K-SVD

Paola Ardon, Jose Bernal, Rodrigo Daudt, Seperh Mohaimanian, Èric Pairet, Armine Vardazaryan

University of Burgundy

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Outline

Introduction

Motivation:

- Image interpretation for medical diagnosis
- Optical Coherence Tomography (OCT)
- More pleasant images

Objective:

- Compare different denoising methods
- Evaluate K-SVD performance and efficiency
- Use denoised images for segmentation

Denoising algorithms

- Mean filter
- Median filter
- Local statistics
- Hard and soft thresholding in wavelet domain
- K-SVD

Mean filter

Characteristics:

- NxN kernel
- All elements have value N^{-2}
- 2D convolution between image and kernel

$$\hat{f}(x,y) = \frac{1}{N^2} \sum_{(u,v)\in N(x,y)} g(x,y)$$

Median filter

Characteristics:

- *NxN* kernel
- Select 50th percentile element from elements

$$\hat{f}(x, y) = \underset{(u,v)\in N(x,y)}{\text{median}} g(x, y)$$

Local statistics

General characteristics:

- Based on local mean and variance
- Assumes the sample mean and variance of a range of pixels is equal to the mean and variance of all the pixels in that fixed range

Advantages:

- Non-recursive
- No transformation needed
- Great for real time processing

Hard and soft thresholding in wavelet domain

Advantages:

- More precise information
- Popular on signal denoising

Disadvantages:

- Not good with functions with discontinuities - Pseudo-Gibbs phenomena

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Main ideas:

- Sparse representation
- Redundant dictionary
- Dictionary can be pretrained or trained on noisy image
- If dictionary is trained using noisy image, training and denoising fuse into one iterated process

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This denoising method uses two main tools:

- K-SVD is a tool used to build overcomplete dictionaries for sparse representations. According to the authors, the K-SVD dictionary learning process has in it a noise rejection capability
- Orthonormal matching pursuit (OMP) is a “simple and efficient” algorithm to find good solutions (not optimal) for the complex mathematical problems in this denoising algorithm

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Sparseland model:

- Consider image patches of size $\sqrt{n} \times \sqrt{n}$
- Reorder pixels into a vector $x \in \mathbb{R}^n$
- Dictionary $D \in \mathbb{R}^{n \cdot k}$, where $k > n$ (redundancy)
- Tolerance ϵ

$$\hat{\alpha} = \underset{\alpha}{\text{argmin}} \, ||\alpha||_0,$$

subject to

$$||D\alpha - x|| \leq \epsilon \text{ and } ||\hat{\alpha}||_0 \leq L \ll n$$

Sparseland model is defined by (ϵ, L, D) triplet

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- In theory, K-SVD denoising could be applied to a square image of any size
- However, authors insist on the usage of a specific fixed and small size dictionary $D \in \mathbb{R}^{n \cdot k}$
- When training takes place, only small dictionaries can be composed
- A small dictionary implies a locality of the resulting algorithms, which simplifies the overall image treatment

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Assuming a pretrained dictionary, the K-SVD denoising can be described by:

$$\{\hat{\alpha}_{ij}, \hat{X}\} = \underset{\alpha_{ij}, X}{\text{argmin}} \left(\begin{array}{c} \lambda \| X - Y \|_2{}^2 \\ + \sum_{ij} \mu_{ij} ||\alpha_{ij}||_0 \\ + \sum_{ij} ||D\alpha_{ij} - R_{ij} X||_2{}^2 \end{array} \right)$$

- First element keeps $\hat{X}$ from being too different from Y
- Second and third terms make sure that every patch of the reconstructed image $\hat{X}$ has a sparse representation in D with bounded error

K-SVD iterative algorithm

X = Y;
D = Overcomplete DCT dictionary;
while *counter* $<$ J **do**
  Use OMP to obtain sparse representation $\hat{\alpha}$ of X;
  Update dictionary D using K-SVD;
  counter++;
end

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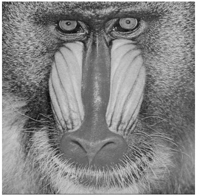
Unanswered questions and topics:

- Is there a universal dictionary that fits all images well?
- If so, what examples should be used to find it?
- State-of-the-art denoising calls for training of different dictionaries and a content aware choice of dictionary (has not been done yet)
- How should k be chosen? How redundant should the dictionary be?

Chosen synthetic images

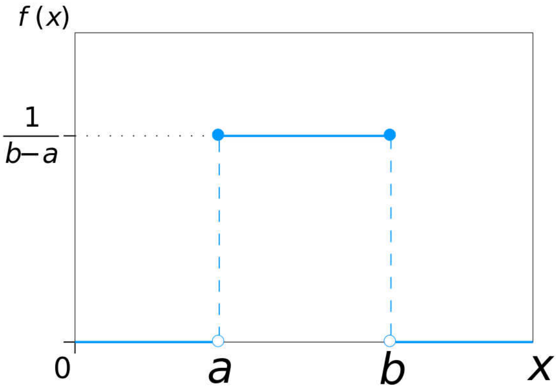

(a) Cameraman


(b) Lena


(c) Baboon

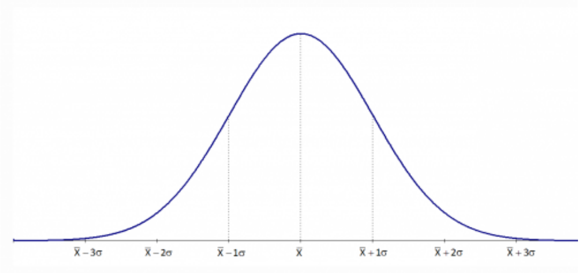
Types of noise

Uniform noise



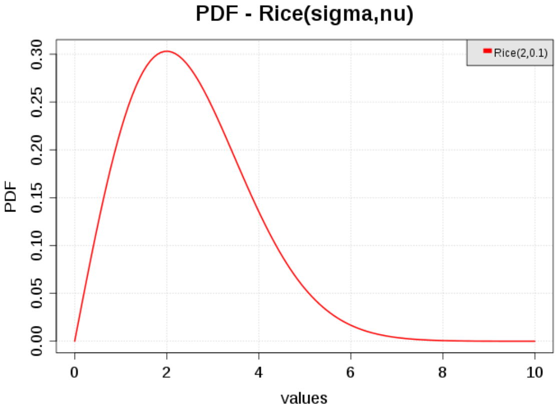
Types of noise

Gaussian noise



Types of noise

Rician noise



Types of noise

Salt and pepper noise

Adding noise

(d) Uniform noise

(e) Gaussian noise

(f) Rician noise

(g) Salt and pepper

- Uniform noise: $\mu = 0$, $\sigma = 0.1$
- Gaussian noise: $\mu = 0$, $\sigma = 0.1$
- Rician noise: $v = 0.05$, $\sigma = 0.1$
- Salt and pepper noise: 5% salt and 5% pepper

Denoising results - Visual comparison

(h) Original

(i) Gaussian noise

(j) Mean filter

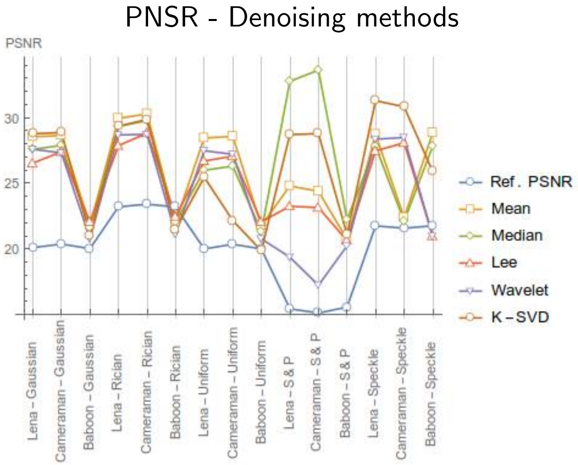
(k) Median filter

(l) Local statistics

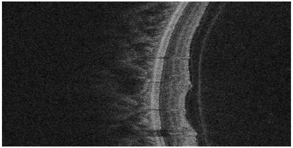
(m) Wavelet

(n) K-SVD

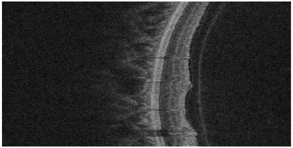
Denoising results



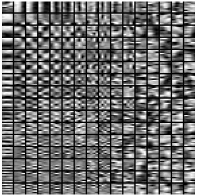
Rethinopathy images



(o) Retinopathy - Original

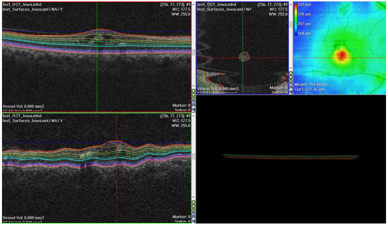


(p) Denoised

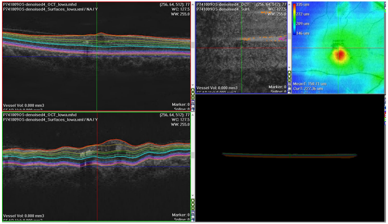


(q) K-SVD Dictionary

Segmentation - OCTExplorer



(r) Original - Noisy image



(s) Denoised image

Final remarks

- Are the results as expected?
- PNSR comparison among algorithms
- Applications of the chosen technique

References

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