$$f_m(\boldsymbol{\delta}) = I_i(\boldsymbol{p}_m) - I_j(w(\boldsymbol{p}_m, D_i(\boldsymbol{p}_m), \boldsymbol{\delta})), \boldsymbol{p}_m \in \boldsymbol{\Omega}_{D_i}, 1 \le m \le n$$

$$W_m(\boldsymbol{\delta}) = \left(2\sigma_I^2 + \left(\frac{\partial_{r_{p_m}}(\boldsymbol{p}_m, \boldsymbol{\delta})}{\partial D_i(\boldsymbol{p}_m)} \right)^2 V_i(\boldsymbol{p}_m) \right)^{-1}$$

$$\boldsymbol{f} = \begin{pmatrix} f_1(\boldsymbol{\delta}) \\ f_2(\boldsymbol{\delta}) \\ \vdots \\ f_n(\boldsymbol{\delta}) \end{pmatrix}, \boldsymbol{W} = \begin{pmatrix} \boldsymbol{W}_1 & 0 & \cdots & 0 \\ 0 & \boldsymbol{W}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{W}_n \end{pmatrix}$$

$$E(\boldsymbol{\delta}) = \boldsymbol{f}^T(\boldsymbol{\delta})\boldsymbol{W}(\boldsymbol{\delta})\boldsymbol{f}(\boldsymbol{\delta})$$

$$\boldsymbol{f}(\boldsymbol{\delta} + \triangle\boldsymbol{\delta}) \approx l(\triangle\boldsymbol{\delta}) = \boldsymbol{f}(\boldsymbol{\delta}) + \boldsymbol{J}(\boldsymbol{\delta})\triangle\boldsymbol{\delta}$$

$$\frac{1}{2}E(\boldsymbol{\delta}) \quad \approx L(\triangle\boldsymbol{\delta}) = \frac{1}{2}l^T(\triangle\boldsymbol{\delta})l(\triangle\boldsymbol{\delta})$$

$$= \frac{1}{2}\boldsymbol{f}^T\boldsymbol{W}\boldsymbol{f} + \triangle\boldsymbol{\delta}^T\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{f} + \frac{1}{2}\triangle\boldsymbol{\delta}^T\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}\triangle\boldsymbol{\delta}$$

$$= E + \triangle\boldsymbol{\delta}^T\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{f} + \frac{1}{2}\triangle\boldsymbol{\delta}^T\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}\triangle\boldsymbol{\delta}$$

$$L^{'} = \boldsymbol{J}^T\boldsymbol{W}\boldsymbol{f} + \boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}\triangle\boldsymbol{\delta} = 0$$

$$\triangle\boldsymbol{\delta} = -(\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J})^{-1}\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{f}$$

(1)

$$\begin{cases} \boldsymbol{p} := \begin{pmatrix} \boldsymbol{p}_x \\ \boldsymbol{p}_y \\ d_p \end{pmatrix}, \boldsymbol{P} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \boldsymbol{p}_x/d_p \\ \boldsymbol{p}_y/d_p \\ 1/d_p \end{pmatrix} \\ \\ w(\boldsymbol{p}, D_i(\boldsymbol{p}), \boldsymbol{\delta}_{ji}) := \begin{pmatrix} x^{'}/z^{'} \\ y^{'}/z^{'} \end{pmatrix}, \begin{pmatrix} x^{'} \\ y^{'} \\ z^{'} \\ 1 \end{pmatrix} = exp(\boldsymbol{\delta}) \begin{pmatrix} \boldsymbol{p}_x/d_p \\ \boldsymbol{p}_y/d_p \\ 1/d_p \\ 1 \end{pmatrix} \end{cases}$$

(2.10)

$$\boldsymbol{q} = \begin{pmatrix} x^{'} \\ y^{'} \\ z^{'} \end{pmatrix}, w_m = (\boldsymbol{p}_m, D_i(\boldsymbol{p}_m), \boldsymbol{\delta}) = \begin{pmatrix} x^{'}/z^{'} \\ y^{'}/z^{'} \end{pmatrix}$$

$$\frac{\partial f_m}{\partial \delta} = -\frac{\partial I_j(w_m)}{\partial \boldsymbol{\delta}} = -\frac{\partial I_j}{\partial w_m} \frac{\partial w_m}{\partial \boldsymbol{q}} \frac{\partial \boldsymbol{q}}{\partial \boldsymbol{\delta}}$$

$$= -\begin{pmatrix} g_x & g_y \end{pmatrix} \begin{pmatrix} \frac{1}{z^{'}} & 0 & -\frac{x^{'}}{z^{'}} \\ 0 & \frac{1}{z^{'}} & -\frac{y^{'}}{z^{'}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & -z^{'} & y^{'} \\ 0 & 1 & 0 & z^{'} & 0 & -x^{'} \\ 0 & 0 & 1 & -y^{'} & x^{'} & 0 \end{pmatrix}$$

$$= -\begin{pmatrix} \frac{g_x}{z^{'}} & \frac{g_y}{z^{'}} & -\frac{x^{'}g_x}{z^{'2}} - \frac{y^{'}g_y}{z^{'2}} & -\frac{x^{'}y^{'}g_x}{z^{'2}} - (1 + \frac{y^{'2}}{z^{'2}})g_y & (1 + \frac{x^{'2}}{z^{'2}})g_x + \frac{x^{'}y^{'}g_y}{z^{'2}} & -\frac{y^{'}g_x}{z^{'}} + \frac{x^{'}g_y}{z^{'}} \end{pmatrix}$$

$$\boldsymbol{J} = \begin{pmatrix} \frac{\partial f_1}{\partial \delta} & \frac{\partial f_2}{\partial \delta} & \cdots & \frac{\partial f_N}{\partial \delta} \end{pmatrix}^T = \begin{pmatrix} \boldsymbol{J}_1 & \boldsymbol{J}_2 & \cdots & \boldsymbol{J}_N \end{pmatrix}^T$$

$$(2)$$

$$\begin{pmatrix} x^{'} \\ y^{'} \\ z^{'} \\ 1 \end{pmatrix} = exp(\boldsymbol{\delta}) \begin{pmatrix} \boldsymbol{p}_x/d_p \\ \boldsymbol{p}_y/d_p \\ 1/d_p \\ 1 \end{pmatrix} = \frac{1}{d_p} \begin{pmatrix} \boldsymbol{R}_1 & t_1 \\ \boldsymbol{R}_2 & t_2 \\ \boldsymbol{R}_3 & t_3 \\ \boldsymbol{0} & 1 \end{pmatrix}_{4\times4} \begin{pmatrix} \boldsymbol{P} \\ \\ d_p \end{pmatrix}_{4\times1} = \frac{1}{d_p} \begin{pmatrix} \boldsymbol{R}_1\boldsymbol{P} + t_1 d_p \\ \boldsymbol{R}_2\boldsymbol{P} + t_2 d_p \\ \boldsymbol{R}_3\boldsymbol{P} + t_3 d_p \\ d_p \end{pmatrix}$$

$$w_m = \begin{pmatrix} x^{'}/z^{'} \\ y^{'}/z^{'} \end{pmatrix} = \begin{pmatrix} \frac{\boldsymbol{R}_1\boldsymbol{P} + t_1 d_p}{\boldsymbol{R}_3\boldsymbol{P} + t_3 d_p} \\ \frac{\boldsymbol{R}_2\boldsymbol{P} + t_2 d_p}{\boldsymbol{R}_3\boldsymbol{P} + t_3 d_p} \end{pmatrix}$$

$$\frac{\partial_{r_{p_m}}(\boldsymbol{p}_m, \boldsymbol{\delta})}{\partial D_i(\boldsymbol{p}_m)} := \frac{\partial f_m}{\partial d_{p_m}} = -\frac{\partial I_j}{\partial w_m} \frac{\partial w_m}{\partial d_{p_m}}$$

$$= -\begin{pmatrix} g_x & g_y \end{pmatrix} \begin{pmatrix} \frac{t_1 z^{'} - x^{'} t_3}{z^{'2}} \\ \frac{t_2 z^{'} - y^{'} t_3}{z^{'2}} \end{pmatrix} = -(g_x \frac{t_1 z^{'} - x^{'} t_3}{z^{'2}} + g_y \frac{t_2 z^{'} - y^{'} t_3}{z^{'2}})$$

$$W_m := \left(2\sigma_I^2 + w_m^2 V_m)\right)^{-1}$$

$$(3)$$

$$\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J} = \begin{pmatrix} \boldsymbol{J}_1^T & \boldsymbol{J}_2^T & \cdots & \boldsymbol{J}_N^T \end{pmatrix}_{6\times N} \begin{pmatrix} \boldsymbol{W}_1 & 0 & \cdots & 0 \\ 0 & \boldsymbol{W}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{W}_N \end{pmatrix}_{N\times N} \begin{pmatrix} \boldsymbol{J}_1 \\ \boldsymbol{J}_2 \\ \vdots \\ \boldsymbol{J}_N \end{pmatrix}_{N\times 6}$$

$$= \sum_{i=1}^{N} (J_i^T W_i J_i)_{6\times 6} = \boldsymbol{A}_{6\times 6}$$

$$\boldsymbol{J}^T\boldsymbol{W} = \sum_{i=1}^{N} (J_i^T W_i)_{6\times 1} = \boldsymbol{b}_{6\times 1}$$

$$\boldsymbol{A}\boldsymbol{\delta}^* = \boldsymbol{b}$$

$$\Rightarrow \boldsymbol{L}\boldsymbol{D}\boldsymbol{L}^T\boldsymbol{\delta}^* = \boldsymbol{b}$$

$$\Rightarrow (\boldsymbol{L}\boldsymbol{D}^{1/2})(\boldsymbol{L}\boldsymbol{D}^{1/2})^T\boldsymbol{\delta}^* = \boldsymbol{b}$$

$$\Rightarrow \boldsymbol{G}\boldsymbol{G}^T\boldsymbol{\delta}^* = \boldsymbol{b}$$

$$\Rightarrow \boldsymbol{G}\boldsymbol{\delta}^{*'} = \boldsymbol{b}$$

$$\Rightarrow \boldsymbol{G}^T\boldsymbol{\delta}^* = \boldsymbol{\delta}^{*'}$$

$$\text{(4)}$$

$$cost = \sum_{i=j=1}^{5} (A(i,j) - B(i,j))^2 \tag{4.1}$$

$$\vec{s_1} = \vec{n_1} \times \vec{n_2} = \boldsymbol{t} \times \vec{n_1} \times \overrightarrow{Op}$$

$$= \begin{pmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ 1 \end{pmatrix} \tag{4.2}$$

$$= \begin{pmatrix} -at_3 + t_1 \\ -bt_3 + t_2 \\ 0 \end{pmatrix} = \begin{pmatrix} epx \\ epy \\ 0 \end{pmatrix}$$

$$d_2\boldsymbol{p}_{best} = \boldsymbol{R}(d_1\boldsymbol{p}_c) + \boldsymbol{t}$$

$$\Rightarrow \qquad 0 = [\boldsymbol{p}_{best}]_\times(d_1\boldsymbol{R}\boldsymbol{p}_c + \boldsymbol{t})$$

$$\Rightarrow \qquad d_1[\boldsymbol{p}_{best}]_\times\boldsymbol{R}\boldsymbol{p}_c = -[\boldsymbol{p}_{best}]_\times\boldsymbol{t}$$

$$\Rightarrow \qquad d_1\begin{pmatrix} 0 & -1 & b_1 \\ 1 & 0 & -a_1 \\ -b_1 & a_1 & 0 \end{pmatrix}\begin{pmatrix} R_{row0} \\ R_{row1} \\ R_{row2} \end{pmatrix}\boldsymbol{p}_c = \begin{pmatrix} 0 & 1 & -b_1 \\ -1 & 0 & a_1 \\ b_1 & -a_1 & 0 \end{pmatrix}\begin{pmatrix} t_0 \\ t_1 \\ t_2 \end{pmatrix}$$

$$\Rightarrow \qquad d_1\begin{pmatrix} b_1R_{row2} - R_{row1} \\ R_{row0} - a_1R_{row2} \\ a_1R_{row1} - b_1R_{row0} \end{pmatrix}\boldsymbol{p}_c = \begin{pmatrix} t_1 - b_1t_2 \\ a_1t_2 - t_0 \\ b_1t_0 - a_1t_1 \end{pmatrix}$$

$$\Rightarrow \qquad d_1 = \frac{t_1 - b_1t_2}{(b_1R_{row2} - R_{row1})\boldsymbol{p}_c}$$

$$\Rightarrow \qquad = \frac{a_1t_2 - t_0}{(R_{row0} - a_1R_{row2})\boldsymbol{p}_c}$$

$$\Rightarrow \qquad = \frac{b_1t_0 - a_1t_1}{(a_1R_{row1} - b_1R_{row0})\boldsymbol{p}_c} \tag{4.4}$$

$$L := \left\{ \boldsymbol{x} = \boldsymbol{p}_{close} + \lambda\begin{pmatrix} l_x \\ l_y \end{pmatrix} \Big| \lambda = k|\mathbf{S}|, k \in \mathbf{N}^+ \right\} \tag{4.4}$$

$$\mathcal{N}(\frac{\sigma_p^2 d_o + \sigma_o^2 d_p}{\sigma_p^2 + \sigma_o^2}, \frac{\sigma_p^2\sigma_o^2}{\sigma_p^2 + \sigma_o^2}) \tag{2.14}$$

$$d_new = (d_p^{-1} - t_z)^{-1}$$

$$\sigma_{new}^2 = \left(\frac{d_{new}}{d_p}\right)^4\sigma_p^2 \tag{2.14}$$

$$f := \begin{cases} X_{max} = MAX\{x_1, x_2, \ldots, x_m\} \\ X_{min} = MIN\{x_1, x_2, \ldots, x_m\} \\ H_{sum} = \dfrac{X_{max} - X_{min}}{H} \\ Z_{max} = MAX\{z_1, z_2, \ldots, z_m\} \\ Z_{min} = MIN\{z_1, z_2, \ldots, z_m\} \\ V_{sum} = \dfrac{Z_{max} - Z_{min}}{V} \\ \boldsymbol{p}_i = [x_i, z_i]^T \in \boldsymbol{P}, \boldsymbol{q}_i = [h_i, v_i]^T \in \boldsymbol{Q} \\ h_i = \left\lfloor\dfrac{x_i}{H}\right\rfloor, v_i = \left\lfloor\dfrac{z_i}{V}\right\rfloor \end{cases} \tag{3.2}$$

Algorithm 1 : Time optimized A* Algorithm

Require: **Start**,**End**,**Q**

Ensure: Father node of **End**

 1: **function** ASTARSEARCH(**Start**,**End**,**Q**)
 2:     **open** = binary heap containing **Start** node
 3:     **closed** = empty set
 4:     $movecost(x, y) =$ distance from node x to node y
 5:     **while** **End** node not in **open** **do**
 6:         i = node with min $f(i)$ in **open**
 7:         remove i from **open**
 8:         add i to **closed**
 9:         $count = 0$
10:         **for** j = neighbor node of i and not in **closed** and reachable in **Q** **do**
11:             $count++$
12:             $cost = g(i) + movecost(i, j)$
13:             **if** j in **open** and $cost < g(j)$ **then**
14:                 remove j from **open**
15:             **end if**
16:             **if** j not in **open** and not in **closed** **then**
17:                 add j into **open**
18:                 $f(j) = g(j) + h(j)$
19:                 set father node of j is i
20:             **end if**
21:         **end for**
22:         **if** $count == 0$ **then**
23:             can't find path
24:             break out
25:         **end if**
26:     **end while**
27: **end function**

$$\ddot{X}_{t-1} \sim \mathcal{N}(0, k_1^2), \ddot{Z}_{t-1} \sim \mathcal{N}(0, k_2^2), \ddot{\theta}_{t-1} \sim \mathcal{N}(0, k_3^2),$$

$$\boldsymbol{R}_t = \begin{pmatrix} \frac{1}{4}T^4k_1^2 & \frac{1}{2}T^3k_1^2 & 0 & 0 & 0 & 0 \\ \frac{1}{2}T^3k_1^2 & T^2k_1^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4}T^4k_2^2 & \frac{1}{2}T^3k_2^2 & 0 & 0 \\ 0 & 0 & \frac{1}{2}T^3k_2^2 & T^2k_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4}T^4k_3^2 & \frac{1}{2}T^3k_3^2 \\ 0 & 0 & 0 & 0 & \frac{1}{2}T^3k_3^2 & T^2k_3^2 \end{pmatrix} \tag{3.4}$$

$$\begin{pmatrix} cov(\varepsilon_1, \varepsilon_1) & cov(\varepsilon_1, \varepsilon_2) & \cdots & cov(\varepsilon_1, \varepsilon_6) \\ cov(\varepsilon_2, \varepsilon_1) & cov(\varepsilon_2, \varepsilon_2) & \cdots & cov(\varepsilon_2, \varepsilon_6) \\ \vdots & \ddots & \vdots \\ cov(\varepsilon_6, \varepsilon_1) & cov(\varepsilon_6, \varepsilon_2) & \cdots & cov(\varepsilon_6, \varepsilon_6) \end{pmatrix} \tag{3.4}$$

Algorithm 2 : Linear Kalman Filter

Require: $\boldsymbol{\mu}_{t-1}, \boldsymbol{\Sigma}_{t-1}, \boldsymbol{z}_t$
Ensure: $\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t$
1: **function** Filter($\boldsymbol{\mu}_{t-1}, \boldsymbol{\Sigma}_{t-1}, \boldsymbol{z}_t$)
2:     predict $\overline{\boldsymbol{\mu}}_t = \boldsymbol{A}\boldsymbol{\mu}_{t-1}$
3:                  $\overline{\boldsymbol{\Sigma}}_t = \boldsymbol{A}\boldsymbol{\Sigma}_{t-1}\boldsymbol{A}^T + \boldsymbol{R}_t$
4:     update $\boldsymbol{K}_t = \overline{\boldsymbol{\Sigma}}_t(\overline{\boldsymbol{\Sigma}}_t + \boldsymbol{Q}_t)^{-1}$
5:                  $\boldsymbol{\mu}_t = \overline{\boldsymbol{\mu}}_t + \boldsymbol{K}_t(\boldsymbol{z}_t - \overline{\boldsymbol{\mu}}_t)$
6:                  $\boldsymbol{\Sigma}_t = (\boldsymbol{E} - \boldsymbol{K}_t)\overline{\boldsymbol{\Sigma}}_t$
7: **end function**

$$\boldsymbol{\mathcal{F}} = \{\boldsymbol{I}_1, \boldsymbol{I}_2, \boldsymbol{I}_3, \boldsymbol{I}_4\} \xi \Pi \frac{\mathbf{p}}{\partial \mathbf{p}_w}, \begin{pmatrix} f_x z^{-1} & 0 & -x f_x z^{-2} & 0 \\ 0 & f_y z^{-1} & -y f_y z^{-2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & z^{-2} & 0 \end{pmatrix} \tag{3.4}$$

$$\left(\sum_{j \in obs^t(\mathbf{P})} E_{ij}^{\mathbf{P}} + \lambda E_{is}^{\mathbf{P}} \right)$$

$$
\begin{aligned}
E(\delta) &= \color{red}{E_{1L2L}^{\mathbf{P}_1} + E_{2L1L}^{\mathbf{P}_2} + E_{2L3L}^{\mathbf{P}_2} + E_{2L3L}^{\mathbf{P}_3} + E_{3L1L}^{\mathbf{P}_4} + E_{3L2L}^{\mathbf{P}_4} + E_{4L3L}^{\mathbf{P}_5}} \\
&\quad \color{blue}{+ E_{1L1R}^{\mathbf{P}_1} + E_{2L2R}^{\mathbf{P}_2} + E_{4L4R}^{\mathbf{P}_5}} \\
&= E_d(\delta) + E_s(\delta)
\end{aligned}
$$

$$
E_s(\delta) = \begin{pmatrix} r_{\mathbf{P}_1}^s \\ r_{\mathbf{P}_2}^s \\ r_{\mathbf{P}_5}^s \end{pmatrix}^T \begin{pmatrix} \lambda w_{\mathbf{P}_1} & 0 & 0 \\ 0 & \lambda w_{\mathbf{P}_2} & 0 \\ 0 & 0 & \lambda w_{\mathbf{P}_5} \end{pmatrix} \begin{pmatrix} r_{\mathbf{P}_1}^s \\ r_{\mathbf{P}_2}^s \\ r_{\mathbf{P}_5}^s \end{pmatrix} = (\mathbf{r}^s)^T \mathbf{W} \mathbf{r}^s
$$

$$
J_s = \begin{pmatrix}
\frac{\partial r_{\mathbf{P}_1}^s}{\partial \xi_1} \cdots & \frac{\partial r_{\mathbf{P}_1}^s}{\partial \xi_4} & \frac{\partial r_{\mathbf{P}_1}^s}{\partial d_{\mathbf{P}_1}} \cdots & \frac{\partial r_{\mathbf{P}_1}^s}{\partial d_{\mathbf{P}_5}} & \frac{\partial r_{\mathbf{P}_1}^s}{\partial a_1^L} \cdots & \frac{\partial r_{\mathbf{P}_1}^s}{\partial b_4^R} & \frac{\partial r_{\mathbf{P}_1}^s}{\partial f_x} \cdots & \frac{\partial r_{\mathbf{P}_1}^s}{\partial c_y} \\
\frac{\partial r_{\mathbf{P}_2}^s}{\partial \xi_1} \cdots & \frac{\partial r_{\mathbf{P}_2}^s}{\partial \xi_4} & \frac{\partial r_{\mathbf{P}_2}^s}{\partial d_{\mathbf{P}_1}} \cdots & \frac{\partial r_{\mathbf{P}_2}^s}{\partial d_{\mathbf{P}_5}} & \frac{\partial r_{\mathbf{P}_2}^s}{\partial a_1^L} \cdots & \frac{\partial r_{\mathbf{P}_2}^s}{\partial b_4^R} & \frac{\partial r_{\mathbf{P}_2}^s}{\partial f_x} \cdots & \frac{\partial r_{\mathbf{P}_2}^s}{\partial c_y} \\
\frac{\partial r_{\mathbf{P}_5}^s}{\partial \xi_1} \cdots & \frac{\partial r_{\mathbf{P}_5}^s}{\partial \xi_4} & \frac{\partial r_{\mathbf{P}_5}^s}{\partial d_{\mathbf{P}_1}} \cdots & \frac{\partial r_{\mathbf{P}_5}^s}{\partial d_{\mathbf{P}_5}} & \frac{\partial r_{\mathbf{P}_5}^s}{\partial a_1^L} \cdots & \frac{\partial r_{\mathbf{P}_5}^s}{\partial b_4^R} & \frac{\partial r_{\mathbf{P}_5}^s}{\partial f_x} \cdots & \frac{\partial r_{\mathbf{P}_5}^s}{\partial c_y}
\end{pmatrix} \tag{2.2}
$$

$$
E_d(\delta) = \begin{pmatrix} (r_{\mathbf{P}_1}^d)_{12} \\ (r_{\mathbf{P}_1}^d)_{21} \\ \vdots \\ (r_{\mathbf{P}_5}^d)_{43} \end{pmatrix}^T \begin{pmatrix} w_{\mathbf{P}_1} & 0 & \dots & 0 \\ 0 & w_{\mathbf{P}_1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & w_{\mathbf{P}_5} \end{pmatrix} \begin{pmatrix} (r_{\mathbf{P}_1}^d)_{12} \\ (r_{\mathbf{P}_1}^d)_{21} \\ \vdots \\ (r_{\mathbf{P}_5}^d)_{43} \end{pmatrix} = (\mathbf{r}^d)^T \mathbf{W} \mathbf{r}^d
$$

$$J_d = \begin{pmatrix} \frac{\partial (r_{\mathbf{p}_1}^d)_{12}}{\partial \boldsymbol{\xi}_1} \cdots & \frac{\partial (r_{\mathbf{p}_1}^d)_{12}}{\partial \boldsymbol{\xi}_4} & \frac{\partial (r_{\mathbf{p}_1}^d)_{12}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial (r_{\mathbf{p}_1}^d)_{12}}{\partial d_{\mathbf{p}_5}} & \frac{\partial (r_{\mathbf{p}_1}^d)_{12}}{\partial a_1^L} \cdots & \frac{\partial (r_{\mathbf{p}_1}^d)_{12}}{\partial b_4^R} & \frac{\partial (r_{\mathbf{p}_1}^d)_{12}}{\partial f_x} \cdots & \frac{\partial (r_{\mathbf{p}_1}^d)_{12}}{\partial c_y} \\ \frac{\partial (r_{\mathbf{p}_1}^d)_{21}}{\partial \boldsymbol{\xi}_1} \cdots & \frac{\partial (r_{\mathbf{p}_1}^d)_{21}}{\partial \boldsymbol{\xi}_4} & \frac{\partial (r_{\mathbf{p}_1}^d)_{21}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial (r_{\mathbf{p}_1}^d)_{21}}{\partial d_{\mathbf{p}_5}} & \frac{\partial (r_{\mathbf{p}_1}^d)_{21}}{\partial a_1^L} \cdots & \frac{\partial (r_{\mathbf{p}_4}^d)_{21}}{\partial b_4^R} & \frac{\partial (r_{\mathbf{p}_1}^d)_{21}}{\partial f_x} \cdots & \frac{\partial (r_{\mathbf{p}_1}^d)_{21}}{\partial c_y} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial (r_{\mathbf{p}_5}^d)_{43}}{\partial \boldsymbol{\xi}_1} \cdots & \frac{\partial (r_{\mathbf{p}_5}^d)_{43}}{\partial \boldsymbol{\xi}_4} & \frac{\partial (r_{\mathbf{p}_5}^d)_{43}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial (r_{\mathbf{p}_5}^d)_{43}}{\partial d_{\mathbf{p}_5}} & \frac{\partial (r_{\mathbf{p}_5}^d)_{43}}{\partial a_1^L} \cdots & \frac{\partial (r_{\mathbf{p}_5}^d)_{43}}{\partial b_4^R} & \frac{\partial (r_{\mathbf{p}_5}^d)_{43}}{\partial f_x} \cdots & \frac{\partial (r_{\mathbf{p}_5}^d)_{43}}{\partial c_y} \end{pmatrix}$$
$$(2.2)$$

$$(\mathbf{J}_s^T \lambda \mathbf{W}^s \mathbf{J}_s + \mathbf{J}_d^T \mathbf{W}^d \mathbf{J}_d)\delta^* = -(\mathbf{J}_s^T \lambda \mathbf{W}^s \mathbf{r}_s + \mathbf{J}_d^T \mathbf{W}^d \mathbf{r}_d)$$
$$\mathbf{J}_s \in \mathbb{R}^{49 \times 3}, \mathbf{W}^s \in \mathbb{R}^{3 \times 3}, \mathbf{J}_s \in \mathbb{R}^{49 \times 7}, \mathbf{W}^s \in \mathbb{R}^{7 \times 7},$$
$$(2.2)$$

$$\mathbf{p}' = d_{\mathbf{p}}^{iR} \mathbf{K}(\mathbf{T}_{RL}((d_{\mathbf{p}}^{iL})^{-1} \mathbf{K}^{-1} \mathbf{p}))$$
$$\frac{\partial \mathbf{p}'}{\partial d_{\mathbf{p}}^{iL}} = 0?$$
$$(2.2)$$

$$\frac{\partial (r_{\mathbf{p}}^d)}{\partial \boldsymbol{\xi_i}} = \frac{\partial I_j^L(\mathbf{p}')}{\partial \boldsymbol{\xi_i}} = \frac{\partial I_j^L(\mathbf{p}')}{\partial \mathbf{p}'} \frac{\partial \mathbf{p}'}{\partial \mathbf{p}'_w} \frac{\partial \mathbf{p}'_w}{\partial \boldsymbol{\xi_i}}$$
$$\mathbf{p}'_w = d_{\mathbf{p}}^{jL} \mathbf{K}(\mathbf{T}_j \mathbf{T}_i^{-1}((d_{\mathbf{p}}^{iL})^{-1} \mathbf{K}^{-1} \mathbf{p}))$$
$$(2.2)$$

$$\begin{cases} \mathbf{p}_w = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} f_x^{-1}(d_{\mathbf{p}}^{iL})^{-1}(u^i - c_x) \\ f_y^{-1}(d_{\mathbf{p}}^{iL})^{-1}(v^i - c_y) \\ d_{\mathbf{p}}^{iL} \\ 1 \end{pmatrix} \\ \mathbf{p}'_w = d_{\mathbf{p}}^{jL} \mathbf{K}(\mathbf{T}_j \mathbf{T}_i^{-1}((d_{\mathbf{p}}^{iL})^{-1} \mathbf{K}^{-1} \mathbf{p})) \end{cases}$$
$$(2.2)$$

$$\xi^\wedge = \begin{pmatrix} \boldsymbol{\rho} \\ \boldsymbol{\phi} \end{pmatrix}^\wedge = \begin{pmatrix} \phi^\wedge & \boldsymbol{\rho} \\ \mathbf{0}^T & \mathbf{0}^T \end{pmatrix} \in \mathbb{R}^{4 \times 4}$$

$$\boldsymbol{\epsilon} \in \mathbb{R}^3, \begin{pmatrix} \boldsymbol{\epsilon} \\ 1 \end{pmatrix}^\odot = \begin{pmatrix} \mathbf{E} & -\boldsymbol{\epsilon}^\wedge \\ \mathbf{0}^T & \mathbf{0}^T \end{pmatrix} \in \mathbb{R}^{4 \times 6}$$

$$\frac{\partial (exp(\xi^\wedge)\mathbf{p}_w)}{\partial \boldsymbol{\xi}} = \frac{\partial (\mathbf{T}\mathbf{p}_w)}{\partial \boldsymbol{\xi}} = (\mathbf{T}\mathbf{p}_w)^\odot$$

$$\mathbf{T}\mathbf{p}_w = exp(\xi^\wedge)\mathbf{p}_w \approx (\mathbf{E} + \xi^\wedge)\mathbf{p}_w$$
$$(2.2)$$

$$\frac{\partial (exp(\xi^\wedge)\mathbf{p}_w)}{\partial \boldsymbol{\xi}} \approx \frac{\partial (\mathbf{E} + \xi^\wedge)}{\partial \boldsymbol{\xi}} = \mathbf{0} + \frac{\partial (\xi^\wedge \mathbf{p}_w)}{\partial \boldsymbol{\xi}} \approx (\mathbf{T}\mathbf{p}_w)^\odot$$

$$since, \frac{\partial (\mathbf{T}\mathbf{p}_w)}{\partial \boldsymbol{\xi}} = (\mathbf{T}^{-1}\mathbf{p}_w)^\odot = \frac{\partial (exp(-\xi^\wedge)\mathbf{p}_w)}{\partial \boldsymbol{\xi}}$$

$$= \frac{\partial (\mathbf{E} - \xi^\wedge)}{\partial \boldsymbol{\xi}} = -(\mathbf{T}\mathbf{p}_w)^\odot$$

Algorithm 3 Levenberg-Marquardt锟斤拷锟斤拷锟脚伙拷锟斤拷锟斤拷位锟斤拷

Require: $\mathcal{K}_i,\mathcal{I}_j,\boldsymbol{\delta}_{i(i-1)}$,k_{max}(锟斤拷锟斤拷锟斤拷锟斤拷锟斤拷锟斤拷)
Ensure: $\boldsymbol{\delta}_{ji}$

1: **function** TRACKFRAME($Array, left, middle, right$)
2:     $v \leftarrow 2$
3:     $\boldsymbol{\delta} \leftarrow \boldsymbol{\delta}_{i(i-1)}$
4:     $\boldsymbol{f}_i$
5:     **for** $i = 0 \rightarrow k_{max} - 1$ **do**
6:         $\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}$
7:         **while** $true$ **do**
8:             slove $\boldsymbol{\delta}^*$
9:             update
10:             $\boldsymbol{f}_i$
11:             $error$=caclweight
12:             **if** $error < lastError$ **then**
13:                 $lastResidual=lastError=error$
14:                 **if** $\lambda <= 0.2$ **then**
15:                     λ=0
16:                 **else**
17:                     $\lambda=\frac{1}{2}\lambda$
18:                 **end if**
19:                 break
20:             **else**
21:                 **if** 锟斤拷锟斤拷$< \varepsilon_1$ **then**
22:                     $i=k_{max} - 1$
23:                     break
24:                 **end if**
25:                 **if** $\lambda==0$ **then**
26:                     λ=0.2
27:                 **else**
28:                     λ=2*inctry
29:                 **end if**
30:             **end if**
31:         **end while**
32:     **end for**
33:     **return** $\boldsymbol{\delta}$
34: **end function**