

3. Stochastic matrix

Given a positive integer $n \geq 3$, randomly generate a matrix $A \in \mathbf{R}^{n \times n}$ such that:

- (i) $a_{ij} \geq 0$ for all i, j
- (ii) $\sum_{i=1}^n a_{ij} = 1$ for $j = 1, 2, \dots, n$

For example, when $n = 3$:

$$A = \begin{pmatrix} 0.2 & 0.1 & 0.4 \\ 0.5 & 0.6 & 0.3 \\ 0.3 & 0.3 & 0.3 \end{pmatrix}$$

Note: The answer is not unique, but needs to satisfy the conditions.

Scoring Criteria: 80% of your matrix must not be 0 to get a score.

Input Format

A positive integer $n \geq 3$

Output Format

An $n \times n$ numpy array representing the stochastic matrix A where each column sums to 1 and all entries are non-negative.

Constraints

- $3 \leq n \leq 3000$
- All matrix entries $a_{ij} \geq 0$
- Each column sum equals 1: $\sum_{i=1}^n a_{ij} = 1$ for all j
- At least 80% of matrix entries must be non-zero

Sample Input

```
n = 3
```

Sample Output

```
[[0.2 0.1 0.4]
 [0.5 0.6 0.3]
 [0.3 0.3 0.3]]
```

Implementation

Goal: Fill in the following function:

```
def stochastic_matrix(n):  
    ...  
    return ... # Return the stochastic matrix A  
exec("\n".join(iter(input, "#Exit"))) # Don't remove this line
```

Hint

- NumPy has `np.random.rand(n, n)` to create an $n \times n$ matrix with random values