$$
\begin{aligned}
&f_m(\boldsymbol{\delta}) = I_i(\boldsymbol{p}_m) - I_j(w(\boldsymbol{p}_m, D_i(\boldsymbol{p}_m), \boldsymbol{\delta})), \boldsymbol{p}_m \in \boldsymbol{\Omega}_{D_i}, 1 \leq m \leq n \\
&W_m(\boldsymbol{\delta}) = \left(2\sigma_I^2 + \left(\frac{\partial_{r_{p_m}}(\boldsymbol{p}_m, \boldsymbol{\delta})}{\partial D_i(\boldsymbol{p}_m)}\right)^2 V_i(\boldsymbol{p}_m)\right)^{-1} \\
&\boldsymbol{f} = \begin{pmatrix} f_1(\boldsymbol{\delta}) \\ f_2(\boldsymbol{\delta}) \\ \vdots \\ f_n(\boldsymbol{\delta}) \end{pmatrix}, \boldsymbol{W} = \begin{pmatrix} \boldsymbol{W}_1 & 0 & \cdots & 0 \\ 0 & \boldsymbol{W}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{W}_n \end{pmatrix} \\
&E(\boldsymbol{\delta}) = \boldsymbol{f}^T(\boldsymbol{\delta})\boldsymbol{W}(\boldsymbol{\delta})\boldsymbol{f}(\boldsymbol{\delta}) \\
&\boldsymbol{f}(\boldsymbol{\delta} + \triangle\boldsymbol{\delta}) \approx l(\triangle\boldsymbol{\delta}) = \boldsymbol{f}(\boldsymbol{\delta}) + \boldsymbol{J}(\boldsymbol{\delta})\triangle\boldsymbol{\delta} \\
&\frac{1}{2}E(\boldsymbol{\delta}) \quad \approx L(\triangle\boldsymbol{\delta}) = \frac{1}{2}l^T(\triangle\boldsymbol{\delta})l(\triangle\boldsymbol{\delta}) \\
&\qquad\qquad\qquad\qquad = \frac{1}{2}\boldsymbol{f}^T\boldsymbol{W}\boldsymbol{f} + \triangle\boldsymbol{\delta}^T\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{f} + \frac{1}{2}\triangle\boldsymbol{\delta}^T\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}\triangle\boldsymbol{\delta} \\
&\qquad\qquad\qquad\qquad = E + \triangle\boldsymbol{\delta}^T\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{f} + \frac{1}{2}\triangle\boldsymbol{\delta}^T\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}\triangle\boldsymbol{\delta} \\
&L^{'} = \boldsymbol{J}^T\boldsymbol{W}\boldsymbol{f} + \boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J}\triangle\boldsymbol{\delta} = 0 \\
&\triangle\boldsymbol{\delta} = -(\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J})^{-1}\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{f}
\end{aligned}
\tag{1}
$$

$$
\begin{cases}
\boldsymbol{p} := \begin{pmatrix} \boldsymbol{p}_x \\ \boldsymbol{p}_y \\ d_p \end{pmatrix}, \boldsymbol{P} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} \boldsymbol{p}_x/d_p \\ \boldsymbol{p}_y/d_p \\ 1/d_p \end{pmatrix} \\
w(\boldsymbol{p}, D_i(\boldsymbol{p}), \boldsymbol{\delta}_{ji}) := \begin{pmatrix} x^{'}/z^{'} \\ y^{'}/z^{'} \end{pmatrix}, \begin{pmatrix} x^{'} \\ y^{'} \\ z^{'} \\ 1 \end{pmatrix} = exp(\boldsymbol{\delta}) \begin{pmatrix} \boldsymbol{p}_x/d_p \\ \boldsymbol{p}_y/d_p \\ 1/d_p \\ 1 \end{pmatrix}
\end{cases}
\tag{2.10}
$$

$$
\begin{aligned}
&\boldsymbol{q} = \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}, w_m = (\boldsymbol{p}_m, D_i(\boldsymbol{p}_m), \boldsymbol{\delta}) = \begin{pmatrix} x'/z' \\ y'/z' \end{pmatrix} \\
&\frac{\partial f_m}{\partial \delta} = -\frac{\partial I_j(w_m)}{\partial \boldsymbol{\delta}} = -\frac{\partial I_j}{\partial w_m}\frac{\partial w_m}{\partial \boldsymbol{q}}\frac{\partial \boldsymbol{q}}{\partial \boldsymbol{\delta}} \\
&= -\begin{pmatrix} g_x & g_y \end{pmatrix} \begin{pmatrix} \frac{1}{z'} & 0 & -\frac{x'}{z'} \\ 0 & \frac{1}{z'} & -\frac{y'}{z'} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 & -z' & y' \\ 0 & 1 & 0 & z' & 0 & -x' \\ 0 & 0 & 1 & -y' & x' & 0 \end{pmatrix} \\
&= -\begin{pmatrix} \frac{g_x}{z'} & \frac{g_y}{z'} & -\frac{x' g_x}{z'^2} - \frac{y' g_y}{z'^2} & -\frac{x' y' g_x}{z'^2} - (1 + \frac{y'^2}{z'^2}) g_y & (1 + \frac{x'^2}{z'^2}) g_x + \frac{x' y' g_y}{z'^2} & -\frac{y' g_x}{z'} + \frac{x' g_y}{z'} \end{pmatrix} \\
&\boldsymbol{J} = \begin{pmatrix} \frac{\partial f_1}{\partial \delta} & \frac{\partial f_2}{\partial \delta} & \cdots & \frac{\partial f_N}{\partial \delta} \end{pmatrix}^T = \begin{pmatrix} \boldsymbol{J}_1 & \boldsymbol{J}_2 & \cdots & \boldsymbol{J}_N \end{pmatrix}^T
\end{aligned} \tag{2}
$$

$$
\begin{aligned}
&\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = exp(\boldsymbol{\delta}) \begin{pmatrix} \boldsymbol{p}_x/d_p \\ \boldsymbol{p}_y/d_p \\ 1/d_p \\ 1 \end{pmatrix} = \frac{1}{d_p} \begin{pmatrix} \boldsymbol{R}_1 & t_1 \\ \boldsymbol{R}_2 & t_2 \\ \boldsymbol{R}_3 & t_3 \\ \boldsymbol{0} & 1 \end{pmatrix}_{4\times 4} \begin{pmatrix} \boldsymbol{P} \\ \\ d_p \end{pmatrix}_{4\times 1} = \frac{1}{d_p} \begin{pmatrix} \boldsymbol{R}_1 \boldsymbol{P} + t_1 d_p \\ \boldsymbol{R}_2 \boldsymbol{P} + t_2 d_p \\ \boldsymbol{R}_3 \boldsymbol{P} + t_3 d_p \\ d_p \end{pmatrix} \\
&w_m = \begin{pmatrix} x'/z' \\ y'/z' \end{pmatrix} = \begin{pmatrix} \frac{\boldsymbol{R}_1 \boldsymbol{P} + t_1 d_p}{\boldsymbol{R}_3 \boldsymbol{P} + t_3 d_p} \\ \frac{\boldsymbol{R}_2 \boldsymbol{P} + t_2 d_p}{\boldsymbol{R}_3 \boldsymbol{P} + t_3 d_p} \end{pmatrix} \\
&\frac{\partial_{r_{p_m}}(\boldsymbol{p}_m, \boldsymbol{\delta})}{\partial D_i(\boldsymbol{p}_m)} := \frac{\partial f_m}{\partial d_{p_m}} = -\frac{\partial I_j}{\partial w_m}\frac{\partial w_m}{\partial d_{p_m}} \\
&= -\begin{pmatrix} g_x & g_y \end{pmatrix} \begin{pmatrix} \frac{t_1 z' - x' t_3}{z'^2} \\ \frac{t_2 z' - y' t_3}{z'^2} \end{pmatrix} = -(g_x \frac{t_1 z' - x' t_3}{z'^2} + g_y \frac{t_2 z' - y' t_3}{z'^2}) \\
&W_m := \left(2\sigma_I^2 + w_m^2 V_m)\right)^{-1}
\end{aligned} \tag{3}
$$

$$
\begin{aligned}
\boldsymbol{J}^T\boldsymbol{W}\boldsymbol{J} &= \begin{pmatrix} \boldsymbol{J}_1^T & \boldsymbol{J}_2^T & \cdots & \boldsymbol{J}_N^T \end{pmatrix}_{6\times N} \begin{pmatrix} \boldsymbol{W}_1 & 0 & \cdots & 0 \\ 0 & \boldsymbol{W}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \boldsymbol{W}_N \end{pmatrix}_{N\times N} \begin{pmatrix} \boldsymbol{J}_1 \\ \boldsymbol{J}_2 \\ \vdots \\ \boldsymbol{J}_N \end{pmatrix}_{N\times 6} \\
&= \sum_{i=1}^{N} (J_i^T W_i J_i)_{6\times 6} = \boldsymbol{A}_{6\times 6} \\
\boldsymbol{J}^T\boldsymbol{W} &= \sum_{i=1}^{N} (J_i^T W_i)_{6\times 1} = \boldsymbol{b}_{6\times 1} \\
& \boldsymbol{A}\boldsymbol{\delta}^* = \boldsymbol{b} \\
& \Rightarrow \boldsymbol{L}\boldsymbol{D}\boldsymbol{L}^T\boldsymbol{\delta}^* = \boldsymbol{b} \\
& \Rightarrow (\boldsymbol{L}\boldsymbol{D}^{1/2})(\boldsymbol{L}\boldsymbol{D}^{1/2})^T\boldsymbol{\delta}^* = \boldsymbol{b} \\
& \Rightarrow \boldsymbol{G}\boldsymbol{G}^T\boldsymbol{\delta}^* = \boldsymbol{b} \\
& \Rightarrow \boldsymbol{G}\boldsymbol{\delta}^{*'} = \boldsymbol{b} \\
& \Rightarrow \boldsymbol{G}^T\boldsymbol{\delta}^* = \boldsymbol{\delta}^{*'}
\end{aligned} \tag{4}
$$

$$
cost = \sum_{i=j=1}^{5} (A(i,j) - B(i,j))^2 \tag{4.1}
$$

$$
\begin{aligned}
\overrightarrow{s_1} &= \overrightarrow{n_1} \times \overrightarrow{n_2} = \boldsymbol{t} \times \overrightarrow{n_1} \times \overrightarrow{Op} \\
&= \begin{pmatrix} 0 & -t_3 & t_2 \\ t_3 & 0 & -t_1 \\ -t_2 & t_1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} -at_3 + t_1 \\ -bt_3 + t_2 \\ 0 \end{pmatrix} = \begin{pmatrix} epx \\ epy \\ 0 \end{pmatrix}
\end{aligned} \tag{4.2}
$$

$$
\begin{aligned}
& & d_2\boldsymbol{p}_{best} &= \boldsymbol{R}(d_1\boldsymbol{p}_c) + \boldsymbol{t} \\
&\Rightarrow & 0 &= [\boldsymbol{p}_{best}]_\times (d_1\boldsymbol{R}\boldsymbol{p}_c + \boldsymbol{t}) \\
&\Rightarrow & d_1[\boldsymbol{p}_{best}]_\times \boldsymbol{R}\boldsymbol{p}_c &= -[\boldsymbol{p}_{best}]_\times \boldsymbol{t} \\
&\Rightarrow & d_1 \begin{pmatrix} 0 & -1 & b_1 \\ 1 & 0 & -a_1 \\ -b_1 & a_1 & 0 \end{pmatrix} \begin{pmatrix} R_{row0} \\ R_{row1} \\ R_{row2} \end{pmatrix} \boldsymbol{p}_c &= \begin{pmatrix} 0 & 1 & -b_1 \\ -1 & 0 & a_1 \\ b_1 & -a_1 & 0 \end{pmatrix} \begin{pmatrix} t_0 \\ t_1 \\ t_2 \end{pmatrix} \\
&\Rightarrow & d_1 \begin{pmatrix} b_1R_{row2} - R_{row1} \\ R_{row0} - a_1R_{row2} \\ a_1R_{row1} - b_1R_{row0} \end{pmatrix} \boldsymbol{p}_c &= \begin{pmatrix} t_1 - b_1t_2 \\ a_1t_2 - t_0 \\ b_1t_0 - a_1t_1 \end{pmatrix} \\
&\Rightarrow & d_1 &= \frac{t_1 - b_1t_2}{(b_1R_{row2} - R_{row1})\boldsymbol{p}_c} \\
&\Rightarrow & &= \frac{a_1t_2 - t_0}{(R_{row0} - a_1R_{row2})\boldsymbol{p}_c} \\
&\Rightarrow & &= \frac{b_1t_0 - a_1t_1}{(a_1R_{row1} - b_1R_{row0})\boldsymbol{p}_c}
\end{aligned} \tag{4.4}
$$

$$
L := \left\{ \boldsymbol{x} = \boldsymbol{p}_{close} + \lambda \begin{pmatrix} l_x \\ l_y \end{pmatrix} | \lambda = k|\mathbf{S}|, k \in \mathbf{N}^+ \right\} \tag{4.4}
$$

$$
\mathcal{N}\left(\frac{\sigma_p^2 d_o + \sigma_o^2 d_p}{\sigma_p^2 + \sigma_o^2}, \frac{\sigma_p^2\sigma_o^2}{\sigma_p^2 + \sigma_o^2}\right) \tag{2.14}
$$

$$
\begin{aligned}
d_new &= (d_p^{-1} - t_z)^{-1} \\
\sigma_{new}^2 &= \left(\frac{d_{new}}{d_p}\right)^4 \sigma_p^2
\end{aligned} \tag{2.14}
$$

$$
f := \begin{cases}
X_{max} = MAX\{x_1, x_2, \ldots, x_m\} \\
X_{min} = MIN\{x_1, x_2, \ldots, x_m\} \\
H_{sum} = \dfrac{X_{max} - X_{min}}{H} \\
Z_{max} = MAX\{z_1, z_2, \ldots, z_m\} \\
Z_{min} = MIN\{z_1, z_2, \ldots, z_m\} \\
V_{sum} = \dfrac{Z_{max} - Z_{min}}{V} \\
\boldsymbol{p}_i = [x_i, z_i]^T \in \boldsymbol{P}, \boldsymbol{q}_i = [h_i, v_i]^T \in \boldsymbol{Q} \\
h_i = \left\lfloor \dfrac{x_i}{H} \right\rfloor, v_i = \left\lfloor \dfrac{z_i}{V} \right\rfloor
\end{cases} \tag{3.2}
$$

Algorithm 1 : Time optimized A* Algorithm

Require: ***Start***,***End***,***Q***

Ensure: Father node of ***End***

```
function ASTARSEARCH(Start,End,Q)
    open = binary heap containing Start node
    closed = empty set
    movecost(x, y) = distance from node x to node y
    while End node not in open do
        i = node with min f(i) in open
        remove i from open
        add i to closed
        count = 0
        for j = neighbor node of i and not in closed and reachable in Q do
            count++
            cost = g(i) + movecost(i, j)
            if j in open and cost < g(j) then
                remove j from open
            end if
            if j not in open and not in closed then
                add j into open
                f(j) = g(j) + h(j)
                set father node of j is i
            end if
        end for
        if count == 0 then
            can't find path
            break out
        end if
    end while
end function
```

$$\ddot{X}_{t-1} \sim \mathcal{N}(0, k_1^2), \ddot{Z}_{t-1} \sim \mathcal{N}(0, k_2^2), \ddot{\theta}_{t-1} \sim \mathcal{N}(0, k_3^2),$$

$$\boldsymbol{R}_t = \begin{pmatrix} \frac{1}{4}T^4k_1^2 & \frac{1}{2}T^3k_1^2 & 0 & 0 & 0 & 0 \\ \frac{1}{2}T^3k_1^2 & T^2k_1^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{4}T^4k_2^2 & \frac{1}{2}T^3k_2^2 & 0 & 0 \\ 0 & 0 & \frac{1}{2}T^3k_2^2 & T^2k_2^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{4}T^4k_3^2 & \frac{1}{2}T^3k_3^2 \\ 0 & 0 & 0 & 0 & \frac{1}{2}T^3k_3^2 & T^2k_3^2 \end{pmatrix} \tag{3.4}$$

$$\begin{pmatrix} cov(\varepsilon_1, \varepsilon_1) & cov(\varepsilon_1, \varepsilon_2) & \cdots & cov(\varepsilon_1, \varepsilon_6) \\ cov(\varepsilon_2, \varepsilon_1) & cov(\varepsilon_2, \varepsilon_2) & \cdots & cov(\varepsilon_2, \varepsilon_6) \\ \vdots & \ddots & \vdots & \\ cov(\varepsilon_6, \varepsilon_1) & cov(\varepsilon_6, \varepsilon_2) & \cdots & cov(\varepsilon_6, \varepsilon_6) \end{pmatrix} \tag{3.4}$$

Algorithm 2 : Linear Kalman Filter

Require: $\boldsymbol{\mu}_{t-1}, \boldsymbol{\Sigma}_{t-1}, \boldsymbol{z}_t$

Ensure: $\boldsymbol{\mu}_t, \boldsymbol{\Sigma}_t$

```
function FILTER(μ_{t-1}, Σ_{t-1}, z_t)
    predict  μ̄_t = A μ_{t-1}
             Σ̄_t = A Σ_{t-1} A^T + R_t
    update   K_t = Σ̄_t (Σ̄_t + Q_t)^{-1}
             μ_t = μ̄_t + K_t (z_t − μ̄_t)
             Σ_t = (E − K_t) Σ̄_t
end function
```

$$\boldsymbol{\mathcal{F}} = \{\boldsymbol{I}_1, \boldsymbol{I}_2, \boldsymbol{I}_3, \boldsymbol{I}_4\}\xi\Pi\frac{\mathbf{p}}{\partial \mathbf{p}_w}, \begin{pmatrix} f_x z^{-1} & 0 & -x f_x z^{-2} & 0 \\ 0 & f_y z^{-1} & -y f_y z^{-2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & z^{-2} & 0 \end{pmatrix} \tag{3.4}$$

$$\Big(\sum_{j \in obs^t(\mathbf{P})} E_{ij}^{\mathbf{p}} + \lambda E_{is}^{\mathbf{p}}\Big)$$

$$\begin{aligned} E(\delta) &= E_{1L2L}^{\mathbf{p}_1} + E_{2L1L}^{\mathbf{p}_2} + E_{2L3L}^{\mathbf{p}_2} + E_{2L3L}^{\mathbf{p}_3} + E_{3L1L}^{\mathbf{p}_4} + E_{3L2L}^{\mathbf{p}_4} + E_{4L3L}^{\mathbf{p}_5} \\ &\quad + E_{1L1R}^{\mathbf{p}_1} + E_{2L2R}^{\mathbf{p}_2} + E_{4L4R}^{\mathbf{p}_5} \\ &= E_d(\delta) + E_s(\delta) \\ E_s(\delta) &= \begin{pmatrix} r_{\mathbf{p}_1}^s \\ r_{\mathbf{p}_2}^s \\ r_{\mathbf{p}_5}^s \end{pmatrix}^T \begin{pmatrix} \lambda w_{\mathbf{p}_1} & 0 & 0 \\ 0 & \lambda w_{\mathbf{p}_2} & 0 \\ 0 & 0 & \lambda w_{\mathbf{p}_5} \end{pmatrix} \begin{pmatrix} r_{\mathbf{p}_1}^s \\ r_{\mathbf{p}_2}^s \\ r_{\mathbf{p}_5}^s \end{pmatrix} = (\mathbf{r}^s)^T \mathbf{W} \mathbf{r}^s \\ J_s &= \begin{pmatrix} \frac{\partial r_{\mathbf{p}_1}^s}{\partial \boldsymbol{\xi}_1} \cdots & \frac{\partial r_{\mathbf{p}_1}^s}{\partial \boldsymbol{\xi}_4} & \frac{\partial r_{\mathbf{p}_1}^s}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial r_{\mathbf{p}_1}^s}{\partial d_{\mathbf{p}_5}} & \frac{\partial r_{\mathbf{p}_1}^s}{\partial a_1^L} \cdots & \frac{\partial r_{\mathbf{p}_1}^s}{\partial b_4^R} & \frac{\partial r_{\mathbf{p}_1}^s}{\partial f_x} \cdots & \frac{\partial r_{\mathbf{p}_1}^s}{\partial c_y} \\ \frac{\partial r_{\mathbf{p}_2}^s}{\partial \boldsymbol{\xi}_1} \cdots & \frac{\partial r_{\mathbf{p}_2}^s}{\partial \boldsymbol{\xi}_4} & \frac{\partial r_{\mathbf{p}_2}^s}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial r_{\mathbf{p}_2}^s}{\partial d_{\mathbf{p}_5}} & \frac{\partial r_{\mathbf{p}_2}^s}{\partial a_1^L} \cdots & \frac{\partial r_{\mathbf{p}_2}^s}{\partial b_4^R} & \frac{\partial r_{\mathbf{p}_2}^s}{\partial f_x} \cdots & \frac{\partial r_{\mathbf{p}_2}^s}{\partial c_y} \\ \frac{\partial r_{\mathbf{p}_5}^s}{\partial \boldsymbol{\xi}_1} \cdots & \frac{\partial r_{\mathbf{p}_5}^s}{\partial \boldsymbol{\xi}_4} & \frac{\partial r_{\mathbf{p}_5}^s}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial r_{\mathbf{p}_5}^s}{\partial d_{\mathbf{p}_5}} & \frac{\partial r_{\mathbf{p}_5}^s}{\partial a_1^L} \cdots & \frac{\partial r_{\mathbf{p}_5}^s}{\partial b_4^R} & \frac{\partial r_{\mathbf{p}_5}^s}{\partial f_x} \cdots & \frac{\partial r_{\mathbf{p}_5}^s}{\partial c_y} \end{pmatrix} \\ E_d(\delta) &= \begin{pmatrix} (r_{\mathbf{p}_1}^d)_{12} \\ (r_{\mathbf{p}_1}^d)_{21} \\ \vdots \\ (r_{\mathbf{p}_5}^d)_{43} \end{pmatrix}^T \begin{pmatrix} w_{\mathbf{p}_1} & 0 & \dots & 0 \\ 0 & w_{\mathbf{p}_1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & w_{\mathbf{p}_5} \end{pmatrix} \begin{pmatrix} (r_{\mathbf{p}_1}^d)_{12} \\ (r_{\mathbf{p}_1}^d)_{21} \\ \vdots \\ (r_{\mathbf{p}_5}^d)_{43} \end{pmatrix} = (\mathbf{r}^d)^T \mathbf{W} \mathbf{r}^d \end{aligned} \tag{2.2}$$

$$J_d = \begin{pmatrix} \frac{\partial(r^d_{\mathbf{p}_1})_{12}}{\partial\boldsymbol{\xi}_1} \cdots & \frac{\partial(r^d_{\mathbf{p}_1})_{12}}{\partial\boldsymbol{\xi}_4} & \frac{\partial(r^d_{\mathbf{p}_1})_{12}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial(r^d_{\mathbf{p}_1})_{12}}{\partial d_{\mathbf{p}_5}} & \frac{\partial(r^d_{\mathbf{p}_1})_{12}}{\partial a^L_1} \cdots & \frac{\partial(r^d_{\mathbf{p}_1})_{12}}{\partial b^R_4} & \frac{\partial(r^d_{\mathbf{p}_1})_{12}}{\partial f_x} \cdots & \frac{\partial(r^d_{\mathbf{p}_1})_{12}}{\partial c_y} \\ \frac{\partial(r^d_{\mathbf{p}_1})_{21}}{\partial\boldsymbol{\xi}_1} \cdots & \frac{\partial(r^d_{\mathbf{p}_1})_{21}}{\partial\boldsymbol{\xi}_4} & \frac{\partial(r^d_{\mathbf{p}_1})_{21}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial(r^d_{\mathbf{p}_1})_{21}}{\partial d_{\mathbf{p}_5}} & \frac{\partial(r^d_{\mathbf{p}_1})_{21}}{\partial a^L_1} \cdots & \frac{\partial(r^d_{\mathbf{p}_1})_{21}}{\partial b^R_4} & \frac{\partial(r^d_{\mathbf{p}_1})_{21}}{\partial f_x} \cdots & \frac{\partial(r^d_{\mathbf{p}_1})_{21}}{\partial c_y} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial(r^d_{\mathbf{p}_5})_{43}}{\partial\boldsymbol{\xi}_1} \cdots & \frac{\partial(r^d_{\mathbf{p}_5})_{43}}{\partial\boldsymbol{\xi}_4} & \frac{\partial(r^d_{\mathbf{p}_5})_{43}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial(r^d_{\mathbf{p}_5})_{43}}{\partial d_{\mathbf{p}_5}} & \frac{\partial(r^d_{\mathbf{p}_5})_{43}}{\partial a^L_1} \cdots & \frac{\partial(r^d_{\mathbf{p}_5})_{43}}{\partial b^R_4} & \frac{\partial(r^d_{\mathbf{p}_5})_{43}}{\partial f_x} \cdots & \frac{\partial(r^d_{\mathbf{p}_5})_{43}}{\partial c_y} \end{pmatrix} \tag{2.2}$$

$$\begin{aligned} &(\mathbf{J}_s^T \lambda \mathbf{W}^s \mathbf{J}_s + \mathbf{J}_d^T \mathbf{W}^d \mathbf{J}_d)\delta^* = -(\mathbf{J}_s^T \lambda \mathbf{W}^s \mathbf{r}_s + \mathbf{J}_d^T \mathbf{W}^d \mathbf{r}_d) \\ &\mathbf{J}_s \in \mathbb{R}^{49\times 3}, \mathbf{W}^s \in \mathbb{R}^{3\times 3}, \mathbf{J}_s \in \mathbb{R}^{49\times 7}, \mathbf{W}^s \in \mathbb{R}^{7\times 7}, \end{aligned} \tag{2.2}$$

$$\begin{aligned} &\mathbf{p}^{'} = d^{iR}_{\mathbf{p}} \mathbf{K}(\mathbf{T}_{RL}((d^{iL}_{\mathbf{p}})^{-1}\mathbf{K}^{-1}\mathbf{p})) \\ &\frac{\partial \mathbf{p}^{'}}{\partial d^{iL}_{\mathbf{p}}} = 0? \end{aligned} \tag{2.2}$$

$$\begin{aligned} &\frac{\partial(r^d_{\mathbf{p}})}{\partial \boldsymbol{\xi_i}} = \frac{\partial I^L_j(\mathbf{p}^{'})}{\partial \boldsymbol{\xi_i}} = \frac{\partial I^L_j(\mathbf{p}^{'})}{\partial \mathbf{p}^{'}} \frac{\partial \mathbf{p}^{'}}{\partial \mathbf{p}^{'}_w} \frac{\partial \mathbf{p}^{'}_w}{\partial \boldsymbol{\xi_i}} \\ &\mathbf{p}^{'}_w = d^{jL}_{\mathbf{p}} \mathbf{K}(\mathbf{T}_j \mathbf{T}_i^{-1}((d^{iL}_{\mathbf{p}})^{-1}\mathbf{K}^{-1}\mathbf{p})) \end{aligned} \tag{2.2}$$

$$\begin{cases} \mathbf{p}_w = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} f_x^{-1}(d^{iL}_{\mathbf{p}})^{-1}(u^i - c_x) \\ f_y^{-1}(d^{iL}_{\mathbf{p}})^{-1}(v^i - c_y) \\ d^{iL}_{\mathbf{p}} \\ 1 \end{pmatrix} \\ \mathbf{p}^{'}_w = d^{jL}_{\mathbf{p}} \mathbf{K}(\mathbf{T}_j \mathbf{T}_i^{-1}((d^{iL}_{\mathbf{p}})^{-1}\mathbf{K}^{-1}\mathbf{p})) \end{cases} \tag{2.2}$$

$$\begin{aligned} &\xi^\wedge = \begin{pmatrix} \boldsymbol{\rho} \\ \boldsymbol{\phi} \end{pmatrix}^\wedge = \begin{pmatrix} \boldsymbol{\phi}^\wedge & \boldsymbol{\rho} \\ \mathbf{0}^T & \mathbf{0}^T \end{pmatrix} \in \mathbb{R}^{4\times 4} \\ &\boldsymbol{\epsilon} \in \mathbb{R}^3, \begin{pmatrix} \boldsymbol{\epsilon} \\ 1 \end{pmatrix}^\odot = \begin{pmatrix} \mathbf{E} & -\boldsymbol{\epsilon}^\wedge \\ \mathbf{0}^T & \mathbf{0}^T \end{pmatrix} \in \mathbb{R}^{4\times 6} \\ &\frac{\partial(exp(\xi^\wedge)\mathbf{p}_w)}{\partial \boldsymbol{\xi}} = \frac{\partial(\mathbf{T}\mathbf{p}_w)}{\partial \boldsymbol{\xi}} = (\mathbf{T}\mathbf{p}_w)^\odot \\ &\mathbf{T}\mathbf{p}_w = exp(\xi^\wedge)\mathbf{p}_w \approx (\mathbf{E} + \xi^\wedge)\mathbf{p}_w \\ &\frac{\partial(exp(\xi^\wedge)\mathbf{p}_w)}{\partial \boldsymbol{\xi}} \approx \frac{\partial(\mathbf{E} + \xi^\wedge)}{\partial \boldsymbol{\xi}} = \mathbf{0} + \frac{\partial(\xi^\wedge \mathbf{p}_w)}{\partial \boldsymbol{\xi}} \approx (\mathbf{T}\mathbf{p}_w)^\odot \\ &since, \frac{\partial(\mathbf{T}\mathbf{p}_w)}{\partial \boldsymbol{\xi}} = (\mathbf{T}^{-1}\mathbf{p}_w)^\odot = \frac{\partial(exp(-\xi^\wedge)\mathbf{p}_w)}{\partial \boldsymbol{\xi}} \\ &= \frac{\partial(\mathbf{E} - \xi^\wedge)}{\partial \boldsymbol{\xi}} = -(\mathbf{T}\mathbf{p}_w)^\odot \end{aligned} \tag{2.2}$$

$$
\begin{aligned}
&\frac{\partial (r_{\mathbf{p}}^{d})_{ij}}{\partial d_{\mathbf{p}}^{iL}} = \frac{\partial (I_j^L(\mathbf{p}^{'}))}{\partial d_{\mathbf{p}}^{iL}} = \frac{\partial (I_j^L(\mathbf{p}^{'}))}{\partial \mathbf{p}^{'}} \frac{\partial \mathbf{p}^{'}}{\partial d_{\mathbf{p}}^{iL}} \\
&\mathbf{p}^{'} = d_{\mathbf{p}}^{jL} \mathbf{K}(\mathbf{T}_j \mathbf{T}_i^{-1} \mathbf{p}_w) \\
&assume : \mathbf{T}_j \mathbf{T}_i^{-1} = \begin{pmatrix} r_{11}^{ji} & r_{12}^{ji} & r_{13}^{ji} & t_1^{ji} \\ r_{21}^{ji} & r_{22}^{ji} & r_{23}^{ji} & t_2^{ji} \\ r_{31}^{ji} & r_{32}^{ji} & r_{33}^{ji} & t_3^{ji} \\ 0 & 0 & 0 & 1 \end{pmatrix}
\end{aligned}
\tag{2.2}
$$

Algorithm 3 Levenberg-Marquardt锟斤拷锟斤拷锟脚伙拷锟斤拷锟斤拷位锟斤拷

Require: $\mathcal{K}_i$,$\boldsymbol{\mathcal{I}}_j$,$\boldsymbol{\delta}_{i(i-1)}$,k_{max}(锟斤拷锟斤拷锟斤拷锟斤拷锟斤拷锟斤拷)

Ensure: $\boldsymbol{\delta}_{ji}$

```
function TrackFrame(Array, left, middle, right)
    v ← 2
    δ ← δ_{i(i−1)}
    f_i
    for i = 0 → k_max − 1 do
        J^T W J
        while true do
            slove δ*
            update
            f_i
            error=caclweight
            if error < lastError then
                lastResidual=lastError=error
                if λ <= 0.2 then
                    λ=0
                else
                    λ=1/2 λ
                end if
                break
            else
                if 锟斤拷锟斤拷< ε_1 then
                    i=k_max − 1
                    break
                end if
                if λ==0 then
                    λ=0.2
                else
                    λ=2*inctry
                end if
            end if
        end while
    end for
    return δ
end function
```