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Brief paper

An adaptive controller for nonlinear teleoperators[☆]

Emmanuel Nuño[a,b,*], Romeo Ortega[c], Luis Basañez[b]

[a] Department of Computer Science, University of Guadalajara, Av. Revolución 1500, 44430 Guadalajara, Mexico

[b] Institute of Industrial and Control Engineering, Technical University of Catalonia, Av. Diagonal 647, 08028 Barcelona, Spain

[c] Laboratoire des Signaux et Systèmes, SUPÉLEC, Plateau de Moulon, 91190 Gif-sur-Yvette, France

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ABSTRACT

In Chopra et al (2008) [Chopra, N., Spong, M. W., & Lozano, R. (2008). Synchronization of bilateral teleoperators with time-delay. Automatica, 44(8), 2142–2148], an adaptive controller for teleoperators with time-delays, which ensures synchronization of positions and velocities of the master and slave manipulators, and does not rely on the use of the ubiquitous scattering transformation, is proposed. In this paper it is shown that this controller will tend to drive to zero the positions of the joints where gravity forces are non-zero. Hence, the scheme is, in general, applicable only to systems without gravity. We also prove in this paper that this limitation can be obviated, replacing the positions and velocities – that are used in the coordinating torques and the adaptation laws – by their *errors*. Simulation results illustrate the performance of both schemes.

1. Introduction

A major breakthrough in the problem of control of bilateral teleoperators, with guaranteed stability properties, was the use of scattering signals to transform the transmission delays into a passive transmission line. Under the reasonable assumption that the human operator and the contact environment define passive (force to velocity) maps, stability of the overall system is then ensured (Anderson & Spong, 1989; Niemeyer & Slotine, 1991). However, most of the scattering-based approaches cannot ensure position tracking. PD-like schemes that overcome this obstacle have been reported in Nuño, Ortega, Barabanov, and Basañez (2008), Nuño, Basañez, Ortega, and Spong (2009). Chopra and Spong (2006) proposed to formulate the teleoperation problem in terms of synchronization, which also avoids the scattering transformation. An adaptive version of this scheme that aims at synchronizing the local and remote positions and velocities is proposed in Chopra, Spong, and Lozano (2008).

The present paper presents an extension to general nonlinear teleoperators of the adaptive controller of Chopra et al. (2008). First, it is proved that Chopra's scheme is, in general, only applicable to systems without gravity, and, in order to overcome this obstacle, a new adaptive controller is proposed. The main, simple but essential, difference between the proposed controller and the one in Chopra et al. (2008) is the use of the position and velocity *errors* in the coordinating torques and in the robot dynamics parametrization—and, consequently, in the adaptation laws. The new adaptive controller ensures asymptotic convergence of position errors and velocities to zero.

Notation. $\mathbb{R} := (-\infty, \infty)$, $\mathbb{R}_{>0} := (0, \infty)$, $\mathbb{R}_{\geq 0} := [0, \infty)$. $\lambda_m\{\mathbf{A}\}$ and $\lambda_M\{\mathbf{A}\}$ represent the minimum and maximum eigenvalues of matrix $\mathbf{A}$, respectively. $|\mathbf{x}|$ stands for the standard Euclidean norm of vector $\mathbf{x}$. For any function $\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n$, the $\mathcal{L}_\infty$-norm is defined as $\|\mathbf{f}\|_\infty = \sup_{t\geq 0} |\mathbf{f}(t)|$, and the $\mathcal{L}_2$-norm as $\|\mathbf{f}\|_2^2 = \int_0^\infty |\mathbf{f}(t)|^2 \mathrm{d}t$. The $\mathcal{L}_\infty$ and $\mathcal{L}_2$ spaces are defined as the sets $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_\infty < \infty\}$ and $\{\mathbf{f} : \mathbb{R}_{\geq 0} \to \mathbb{R}^n : \|\mathbf{f}\|_2 < \infty\}$, respectively.

1.1. Dynamic model of the teleoperator

The local and remote manipulators are modeled as a pair of n-Degrees of Freedom (DOF) serial links. Their corresponding nonlinear dynamics, together with the human operator and environment interactions, are given by

$$\begin{aligned}
&\mathbf{M}_l(\mathbf{q}_l)\ddot{\mathbf{q}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\dot{\mathbf{q}}_l + \mathbf{g}_l(\mathbf{q}_l) = \boldsymbol{\tau}_h - \boldsymbol{\tau}_l \\
&\mathbf{M}_r(\mathbf{q}_r)\ddot{\mathbf{q}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\dot{\mathbf{q}}_r + \mathbf{g}_r(\mathbf{q}_r) = \boldsymbol{\tau}_r - \boldsymbol{\tau}_e,
\end{aligned} \tag{1}$$

where $\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i \in \mathbb{R}^n$ are the joint positions, velocities and accelerations; $\mathbf{M}_i(\mathbf{q}_i) \in \mathbb{R}^{n\times n}$ are the inertia matrices; $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) \in \mathbb{R}^{n\times n}$

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* Corresponding author at: Institute of Industrial and Control Engineering, Technical University of Catalonia, Av. Diagonal 647, 08028 Barcelona, Spain. Tel.: +34 93 401 2597, +34 934011706; fax: +34 93 401 6605, +34 934016605.

E-mail addresses: emmanuel.nuno@upc.edu (E. Nuño), ortega@lss.supelec.fr (R. Ortega), luis.basanez@upc.edu (L. Basañez).

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are the Coriolis and centrifugal effects, which are defined using the Christoffel symbols of the first kind; $\mathbf{g}_i(\mathbf{q}_i) \in \mathbb{R}^n$ are the gravitational forces; $\boldsymbol{\tau}_i \in \mathbb{R}^n$ are the control signals; and $\boldsymbol{\tau}_h \in \mathbb{R}^n$, $\boldsymbol{\tau}_e \in \mathbb{R}^n$ are the forces exerted by the human and the environment. The subscript i stands for l or r, which denote local or remote robot manipulators, respectively. It is assumed that the manipulators are composed by actuated revolute joints and that friction can be neglected.

These dynamical models have some important well-known properties (Kelly, Santibáñez, & Loria, 2005; Spong, Hutchinson, & Vidyasagar, 2005).

P1. The inertia matrix is lower and upper bounded, i.e., $0 < \lambda_m\{\mathbf{M}_i\}\mathbf{I} \leq \mathbf{M}_i(\mathbf{q}_i) \leq \lambda_M\{\mathbf{M}_i\}\mathbf{I} < \infty$.
P2. The Coriolis and inertia matrices are related as $\dot{\mathbf{M}}_i(\mathbf{q}_i) = \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i) + \mathbf{C}_i^\top(\mathbf{q}_i, \dot{\mathbf{q}}_i)$.
P3. The Coriolis forces are bounded as $\forall \mathbf{q}_i, \dot{\mathbf{q}}_i \in \mathbb{R}^n \exists k_{c_i} \in \mathbb{R}_{>0}$ such that $|\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i| \leq k_{c_i}|\dot{\mathbf{q}}_i|^2$.
P4. The dynamics are linearly parameterizable. Thus, $\mathbf{M}_i(\mathbf{q}_i)\ddot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\dot{\mathbf{q}}_i + \mathbf{g}_i(\mathbf{q}_i) = \mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i)\boldsymbol{\theta}_i$ where $\mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \ddot{\mathbf{q}}_i) \in \mathbb{R}^{n\times p}$ are matrices of known functions and $\boldsymbol{\theta}_i \in \mathbb{R}^p$ are constant vectors of the manipulator physical parameters (link masses, moments of inertia, etc.).

2. Previous results

In this section we briefly review the results reported in Chopra et al. (2008) and point out the constraint it imposes on the gravity.

2.1. A synchronization result

Let $\mathbf{e}_i \in \mathbb{R}^n$ denote the position error vectors, defined, for a constant time-delay T, by

$$\mathbf{e}_l = \mathbf{q}_r(t-T) - \mathbf{q}_l; \qquad \mathbf{e}_r = \mathbf{q}_l(t-T) - \mathbf{q}_r. \tag{2}$$

The control objective of Chopra et al. (2008) is to drive the coordination errors, $\mathbf{e}_i, \dot{\mathbf{e}}_i$, to zero independently of the constant time-delay T and without using the scattering transformation. In this case, it is said that the manipulators synchronize. In order to achieve such objective the following adaptive controllers are proposed

$$\begin{aligned} \boldsymbol{\tau}_l &= \hat{\mathbf{M}}_l(\mathbf{q}_l)\lambda\dot{\mathbf{q}}_l + \hat{\mathbf{C}}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\lambda\mathbf{q}_l - \hat{\mathbf{g}}_l(\mathbf{q}_l) - \bar{\boldsymbol{\tau}}_l \\ \boldsymbol{\tau}_r &= \bar{\boldsymbol{\tau}}_r - \hat{\mathbf{M}}_r(\mathbf{q}_r)\lambda\dot{\mathbf{q}}_r - \hat{\mathbf{C}}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\lambda\mathbf{q}_r + \hat{\mathbf{g}}_r(\mathbf{q}_r) \end{aligned} \tag{3}$$

where $\hat{\mathbf{M}}_i, \hat{\mathbf{C}}_i, \hat{\mathbf{g}}_i$ are the estimates of the inertia and Coriolis matrices and the gravity forces, respectively. $\lambda \in \mathbb{R}_{>0}$,[1] and $\bar{\boldsymbol{\tau}}_i$ are the coordinating torques given by

$$\bar{\boldsymbol{\tau}}_l = K[\mathbf{s}_r(t-T) - \mathbf{s}_l]; \qquad \bar{\boldsymbol{\tau}}_r = K[\mathbf{s}_l(t-T) - \mathbf{s}_r] \tag{4}$$

where $K \in \mathbb{R}_{>0}$. The *synchronization signals* are defined as

$$\mathbf{s}_i = \dot{\mathbf{q}}_i + \lambda\mathbf{q}_i, \tag{5}$$

which is similar to the Slotine–Li variable without the desired trajectory $\mathbf{q}_d, \dot{\mathbf{q}}_d$ (Ortega & Spong, 1989). From P4, the controllers (3) can be also written as $\boldsymbol{\tau}_l = \mathbf{Y}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\hat{\boldsymbol{\theta}}_l - \bar{\boldsymbol{\tau}}_l$ and $\boldsymbol{\tau}_r = \bar{\boldsymbol{\tau}}_r - \mathbf{Y}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\hat{\boldsymbol{\theta}}_r$, where $\mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ are matrices of known functions, and $\hat{\boldsymbol{\theta}}_i$ are the physical estimated parameters. Replacing (3) in (1) and adding the term

$$\mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\boldsymbol{\theta}_i = \mathbf{M}_i(\mathbf{q}_i)\lambda\dot{\mathbf{q}}_i + \mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\lambda\mathbf{q}_i - \mathbf{g}_i(\mathbf{q}_i), \tag{6}$$

at each side, yields

$$\begin{aligned} \mathbf{M}_l(\mathbf{q}_l)\dot{\mathbf{s}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\mathbf{s}_l &= \mathbf{Y}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\tilde{\boldsymbol{\theta}}_l + \bar{\boldsymbol{\tau}}_l + \boldsymbol{\tau}_h \\ \mathbf{M}_r(\mathbf{q}_r)\dot{\mathbf{s}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\mathbf{s}_r &= \mathbf{Y}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\tilde{\boldsymbol{\theta}}_r + \bar{\boldsymbol{\tau}}_r - \boldsymbol{\tau}_e \end{aligned} \tag{7}$$

where $\tilde{\boldsymbol{\theta}}_i = \boldsymbol{\theta}_i - \hat{\boldsymbol{\theta}}_i$ are the estimation errors. The parameter update laws are given by

$$\dot{\hat{\boldsymbol{\theta}}}_l = \boldsymbol{\Gamma}_l\mathbf{Y}_l^\top(\mathbf{q}_l, \dot{\mathbf{q}}_l)\mathbf{s}_l; \qquad \dot{\hat{\boldsymbol{\theta}}}_r = \boldsymbol{\Gamma}_r\mathbf{Y}_r^\top(\mathbf{q}_r, \dot{\mathbf{q}}_r)\mathbf{s}_r \tag{8}$$

where $\boldsymbol{\Gamma}_i$ are constant positive definite matrices.

Proposition 1 (Chopra et al., 2008). *Consider the bilateral teleoperator (1) in free motion ($\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$) controlled by (3) using the parameter update law (8) and coordinating torques (4), (5). Then, for any constant time-delay T, all signals in the system are bounded and $|\mathbf{e}_i|, |\dot{\mathbf{e}}_i| \to 0$ as $t \to \infty$.*

2.2. A practical limitation

In this subsection it is shown that the controller of Chopra et al. (2008) will tend to drive to zero the positions of the joints where gravity forces are non-zero. It should be underscored that this deleterious effect is due to the inclusion of the adaptation law, and hence it is absent in the known parameter case. For the sake of clarity, an informal discussion is first presented, leaving the formal result – i.e., Proposition 2 – to the end of the subsection. Consistently to the convergence claim of Proposition 1, we will study the constant position equilibria of the closed-loop system (1)–(5), (8), whose state vector is $(\mathbf{q}_l, \mathbf{q}_r, \dot{\mathbf{q}}_l, \dot{\mathbf{q}}_r, \hat{\theta}_l, \hat{\theta}_r)$.[2] Evaluating (5), (8) at the equilibrium we get

$$\mathbf{q}^\top\mathbf{Y}(\mathbf{q}, \mathbf{0}) = \mathbf{0}. \tag{9}$$

Notice that the equilibrium constraint (9) is imposed even if the manipulators are not in free motion. It will be shown that this constraint implies a restriction on the gravity forces. Indeed, from (6) we get

$$\mathbf{Y}(\mathbf{q}, \mathbf{0})\boldsymbol{\theta} = -\mathbf{g}(\mathbf{q}), \tag{10}$$

which establishes a relationship between $\mathbf{Y}(\mathbf{q}, \mathbf{0})$ and $\mathbf{g}(\mathbf{q})$. Before considering the general case let us analyze the implications of the equilibrium constraint (9) in some simple examples. Consider the five-bar linkage system studied in Spong et al. (2005), whose gravity forces are $\mathbf{g}(\mathbf{q}) = [a_1\cos(q_1), a_2\cos(q_2)]^\top$, with $a_i \in \mathbb{R}_{>0}$. Selecting a minimal parametrization we get

$$\mathbf{Y}(\mathbf{q}, \mathbf{0}) = -\begin{bmatrix} 0 & 0 & \cos(q_1) & 0 \\ 0 & 0 & 0 & \cos(q_2) \end{bmatrix},$$

and the constraint (9) is $q_1\cos(q_1) = q_2\cos(q_2) = 0$. Hence, the controller tends to drive to zero the position of the joints where gravity forces are non-zero. A similar situation happens for of a 2-DOF manipulator with rotational joints, whose minimal parametrization is

$$\mathbf{Y}(\mathbf{q}, \mathbf{0}) = -\begin{bmatrix} 0 & \cdots 0 & \cos(q_1) & \cos(q_1+q_2) \\ 0 & \cdots 0 & 0 & \cos(q_1+q_2) \end{bmatrix},$$

and the equilibrium constraint (9) becomes $q_1\cos(q_1) = 0$, $q_2\cos(q_1+q_2) = 0$, implying, again, that the position of the second joint will go to zero if the corresponding gravity force is not zero. It can be easily shown that similar restrictions apply for the 3-DOF and the Puma manipulators.

The question that arises naturally is whether there exists manipulators for which (9) *does not* impose a constraint. That is, is there a manipulator with gravity forces that satisfy (9) *for all* $\mathbf{q}$? In the proposition below it is proven that the answer to this question is negative.

[1] In Chopra et al. (2008) λ is a positive definite matrix. As will become clear below, taking it to be a scalar, does not change our main argument. See Remark (i) in Section 2.2.

[2] To avoid cluttering, and with some obvious abuse of notation, we will not distinguish the constant equilibria from the variable itself, that is, we omit the standard upperbar notation. Also, when clear from the context, the subindex i, that identifies the local and remote manipulator, is omitted.

Proposition 2. *Consider the bilateral teleoperator (1) in closed-loop with (3), (4), (5), (8). The set of achievable (constant) equilibrium positions is strictly contained in the set* $\{(\mathbf{q}_l, \mathbf{q}_r) \in \mathbb{R}^{2n} \mid \mathbf{q}_i^\top \mathbf{Y}_i(\mathbf{q}_i, \mathbf{0}) = \mathbf{0},\ i = l, r\}$. *Moreover, for all manipulators of the form (1), composed by kinematic open chains of revolute or prismatic joints*[3] *there is no (non-zero) gravity force that satisfies (9), (10)* for all $\mathbf{q}$, *therefore, this set is a* strict *subset of* $\mathbb{R}^{2n}$.

Proof. First, notice that (9) and (10) imply

$$\mathbf{q}^\top \mathbf{g}(\mathbf{q}) = 0. \tag{11}$$

Now, since gravity forces are the gradient of the potential energy function $U(\mathbf{q})$, that is $\mathbf{g}(\mathbf{q}) = \frac{\partial}{\partial \mathbf{q}} U(\mathbf{q})$, (11) becomes the partial differential equation (PDE)

$$\mathbf{q}^\top \frac{\partial}{\partial \mathbf{q}} U(\mathbf{q}) = 0. \tag{12}$$

It will be proved that the only potential energy function $U(\mathbf{q})$ that satisfies this PDE is $U(\mathbf{q}) =$ constant, that is $\mathbf{g}(\mathbf{q}) = \mathbf{0}$. Toward this end, recall that for manipulators with prismatic and revolute joints, $U(\mathbf{q})$ is a polynomial function in the arguments q_i, $\sin(q_i)$ and $\cos(q_i)$, that is, a function of the form

$$P(\mathbf{q}) = \sum_{j=1}^{n} a_j \prod_{i=1}^{n} q_i^{b_i} \prod_{i=1}^{n} \sin^{c_i}(q_i) \prod_{i=1}^{n} \cos^{d_i}(q_i), \tag{13}$$

where a_j are real numbers and b_j, c_j, d_j are nonnegative integers (Kelly et al., 2005; Spong et al., 2005). It will be now proved that the only solution of (12) of the form (13) is the constant solution. Notice that the function $P(\mathbf{q})$ has a limit at zero, that is, $P(0) = \lim_{|\mathbf{q}|\to 0} P(\mathbf{q})$. It is possible to prove that all solutions of the partial differential equation (12) are homogeneous, that is, they satisfy $U(\mathbf{q}) = U(r\mathbf{q})$ for any $r \in \mathbb{R}$.[4] Indeed, evaluating the time derivative of $U(\mathbf{q})$ we get $\dot{U} = \frac{\partial}{\partial \mathbf{q}} U^\top(\mathbf{q})\dot{\mathbf{q}}$, which, in view of (12), is equal to zero along the flow of the system $\dot{\mathbf{q}} = \mathbf{q}$, *i.e.* along $\mathbf{q}(t) = e^t \mathbf{q}(0)$. Hence,

$$U(\mathbf{q}(t_1)) = U(e^{(t_2-t_1)}\mathbf{q}(t_1)), \quad \forall t_1, t_2$$

establishing the claim of homogeneity. Now, it is clear that for any function $P(\mathbf{q})$, which has a limit at zero, the following identity holds $\lim_{|\mathbf{q}|\to 0} P(\mathbf{q}) = \lim_{r\to 0} P(r\mathbf{q})$.

On the other hand, since $P(\mathbf{q})$ is homogeneous $\lim_{r\to 0} P(r\mathbf{q}) = \lim_{r\to 0} P(\mathbf{q}) = P(\mathbf{q})$. Hence, $P(\mathbf{q}) = P(\mathbf{0})$. The proof is concluded noting that the only polynomial function that is constant for all $\mathbf{q}$ is the constant function, and consequently $\mathbf{g}(\mathbf{q}) = \mathbf{0}$. □

The following remarks are in order.

(i) In the derivations above it has been assumed that the tuning parameter λ is a scalar. If it is a matrix, (9) becomes $\mathbf{q}^\top \boldsymbol{\lambda} \mathbf{Y}(\mathbf{q}, \mathbf{0}) = \mathbf{0}$, leaving the scenario discussed above, essentially, unmodified.

(ii) Proposition 1 suggests that the trajectories will converge to constant positions where $\mathbf{q}_l = \mathbf{q}_r$. In that case, if the local and remote manipulators are kinematically different one from the other, and if link j, on the local, is not affected by gravity but that same link on the remote is, then the solution is $q_j = 0$ for both. Thus, possible equilibria with $q_j \neq 0$ are restricted to manipulators that are kinematically similar.

(iii) The experiments in Chopra et al. (2008) yield the desired behavior because the manipulators are composed by two revolute joints whose DOFs lie on the horizontal plane. Thus, in such a case $\mathbf{g}_i(\mathbf{q}_i) = \mathbf{0}$.

[3] For the sake of generality we consider general joints in this proposition.

[4] This conclusion also follows noting that all solutions of (12) are of the form $U(\mathbf{q}) = f\left(\frac{q_2}{q_1}, \frac{q_3}{q_1}, \ldots, \frac{q_n}{q_1}\right)$, where $f : \mathbb{R}^{n-1} \to \mathbb{R}$ is an arbitrary C^1 map, see Ibragimov (1999).

3. A new adaptive controller

The new proposed controllers are given by

$$\begin{aligned}\boldsymbol{\tau}_l &= -\hat{\mathbf{M}}_l(\mathbf{q}_l)\boldsymbol{\lambda}\dot{\mathbf{e}}_l - \hat{\mathbf{C}}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\boldsymbol{\lambda}\mathbf{e}_l - \hat{\mathbf{g}}_l(\mathbf{q}_l) + \bar{\boldsymbol{\tau}}_l \\ \boldsymbol{\tau}_r &= \hat{\mathbf{M}}_r(\mathbf{q}_r)\boldsymbol{\lambda}\dot{\mathbf{e}}_r + \hat{\mathbf{C}}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\boldsymbol{\lambda}\mathbf{e}_r + \hat{\mathbf{g}}_r(\mathbf{q}_r) - \bar{\boldsymbol{\tau}}_r,\end{aligned} \tag{14}$$

and for $\mathbf{Y}_i\hat{\boldsymbol{\theta}}_i = -\hat{\mathbf{M}}_i(\mathbf{q}_i)\boldsymbol{\lambda}\dot{\mathbf{e}}_i - \hat{\mathbf{C}}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)\boldsymbol{\lambda}\mathbf{e}_i - \hat{\mathbf{g}}_i(\mathbf{q}_i)$, they can be written as $\boldsymbol{\tau}_l = \mathbf{Y}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l, \mathbf{e}_l, \dot{\mathbf{e}}_l)\hat{\boldsymbol{\theta}}_l + \bar{\boldsymbol{\tau}}_l$ and $\boldsymbol{\tau}_r = -\mathbf{Y}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r, \mathbf{e}_r, \dot{\mathbf{e}}_r)\hat{\boldsymbol{\theta}}_r - \bar{\boldsymbol{\tau}}_r$.

Let us define the *synchronizing signal* $\boldsymbol{\epsilon}_i$ as

$$\boldsymbol{\epsilon}_i = \dot{\mathbf{q}}_i - \boldsymbol{\lambda}\mathbf{e}_i, \tag{15}$$

where $\mathbf{e}_i$ has been previously defined in (2) and $\boldsymbol{\lambda}$ is a diagonal positive definite matrix. Substituting the controllers (14) on the teleoperator dynamics (1) and using (15), yields

$$\begin{aligned}\mathbf{M}_l(\mathbf{q}_l)\dot{\boldsymbol{\epsilon}}_l + \mathbf{C}_l(\mathbf{q}_l, \dot{\mathbf{q}}_l)\boldsymbol{\epsilon}_l &= \mathbf{Y}_l\tilde{\boldsymbol{\theta}}_l - \bar{\boldsymbol{\tau}}_l + \boldsymbol{\tau}_h \\ \mathbf{M}_r(\mathbf{q}_r)\dot{\boldsymbol{\epsilon}}_r + \mathbf{C}_r(\mathbf{q}_r, \dot{\mathbf{q}}_r)\boldsymbol{\epsilon}_r &= \mathbf{Y}_r\tilde{\boldsymbol{\theta}}_r - \bar{\boldsymbol{\tau}}_r - \boldsymbol{\tau}_e.\end{aligned} \tag{16}$$

The dynamics of the estimations of the uncertain parameters are given by

$$\dot{\hat{\boldsymbol{\theta}}}_i = \boldsymbol{\Gamma}_i \mathbf{Y}_i^\top \boldsymbol{\epsilon}_i. \tag{17}$$

where $\boldsymbol{\Gamma}_i$ are positive definite matrices. The torques $\bar{\boldsymbol{\tau}}_i$ are

$$\bar{\boldsymbol{\tau}}_i = \mathbf{K}_i\boldsymbol{\epsilon}_i - \mathbf{B}\dot{\mathbf{e}}_i \tag{18}$$

where $\mathbf{K}_i$ are positive definite matrices and $\mathbf{B}$ is diagonal positive definite.

The following proposition discusses the asymptotic stability of the teleoperator when the human operator and the environment do not exert any forces on the corresponding manipulators. It should be underscored that asymptotic stability is ensured despite any constant time-delay.

Proposition 3. *Consider the bilateral teleoperator (1) in free motion* ($\boldsymbol{\tau}_h = \boldsymbol{\tau}_e = \mathbf{0}$) *controlled by (14) using the parameter update law (17) and coordinating torques (18) together with (15). Then, for any constant time-delay T, all signals in the system are bounded. Moreover, position errors and velocities asymptotically converge to zero,* i.e., $|\mathbf{e}_i| \to |\dot{\mathbf{q}}_i| \to 0$ *as* $t \to \infty$.

Proof. Let us propose a Lyapunov–Krasovskiĭ candidate function V as the following

$$V = \frac{1}{2}\sum_{i\in\{l,r\}} \left[\boldsymbol{\epsilon}_i^\top \mathbf{M}_i\boldsymbol{\epsilon}_i + \tilde{\boldsymbol{\theta}}_i^\top \boldsymbol{\Gamma}_i^{-1}\tilde{\boldsymbol{\theta}}_i + \mathbf{e}_i^\top \boldsymbol{\lambda}\mathbf{B}\mathbf{e}_i + \int_{t-T}^{t} \dot{\mathbf{q}}_i^\top \mathbf{B}\dot{\mathbf{q}}_i \mathrm{d}\sigma\right].$$

This function is positive definite and radially unbounded in $\boldsymbol{\epsilon}_i, \tilde{\boldsymbol{\theta}}_i, \mathbf{e}_i$. Its time derivative along (16)–(18), using P2, is given by

$$\dot{V} = \sum_{i\in\{l,r\}} \left[-\boldsymbol{\epsilon}_i^\top \mathbf{K}_i\boldsymbol{\epsilon}_i + \dot{\mathbf{q}}_i^\top \mathbf{B}\dot{\mathbf{e}}_i - \frac{1}{2}\dot{\mathbf{q}}_i^\top(t-T)\mathbf{B}\dot{\mathbf{q}}_i(t-T) + \frac{1}{2}\dot{\mathbf{q}}_i^\top \mathbf{B}\dot{\mathbf{q}}_i\right].$$

Notice that, for $i = l$, $\dot{\mathbf{q}}_l^\top \mathbf{B}\dot{\mathbf{e}}_l = \dot{\mathbf{q}}_l^\top \mathbf{B}(\dot{\mathbf{q}}_r(t-T) - \dot{\mathbf{q}}_l)$. Hence, when $i = r$ and gathering the crossed terms $-\frac{1}{2}[\dot{\mathbf{q}}_l^\top \mathbf{B}\dot{\mathbf{q}}_l - 2\dot{\mathbf{q}}_l^\top \mathbf{B}\dot{\mathbf{q}}_r(t-T) + \dot{\mathbf{q}}_r^\top(t-T)\mathbf{B}\dot{\mathbf{q}}_r(t-T)]$, yields $\dot{V} = -\sum_{i\in\{l,r\}}\left[\boldsymbol{\epsilon}_i^\top \mathbf{K}_i\boldsymbol{\epsilon}_i + \frac{1}{2}\dot{\mathbf{e}}_i^\top \mathbf{B}\dot{\mathbf{e}}_i\right]$.

Due to $V \geq 0$ and $\dot{V} \leq 0$, $\boldsymbol{\epsilon}_i, \dot{\mathbf{e}}_i \in \mathcal{L}_2$ and $\boldsymbol{\epsilon}_i, \tilde{\boldsymbol{\theta}}_i, \mathbf{e}_i \in \mathcal{L}_\infty$. From (15) it can be shown that $\dot{\mathbf{q}}_i \in \mathcal{L}_\infty$, implying that $\dot{\mathbf{e}}_i \in \mathcal{L}_\infty$. All these bounded signals together with P1 and P3 guarantee that $\mathbf{Y}_i \in \mathcal{L}_\infty$. Now, from (16), using P1 and P3 together with boundedness of $\bar{\boldsymbol{\tau}}_i, \mathbf{Y}_i, \tilde{\boldsymbol{\theta}}_i, \boldsymbol{\epsilon}_i, \dot{\mathbf{q}}_i$, it can be concluded that $\dot{\boldsymbol{\epsilon}}_i \in \mathcal{L}_\infty$. Hence, $\boldsymbol{\epsilon}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$, $\dot{\boldsymbol{\epsilon}}_i \in \mathcal{L}_\infty$ support that $|\boldsymbol{\epsilon}_i| \to 0$, and $\dot{\mathbf{e}}_i, \dot{\mathbf{e}}_i \in \mathcal{L}_\infty$

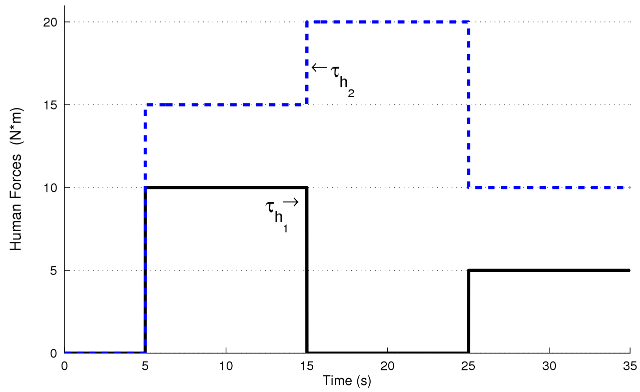

Fig. 1. Generalized human forces.

imply that $\ddot{\mathbf{q}}_i \in \mathcal{L}_\infty$, hence $\ddot{\mathbf{e}}_i \in \mathcal{L}_\infty$. This last, and the fact that $\dot{\mathbf{e}}_i \in \mathcal{L}_\infty \cap \mathcal{L}_2$ prove that $|\dot{\mathbf{e}}_i| \to 0$.

Now, $\mathbf{e}_i, \dot{\mathbf{e}}_i, \ddot{\mathbf{e}}_i \in \mathcal{L}_\infty$ and $|\dot{\mathbf{e}}_i| \to 0$ imply that $\lim_{t\to\infty} \int_0^t \dot{\mathbf{e}}_i \mathrm{d}\sigma = \mathbf{e}_i - \mathbf{e}_i(0) = k_i < \infty$. On the other hand,

$$\lim_{t\to\infty} |\boldsymbol{\epsilon}_i| = \lim_{t\to\infty} |\dot{\mathbf{q}}_i - \boldsymbol{\lambda}\mathbf{e}_i| = \lim_{t\to\infty} |\dot{\mathbf{q}}_i - \boldsymbol{\lambda}(k_i + \mathbf{e}_i(0))| = 0$$

imply that when $t \to \infty$, $\dot{\mathbf{q}}_i \to \boldsymbol{\lambda}(k_i + \mathbf{e}_i(0))$ that is constant. This and $|\dot{\mathbf{e}}_i| \to 0$ ensure that $|\dot{\mathbf{q}}_i| \to |\dot{\mathbf{q}}_c|$, for $\dot{\mathbf{q}}_c$ any constant vector, and $\mathbf{q}_l - \mathbf{q}_r(t-T) \to \mathbf{q}_l - \mathbf{q}_r$. Thus, in the limit, $\boldsymbol{\epsilon}_l = \dot{\mathbf{q}}_c - \boldsymbol{\lambda}\mathbf{e}_l = \dot{\mathbf{q}}_c + \boldsymbol{\lambda}\mathbf{e}_r$ and $\boldsymbol{\epsilon}_r = \dot{\mathbf{q}}_c - \boldsymbol{\lambda}\mathbf{e}_r$. Hence $\boldsymbol{\epsilon}_l + \boldsymbol{\epsilon}_l = 2\dot{\mathbf{q}}_c$, the fact that $|\boldsymbol{\epsilon}_i| \to 0$ proves that $|\dot{\mathbf{q}}_c| \to 0$. Hence, $|\dot{\mathbf{q}}_i| \to |\mathbf{e}_i| \to 0$. $\square$

4. Simulations

In the performed simulations the local and remote manipulators are modeled as a pair of 2-DOF serial links with revolute joints. Their corresponding nonlinear dynamics are modeled by (1) where the elements of the inertia matrices $\mathbf{M}_i(\mathbf{q}_i)$ are $M_{i_{11}} = \alpha_i + 2\beta_i c_{2_i}$, $M_{i_{12}} = M_{i_{21}} = \delta_i + \beta_i c_{2_i}$ and $M_{i_{22}} = \delta_i$; the elements of the Coriolis and centrifugal matrices $\mathbf{C}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$ are $C_{i_{11}} = 2C_{i_{12}} = -2\beta_i s_{2_i}\dot{q}_{2_i}$, $C_{i_{21}} = \beta_i s_{2_i}\dot{q}_{1_i}$ and $C_{i_{21}} = 0$; and, finally, the elements of the gravity forces $\mathbf{g}_i(\mathbf{q}_i)$ are $g_{i_1} = \frac{1}{l_{2_i}} g\delta_i c_{12_i} + \frac{1}{l_{1_i}}(\alpha_i - \delta_i)c_{1_i}$ and $g_{i_2} = \frac{1}{l_{2_i}} g\delta_i c_{12_i}$. In which c_{2_i}, s_{2_i} and c_{12_i} are the short notations for $\cos(q_{2_i})$, $\sin(q_{2_i})$ and $\cos(q_{1_i} + q_{2_i})$, respectively. q_{k_i} is the articular position of link k of manipulator i, with $k \in \{1, 2\}$. $\dot{q}_{1_i}$ and $\dot{q}_{2_i}$ are the respective revolute velocities of the two links. $\alpha_i = l_{2_i}^2 m_{2_i} + l_{1_i}^2(m_{1_i} + m_{2_i})$, $\beta_i = l_{1_i} l_{2_i} m_{2_i}$ and $\delta_i = l_{2_i}^2 m_{2_i}$. l_{k_i} and m_{k_i} are the respective lengths and masses of each link.

The following parametrization is proposed for both manipulators

$$\mathbf{Y}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) = \begin{bmatrix} \ddot{q}_1 & Y_{21} & \ddot{q}_2 & gc_{12} & gc_1 \\ 0 & c_2\ddot{q}_1 + s_2\dot{q}_1^2 & \ddot{q}_1 + \ddot{q}_2 & gc_{12} & 0 \end{bmatrix},$$

$$\boldsymbol{\theta} = \begin{bmatrix} \alpha & \beta & \delta & \frac{1}{l_2}\delta & \frac{1}{l_1}(\alpha - \delta) \end{bmatrix}^\top,$$

where $Y_{21} = 2c_2\ddot{q}_1 + c_2\ddot{q}_2 - s_2\dot{q}_2^2 - 2s_2\dot{q}_1\dot{q}_2$. The physical parameters for the manipulators are: the length of links l_{1_i} and l_{2_i}, for both manipulators, is 0.38 m; the masses for the links are $m_{1_l} = 3.9473$ kg, $m_{2_l} = 0.6232$ kg, $m_{1_r} = 3.2409$ kg and $m_{2_r} = 0.3185$ kg.

The initial conditions are $\ddot{\mathbf{q}}_i(0) = \dot{\mathbf{q}}_i(0) = \mathbf{0}$ and $\mathbf{q}_l^\top(0) = [-1/3\pi; 1/3\pi]$, $\mathbf{q}_r^\top(0) = [0; 0]$.

4.1. Controller of Chopra et al. (2008)

For simplicity, as an illustrative example, let us take $\lambda = 1$ in (6), such that matrices $\mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i)$, for this scheme, become

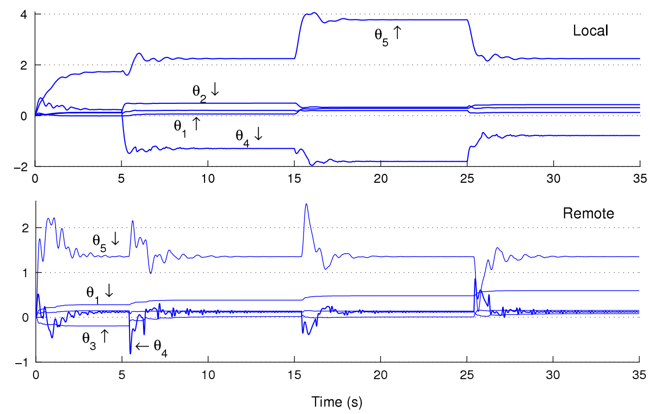

Fig. 2. Parameter estimation of the local and remote manipulators. Using the scheme of Chopra et al. (2008).

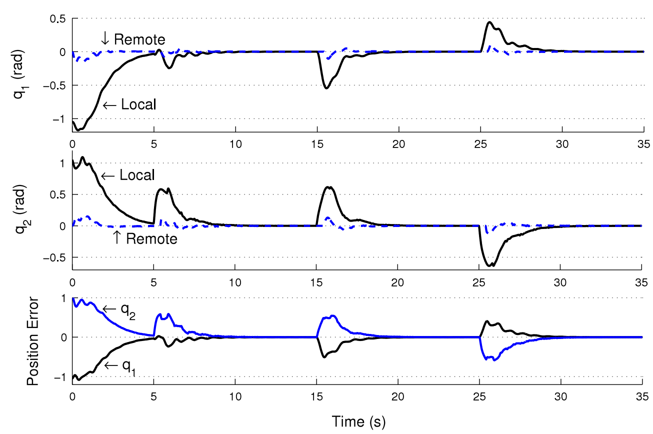

Fig. 3. Joint position of the local and remote manipulators. Using the scheme of Chopra et al. (2008).

$$\begin{bmatrix} \dot{q}_{1_i} & Y_{12_i} & \dot{q}_{2_i} & -gc_{12_i} & -gc_{1_i} \\ 0 & c_{2_i}\dot{q}_{1_i} + s_{2_i}\dot{q}_{1_i}q_{1_i} & \dot{q}_{1_i} + \dot{q}_{2_i} & -gc_{12_i} & 0 \end{bmatrix}$$

where $Y_{12_i} = 2c_{2_i}\dot{q}_{1_i} + c_{2_i}\dot{q}_{2_i} - s_{2_i}\dot{q}_{2_i}q_{2_i} - 2s_{2_i}\dot{q}_{2_i}q_{1_i}$ and the estimated parameters are given by

$$\hat{\boldsymbol{\theta}}_i = \begin{bmatrix} \hat{\alpha}_i & \hat{\beta}_i & \hat{\delta}_i & \frac{1}{\hat{l}_{2_i}}\hat{\delta}_i & \frac{1}{\hat{l}_{1_i}}(\hat{\alpha}_i - \hat{\delta}_i) \end{bmatrix}^\top. \quad (19)$$

The controllers' gains are $K = 3$ and $\boldsymbol{\Gamma}_l = 0.25\mathbf{I}$ and $\boldsymbol{\Gamma}_r = \mathbf{I}$. The time-delays in both paths is set to 0.4 s.

The simulations, in Figs. 2–3, show the system response when the human operator exerts the generalized (piecewise constant) forces shown in Fig. 1 to the local manipulator. It can be seen that the parameter estimations converge to constant values and positions asymptotically converge to zero, as stated by Proposition 2. It can be interpreted that when the human injects force on the local manipulator, the parameter estimation law exerts opposite forces that prevent the teleoperator to move from the equilibrium $\mathbf{q}_i = \mathbf{0}$.

4.2. Proposed adaptive controller

Using $\boldsymbol{\lambda} = \mathbf{I}$, yields $\mathbf{Y}_i(\mathbf{q}_i, \dot{\mathbf{q}}_i, \mathbf{e}_i, \dot{\mathbf{e}}_i)$ as

$$\begin{bmatrix} -\dot{e}_{1_i} & Y_{12_i} & -\dot{e}_{2_i} & -gc_{12_i} & -gc_{1_i} \\ 0 & -c_{2_i}\dot{e}_{1_i} - s_{2_i}\dot{q}_{1_i}e_{1_i} & -\dot{e}_{1_i} - \dot{e}_{2_i} & -gc_{12_i} & 0 \end{bmatrix}$$

where $Y_{12_i} = -2c_{2_i}\dot{e}_{1_i} - c_{2_i}\dot{e}_{2_i} + s_{2_i}\dot{q}_{2_i}e_{2_i} + 2s_{2_i}\dot{q}_{2_i}e_{1_i}$ and the estimated parameters follow (19). The initial conditions and estimation gains are the same as in the previous set of simulations. The controllers' gains, in (18), are $\mathbf{K}_i = 3\mathbf{I}$ and $\mathbf{B} = \mathbf{I}$.

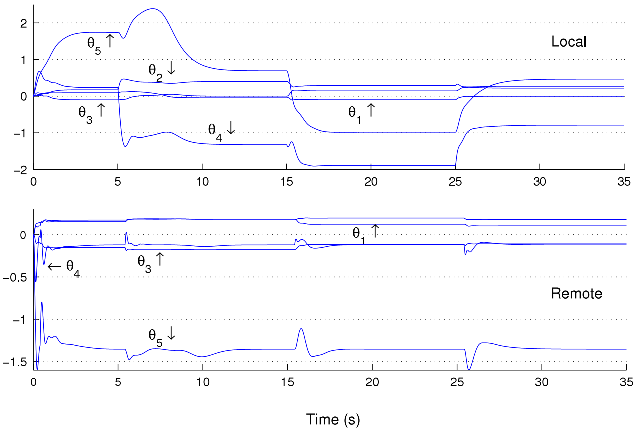

Fig. 4. Parameter estimation of the local and remote manipulators. Using the proposed controller.

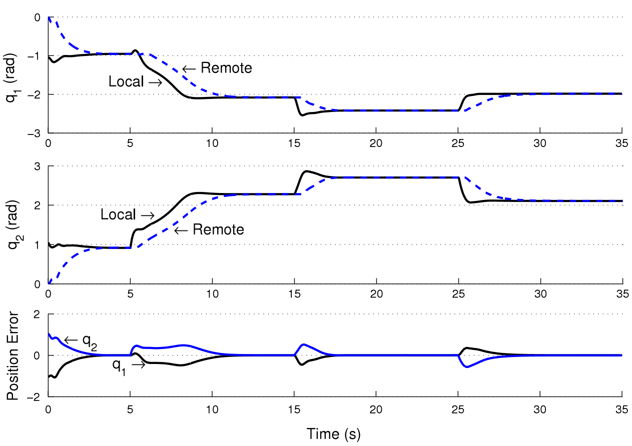

Fig. 5. Joint position of the local and remote manipulators. Using the proposed controller.

The simulations with the proposed controller are shown in Figs. 4–5, in which it can be seen that position errors asymptotically converge to zero but positions converge to values different than zero, *i.e.*, $q_{l_j} = q_{r_j} \neq 0$. When the human exerts force, the local manipulator moves consequently and the remote manipulator follows the corresponding trajectory.

5. Conclusions

In this work an adaptive controller for general nonlinear teleoperators with time-delay is proposed. This controller can be seen as an extension of a previous scheme presented in Chopra et al. (2008). It is also proved that the previous scheme can be only used, in general, on manipulators exempted from gravity forces.

The proposed controller assures that, in free motion, for any constant time-delay, all signals in the system are bounded, and position errors and velocities asymptotically converge to zero. The closed-loop teleoperator can be easily proved to be input-to-state stable from human and environment input forces to local and remote manipulator states. The simulations performed confirm the above conclusions.

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Emmanuel Nuño was born in Guadalajara, México in 1980. He obtained his B.Sc. in Communications and Electronics Engineering from the University of Guadalajara in 2002. He received the Ph.D. degree in Electrical Engineering from the Technical University of Catalonia (UPC), Spain, in 2008.

In 2007 he held two research internships, at the Laboratoire des Signaux et Systemes at SUPELEC and at the Coordinated Science Laboratory at University of Illinois, Urbana-Champaign. In 2008 he joined the Department of Computer Science of the University of Guadalajara. Currently he is a postdoctoral researcher at the Institute of Industrial and Control Engineering (IOC) at the Technical University of Catalonia.

His research interest include nonlinear control of robot manipulators, control of teleoperators with time-delays and synchronization of multiple Euler–Lagrange systems.

Romeo Ortega was born in Mexico. He obtained his B.Sc. in Electrical and Mechanical Engineering from the National University of Mexico, Master of Engineering from Polytechnical Institute of Leningrad, USSR, and the Docteur D'Etat from the Polytechnical Institute of Grenoble, France in 1974, 1978 and 1984 respectively.

He then joined the National University of Mexico, where he worked until 1989. He was a Visiting Professor at the University of Illinois in 1987–1988 and at the McGill University in 1991–1992, and a Fellow of the Japan Society for Promotion of Science in 1990–1991. He has been a member of the French National Researcher Council (CNRS) since June 1992. Currently he is in the Laboratoire de Signaux et Systemes (SUPELEC) in Paris. His research interests are in the fields of nonlinear and adaptive control, with special emphasis on applications.

Dr Ortega is a Fellow Member of the IEEE. He has served as chairman in several IFAC and IEEE committees and editorial boards. Currently he chairs the Automatica Paper Prize Award Committee (2009–2012) is an Editor at Large of IEEE Transactions on Automatic Control, and an Associate Editor of Systems and Control Letters and International Journal of Adaptive Control and Signal Processing.

Luis Basañez received the Ph.D. degree in Electrical Engineering from the Technical University of Catalonia (UPC), Barcelona, Spain, in 1975. From 1976 to 1987, he was the Vice Director of the Institute of Cybernetics (UPC), where he was the Director from 1987 to 1990. Since 1986, he has been a Full Professor of System Engineering and Automatic Control at the UPC, where since 1990 he is the Head of the Robotics Division, Institute of Industrial and Control Engineering. His current research interests include task planning, multirobot systems coordination, teleoperation, sensor integration, and active perception.

Dr. Basañez was an Executive Committee Member of the International Federation of Robotics (IFR) from 1987 to 1992, and he is currently the Spanish delegate at the IFR. In 2005, he was elected Fellow of the International Federation of Automatic Control (IFAC), and since this year, he is a Member of the IFAC Council.