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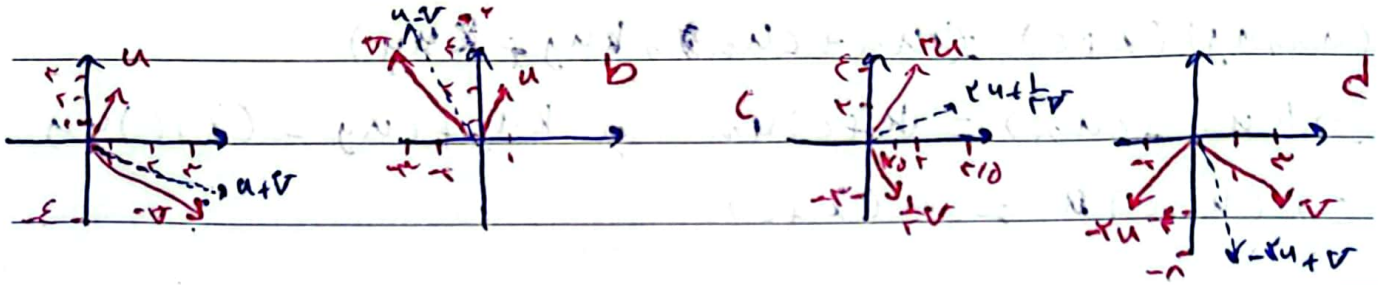
1- $u=(1,2)$ $v=(-4,3)$

a) $u+v \xrightarrow[v=(-4,3)]{u=(1,2)} (1,2)+(-4,3)=(-3,5)$

b) $u-v = u+(-v) = (1,2)+(4,-3) = (5,-1)$

c) $2u+\frac{1}{2}v = (2\times(1,2))+(\frac{1}{2}\times(-4,3)) = (2,4)+(-2,\frac{3}{2}) = (0,\frac{11}{2})$

d) $-2u+v = (-2\times(1,2))+(-4,3) = (-2,-4)+(-4,3) = (-6,-1)$

a 

3- a) $u+v=v+u$

$\Rightarrow (u_x,u_y)+(v_x,v_y) = (v_x,v_y)+(u_x,u_y)$

$= (u_x+v_x, u_y+v_y) = (v_x+u_x, v_y+u_y)$

$\xrightarrow[u_y+v_y=B]{u_x+v_x=A} (A,B)=(A,B) \Rightarrow u+v=v+u$

b) $u+(v+w)=(u+v)+w$

$\Rightarrow (u_x,u_y)+((v_x,v_y)+(w_x,w_y)) = ((u_x,u_y)+(v_x,v_y))+(w_x,w_y)$

$= (u_x+v_x+w_x, u_y+v_y+w_y) = (u_x+v_x+w_x, u_y+v_y+w_y)$

$\xrightarrow[u_y+v_y+w_y=B]{u_x+v_x+w_x=A} (A,B)=(A,B) \Rightarrow u+(v+w)=(u+v)+w$

c) $(ck)u = c(ku)$

$\Rightarrow (ck)(u_x,u_y) = c(k(u_x,u_y)) = ck(u_x,u_y) = (kcu_x, kcu_y)$

$\Rightarrow (ck)(u_x,u_y) = (ckv_x, cku_y) \Rightarrow (ck)u = c(ku)$

d) $k(u+v) = ku+kv$

$\Rightarrow k((u_x,u_y)+(v_x,v_y)) = k(u_x,u_y)+k(v_x,v_y)$

$= (ku_x+kv_x, ku_y+kv_y) = k(u_x+v_x)+k(u_y,v_y)$

$\Rightarrow k(u+v) = ku+kv$

e) $u(k+c) = ku+cu$

$(u_x,u_y)(k+c) = (ku_x+cu_x, ku_y+cu_y)$

$\Rightarrow (ku_x+cu_x) = (k+c)u_x \quad , \quad ku_y+cu_y = (k+c)u_y$

$\Rightarrow (k+c)u = u(k+c)$

۴- $2((1,3,2)-x)-(-2,0,4) = -2(1,2,2)$

$(2,4,4)-2x+(2,0,-4) = (-2,-4,-4)$

$(4,4,2)-2x = (-2,-4,-4)$

$(4,4,2)+(2,4,4) = 2x \Rightarrow (6,8,6) = 2x$

$\Rightarrow x = (3,4,3)$

۵- $u = (-1,2,2) \quad v = (2,-4,1)$

$|u| = \sqrt{1+9+4} = \sqrt{14} \quad \Rightarrow \|u\| = \left(\frac{-\sqrt{14}}{14}, \frac{3\sqrt{14}}{14}, \frac{2\sqrt{14}}{7}\right)$ بردار یکه

$|v| = \sqrt{4+16+1} = \sqrt{21} \quad$ بردار یکه $\|v\| = \left(\frac{2\sqrt{21}}{21}, \frac{-4\sqrt{21}}{21}, \frac{\sqrt{21}}{21}\right)$

۶- $u = (u_x,u_y,u_z) \quad v = (v_x,v_y,v_z) \quad w = (w_x,w_y,w_z)$

a) $u.v = v.u$

$\Rightarrow (u_x,u_y,u_z).(v_x,v_y,v_z) = u_xv_x+u_yv_y+u_zv_z$

$= v_xu_x+v_yu_y+v_zu_z \Rightarrow u.v = v.u$

b) $u.(v,w) = u.v + u.w$ ۳

$u_x v_x + u_y v_y + u_z v_z + u_x w_x + u_y w_y + u_z w_z$ ①

$= u_x (v_x + w_x) + u_y (v_y + w_y) + u_z (v_z + w_z)$ ②

① و ② $\Rightarrow u.(v.w) = u.v + u.w$

c) $k(u.v) = (ku).v = u.(kv)$

$k(u_x v_x + u_y v_y) = ku_x v_x + ku_y v_y = (ku).v$

①

① $\Rightarrow u.(kv)$

$\Rightarrow k(u.v) = (ku).v = u.(kv)$

d) $v.v = \|v\|^2$

$v_x v_x + v_y v_y = v_x^2 + v_y^2$ ①

$\|v\|^2 = (\sqrt{v_x^2 + v_y^2})^2 = v_x^2 + v_y^2$ ②

① = ② $\Rightarrow v.v = \|v\|^2$

e) $0.v = 0$

$0.(v_x, v_y, v_z) = 0 v_x + 0 v_y + 0 v_z = 0$

$u \times ku = 0$ ۱۷

$(u_x, u_y, u_z) \times (ku_x, ku_y, ku_z)$

$$\begin{vmatrix} i & j & k \\ u_x & u_y & u_z \\ ku_x & ku_y & ku_z \end{vmatrix} = \cancel{(u_y \times ku_z) i - (u_x \times ku_z)}$$

$= (u_y \times ku_z - u_z \times ku_y)\hat{i} - (u_x \times ku_z - u_z ku_x)\hat{j} + (u_x \times ku_y - u_y ku_x)\hat{k}$

(0) (0) (0)

$0\hat{i} - 0\hat{j} + 0\hat{k} = \vec{0}$