrated that the number of extracted features (lines or circles) converged to the true number with increase of scale, and the error between the extracted and true feature coordinates converged to a minimum. It has been shown that with increase of scale, the algorithm automatically reduces noise, only within the model-compliant regions of the range scans, yielding superior, postsmoothing, segmentation possibilities.

References

[1] P. Perona and J. Malik, "Scale-space and edge detection using anisotropic diffusion," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 12, no. 7, pp. 629–639, Jul. 1990.

[2] P. V. C. Hough, "Machine analysis of bubble chamber pictures," presented at the Int. Conf. High Energy Accelerators Instrum., Geneva, Switzerland, 1959.

[3] J. Forsberg, U. Larsson, and A. Wernersson, "Mobile robot navigation using the range-weighted Hough transform," *IEEE Robot. Autom. Mag.*, vol. 2, no. 1, pp. 18–26, Mar. 1995.

[4] R. H. T. Chan and P. K. S. Tam, "A new Hough transform based position estimation algorithm," *Intell. Inf. Syst.*, vol. 29, pp. 140–144, 1994.

[5] M. A. Fischler and R. C. Bolles, "Random sample consensus: A paradigm for model fitting with applications to image analysis and automated cartography," *Commun. Assoc. Comput. Mach.*, vol. 24, no. 6, pp. 381–395, Jun. 1981.

[6] R. Martinez-Cantin, J. A. Castellanos, J. D. Tardos, and J. M. M. Montiel, "Adaptive scale robust segmentation for 2D laser scanner," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, 2006, pp. 796–801.

[7] V. Nguyen, A. Martinelli, N. Tomatis, and R. Siegwart, "A comparison of line extraction algorithms using 2d laser rangefinders for indoor mobile robots," in *IEEE Int. Conf. Intell. Robots Syst.*, 2005, pp. 1929–1934.

[8] S. L. Horowitz and T. Pavlidis, "Picture segmentation by a tree traversal algorithm," *J. ACM*, vol. 23, no. 2, pp. 368–388, 1976.

[9] M. Adams. *Sensor Modelling, Design and Data Processing for Autonomous Navigation*. Singapore: World Scientific, 1999.

[10] S. Zhang, L. Xie, M. Adams, and F. Tang, "Geometrical feature extraction using 2d range scanners," presented at the Int. Conf. Cont. Autom., Montreal, Canada, Jun. 2003.

[11] S. I. Roumeliotis and G. A. Bekey, "Segments: A layered, dual-Kalman filter algorithm for indoor feature extraction," in *Proc. IEEE/RSJ Int. Conf. Intell. Robots Syst.*, 2000, pp. 454–461.

[12] C. Tomasi and R. Manduchi, "Bilateral filtering for gray and color images," in *Proc. Sixth Int. Conf. Comput. Vis.*, 1998, p. 839.

[13] D. Comaniciu and P. Meer-Mean, "Shift analysis and applications," in *Proc. IEEE Int. Conf. Comput. Vis.*, 1999, pp. 1197–1203.

[14] M. J. Black, G. Sapiro, D. H. Marimont, and D. Heeger, "Robust anisotropic diffusion," *IEEE Trans. Image Process.*, vol. 7, no. 3, pp. 421–432, Mar. 1998.

[15] F. Tang, M. Adams, J. Ibanez-Guzman, and W. S. Wijesoma, "Pose invariant, robust feature extraction from range data with a modified scale space approach," in *Proc. IEEE Int. Conf. Robot. Autom.*, 2004, pp. 3173–3179.

[16] Y. L. You, W. Xu, A. Tannenbaum, and M. Kaveh, "Behavioral analysis of anisotropic diffusion in image processing," *IEEE Trans. Image Process.*, vol. 5, no. 11, pp. 1539–1553, Nov. 1996.

[17] O. Chum and J. Matas, "Randomized RANSAC with Td,d Test," presented at the Br. Mach. Vis. Conf., Cardiff, U.K., Sep. 2002.

A Globally Stable PD Controller for Bilateral Teleoperators

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***Abstract*—In a recent scheme, with delayed derivative action [Lee and Spong, *IEEE Trans. Robot.*, vol. 22, no. 2, pp. 269–281, Apr. 2006], it is claimed that a simple proportional derivative (PD) scheme yields a stable operation. Unfortunately, the stability proof hinges upon unverifiable assumptions on the human and contact environment operators, namely, that they define $\mathcal{L}_\infty$–stable maps *from velocity to force*. In this short paper, we prove that it is indeed possible to achieve stable behavior with simple PD-like schemes—even without the delayed derivative action—under the classical assumption of passivity of the terminal operators.**

***Index Terms*—Bilateral teleoperation, communication delays, passivity, proportional derivative (PD) control.**

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I. Introduction

In bilateral teleoperation, the master and the slave manipulators are connected with a communication channel that often involves large distances or imposes limited data transfer between the local and the remote sites. Such situations can result in substantial delays between the time a command is introduced by the operator, and the time the command is executed by the remote robot. This time delay affects the overall stability of the system [2].

In their ground-breaking work [3], Anderson and Spong proposed to send the scattering signals to transform the transmission delays into a passive (virtual) transmission line. The transmission line is then interconnected with the master and slave robots, which define passive force to velocity operators, while the human operator and the contact environment constitute the terminations to the transmission line. Since power-preserving interconnection of passive systems is again passive, $\mathcal{L}_2$-stability of the overall system is ensured under the reasonable assumption that the human operator and the environment define passive (force to velocity) maps. This robust and physically appealing scheme has ever since dominated the field. See [4] for the application of the dual idea, that is, transform a real transmission line into pure delays, a classical problem of electrical systems.

One of the major drawbacks of the basic scattering/wave variable method is that there is no guarantee of *position coordination* (i.e., the position of the slave converges to the position of the master) in free motion. It is shown in [5] that this objective is achieved adding a term proportional to the delayed position error. In a recent publication [1], it is claimed that a simple PD-like, with delayed derivative action, that does not require scattering transformations, yields a stable operation including the position coordination. Unfortunately, the stability analysis hinges upon unverifiable assumptions on the human and contact environment operators, namely, that they define $\mathcal{L}_\infty$-stable maps *from velocity to force*. Notice that even the simplest scenario of a linear spring–damper system does not verify this assumption because the transfer function from velocity to force contains a derivative operator that is not $\mathcal{L}_\infty$-stable.

In a recent interesting paper [6], teleoperation is viewed as a synchronization problem, where the objective is to synchronize a weighted sum of velocities and positions of the master and slave, called $\mathbf{r}_m$ and $\mathbf{r}_s$, respectively. Scattering signals are avoided in this paper and an adaptive scheme for the master and slave manipulators is used instead. It is assumed in [6] that the human and the environment define passive maps from forces to the $\mathbf{r}_m$ and $\mathbf{r}_s$ signals. Although this assumption holds in simple scenarios, like linear spring–damper models, it is not clear how it is related to the original physically motivated condition of passivity of the force to velocity map.

In this paper, we prove that it is indeed possible to achieve stable behavior of nonlinear teleoperators with simple PD-like schemes under the classical assumption of passivity of the terminal operators provided sufficiently large damping is injected (via velocity feedback) to both manipulator subsystems. In this paper, we consider two schemes: controlling the master and the slave with the (delayed) position errors or the slave with position and the master with (delayed) force. In both cases, we prove that all signals remain bounded and that the velocities belong to $\mathcal{L}_2$ for any passive external interaction. Furthermore, velocities converge to zero if the forces applied by the human and the environment are bounded. We also prove that by adding gravity compensation (and a mild assumption on the inertia matrices) we achieve position coordination. Furthermore, we show that the bound that relates the controller gains and the injected damping is *tight*—in the sense that, if it is violated, the controller may induce an unstable behavior. As an immediate corollary of our research, we prove stability of the scheme proposed in [1]—even without the delayed derivative action.

II. Modeling The n-DOF Teleoperator System

The master and the slave are modeled as a pair of n-DOF serial links with revolute joints. Their corresponding nonlinear dynamics are described by

$$\begin{aligned}\mathbf{M}_m(\mathbf{q}_m)\ddot{\mathbf{q}}_m + \mathbf{C}_m(\mathbf{q}_m,\dot{\mathbf{q}}_m)\dot{\mathbf{q}}_m + \mathbf{g}_m(\mathbf{q}_m) &= \boldsymbol{\tau}_m - \boldsymbol{\tau}_h\\ \mathbf{M}_s(\mathbf{q}_s)\ddot{\mathbf{q}}_s + \mathbf{C}_s(\mathbf{q}_s,\dot{\mathbf{q}}_s)\dot{\mathbf{q}}_s + \mathbf{g}_s(\mathbf{q}_s) &= \boldsymbol{\tau}_e - \boldsymbol{\tau}_s \end{aligned}\tag{1}$$

where $i = m$ for the master and $i = s$ for the slave, and $\ddot{\mathbf{q}}_i, \dot{\mathbf{q}}_i, \mathbf{q}_i \in \mathbb{R}^n$ are the acceleration, velocity, and joint position, respectively. $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ are the inertia matrices, $\mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$ the Coriolis and centrifugal effects, defined via the Christoffel symbols of the first kind, $\mathbf{g}_i \in \mathbb{R}^n$ the vectors of gravitational forces, $\boldsymbol{\tau}_i \in \mathbb{R}^n$ are the control signals, and $\boldsymbol{\tau}_h \in \mathbb{R}^n$ and $\boldsymbol{\tau}_e \in \mathbb{R}^n$ are the forces exerted by the human operator and the environment interaction, respectively.

We use the following well-known properties of the dynamical model of robotic manipulators with rotational joints [7].

P1: $\dot{\mathbf{M}}_i(\mathbf{q}_i) = \mathbf{C}_i(\mathbf{q}_i,\dot{\mathbf{q}}_i) + \mathbf{C}_i^\top(\mathbf{q}_i,\dot{\mathbf{q}}_i)$.
P2: $\exists k_{u_i} \in \mathbb{R}_+$ such that $U_i(\mathbf{q}_i) \geq k_{u_i}$ where $U_i(\mathbf{q}_i)$ is the potential energy of the manipulator that satisfies $\mathbf{g}_i(\mathbf{q}_i) = \partial U_i(\mathbf{q}_i)/\partial \mathbf{q}_i$.
P3: $\exists \alpha_i, \beta_i \in \mathbb{R}_+$ such that $\alpha_i I \geq \mathbf{M}_i(\mathbf{q}_i) \geq \beta_i I$.
P4: For all $\mathbf{q}_i, \mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, $\exists k_{c_i} \in \mathbb{R}_+$ such that $|\mathbf{C}_i(\mathbf{q}_i,\mathbf{x})\mathbf{y}| \leq k_{c_i}|\mathbf{x}||\mathbf{y}|$, where $|\cdot|$ is the Euclidean norm.

We assume that the time delay imposed by the communication channel is constant and equal to $T \geq 0$. Also, following standard considerations, we assume that the human operator and the environment define passive (force to velocity) maps, that is, there exists $\kappa_i \in \mathbb{R}_+$ such that

$$\int_0^t \dot{\mathbf{q}}_m^\top(\sigma)\boldsymbol{\tau}_h(\sigma)d\sigma \geq -\kappa_m, \quad \int_0^t (-\dot{\mathbf{q}}_s^\top(\sigma))\boldsymbol{\tau}_e(\sigma)d\sigma \geq -\kappa_s \tag{2}$$

for all $t \geq 0$.

III. Control VIA Proportional Position Errors Plus Damping Injection

In this section, we assume that the forces applied on both sides are proportional to the position errors between the master and the slave plus a damping injection term. The control laws are then given by [1]

$$\begin{aligned}\boldsymbol{\tau}_m &= K_m[\mathbf{q}_s(t-T) - \mathbf{q}_m] - B_m\dot{\mathbf{q}}_m\\ \boldsymbol{\tau}_s &= K_s[\mathbf{q}_s - \mathbf{q}_m(t-T)] + B_s\dot{\mathbf{q}}_s\end{aligned}\tag{3}$$

where K_m, K_s, B_m, and B_s are positive constants. The use of scalar gains is made for simplicity; the case when these gains are positive definite, diagonal matrices can be treated with slight modifications to the proof.

Before giving the stability result, we present a lemma that will be instrumental for the analysis. The proof of the lemma is established with a direct application of Young's and Schwartz's inequalities and is given, in the Appendix for completeness.

Lemma 1: For any vector signals $\mathbf{x}, \mathbf{y}$ and any $T, \alpha > 0$, we have

$$2\int_0^t \mathbf{x}^\top(s)\left[\int_0^T \mathbf{y}(s-\sigma)d\sigma\right]ds \,\leq\, \alpha\|\mathbf{x}\|_2^2 + \frac{T^2}{\alpha}\|\mathbf{y}\|_2^2 \tag{4}$$

where $\|\cdot\|_2$ is the $\mathcal{L}_2$-norm of the signal.

[1]To avoid cluttering the notation, we will omit the argument of all time signals except for the case when it appears delayed.

Proposition 1: Consider the teleoperator system (1) controlled by (3) with $\boldsymbol{\tau}_h, \boldsymbol{\tau}_e$ verifying (2). Fix the damping injection and proportional gains such that

$$B_m B_s > T^2 K_m K_s. \tag{5}$$

Then, the following hold.

1) Velocities and position error are bounded (i.e., $\dot{\mathbf{q}}_i, \mathbf{q}_m - \mathbf{q}_s \in \mathcal{L}_\infty$), and moreover, $\dot{\mathbf{q}}_i \in \mathcal{L}_2$.
2) Assume additionally that
   A1) The human operator stands still and the slave robot is not in contact with the environment [i.e., $\boldsymbol{\tau}_h(t) \equiv 0$ and $\boldsymbol{\tau}_e(t) \equiv 0$].
   A2) A gravity compensation term is added to the controllers, that is,

$$\begin{aligned} \boldsymbol{\tau}_m &= K_m[\mathbf{q}_s(t-T) - \mathbf{q}_m] - B_m\dot{\mathbf{q}}_m + \mathbf{g}_m(\mathbf{q}_m) \\ \boldsymbol{\tau}_s &= K_s[\mathbf{q}_s - \mathbf{q}_m(t-T)] + B_s\dot{\mathbf{q}}_s - \mathbf{g}_s(\mathbf{q}_s). \end{aligned} \tag{6}$$

   A3) The terms $\partial^2 M_i^{jk}/\partial q_i^r \partial q_i^l$ are bounded.

Under these conditions, the master and slave velocities asymptotically converge to zero and position coordination is achieved, that is,

$$\lim_{t\to\infty} |\mathbf{q}_m(t) - \mathbf{q}_s(t-T)| = 0.$$

Proof: Consider the following nonnegative function

$$\begin{aligned} V(\mathbf{q}_i, \dot{\mathbf{q}}_i, t) = &\frac{1}{2}\dot{\mathbf{q}}_m^\top \mathbf{M}_m(\mathbf{q}_m)\dot{\mathbf{q}}_m + \frac{K_m}{2K_s}\dot{\mathbf{q}}_s^\top \mathbf{M}_s(\mathbf{q}_s)\dot{\mathbf{q}}_s + \\ &+ \frac{K_m}{2}|\mathbf{q}_m - \mathbf{q}_s|^2 + \int_0^t \left(\dot{\mathbf{q}}_m^\top \boldsymbol{\tau}_h - \frac{K_m}{K_s}\dot{\mathbf{q}}_s^\top \boldsymbol{\tau}_e\right)d\sigma + \\ &+ U_m(\mathbf{q}_m) + U_s(\mathbf{q}_s) - k_{u_m} - k_{u_s} + \kappa_m + \frac{K_m\kappa_s}{K_s}. \end{aligned} \tag{7}$$

Using (2) and the properties P1 and P2 of the robot manipulators, we obtain

$$\dot{V} = \dot{\mathbf{q}}_m^\top[\boldsymbol{\tau}_m + K_m(\mathbf{q}_m - \mathbf{q}_s)] - \frac{K_m}{K_s}\dot{\mathbf{q}}_s^\top[\boldsymbol{\tau}_s + K_s(\mathbf{q}_m - \mathbf{q}_s)]$$

substituting the control laws $\boldsymbol{\tau}_i$ and noting that

$$\mathbf{q}_i(t-T) - \mathbf{q}_i(t) = \int_0^T \dot{\mathbf{q}}_i(t-\sigma)d\sigma \tag{8}$$

we get

$$\begin{aligned} \frac{1}{K_m}\dot{V} = &-\frac{B_m}{K_m}|\dot{\mathbf{q}}_m|^2 - \frac{B_s}{K_s}|\dot{\mathbf{q}}_s|^2 + \\ &+ \dot{\mathbf{q}}_m^\top \int_0^T \dot{\mathbf{q}}_s(t-\sigma)d\sigma + \dot{\mathbf{q}}_s^\top \int_0^T \dot{\mathbf{q}}_m(t-\sigma)d\sigma. \end{aligned} \tag{9}$$

We will now invoke Lemma 1 to obtain a bound on the integral of $\dot{V}$. Toward this end, we integrate (9) from 0 to t and apply Lemma 1 to the third and fourth right-hand terms, yielding

$$\begin{aligned} V(t) - V(0) \le &-\left[B_m - \frac{K_m}{2}\left(\alpha_m + \frac{T^2}{\alpha_s}\right)\right]\|\dot{\mathbf{q}}_m\|_2^2 - \\ &- K_m\left[\frac{B_s}{K_s} - \frac{1}{2}\left(\alpha_s + \frac{T^2}{\alpha_m}\right)\right]\|\dot{\mathbf{q}}_s\|_2^2. \end{aligned} \tag{10}$$

Note that $B_m > K_m/2[\alpha_m + T^2/\alpha_s]$ and $B_s > K_s/2[\alpha_s + T^2/\alpha_m]$ have a positive solution for α_m and α_s if $B_m B_s > T^2 K_m K_s$. Thus, satisfying (5) with the nonnegativity of V proves that $\dot{\mathbf{q}}_i \in \mathcal{L}_2$. Furthermore, since V is bounded, from (7) and Property P3, we conclude that $\dot{\mathbf{q}}_i, \mathbf{q}_m - \mathbf{q}_s \in \mathcal{L}_\infty$, and part 1) of Proposition 1 is proved.

We now proceed to establish 2). First, we repeat the calculations done before with the new function

$$\begin{aligned} \tilde{V}(\mathbf{q}_i, \dot{\mathbf{q}}_i, t) = &\frac{1}{2}\dot{\mathbf{q}}_m^\top \mathbf{M}_m(\mathbf{q}_m)\dot{\mathbf{q}}_m + \frac{K_m}{2K_s}\dot{\mathbf{q}}_s^\top \mathbf{M}_s(\mathbf{q}_s)\dot{\mathbf{q}}_s + \\ &+ \frac{K_m}{2}|\mathbf{q}_m - \mathbf{q}_s|^2 + \int_0^t \left(\dot{\mathbf{q}}_m^\top \boldsymbol{\tau}_h - \frac{K_m}{K_s}\dot{\mathbf{q}}_s^\top \boldsymbol{\tau}_e\right)d\sigma + \\ &+ \kappa_m + \frac{K_m\kappa_s}{K_s} \end{aligned} \tag{11}$$

where we have removed the terms associated to the potential energy, which satisfies the bound (10). Then, we will prove that $\dot{\mathbf{q}}_i$ are uniformly continuous and, since they belong to $\mathcal{L}_2$, will converge to zero. Note that

$$\mathbf{q}_m - \mathbf{q}_s(t-T) = \mathbf{q}_m - \mathbf{q}_s + \mathbf{q}_s - \mathbf{q}_s(t-T) \tag{12}$$

and

$$\begin{aligned} \mathbf{q}_s - \mathbf{q}_s(t-T) = \int_0^T \dot{\mathbf{q}}_s(t-\sigma)d\sigma &\le \left(T\int_0^T |\dot{\mathbf{q}}_s(t-\sigma)|^2 d\sigma\right)^{1/2} \\ &\le T^{1/2}\|\dot{\mathbf{q}}_s\|_2 \end{aligned} \tag{13}$$

where the first bound is obtained applying Schwartz inequality. From (12) and (13), the facts that $\dot{\mathbf{q}}_s \in \mathcal{L}_2$ and $\mathbf{q}_m - \mathbf{q}_s \in \mathcal{L}_\infty$, we conclude that $\mathbf{q}_m - \mathbf{q}_s(t-T) \in \mathcal{L}_\infty$. Doing similar computations, we can show that $\mathbf{q}_m(t-T) - \mathbf{q}_s \in \mathcal{L}_\infty$.

Now, under Assumptions $A1$) and $A2$), the dynamics take the form

$$\begin{aligned} \ddot{\mathbf{q}}_m &= -\mathbf{M}_m^{-1}[(B_m + \mathbf{C}_m)\dot{\mathbf{q}}_m - K_m[\mathbf{q}_s(t-T) - \mathbf{q}_m]] \\ \ddot{\mathbf{q}}_s &= -\mathbf{M}_s^{-1}[(B_s + \mathbf{C}_s)\dot{\mathbf{q}}_s - K_s[\mathbf{q}_s - \mathbf{q}_m(t-T)]] \end{aligned} \tag{14}$$

where the arguments of $\mathbf{M}_i$ and $\mathbf{C}_i$ are omitted for simplicity. From the derivations earlier, and invoking Properties P3 and P4, we see that $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$, which together with $\dot{\mathbf{q}}_i \in \mathcal{L}_2 \cap \mathcal{L}_\infty$ proves the claim that $\dot{\mathbf{q}}_i \to 0$.

From (14) and convergence of speeds, we note that the claim of position coordination will be established, if we can prove that $\ddot{\mathbf{q}}_i \to 0$. Toward this end, we will prove uniform continuity of these signals and use Barbălat's lemma. Differentiating (14), we recover two types of terms: one consisting of $d/dt\,\mathbf{M}_i^{-1}$ times a bounded signal and the second one, the product of $\mathbf{M}_i^{-1}$ times the derivative of the term in brackets. For the first term, we have

$$d/dt\,\mathbf{M}_i^{-1} = -\mathbf{M}_i^{-1}\dot{\mathbf{M}}_i\mathbf{M}_i^{-1} = -\mathbf{M}_i^{-1}(\mathbf{C}_i + \mathbf{C}_i^\top)\mathbf{M}_i^{-1}$$

which is bounded because of Properties P3 and P4. The derivative of the term in brackets is also bounded under Assumption A.3.[2] Consequently, $d/dt\,\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$ and $\ddot{\mathbf{q}}_i$ are uniformly continuous. Because of continuity of these signals, the integral exists and is given by

$$\int_0^t \ddot{\mathbf{q}}_i(\sigma)d\sigma = \dot{\mathbf{q}}_i(t) - \dot{\mathbf{q}}_i(0).$$

Taking the limit as $t \to \infty$ and using the fact that $\dot{\mathbf{q}}_i \to 0$, we get $\int_0^\infty \ddot{\mathbf{q}}_i(\sigma)d\sigma = -\dot{\mathbf{q}}_i(0)$, which is clearly bounded. Barbălat's lemma

[2]We recall that the matrices $\mathbf{C}_i$ are defined using the Christoffel symbols. Therefore, Assumption A3 ensures that the terms $\partial^2 C_i^{jk}/\partial q_i^r \partial q_i^l$ are bounded.

then allows to conclude that $\ddot{\mathbf{q}}_i \to 0$ as required. This completes the proof. ■

The next proposition, which provides a partial converse to Proposition 1, shows that the bound on the gains (5) is tight.

Proposition 2: Consider Proposition 1, and assume that the damping injection and proportional gains of the controller (5) *do not satisfy* condition (5), that is,

$$B_m B_s < T^2 K_m K_s \tag{15}$$

where B_i, K_i are strictly positive. Then, for all $T > 0$, there exist teleoperator systems of the form (1) such that the closed-loop system is *unstable* for some values of B_i, K_i satisfying (15).

Proof: Consider a linear 1 DOF symmetric teleoperator of the form (1) without external interaction and controlled by (6). Assume, without loss of generality, that $M_m = M_s = 1$, the dynamics are then given by

$$\begin{aligned}\ddot{q}_m + B_m \dot{q}_m &= K_m (q_s(t-T) - q_m)\\ \ddot{q}_s + B_s \dot{q}_s &= K_s (q_m(t-T) - q_s).\end{aligned}$$

Take $B_s = B_m$ and $K_s = K_m$. The characteristic quasi-polynomial of the system contains the factor

$$s^2 + B_m s + K_m + K_m e^{-sT} = 0. \tag{16}$$

From [8], we get the following necessary condition for stability of the quasi-polynomial (16)

$$\frac{K_m - B_m^2}{B_m \sqrt{2K_m - B_m^2}} \tan\left(T\sqrt{2K_m - B_m^2}\right) \leq 1. \tag{17}$$

Fix $T > 0$ and let us look at a line in the plane $K_m - B_m$ defined by $B_m = \ell T K_m$ with the new parameter $\ell \in (0,1)$, which is contained in the region defined by (15). Evaluating (17) on this line yields the condition $f_{T,\ell}(K_m) \leq 1$, where

$$f_{T,\ell}(K_m) = \frac{1 - \ell^2 T^2 K_m}{\ell T \sqrt{2K_m - \ell^2 T^2 K_m^2}} \tan\left(T\sqrt{2K_m - \ell^2 T^2 K_m^2}\right).$$

Applying L'Hopital's rule, it is possible to show that

$$\lim_{K_m \to 0} f_{T,\ell}(K_m) = \frac{1}{\ell}.$$

Due to the fact that $\ell \in (0,1)$, and from continuity of $f_{T,\ell}(K_m)$ in a neighborhood of zero, $f_{T,\ell}(K_m) > 1$ for all sufficiently small K_m. Hence, for all T, there exists (sufficiently small) K_m and B_m, satisfying (15), such that the system will be unstable. ■

A corollary of Proposition 1 is the proof that we can add a delayed derivative error term to (3), which is the controller proposed in [1], without modifying its stability properties. The proof follows *verbatim* the steps of the proof of Proposition 1—with the same nonnegative function (7). From the evaluation of the time derivative of this function, no clear advantage for the added complication is, however, observed.

Corollary 1: Proposition 1 holds if the controller (3) is replaced by

$$\begin{aligned}\boldsymbol{\tau}_m &= K_{dm}[\dot{\mathbf{q}}_s(t-T) - \dot{\mathbf{q}}_m] + K_m[\mathbf{q}_s(t-T) - \mathbf{q}_m] - B_m \dot{\mathbf{q}}_m\\ \boldsymbol{\tau}_s &= K_{ds}[\dot{\mathbf{q}}_s - \dot{\mathbf{q}}_m(t-T)] + K_s[\mathbf{q}_s - \mathbf{q}_m(t-T)] + B_s \dot{\mathbf{q}}_s\end{aligned} \tag{18}$$

where K_{dm}, K_{ds} are arbitrary nonnegative constants and K_m, K_s, B_m, B_s satisfy condition (5).

IV. Control via Proportional Position Error and Force Feedback Plus Damping Injection

In this section, we prove that we can also control the teleoperator reflecting the force generated at the slave to the master while controlling the slave with a proportional position error term—provided damping is injected to both manipulators.

Proposition 3: Consider the teleoperator system (1) controlled by

$$\begin{aligned}\boldsymbol{\tau}_m &= \boldsymbol{\tau}_s(t-T) - B_m \dot{\mathbf{q}}_m\\ \boldsymbol{\tau}_s &= K_s[\mathbf{q}_s - \mathbf{q}_m(t-T)] + B_s \dot{\mathbf{q}}_s.\end{aligned} \tag{19}$$

Fix the damping injection and proportional gains such that

$$\frac{B_m}{K_s} > T^2\left(3 + \frac{1}{2\epsilon}\right) + \epsilon + 1, \qquad \frac{B_s}{K_s} > T^2 + \epsilon \tag{20}$$

for some $\epsilon > 0$.

1) Velocities and position error are bounded (i.e., $\dot{\mathbf{q}}_{\mathrm{i}}, \mathbf{q}_{\mathrm{m}} - \mathbf{q}_{\mathrm{s}} \in \mathcal{L}_\infty$), and moreover, $\dot{\mathbf{q}}_i \in \mathcal{L}_2$.
2) Under the Assumptions $A1$), $A2$), and $A3$) of Proposition 1, the master, and slave velocities asymptotically converge to zero and position coordination is achieved, that is,

$$\lim_{t\to\infty} |\mathbf{q}_m(t) - \mathbf{q}_s(t-T)| = 0.$$

Proof: Let us propose the following nonnegative function

$$\begin{aligned}V(\mathbf{q}_i, \dot{\mathbf{q}}_i, t) =& \frac{1}{2}\dot{\mathbf{q}}_m^\top \mathbf{M}_m(\mathbf{q}_m)\dot{\mathbf{q}}_m + \frac{1}{2}\dot{\mathbf{q}}_s^\top \mathbf{M}_s(\mathbf{q}_s)\dot{\mathbf{q}}_s +\\ &+ \frac{K_s}{2}|(\mathbf{q}_m - \mathbf{q}_s)|^2 + \int_0^t (\dot{\mathbf{q}}_m^\top \boldsymbol{\tau}_h - \dot{\mathbf{q}}_s^\top \boldsymbol{\tau}_e)d\sigma +\\ &+ U_m(\mathbf{q}_m) + U_s(\mathbf{q}_s) - k_{u_m} - k_{u_s} + \kappa_m + \kappa_s.\end{aligned}$$

Using (2) and the properties P1, P2 of the robot manipulators, and evaluating $\dot{V}$ along the system trajectories, we obtain

$$\dot{V} = \dot{\mathbf{q}}_m^\top[\boldsymbol{\tau}_m + K_s(\mathbf{q}_m - \mathbf{q}_s)] - \dot{\mathbf{q}}_s^\top[\boldsymbol{\tau}_s + K_s(\mathbf{q}_m - \mathbf{q}_s)]$$

substituting the control laws (19), we get

$$\begin{aligned}\frac{1}{K_s}\dot{V} =& -\frac{B_m}{K_s}|\dot{\mathbf{q}}_m|^2 - \frac{B_s}{K_s}|\dot{\mathbf{q}}_s|^2 - \dot{\mathbf{q}}_m^\top[\mathbf{q}_s - \mathbf{q}_s(t-T)] +\\ &+ \dot{\mathbf{q}}_m^\top[\mathbf{q}_m - \mathbf{q}_m(t-2T)] - \dot{\mathbf{q}}_s^\top[\mathbf{q}_m - \mathbf{q}_m(t-T)] +\\ &+ \frac{B_s}{K_s}\dot{\mathbf{q}}_m^\top \dot{\mathbf{q}}_s(t-T).\end{aligned}$$

Now, we use (8) to replace the inner products with the terms in brackets by their integrals. Then, we apply the bound

$$2\dot{\mathbf{q}}_m^\top \dot{\mathbf{q}}_s(t-T) \leq |\dot{\mathbf{q}}_m|^2 + |\dot{\mathbf{q}}_s(t-T)|^2$$

to the last right-hand term, integrate $\dot{V}$, and invoke Lemma 1 with the constant $\alpha = 1$ for all the terms except the fourth right-hand term, which we take equal to ϵ. This yields

$$\begin{aligned}\frac{1}{K_s}[V(t) - V(0)] \leq& -\left[\frac{B_m}{K_s} - \frac{B_s}{2K_s} - 1 - T^2\left(2 + \frac{1}{2\epsilon}\right)\right]\|\dot{\mathbf{q}}_m\|_2^2 -\\ &- \frac{1}{2}\left(\frac{B_s}{K_s} - T^2 - \epsilon\right)\|\dot{\mathbf{q}}_s\|_2^2.\end{aligned}$$

It is easy to show that condition (20) ensures that both terms in parenthesis on the right-hand side of the inequality are positive; hence, nonnegativity of V proves that $\dot{\mathbf{q}}_i \in \mathcal{L}_2$. Taking

$$\begin{aligned}\ddot{\mathbf{q}}_m &= -\mathbf{M}_m^{-1}\left[(B_m + \mathbf{C}_m)\dot{\mathbf{q}}_m - \boldsymbol{\tau}_s(t-T)\right]\\ \ddot{\mathbf{q}}_s &= -\mathbf{M}_s^{-1}\left[(B_s + \mathbf{C}_s)\dot{\mathbf{q}}_s - K_s(\mathbf{q}_m(t-T) - \mathbf{q}_s)\right]\end{aligned}$$

the rest of the proof follows *verbatim* the steps of the proof of Proposition 1.

V. Additional Remarks

After this paper had been completed, we became aware of the recent work [9]. The authors consider a *symmetric* teleoperator, and propose a modification to the scattering-based scheme of [5] to overcome the need of adding a (twice delayed) term $\dot{\mathbf{q}}_s(t-2T)$ in the slave robots force. Mimicking the derivations of the stability proof in [5], it is claimed that the closed-loop system is Lyapunov stable, and that velocities and velocity errors asymptotically converge to zero. Unfortunately, their argument is based on the erroneous conclusion that the Lyapunov-like function used in their calculations [$V(x)$ in their notation] qualifies as a *bona fide* Lyapunov function even though—as in [5] and the present paper—it is only shown to satisfy that its integral from 0 to t is nonpositive, from which we cannot conclude that the derivative is nonpositive too. It is also claimed in [9] that their controller imposes no restriction on the damping injection but this also seems to be unsubstantiated as their critical condition (23) exactly coincides with the one given in [5], namely (5) in this paper, that we proved is *necessary*. Interestingly, the resulting controller scheme exactly coincides with (18) (if we set $K_m = K_s, B_m = B_s, K_{dm} = K_{ds}$), establishing a very interesting connection between PD and scattering-based controllers. It is worth mentioning that the Lyapunov-like functions of [9] and the one used in the present paper [7] are not the same. The former contains an additional term that brings along in the derivative a negative square of the velocity errors [9, eq. (16)].

It may be argued that control schemes that rely on the injection of damping may yield poor performance, particularly in teleoperation, where transparency is a major issue to be addressed. A major challenge for this study is the precise definition of "performance" in this (nonstandard) control application. See [10] and [11] for some results along these lines. The idea that damping injection should dominate the proportional gain has been mentioned for linear systems in [12].

VI. Simulations

In order to show the effectiveness of the proposed scheme, we present in this section some simulations, in which the local and remote manipulators are modeled as a pair of 2 DOF serial links. Their corresponding nonlinear dynamics follow (1). The inertia matrix $\mathbf{M}_i(\mathbf{q}_i)$ is given by

$$\mathbf{M}_i(\mathbf{q}_i) = \begin{bmatrix} \alpha_i + 2\beta_i \cos(q_{2_i}) & \delta_i + \beta_i \cos(q_{2_i}) \\ \delta_i + \beta_i \cos(q_{2_i}) & \delta_i \end{bmatrix} \tag{21}$$

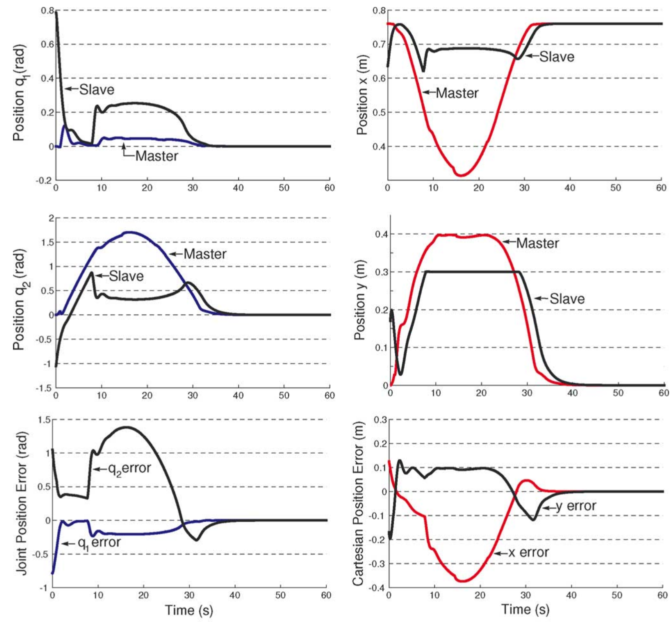
where q_{k_i} is the articular position of each link with $k \in \{1, 2\}$, $\alpha_i = l_{2_i}^2 m_{2_i} + l_{1_i}^2(m_{1_i} + m_{2_i})$, $\beta_i = l_{1_i} l_{2_i} m_{2_i}$, and $\delta_i = l_{2_i}^2 m_{2_i}$. The lengths for both links l_{1_i} and l_{2_i} in each manipulator are 0.38 m. The masses for each link correspond to $m_{1_l} = 3.9473$ kg, $m_{2_l} = 0.6232$ kg, $m_{1_r} = 3.2409$ kg, and $m_{2_r} = 0.3185$ kg, respectively. These values are the same as used in [1]. Coriolis and centrifugal forces are modeled as the vector $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i$, which are

$$\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i = \begin{bmatrix} -\beta_i \sin(q_{2_i})\dot{q}_{2_i}^2 - \beta_i \sin(q_{2_i})\dot{q}_{1_i}\dot{q}_{2_i} \\ \beta_i \sin(q_{2_i})\dot{q}_{1_i}^2 \end{bmatrix}$$

where $\dot{q}_{1_i}$ and $\dot{q}_{2_i}$ are the respective revolute velocities of the two links. The gravity effects for each manipulator are represented by

$$\mathbf{g}_i(\mathbf{q}_i) = \begin{bmatrix} \frac{1}{l_{2_i}} g\delta_i \cos(q_{1_i} + q_{2_i}) + \frac{1}{l_{1_i}}(\alpha_i - \delta_i)\cos(q_{1_i}) \\ \frac{1}{l_{2_i}} g\delta_i \cos(q_{1_i} + q_{2_i}) \end{bmatrix}.$$

It should be clarified that the human exerts a force on the local manipulator's tip, and the remote manipulator interaction with the environment is also measured at its tip. Hence, for the simulations, the following expressions are used $\boldsymbol{\tau}_h = \mathbf{J}_l^\top(\mathbf{q}_l)\mathbf{f}_h$ and $\boldsymbol{\tau}_e = \mathbf{J}_r^\top(\mathbf{q}_r)\mathbf{f}_e$, where $\mathbf{J}_i(\mathbf{q}_i)$ is the Jacobian of the robot manipulator. The simulations have been carried out using MatLab SimuLink.



(a) (b)

Fig. 1. Simulations for the teleoperator controlled by the P controller (3) with $T = 0.7$ s.

The controllers for these simulations are given by (3) and its gains by $K_m = 1.3$, $K_s = 1.7$, $B_m = 1.1$, and $B_s = 2.1$, which clearly fulfill (6). The time delay was set to $T = 0.7$ s. In order to evaluate the stability of the proposed scheme, a high stiff wall (20 000 N/m) at the cartesian coordinate, $y = 0.3$ m, has been included in the environment. The initial positions for the master and the slave differ one from the other, i.e., $\mathbf{q}_m(0) = [0, 0]^T$ and $\mathbf{q}_s(0) = [1/4\pi, -1/3\pi]^T$.

Fig. 1 shows the simulation results, which are composed of the joint (a) and cartesian (b) space measures. Analyzing the plots, we can clearly see that the master and slave initial positions are different; the slave reaches the high stiff wall, located at $y = 0.3$ m, around 8 s and leaves it at 29 s; and, around 40 s, the position error converges to 0. The simulation of the teleoperator with the control laws (18) and (19) have not been reported here because their results are similar to the system response in Fig. 1.

VII. Conclusion

We have shown in this paper that it is possible to control a bilateral teleoperator with simple PD-like schemes—obviating the need for scattering transformations and passivity considerations. As shown in the proofs, the key ingredient is the inclusion of damping that should "dominate" the proportional gains—see (5) and (20)—to ensure that the velocities are in $\mathcal{L}_2$. Adding delayed damping, as proposed in [1], has also been explored but the analysis does not reveal any advantage for this new term. It should be noted that the key property of square integrability of the velocities is given in a secondary lemma in the calculations of [1], while the main proposition seems to implicitly assume this property to ensure that they admit a Fourier transform (in the infinite time horizon). In any case, the present paper proves that the main claim of [1] is correct without the assumption of finite gain of the operators $\dot{\mathbf{q}}_m \to \boldsymbol{\tau}_h$ and $\dot{\mathbf{q}}_s \to \boldsymbol{\tau}_e$, which is explicitly required in Lemma 1. We also performed some simulations that validate the theoretical results of this paper. Encouraging experimental results are currently under way and will be reported in the near future.

APPENDIX

Proof of Lemma 1. For any vector signals $\mathbf{a}, \mathbf{b}$, Schwartz's inequality is defined as

$$\int_0^t \mathbf{a}^\top(s)\mathbf{b}(s)ds \le \left[\int_0^t |\mathbf{a}(s)|^2 ds\right]^{1/2} \left[\int_0^t |\mathbf{b}(s)|^2 ds\right]^{1/2}.$$

On the other hand, Young's inequality for two positive numbers c, d and any $\alpha > 0$ is defined as $2cd \le \alpha c^2 + 1/\alpha\, d^2$. Now, to prove the inequality in Lemma 1, we apply to the left-hand term of the inequality (4) first, Schwartz's [with $\mathbf{a}(s) = \mathbf{x}(s)$ and $\mathbf{b}(s) = \int_0^T \mathbf{y}(s-\sigma)d\sigma$] and then Young's inequalities, yielding

$$2\int_0^t \mathbf{x}^\top(s)\int_0^T \mathbf{y}(s-\sigma)d\sigma ds \le \alpha \int_0^t |\mathbf{x}(s)|^2 ds + \frac{1}{\alpha}\int_0^t \left|\int_0^T \mathbf{y}(s-\sigma)d\sigma\right|^2 ds. \tag{22}$$

We will center our attention on finding a bound on the second right-hand term of (22). Notice that

$$\left|\int_0^T \mathbf{y}(s-\sigma)d\sigma\right|^2 = \sum_{j=1}^n \left[\int_0^T y_j(s-\sigma)d\sigma\right]^2$$

where y_j are the elements of the vector $\mathbf{y}$. Applying Schwartz's with $a(s) = 1$ and $b(s) = y_j(s-\sigma)$, we get

$$\sum_{j=1}^n \left[\int_0^T y_j(s-\sigma)d\sigma\right]^2 \le T\sum_{j=1}^n \int_0^T y_j^2(s-\sigma)d\sigma$$

which is nothing but $T\int_0^T |\mathbf{y}(s-\sigma)|^2 d\sigma$. Hence,

$$\left|\int_0^T \mathbf{y}(s-\sigma)d\sigma\right|^2 \le T\int_0^T |\mathbf{y}(s-\sigma)|^2 d\sigma. \tag{23}$$

Replacing the bound (23) on the second right-hand term of (22) and inverting the integration order, yields $T/\alpha \int_0^T \int_0^t |\mathbf{y}(s-\sigma)|^2 ds d\sigma$.

Finally, note that $\int_0^t |\mathbf{y}(s-\sigma)|^2 ds \le \int_0^t |\mathbf{y}(s)|^2 ds$, due to the fact that we are eliminating some area of the signal for the interval $[-\sigma, 0]$ on the integral on the left. Hence, rewriting

$$\frac{T}{\alpha}\int_0^T \int_0^t |\mathbf{y}(s-\sigma)|^2 ds\, d\sigma \le \frac{T}{\alpha}\int_0^T \left[\int_0^t |\mathbf{y}(s)|^2 ds\right] d\sigma.$$

Note that the integral in brackets does not depend on the integration variable σ. Thus, the bound for the second right term of (22) will finally be given by $T^2/\alpha \int_0^t |\mathbf{y}(s)|^2 ds$. Substituting the bound on (22), we get

$$2\int_0^t \mathbf{x}^\top(s)\int_0^T \mathbf{y}(s-\sigma)d\sigma\, ds \le \alpha\int_0^t |\mathbf{x}(s)|^2 ds + \frac{T^2}{\alpha}\int_0^t |\mathbf{y}(s)|^2\, ds$$

from which we immediately get (4). ■

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REFERENCES

[1] D. Lee and M. Spong, "Passive bilateral teleoperation with constant time delay," *IEEE Trans. Robot.*, vol. 22, no. 2, pp. 269–281, Apr. 2006.

[2] T. Sheridan, "Space teleoperation through time delay: Review and prognosis," *IEEE Trans. Robot. Autom.*, vol. 9, no. 5, pp. 592–606, Oct. 1993.

[3] R. Anderson and M. Spong, "Bilateral control of teleoperators with time delay," *IEEE Trans. Autom. Control*, vol. 34, no. 5, pp. 494–501, May 1989.

[4] A. de Rinaldis, R. Ortega, and M. Spong, "A compensator for attenuation of wave reflections in long cable actuator–plant interconnections with guaranteed stability," *Automatica*, vol. 42, no. 10, pp. 1621–1635, Oct. 2006.

[5] N. Chopra, M. Spong, R. Ortega, and N. Barbanov, "On tracking preformance in bilateral teleoperation," *IEEE Trans. Robot.*, vol. 22, no. 4, pp. 844–847, Aug. 2006.

[6] N. Chopra and M. Spong, "On synchronization of networked passive systems with time delays and application to bilateral teleoperation," in *Proc. IEEE/SICE Int. Conf. Instrum., Control Inf. Technol.*, 2005, pp. 3424–3429.

[7] R. Kelly, V. Santibáñez, and A. Loría, *Control of Robot Manipulators in Joint Space* (Advanced Textbooks in Control and Signal Processing). New York: Springer-Verlag, 2005.

[8] E. Malakhovski and L. Mirkin, "On stability of second-order quasi-polynomials with a single delay," *Automatica*, vol. 42, no. 6, pp. 1041–1047, Jun. 2006.

[9] T. Namerikawa and H. Kawada, "Symmetric impedance matched teleoperation with position tracking," in *Proc. 45th IEEE Conf. Decis. Control*, San Diego, CA, Dec. 2006, pp. 4496–4501.

[10] P. Arcara and C. Melchiorri, "Control schemes for teleoperation with time delay: A comparative study," *Robot. Autonom. Syst.*, vol. 38, pp. 49–64, 2002.

[11] P. Hokayem and M. Spong, "Bilateral teleoperation: An historical survey," *Automatica*, vol. 42, pp. 2035–2057, 2006.

[12] J. Artigas, J. Vilanova, C. Preusche, and G. Hirzinger, "Time domain passivity control-based telepresence with time delay," in *Proc. IEEE Int. Conf. Intell. Robots Syst.*, 2006, pp. 4205–4210.