

Assignment 3

3.1

$$\det K = 99 - 96 = 3$$

$$K = \begin{pmatrix} 11 & 8 \\ 12 & 9 \end{pmatrix} \quad \gcd(3, 26) = 1$$

Plaintext = "Hello"

ABCDEFGHIJKLMNOPQRSTUVWXYZ
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25

hello
7 4 11 14

$$\begin{pmatrix} 7 & 4 \end{pmatrix} \begin{pmatrix} 11 & 8 \\ 12 & 9 \end{pmatrix} = (77 + 48, 56 + 36) = (125, 92) = (21, 14)$$

L L
11 11

$$\begin{pmatrix} 11 & 11 \end{pmatrix} \begin{pmatrix} 11 & 8 \\ 12 & 9 \end{pmatrix} = (121 + 132, 88 + 99) = (253, 187)$$

O A
14 0

$$\begin{pmatrix} 14 & 0 \end{pmatrix} \begin{pmatrix} 11 & 8 \\ 12 & 9 \end{pmatrix} = (154, 32) = (19, 5)$$

V O T F Y I
Cyphertext = (21, 14) (19, 5) (24, 8)

"VOTFYI"

Question on matrix multiplication.

3.2

$$K = \begin{pmatrix} 11 & 8 \\ 3 & 7 \end{pmatrix}$$

$$\begin{vmatrix} 11 & 8 \\ 3 & 7 \end{vmatrix} = 77 - 24 = 53$$

$$\text{Ciphertext} = x_i y_j$$

$$K^{-1} = (53)^{-1} \begin{pmatrix} 7 & -8 \\ -3 & 11 \end{pmatrix}$$

$$= 1 \cdot \begin{pmatrix} 7 & -8 \\ -3 & 11 \end{pmatrix} = \begin{pmatrix} 7 & -8 \\ -3 & 11 \end{pmatrix}$$

$$K^{-1} = \begin{pmatrix} 7 & 18 \\ 23 & 11 \end{pmatrix}$$

$$\begin{matrix} x_i \\ (23 \ 8) \end{matrix} \cdot \begin{pmatrix} 7 & 18 \\ 23 & 11 \end{pmatrix} = (161 + 184, 414 + 88)$$

$$(345, 502) \% 26$$

$$= (7, 8)$$

H I

$$\begin{matrix} y_j \\ (24, 9) \end{matrix} \cdot \begin{pmatrix} 7 & 18 \\ 23 & 11 \end{pmatrix} = (168 + 207, 432 + 99) = (375, 531)$$

$$\text{Plaintext} = \begin{matrix} H & I & L & L & L & L \\ (7, 8) & (11, 11) & & & & \end{matrix}$$