Supplementary Material to: Direct Sparse Visual-Inertial Odometry with Stereo Cameras

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Content

Chapter1 Visual-inertial Preliminaries

In our main paper [Ⅳ], The term $\mathrm{J}_r(\boldsymbol{\xi})$ is the right Jacobian of $SE(3)$ can be calculated by (1.1).

$$
\begin{aligned}
&\mathtt{Exp}(\boldsymbol{\xi}^{\wedge})=\mathbf{T}=\begin{pmatrix}\mathbf{R} & \mathbf{t}\\ \mathbf{0}^T & 0\end{pmatrix}_{4\times4},\mathtt{Exp}(\delta\boldsymbol{\xi}^{\wedge})=\begin{pmatrix}\delta\boldsymbol{\phi}^{\wedge} & \delta\boldsymbol{\rho}\\ \mathbf{0}^T & 0\end{pmatrix}_{4\times4},\mathbf{p}\in\mathbb{R}^3\\
&\frac{\partial(\mathbf{Tp})}{\partial\delta\boldsymbol{\xi}}=\lim_{\delta\boldsymbol{\xi}\to\mathbf{0}}\frac{\mathtt{Exp}(\boldsymbol{\xi}^{\wedge})\mathtt{Exp}(\delta\boldsymbol{\xi}^{\wedge})\mathbf{p}-\mathtt{Exp}(\boldsymbol{\xi}^{\wedge})\mathbf{p}}{\delta\boldsymbol{\xi}}\\
&\approx\lim_{\delta\boldsymbol{\xi}\to\mathbf{0}}\frac{\mathtt{Exp}(\boldsymbol{\xi}^{\wedge})(\mathbf{I}-\delta\boldsymbol{\xi}^{\wedge})\mathbf{p}-\mathtt{Exp}(\boldsymbol{\xi}^{\wedge})\mathbf{p}}{\delta\boldsymbol{\xi}}=\lim_{\delta\boldsymbol{\xi}\to\mathbf{0}}-\frac{\mathtt{Exp}(\boldsymbol{\xi}^{\wedge})\delta\boldsymbol{\xi}^{\wedge}\mathbf{p}}{\delta\boldsymbol{\xi}}\\
&=\lim_{\delta\boldsymbol{\xi}\to\mathbf{0}}-\frac{\begin{pmatrix}\mathbf{R} & \mathbf{t}\\ \mathbf{0}^T & 0\end{pmatrix}\begin{pmatrix}\delta\boldsymbol{\phi}^{\wedge}\mathbf{p}+\delta\boldsymbol{\rho}\\ 1\end{pmatrix}}{\delta\boldsymbol{\xi}}=\lim_{\delta\boldsymbol{\xi}\to\mathbf{0}}-\frac{\begin{pmatrix}\mathbf{R}\delta\boldsymbol{\phi}^{\wedge}\mathbf{p}+\mathbf{R}\delta\boldsymbol{\rho}+\mathbf{t}\\ \mathbf{0}^T\end{pmatrix}}{\delta\boldsymbol{\xi}}\\
&=\lim_{\delta\boldsymbol{\xi}\to\mathbf{0}}-\frac{\begin{pmatrix}-\mathbf{R}\mathbf{p}^{\wedge}\delta\boldsymbol{\phi}+\mathbf{R}\delta\boldsymbol{\rho}+\mathbf{t}\\ \mathbf{0}^T\end{pmatrix}}{\begin{pmatrix}\delta\boldsymbol{\rho}\\ \delta\boldsymbol{\phi}\end{pmatrix}}=\begin{pmatrix}-\mathbf{R} & \mathbf{R}\mathbf{p}^{\wedge}\\ \mathbf{0}^T & \mathbf{0}^T\end{pmatrix}_{4\times6}
\end{aligned}
\tag{1.1}
$$

Homogeneous camera calibration matrices are denoted by $\mathbf{K}$ as (1.2.1). and homogeneous 2D image coordinate point $\mathbf{p}$ is represented by its image coordinate and inverse depth as (1.2.3) relative to its host keyframe i^L. Corresponding homogeneous 3D camera coordinate point $\mathbf{p}_{\mathtt{C}}$ is denoted as (1.2.4). $\Pi_{\mathbf{K}}$ are used to denote camera projection functions. The jacobian of I_i^L, $\Pi_{\mathbf{K}}$ is denoted as (1.5)

$$
\begin{aligned}
&\mathbf{K}=\begin{pmatrix}f_x & 0 & c_x & 0\\ 0 & f_y & c_y & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix},\mathbf{K}^{-1}=\begin{pmatrix}f_x^{-1} & 0 & -f_x^{-1}c_x & 0\\ 0 & f_y^{-1} & -f_y^{-1}c_y & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix},\\
&\mathbf{p}=\begin{pmatrix}u^i\\ v^i\\ 1\\ d_{\mathbf{p}}\end{pmatrix},\mathbf{p}_{\mathtt{C}}=\begin{pmatrix}x\\ y\\ z\\ 1\end{pmatrix},d_{\mathbf{p}}=z^{-1},\mathbf{p}=d_{\mathbf{p}}\mathbf{K}\mathbf{p}_{\mathtt{C}}=\Pi_{\mathbf{K}}(\mathbf{p}_{\mathtt{C}})\\
&\frac{\partial(\mathrm{I}_i^L(\mathbf{p}))}{\partial\mathbf{p}}=(g_x,g_y,0,0),\frac{\partial\mathbf{p}}{\partial\mathbf{p}_{\mathtt{C}}}=\frac{\partial\Pi_{\mathbf{K}}(\mathbf{p}_{\mathtt{C}})}{\partial\mathbf{p}_{\mathtt{C}}}=\begin{pmatrix}f_xz^{-1} & 0 & -xf_xz^{-2} & 0\\ 0 & f_yz^{-1} & -yf_yz^{-2} & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & z^{-2} & 0\end{pmatrix}
\end{aligned}
\tag{1.2}
$$

Chapter2 IMU Error Factors

1.1 Time-closest measurements selection strategy

Because of asynchronous but same frequency for accelerometer and gyroscope data, there will be different quantity samples of these two sensors between two consecutive keyframes. We select time-closest gyroscope measurement for one accelerometer measurement accordind to (Alg.1)

Algorithm 1 Time-closest measurements selection

Input: $gyro\_list$,$acc\_list[s]$(an element in acc_list)
Output: $gyro\_measure$(time closest element in gyro_list)

```
function TIME_CLOSEST_SELECT(gyro_list, i)
    t ← acc_list[s].timestamp, i ← s
    while true do
        if i >= gyro_list.size then
            return gyro_list.back
        else
            t_now ← gyro_list[i].timestamp
            t_next ← gyro_list[i + 1].timestamp
            if t_now < t then
                if t_next > t then
                    t_front ← abs(t_now − t), t_back ← abs(t_next − t)
                    return t_front > t_back ? gyro_list[i + 1] : gyro_list[i]
                else
                    i = i + 1
                end if
            else if t_now > t then
                i = i − 1
            else
                return gyro_list[i]
            end if
        end if
    end while
end function
```

1.2 Errors and covariance calculation pseudo code

In our main paper [Fig. 2], we can get gyroscope, accelerometer data lists whose size is m, n. We have 8 error items to define:

$\Delta\bar{\mathrm{R}}_{ij}, \frac{\partial\Delta\bar{\mathbf{R}}_{ij}}{\partial\mathbf{b}^g}, \frac{\partial\Delta\bar{\mathbf{v}}_{ij}}{\partial\mathbf{b}^a}, \frac{\partial\Delta\bar{\mathbf{p}}_{ij}}{\partial\mathbf{b}^a}$ are pure rotation values and aren't related to accelerometer data.

$\Delta\bar{\mathbf{v}}_{ij}, \frac{\partial\Delta\bar{\mathbf{v}}_{ij}}{\partial\mathbf{b}^g}, \Delta\bar{\mathbf{p}}_{ij}, \frac{\partial\Delta\bar{\mathbf{p}}_{ij}}{\partial\mathbf{b}^g}$ are rotation "plus" translation values and are related to both gyroscope and accelerometer data.

We calculate these error items by recursion. As an example, the recurrence of $\Delta\bar{\mathrm{R}}_{ij}, \frac{\partial\Delta\bar{\mathbf{R}}_{ij}}{\partial\mathbf{b}^g}$ are presented here in (2.1), (2.2).

$$
\begin{aligned}
&\Delta\bar{\mathbf{R}}_{ik} = \begin{cases} \mathbf{I}_{3\times 3}, & k=i \\ \prod_{m=i}^{k-1} \mathbf{Exp}((\tilde{\boldsymbol{\omega}}_m - \bar{\mathbf{b}}_i^g)\Delta t), & k>i \end{cases} \\
&e.g.\quad k:0\to 44, i=0 \\
&\quad \Delta\bar{\mathbf{R}}_{00} = \mathbf{I}_{3\times 3} \\
&\quad \Delta\bar{\mathbf{R}}_{01} = \mathbf{Exp}((\tilde{\boldsymbol{\omega}}_0 - \bar{\mathbf{b}}_0^g)\Delta t) \\
&\quad \Delta\bar{\mathbf{R}}_{02} = \mathbf{Exp}((\tilde{\boldsymbol{\omega}}_0 - \bar{\mathbf{b}}_0^g)\Delta t)\mathbf{Exp}((\tilde{\boldsymbol{\omega}}_1 - \bar{\mathbf{b}}_0^g)\Delta t) \\
&\qquad\qquad \vdots \\
&\quad \Delta\bar{\mathbf{R}}_{0(44)} = \mathbf{Exp}((\tilde{\boldsymbol{\omega}}_0 - \bar{\mathbf{b}}_0^g)\Delta t)\mathbf{Exp}((\tilde{\boldsymbol{\omega}}_1 - \bar{\mathbf{b}}_0^g)\Delta t)\cdots\mathbf{Exp}((\tilde{\boldsymbol{\omega}}_{43} - \bar{\mathbf{b}}_0^g)\Delta t)
\end{aligned}
\tag{2.1}
$$

$$
\begin{aligned}
&\frac{\partial\Delta\bar{\mathbf{R}}_{ik}}{\partial\bar{\mathbf{b}^g}} = \begin{cases} \mathbf{0}_{3\times 3}, & k=i \\ \sum_{m=i}^{k-1} -\Delta\bar{\mathbf{R}}_{m+1k}^T\mathbf{J}_r^m\Delta t, & k>i \end{cases} \\
&= \begin{cases} \mathbf{0}_{3\times 3}, & k=i \\ \mathbf{J}_r^0\Delta t, & k=i+1 \\ \Delta\bar{\mathbf{R}}_{(k-1)k}^T \frac{\partial\Delta\bar{\mathbf{R}}_{i(k-1)}}{\partial\bar{\mathbf{b}^g}} + \mathbf{J}_r^{k-1}\Delta t, & k>i+1 \end{cases} \\
&e.g.\quad i=0,\quad k:0\to 45 \\
&\quad \frac{\partial\Delta\bar{\mathbf{R}}_{00}}{\partial\bar{\mathbf{b}^g}} = \mathbf{0}_{3\times 3} \\
&\quad \frac{\partial\Delta\bar{\mathbf{R}}_{01}}{\partial\bar{\mathbf{b}^g}} = \sum_{m=0}^{0}\Delta\bar{\mathbf{R}}_{(m+1)1}^T\mathbf{J}_r^m\Delta t = \Delta\bar{\mathbf{R}}_{11}^T\mathbf{J}_r^0\Delta t = \mathbf{J}_r^0\Delta t \\
&\quad \frac{\partial\Delta\bar{\mathbf{R}}_{02}}{\partial\bar{\mathbf{b}^g}} = \sum_{m=0}^{1}\Delta\bar{\mathbf{R}}_{(m+1)2}^T\mathbf{J}_r^m\Delta t = \Delta\bar{\mathbf{R}}_{12}^T\mathbf{J}_r^0\Delta t + \Delta\bar{\mathbf{R}}_{22}^T\mathbf{J}_r^1\Delta t = \Delta\bar{\mathbf{R}}_{12}^T\frac{\partial\Delta\bar{\mathbf{R}}_{01}}{\partial\bar{\mathbf{b}^g}} + \mathbf{J}_r^1\Delta t \\
&\quad \frac{\partial\Delta\bar{\mathbf{R}}_{03}}{\partial\bar{\mathbf{b}^g}} = \sum_{m=0}^{2}\Delta\bar{\mathbf{R}}_{(m+1)2}^T\mathbf{J}_r^m\Delta t = \Delta\bar{\mathbf{R}}_{13}^T\mathbf{J}_r^0\Delta t + \Delta\bar{\mathbf{R}}_{23}^T\mathbf{J}_r^1\Delta t + \Delta\bar{\mathbf{R}}_{33}^T\mathbf{J}_r^2\Delta t \\
&\qquad = (\Delta\bar{\mathbf{R}}_{12}\Delta\bar{\mathbf{R}}_{23})^T\mathbf{J}_r^0\Delta t + \Delta\bar{\mathbf{R}}_{23}^T\mathbf{J}_r^1\Delta t + \mathbf{J}_r^2\Delta t \\
&\qquad = \Delta\bar{\mathbf{R}}_{23}^T\Delta\bar{\mathbf{R}}_{12}^T\mathbf{J}_r^0\Delta t + \Delta\bar{\mathbf{R}}_{23}^T\mathbf{J}_r^1\Delta t + \mathbf{J}_r^2\Delta t \\
&\qquad = \Delta\bar{\mathbf{R}}_{23}^T\frac{\partial\Delta\bar{\mathbf{R}}_{02}}{\partial\bar{\mathbf{b}^g}} + \mathbf{J}_r^2\Delta t \\
&\qquad \vdots \\
&\quad \frac{\partial\Delta\bar{\mathbf{R}}_{0(44)}}{\partial\bar{\mathbf{b}^g}} = \sum_{m=0}^{43}\Delta\bar{\mathbf{R}}_{(m+1)44}^T\mathbf{J}_r^m\Delta t \\
&\qquad = \Delta\bar{\mathbf{R}}_{1(44)}^T\mathbf{J}_r^0\Delta t + \Delta\bar{\mathbf{R}}_{2(44)}^T\mathbf{J}_r^1\Delta t + \cdots + \Delta\bar{\mathbf{R}}_{43(44)}^T\mathbf{J}_r^{42}\Delta t + \Delta\bar{\mathbf{R}}_{44(44)}^T\mathbf{J}_r^{43}\Delta t \\
&\qquad = \Delta\bar{\mathbf{R}}_{43(44)}^T\frac{\partial\Delta\bar{\mathbf{R}}_{0(43)}}{\partial\bar{\mathbf{b}^g}} + \mathbf{J}_r^{43}\Delta t \\
&\quad \frac{\partial\Delta\bar{\mathbf{R}}_{0(45)}}{\partial\bar{\mathbf{b}^g}} = \sum_{m=0}^{44}\Delta\bar{\mathbf{R}}_{(m+1)45}^T\mathbf{J}_r^m\Delta t \\
&\qquad = \Delta\bar{\mathbf{R}}_{44(45)}^T\frac{\partial\Delta\bar{\mathbf{R}}_{0(44)}}{\partial\bar{\mathbf{b}^g}} + \mathbf{J}_r^{44}\Delta t
\end{aligned}
\tag{2.2}
$$

Furthermore, in order to calculate conveniently, we introduce a $rotate\_list$ to store all pure rotation values. All error items can be seen in (Alg.2).

Algorithm 1 On-Manifold Preintegeration for IMU

Input: $gyro\_list, acc\_list, m, n, rotate\_list$

Output: $(\Delta\bar{\mathbf{R}}_{ij}, \frac{\partial\Delta\bar{\mathbf{R}}_{ij}}{\partial\mathbf{b}^g}, \frac{\partial\Delta\bar{\mathbf{v}}_{ij}}{\partial\mathbf{b}^a}, \frac{\partial\Delta\bar{\mathbf{p}}_{ij}}{\partial\mathbf{b}^a}), (\Delta\bar{\mathbf{v}}_{ij}, \frac{\partial\Delta\bar{\mathbf{v}}_{ij}}{\partial\mathbf{b}^g}, \Delta\bar{\mathbf{p}}_{ij}, \frac{\partial\Delta\bar{\mathbf{p}}_{ij}}{\partial\mathbf{b}^g}), \Sigma_{ij}$

function IMU_PREINTEGERATION($gyro\_list, acc\_list, m, n, rotate\_list$)
  for all $gyro\_list[i], i : 0 \to m$ **do**
    $last\_r \leftarrow rotate\_list[i-1]$
    $rot.timestamp \leftarrow gyro\_list[i].timestamp$
    $rot.\omega \leftarrow gyro\_list[i].\omega - \bar{\mathbf{b}}_i^g$
    $rot.\Delta\bar{\mathbf{R}}_{ik} \leftarrow last\_r.\Delta\bar{\mathbf{R}}_{ik} * \mathtt{Exp}(rot.\omega * \Delta t)$
    $rot.\Delta\bar{\mathbf{R}}_{(k-1)k} \leftarrow \mathtt{Exp}(rot.\omega * \Delta t)$
    $rot.\frac{\partial\Delta\bar{\mathbf{R}}_{ik}}{\partial\mathbf{b}^g} \leftarrow \Delta\bar{\mathbf{R}}_{(k-1)k}^T * last\_r.\frac{\partial\Delta\bar{\mathbf{R}}_{ik}}{\partial\mathbf{b}^g} - \mathtt{J}_r(rot.\omega * \Delta t) * \Delta t$
    $rot.\frac{\partial\Delta\bar{\mathbf{v}}_{ik}}{\partial\mathbf{b}^a} \leftarrow last\_r.\frac{\partial\Delta\bar{\mathbf{v}}_{ik}}{\partial\mathbf{b}^a} - last\_r.\Delta\bar{\mathbf{R}}_{ik} * \Delta t$
    $rot.\frac{\partial\Delta\bar{\mathbf{p}}_{ik}}{\partial\mathbf{b}^a} \leftarrow last\_r.\frac{\partial\Delta\bar{\mathbf{p}}_{ik}}{\partial\mathbf{b}^a} + last\_r.\frac{\partial\Delta\bar{\mathbf{v}}_{ik}}{\partial\mathbf{b}^a} * \Delta t - \frac{1}{2}last\_r.\Delta\bar{\mathbf{R}}_{ik} * \Delta t^2$
    $rotate\_list.push(rot)$
  end for
  $\Delta\bar{\mathbf{R}}_{ij} = rotate\_list.end.\Delta\bar{\mathbf{R}}_{ik}$
  $\frac{\partial\Delta\bar{\mathbf{R}}_{ij}}{\partial\mathbf{b}^g} = rotate\_list.end.\frac{\partial\Delta\bar{\mathbf{R}}_{ik}}{\partial\mathbf{b}^g}$
  $\frac{\partial\Delta\bar{\mathbf{v}}_{ij}}{\partial\mathbf{b}^a} = rotate\_list.end.\frac{\partial\Delta\bar{\mathbf{v}}_{ik}}{\partial\mathbf{b}^a}$
  $\frac{\partial\Delta\bar{\mathbf{p}}_{ij}}{\partial\mathbf{b}^a} = rotate\_list.end.\frac{\partial\Delta\bar{\mathbf{p}}_{ik}}{\partial\mathbf{b}^a}$
  for all $acc\_list[i], i : 0 \to n$ **do**
    $cls\_r \leftarrow time\_closest\_select(rotate\_list, acc\_list[i])$
    $acc \leftarrow acc\_list[i] - \bar{\mathbf{b}}_i^a$
    $\Delta\bar{\mathbf{v}}_{ij} += cls\_r.\Delta\bar{\mathbf{R}}_{ik} * acc * \Delta t$
    $\frac{\partial\Delta\bar{\mathbf{v}}_{ij}}{\partial\mathbf{b}^g} -= cls\_r.\Delta\bar{\mathbf{R}}_{ik} * acc^\wedge * cls\_r.\frac{\partial\Delta\bar{\mathbf{R}}_{ik}}{\partial\mathbf{b}^g} * \Delta t$
    $\Delta\bar{\mathbf{p}}_{ij} += \Delta\bar{\mathbf{v}}_{ij} * \Delta t + \frac{1}{2}cls\_r.\Delta\bar{\mathbf{R}}_{ik} * acc * \Delta t^2$
    $\frac{\partial\Delta\bar{\mathbf{p}}_{ij}}{\partial\mathbf{b}^g} += cls\_r.\frac{\partial\Delta\bar{\mathbf{v}}_{ik}}{\partial\mathbf{b}^g}\Delta t - \frac{1}{2}cls\_r.\Delta\bar{\mathbf{R}}_{ik} * acc^\wedge * cls\_r.\frac{\partial\Delta\bar{\mathbf{R}}_{ik}}{\partial\mathbf{b}^g} * \Delta t^2$
    $A = \begin{pmatrix} cls\_r.\Delta\bar{\mathbf{R}}_{(k-1)k}^T & \mathbf{0} & \mathbf{0} \\ -cls\_r.\Delta\bar{\mathbf{R}}_{ik} * acc^\wedge * \Delta t & \mathbf{I} & \mathbf{0} \\ -\frac{1}{2}cls\_r.\Delta\bar{\mathbf{R}}_{ik} * acc^\wedge * \Delta t^2 & \Delta t\mathbf{I} & \mathbf{I} \end{pmatrix}$
    $B = \begin{pmatrix} \mathtt{J}_r(rot.\omega * \Delta t) * \Delta t & \mathbf{0} \\ \mathbf{0} & cls\_r.\Delta\bar{\mathbf{R}}_{ik} * \Delta t \\ \mathbf{0} & \frac{1}{2}cls\_r.\Delta\bar{\mathbf{R}}_{ik} * \Delta t^2 \end{pmatrix}$
    $\Sigma_{ij} = A * \Sigma_{ij} * A^T + B * \Sigma_\eta * B^T$
  end for
end function

1.3 Jacobian derivation

The derivation of the Jacobians of $\mathbf{r}_{\Delta\mathbf{R}_{ij}}, \mathbf{r}_{\Delta\mathbf{v}_{ij}}, \mathbf{r}_{\Delta\mathbf{p}_{ij}}$ likes (2.3), (2.4), (2.5).

$$
\begin{aligned}
\frac{\partial \mathbf{r}_{\Delta\mathbf{R}_{ij}}}{\partial \delta \mathbf{p}_i} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta\mathbf{R}_{ij}}}{\partial \delta \boldsymbol{\phi}_i} &= -\mathrm{J}_r^{-1}(\mathbf{r}_{\Delta\mathbf{R}_{ij}})\mathbf{R}_j^T\mathbf{R}_i \\
\frac{\partial \mathbf{r}_{\Delta\mathbf{R}_{ij}}}{\partial \delta \mathbf{v}_i} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta\mathbf{R}_{ij}}}{\partial \delta \mathbf{p}_j} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta\mathbf{R}_{ij}}}{\partial \delta \boldsymbol{\phi}_j} &= \mathrm{J}_r^{-1}(\mathbf{r}_{\Delta\mathbf{R}_{ij}}) \\
\frac{\partial \mathbf{r}_{\Delta\mathbf{R}_{ij}}}{\partial \delta \mathbf{v}_j} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta\mathbf{R}_{ij}}}{\partial \delta \mathbf{b}_i^a} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta\mathbf{R}_{ij}}}{\partial \delta \mathbf{b}_i^g} &= -\mathrm{J}_r^{-1}(\mathbf{r}_{\Delta\mathbf{R}_{ij}})\mathrm{Exp}(\mathbf{r}_{\Delta\mathbf{R}_{ij}})^T\mathrm{J}_r(\frac{\partial \Delta\bar{\mathbf{R}}_{ij}}{\partial \mathbf{b}^g}\delta\mathbf{b}_i^g)\frac{\partial \Delta\bar{\mathbf{R}}_{ij}}{\partial \mathbf{b}^g}
\end{aligned}
\tag{2.3}
$$

$$
\begin{aligned}
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{p}_i} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \boldsymbol{\phi}_i} &= (\mathbf{R}_i^T(\mathbf{v}_j - \mathbf{v}_i - \mathbf{g}\Delta t_{ij}))^\wedge \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{v}_i} &= -\mathbf{R}_i^T \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{p}_j} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \boldsymbol{\phi}_j} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{v}_j} &= \mathbf{R}_i^T \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{b}_i^a} &= -\frac{\partial \Delta \bar{\mathbf{v}}_{ij}}{\partial \mathbf{b}^a} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{v}_{ij}}}{\partial \delta \mathbf{b}_i^g} &= -\frac{\partial \Delta \bar{\mathbf{v}}_{ij}}{\partial \mathbf{b}^g}
\end{aligned}
\tag{2.4}
$$

$$
\begin{aligned}
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{p}_i} &= -\mathbf{I} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \boldsymbol{\phi}_i} &= (\mathbf{R}_i^T(\mathbf{p}_j - \mathbf{p}_i - \mathbf{v}_i\Delta t_{ij} - \frac{1}{2}\mathbf{g}\Delta t_{ij}^2))^\wedge \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{v}_i} &= -\mathbf{R}_i^T \Delta t_{ij} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{p}_j} &= \mathbf{R}_i^T \mathbf{R}_j \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \boldsymbol{\phi}_j} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{v}_j} &= \mathbf{0} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{b}_i^a} &= -\frac{\partial \Delta \bar{\mathbf{p}}_{ij}}{\partial \mathbf{b}^a} \\
\frac{\partial \mathbf{r}_{\Delta \mathbf{p}_{ij}}}{\partial \delta \mathbf{b}_i^g} &= -\frac{\partial \Delta \bar{\mathbf{p}}_{ij}}{\partial \mathbf{b}^g}
\end{aligned}
\tag{2.5}
$$

3.1 Construction residual errors

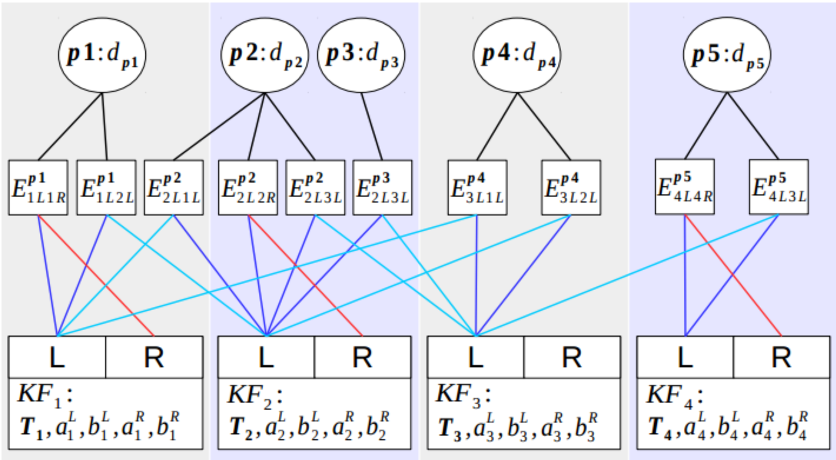

Fig.1

Here, we take [Fig.1] as factor graph to illustrate photometric error optimaztion. According to our main paper [V.B], The parameters we want to optimize are enclosed in (3.1).

$$\boldsymbol{\chi} = \begin{pmatrix} (\phi_1, \ldots, \phi_4)^T \\ (\mathbf{p}_1^T, \ldots, \mathbf{p}_4^T)^T \\ (\mathbf{v}_1^T, \ldots, \mathbf{v}_4^T)^T \\ (\mathbf{b}_1^T, \ldots, \mathbf{b}_4^T)^T \\ (d_{\mathbf{p}_1}, \ldots, d_{\mathbf{p}_5})^T \\ (a_1^L, a_1^R, b_1^L, b_1^R)^T \\ \vdots \\ (a_4^L, a_4^R, b_4^L, b_4^R)^T \end{pmatrix} \begin{array}{c} \in \mathbb{R}^{81}, \\ \boldsymbol{\phi}_i = \mathrm{Log}(\mathbf{R}_i), \\ \boldsymbol{\xi}_i = (\boldsymbol{\phi}_i^T, \mathbf{p}_i^T)^T \end{array} \tag{3.1}$$

In this example, there are 7 dynamic residuals and 3 static residuals, Factor graph of the residuals function is

$$\begin{aligned} E(\boldsymbol{\chi}) &= E^{\mathbf{p}_1}_{1L2L} + E^{\mathbf{p}_2}_{2L1L} + E^{\mathbf{p}_2}_{2L3L} + E^{\mathbf{p}_3}_{2L3L} + E^{\mathbf{p}_4}_{3L1L} + E^{\mathbf{p}_4}_{3L2L} + E^{\mathbf{p}_5}_{4L3L} \\ &\quad + E^{\mathbf{p}_1}_{1L1R} + E^{\mathbf{p}_2}_{2L2R} + E^{\mathbf{p}_5}_{4L4R} \\ &= E_d(\boldsymbol{\chi}) + E_s(\boldsymbol{\chi}) \end{aligned}$$

$$E_d(\boldsymbol{\chi}) = \begin{pmatrix} (r^d_{\mathbf{p}_1})_{12} \\ (r^d_{\mathbf{p}_1})_{21} \\ \vdots \\ (r^d_{\mathbf{p}_5})_{43} \end{pmatrix}^T \begin{pmatrix} w_{\mathbf{p}_1} & 0 & \ldots & 0 \\ 0 & w_{\mathbf{p}_1} & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & w_{\mathbf{p}_5} \end{pmatrix} \begin{pmatrix} (r^d_{\mathbf{p}_1})_{12} \\ (r^d_{\mathbf{p}_1})_{21} \\ \vdots \\ (r^d_{\mathbf{p}_5})_{43} \end{pmatrix} = (\mathbf{r}^d)^T \mathbf{W}^d \mathbf{r}^d \tag{2.2}$$

$$E_s(\boldsymbol{\chi}) = \begin{pmatrix} r^s_{\mathbf{p}_1} \\ r^s_{\mathbf{p}_2} \\ r^s_{\mathbf{p}_5} \end{pmatrix}^T \begin{pmatrix} \lambda w_{\mathbf{p}_1} & 0 & 0 \\ 0 & \lambda w_{\mathbf{p}_2} & 0 \\ 0 & 0 & \lambda w_{\mathbf{p}_5} \end{pmatrix} \begin{pmatrix} r^s_{\mathbf{p}_1} \\ r^s_{\mathbf{p}_2} \\ r^s_{\mathbf{p}_5} \end{pmatrix} = (\mathbf{r}^s)^T \mathbf{W}^s \mathbf{r}^s$$

We first note that $(\mathbf{v}_1^T,\dots,\mathbf{v}_4^T)^T,(\mathbf{b}_1^T,\dots,\mathbf{b}_4^T)^T$ do not appear in the expression of $E_d(\boldsymbol{\chi}), E_s(\boldsymbol{\chi})$, hence the corresponding Jacobians are zero, we omit them for writing simple. The remaining Jacobians can be computed as follows:

$$\mathbf{J}_s=\begin{pmatrix} \frac{\partial r^s_{\mathbf{p}_1}}{\partial \boldsymbol{\xi}_1}\cdots & \frac{\partial r^s_{\mathbf{p}_1}}{\partial \boldsymbol{\xi}_4} & \frac{\partial r^s_{\mathbf{p}_1}}{\partial d_{\mathbf{p}_1}}\cdots & \frac{\partial r^s_{\mathbf{p}_1}}{\partial d_{\mathbf{p}_5}} & \frac{\partial r^s_{\mathbf{p}_1}}{\partial a_1^L}\cdots & \frac{\partial r^s_{\mathbf{p}_1}}{\partial b_4^R} \\ \frac{\partial r^s_{\mathbf{p}_2}}{\partial \boldsymbol{\xi}_1}\cdots & \frac{\partial r^s_{\mathbf{p}_2}}{\partial \boldsymbol{\xi}_4} & \frac{\partial r^s_{\mathbf{p}_2}}{\partial d_{\mathbf{p}_1}}\cdots & \frac{\partial r^s_{\mathbf{p}_2}}{\partial d_{\mathbf{p}_5}} & \frac{\partial r^s_{\mathbf{p}_2}}{\partial a_1^L}\cdots & \frac{\partial r^s_{\mathbf{p}_2}}{\partial b_4^R} \\ \frac{\partial r^s_{\mathbf{p}_5}}{\partial \boldsymbol{\xi}_1}\cdots & \frac{\partial r^s_{\mathbf{p}_5}}{\partial \boldsymbol{\xi}_4} & \frac{\partial r^s_{\mathbf{p}_5}}{\partial d_{\mathbf{p}_1}}\cdots & \frac{\partial r^s_{\mathbf{p}_5}}{\partial d_{\mathbf{p}_5}} & \frac{\partial r^s_{\mathbf{p}_5}}{\partial a_1^L}\cdots & \frac{\partial r^s_{\mathbf{p}_5}}{\partial b_4^R} \end{pmatrix}_{3\times 49}$$

$$\mathbf{J}_d=\begin{pmatrix} \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial \boldsymbol{\xi}_1}\cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial \boldsymbol{\xi}_4} & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial d_{\mathbf{p}_1}}\cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial d_{\mathbf{p}_5}} & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial a_1^L}\cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial b_4^R} & \\ \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial \boldsymbol{\xi}_1}\cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial \boldsymbol{\xi}_4} & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial d_{\mathbf{p}_1}}\cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial d_{\mathbf{p}_5}} & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial a_1^L}\cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial b_4^R} & \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \;\vdots \\ \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial \boldsymbol{\xi}_1}\cdots & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial \boldsymbol{\xi}_4} & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial d_{\mathbf{p}_1}}\cdots & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial d_{\mathbf{p}_5}} & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial a_1^L}\cdots & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial b_4^R} & \end{pmatrix}_{7\times 49} \tag{2.2}$$

Iteration $\delta\boldsymbol{\chi}$ can be calculated by:

$$(\mathbf{J}_s^T\lambda\mathbf{W}^s\mathbf{J}_s+\mathbf{J}_d^T\mathbf{W}^d\mathbf{J}_d)\delta\boldsymbol{\chi}=-(\mathbf{J}_s^T\lambda\mathbf{W}^s\mathbf{r}^s+\mathbf{J}_d^T\mathbf{W}^d\mathbf{r}^d)$$
$$\mathbf{J}_s\in\mathbb{R}^{3\times 49},\mathbf{W}^s\in\mathbb{R}^{3\times 3},\mathbf{J}_s\in\mathbb{R}^{7\times 49},\mathbf{W}^s\in\mathbb{R}^{7\times 7} \tag{2.2}$$

3.2 Jacobian citation

We know for a Lie algebra $\boldsymbol{\rho} \in \mathbb{R}^3, \boldsymbol{\phi} \in \mathbb{R}^3, \boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{\rho} \\ \boldsymbol{\phi} \end{pmatrix} \in \mathbb{R}^6$ and $\mathbf{p}_w$:

$$
\begin{aligned}
&\boldsymbol{\xi}^\wedge = \begin{pmatrix} \boldsymbol{\rho} \\ \boldsymbol{\phi} \end{pmatrix}^\wedge = \begin{pmatrix} \boldsymbol{\phi}^\wedge & \boldsymbol{\rho} \\ \mathbf{0}^T & \mathbf{0}^T \end{pmatrix} \in \mathbb{R}^{4\times 4} \\
&\boldsymbol{\epsilon} \in \mathbb{R}^3, \begin{pmatrix} \boldsymbol{\epsilon} \\ 1 \end{pmatrix}^\odot = \begin{pmatrix} \mathbf{E} & -\boldsymbol{\epsilon}^\wedge \\ \mathbf{0}^T & \mathbf{0}^T \end{pmatrix} \in \mathbb{R}^{4\times 6} \\
&\frac{\partial(exp(\xi^\wedge)\mathbf{p}_w)}{\partial \boldsymbol{\xi}} = \frac{\partial(\mathbf{T}\mathbf{p}_w)}{\partial \boldsymbol{\xi}} = (\mathbf{T}\mathbf{p}_w)^\odot \\
&\mathbf{T}\mathbf{p}_w = exp(\xi^\wedge)\mathbf{p}_w \approx (\mathbf{E} + \xi^\wedge)\mathbf{p}_w \\
&\frac{\partial(exp(\xi^\wedge)\mathbf{p}_w)}{\partial \boldsymbol{\xi}} \approx \frac{\partial(\mathbf{E} + \xi^\wedge)}{\partial \boldsymbol{\xi}} = \mathbf{0} + \frac{\partial(\xi^\wedge \mathbf{p}_w)}{\partial \boldsymbol{\xi}} \approx (\mathbf{T}\mathbf{p}_w)^\odot \\
&since, \frac{\partial(\mathbf{T}\mathbf{p}_w)}{\partial \boldsymbol{\xi}} = (\mathbf{T}^{-1}\mathbf{p}_w)^\odot = \frac{\partial(exp(-\xi^\wedge)\mathbf{p}_w)}{\partial \boldsymbol{\xi}} \\
&= \frac{\partial(\mathbf{E} - \xi^\wedge)}{\partial \boldsymbol{\xi}} = -(\mathbf{T}\mathbf{p}_w)^\odot
\end{aligned}
\tag{2.2}
$$

1.3 Jacobian derivation

1.3.1 Dynamic Parameter

Firstly, if $\mathbf{p}$ is neither observed by frame mL, mR nor hosted by nL, nR:

$$
\frac{\partial(r^d_{\mathbf{p}})_{ij}}{\partial \boldsymbol{\xi_m}} = \frac{\partial(r^d_{\mathbf{p}})_{ij}}{\partial \boldsymbol{\xi_n}} = \mathbf{0}^T, so \quad \frac{\partial(r^d_{\mathbf{p}_1})_{12}}{\partial \boldsymbol{\xi}_3} = \frac{\partial r^d_{(\mathbf{p}_1)12}}{\partial \boldsymbol{\xi}_4} = \ldots = \mathbf{0}^T, \tag{2.2}
$$

$$
\frac{\partial(r^d_{\mathbf{p}})_{ij}}{\partial \boldsymbol{\xi_i}} = \frac{\partial(I^L_j(\mathbf{p}'))}{\partial \boldsymbol{\xi_i}} = \frac{\partial(I^L_j(\mathbf{p}'))}{\partial \mathbf{p}'} \frac{\partial \mathbf{p}'}{\partial \mathbf{p}'_w} \frac{\partial \mathbf{p}'_w}{\partial \boldsymbol{\xi_i}} \tag{2.2}
$$

$$
\mathbf{p}'_w = \mathbf{T}_j \mathbf{T}_i^{-1} \mathbf{p}_w = \mathbf{T}_j \mathbf{T}_i^{-1}((d^{iL}_{\mathbf{p}})^{-1}\mathbf{K}^{-1}\mathbf{p})
$$

otherwise, we follow

For one frame iL, we have $\mathbf{p}$ and $\mathbf{K}$, then we can get

$$
\left\{ \mathbf{p}_w = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} f_x^{-1}(d^{iL}_{\mathbf{p}})^{-1}(u^i - c_x) \\ f_y^{-1}(d^{iL}_{\mathbf{p}})^{-1}(v^i - c_y) \\ (d^{iL}_{\mathbf{p}})^{-1} \\ 1 \end{pmatrix} \right. \tag{2.2}
$$

Secondly, according to

$$(r^d_{\mathbf{p}})_{ij} := I^L_j(\mathbf{p}^{'}) - b^L_j - \frac{e^{a^L_j}}{e^{a^L_i}}(I^L_i(\mathbf{p}) - b^L_i) \tag{2.2}$$

We have:

$$\begin{aligned}
&\frac{\partial \mathbf{p}^{'}_w}{\partial \boldsymbol{\xi_i}} = \mathbf{T}_j \frac{\partial(\mathbf{T}_i^{-1}\mathbf{p}^{'}_w)}{\partial \boldsymbol{\xi_i}} = -\mathbf{T}_j(\mathbf{T}_i\mathbf{p}_w)^{\odot} \\
&\frac{\partial \mathbf{p}^{'}_w}{\partial \boldsymbol{\xi_j}} = \frac{\partial(\mathbf{T}_j\mathbf{T}_i^{-1}\mathbf{p}_w)}{\partial \boldsymbol{\xi_i}} = (\mathbf{T}_j\mathbf{T}_i^{-1}\mathbf{p}_w)^{\odot} \\
&= \begin{pmatrix} x^{'} \\ y^{'} \\ z^{'} \\ 1 \end{pmatrix}^{\odot} = \begin{pmatrix} 1 & 0 & 0 & 0 & z^{'} & -y^{'} \\ 0 & 1 & 0 & -z^{'} & 0 & x^{'} \\ 0 & 1 & 0 & y^{'} & -x^{'} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\
&\Rightarrow \frac{\partial (r^d_{\mathbf{p}})_{ij}}{\partial \boldsymbol{\xi_j}} = \frac{\partial(I^L_j(\mathbf{p}^{'}))}{\partial \mathbf{p}^{'}} \frac{\partial \mathbf{p}^{'}}{\partial \mathbf{p}^{'}_w} \frac{\partial \mathbf{p}^{'}_w}{\partial \boldsymbol{\xi_j}} \\
&= (g^{'}_x, g^{'}_y, 0, 0) \begin{pmatrix} f_x(z^{'})^{-1} & 0 & -x^{'}f_x(z^{'})^{-2} & 0 \\ 0 & f_y(z^{'})^{-1} & -y^{'}f_y(z^{'})^{-2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (z^{'})^{-2} & 0 \end{pmatrix} \\
&\begin{pmatrix} 1 & 0 & 0 & 0 & z^{'} & -y^{'} \\ 0 & 1 & 0 & -z^{'} & 0 & x^{'} \\ 0 & 0 & 1 & y^{'} & -x^{'} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} g^{'}_x f_x (z^{'})^{-1} \\ g^{'}_y f_y (z^{'})^{-1} \\ -(g^{'}_x x^{'} f_x + g^{'}_y y^{'} f_y)(z^{'})^{-2} \\ -g^{'}_y f_y - (g^{'}_x x^{'} y^{'} f_x + g^{'}_y (y^{'})^2 f_y)(z^{'})^{-2} \\ g^{'}_x f_x + (g^{'}_x (x^{'})^2 f_x + g^{'}_y x^{'} y^{'} f_y)(z^{'})^{-2} \\ -g^{'}_x f_x y^{'} (z^{'})^{-1} + g^{'}_y f_y x^{'} (z^{'})^{-1} \end{pmatrix}^T
\end{aligned} \tag{2.2}$$

$$\begin{aligned}
&\frac{\partial (r^d_{\mathbf{p}})_{ij}}{\partial a_i} = \frac{e^{a^L_j}}{e^{a^L_i}}(I^L_i(\mathbf{p}) - b^L_i), \frac{\partial (r^d_{\mathbf{p}})_{ij}}{\partial a_j} = -\frac{e^{a^L_j}}{e^{a^L_i}}(I^L_i(\mathbf{p}) - b^L_i) \\
&\frac{\partial (r^d_{\mathbf{p}})_{ij}}{\partial b_i} = \frac{e^{a^L_j}}{e^{a^L_i}}, \qquad \frac{\partial (r^d_{\mathbf{p}})_{ij}}{\partial b_j} = -1
\end{aligned} \tag{2.2}$$

$$
\begin{aligned}
&\mathbf{p}^{'} = d_{\mathbf{p}}^{jL}\mathbf{K}(\mathbf{T}_j\mathbf{T}_i^{-1}\mathbf{p}_w) \\
&assume:\mathbf{T}_j\mathbf{T}_i^{-1} = \begin{pmatrix} r_{11}^{ji} & r_{12}^{ji} & r_{13}^{ji} & t_1^{ji} \\ r_{21}^{ji} & r_{22}^{ji} & r_{23}^{ji} & t_2^{ji} \\ r_{31}^{ji} & r_{32}^{ji} & r_{33}^{ji} & t_3^{ji} \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
&\mathbf{p}_w^{'} = \begin{pmatrix} r_{11}^{ji}f_x^{-1}(d_{\mathbf{p}}^{iL})^{-1}(u^i-c_x)+r_{12}^{ji}f_y^{-1}(d_{\mathbf{p}}^{iL})^{-1}(v^i-c_y)+r_{13}^{ji}(d_{\mathbf{p}}^{iL})^{-1}+t_1^{ji} \\ r_{21}^{ji}f_x^{-1}(d_{\mathbf{p}}^{iL})^{-1}(u^i-c_x)+r_{22}^{ji}f_y^{-1}(d_{\mathbf{p}}^{iL})^{-1}(v^i-c_y)+r_{23}^{ji}(d_{\mathbf{p}}^{iL})^{-1}+t_2^{ji} \\ r_{31}^{ji}f_x^{-1}(d_{\mathbf{p}}^{iL})^{-1}(u^i-c_x)+r_{32}^{ji}f_y^{-1}(d_{\mathbf{p}}^{iL})^{-1}(v^i-c_y)+r_{33}^{ji}(d_{\mathbf{p}}^{iL})^{-1}+t_3^{ji} \\ 1 \end{pmatrix} \\
&= \begin{pmatrix} \frac{a}{d_{\mathbf{p}}^{iL}}+t_1^{ji} \\ \frac{b}{d_{\mathbf{p}}^{iL}}+t_2^{ji} \\ \frac{c}{d_{\mathbf{p}}^{iL}}+t_3^{ji} \\ 1 \end{pmatrix} = \begin{pmatrix} x^{'} \\ y^{'} \\ z^{'} \\ 1 \end{pmatrix} \Rightarrow d_{\mathbf{p}}^{jL} = (z^{'})^{-1}, \mathbf{p}^{'} = \begin{pmatrix} f_x x^{'} d_{\mathbf{p}}^{jL}+c_x \\ f_y y^{'} d_{\mathbf{p}}^{jL}+c_y \\ 1 \\ d_{\mathbf{p}}^{jL} \end{pmatrix} \\
&\frac{\partial(I_j^L(\mathbf{p}^{'}))}{\partial\mathbf{p}^{'}} = (g_x^{'}, g_y^{'}, 0, 0) \\
&\frac{\partial\mathbf{p}^{'}}{\partial\mathbf{p}_w^{'}} = \begin{pmatrix} f_x(z^{'})^{-1} & 0 & -x^{'}f_x(z^{'})^{-2} & 0 \\ 0 & f_y(z^{'})^{-1} & -y^{'}f_y(z^{'})^{-2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (z^{'})^{-2} & 0 \end{pmatrix}, \frac{\partial\mathbf{p}_w^{'}}{\partial d_{\mathbf{p}}^{iL}} = \begin{pmatrix} -\frac{a}{(d_{\mathbf{p}}^{iL})^2} \\ -\frac{b}{(d_{\mathbf{p}}^{iL})^2} \\ -\frac{c}{(d_{\mathbf{p}}^{iL})^2} \\ 0 \end{pmatrix} \\
&\Rightarrow \frac{\partial(r_{\mathbf{p}}^d)_{ij}}{\partial d_{\mathbf{p}}^{iL}} = \frac{\partial(I_j^L(\mathbf{p}^{'}))}{\partial d_{\mathbf{p}}^{iL}} = \frac{\partial(I_j^L(\mathbf{p}^{'}))}{\partial\mathbf{p}^{'}}\frac{\partial\mathbf{p}^{'}}{\partial\mathbf{p}_w^{'}}\frac{\partial\mathbf{p}_w^{'}}{\partial d_{\mathbf{p}}^{iL}} \\
&= -\frac{g_x^{'}f_x a}{z^{'}(d_{\mathbf{p}}^{iL})^2} - \frac{g_y^{'}f_y b}{z^{'}(d_{\mathbf{p}}^{iL})^2} + \frac{c(g_x^{'}x^{'}f_x+g_y^{'}y^{'}f_y)}{(z^{'}d_{\mathbf{p}}^{iL})^2} \\
&= \frac{c(g_x^{'}x^{'}f_x+g_y^{'}y^{'}f_y)-g_x^{'}f_x a z^{'}-g_y^{'}f_y b z^{'}}{(z^{'}d_{\mathbf{p}}^{iL})^2}
\end{aligned}
\tag{2.2}
$$

1.3.2 Static Parameter

Firstly, For a stereo frame i: inverse depth $d_{\mathbf{p}}^{iL} = d_{\mathbf{p}}^{iR}$, a left frame iL pixel $\mathbf{p}$ is projected to right frame iR with $\mathbf{p}^{'}$:

$$
\begin{aligned}
&\mathbf{p} = \begin{pmatrix} u^i \\ v^i \\ 1 \\ d_{\mathbf{p}}^{iL} \end{pmatrix}, \mathbf{p}_w = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}, d_{\mathbf{p}}^{iL} = z^{-1}, \mathbf{p}_w = (d_{\mathbf{p}}^{iL})^{-1}\mathbf{K}^{-1}\mathbf{p} \\
&= \begin{pmatrix} f_x^{-1}(d_{\mathbf{p}}^{iL})^{-1}(u^i - c_x) \\ f_y^{-1}(d_{\mathbf{p}}^{iL})^{-1}(v^i - c_y) \\ (d_{\mathbf{p}}^{iL})^{-1} \\ 1 \end{pmatrix}, \mathbf{T}_{RL} = \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
&\mathbf{p}^{'} = d_{\mathbf{p}}^{iR}\mathbf{K}(\mathbf{T}_{RL}\mathbf{p}_w) \\
&= d_{\mathbf{p}}^{iL} \begin{pmatrix} f_x & 0 & c_x & 0 \\ 0 & f_y & c_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_x^{-1}(d_{\mathbf{p}}^{iL})^{-1}(u^i - c_x) + t_1 \\ f_y^{-1}(d_{\mathbf{p}}^{iL})^{-1}(v^i - c_y) \\ (d_{\mathbf{p}}^{iL})^{-1} \\ 1 \end{pmatrix} = \begin{pmatrix} u^i + t_1 f_x d_{\mathbf{p}}^{iL} \\ v^i \\ 1 \\ d_{\mathbf{p}}^{iL} \end{pmatrix} \\
&\frac{\partial r_{\mathbf{p}}^s}{\partial d_{\mathbf{p}}^{iL}} = \frac{\partial (I_i^R(\mathbf{p}^{'})) - \frac{e^{a_i^R}}{e^{a_i^L}}(I_i^L(\mathbf{p}))}{\partial d_{\mathbf{p}}^{iL}} = (\frac{\partial (I_i^R(\mathbf{p}^{'}))}{\partial \mathbf{p}^{'}} - \frac{e^{a_i^R}}{e^{a_i^L}}\frac{\partial (I_i^L(\mathbf{p}))}{\partial \mathbf{p}^{'}})\frac{\partial \mathbf{p}^{'}}{\partial d_{\mathbf{p}}^{iL}} \\
&= [(g_x^{iR}, g_y^{iR}, 0, 0) - \mathbf{0}^T] \begin{pmatrix} t_1 f_x \\ 0 \\ 0 \\ 1 \end{pmatrix} = g_x^{iR} t_1 f_x
\end{aligned} \tag{1}
$$

Secondly, according to:

$$
r_{\mathbf{p}}^s := I_i^R(\mathbf{p}^{'}) - b_i^R - \frac{e^{a_i^R}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L) \tag{2.2}
$$

We have:

$$
\begin{aligned}
&\frac{\partial (r_{\mathbf{p}}^s)_{ij}}{\partial a_i} = \frac{e^{a_j^L}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L), \frac{\partial (r_{\mathbf{p}}^s)_{ij}}{\partial a_j} = -\frac{e^{a_j^L}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L) \\
&\frac{\partial (r_{\mathbf{p}}^s)_{ij}}{\partial b_i} = \frac{e^{a_j^L}}{e^{a_i^L}}, \qquad \frac{\partial (r_{\mathbf{p}}^s)_{ij}}{\partial b_j} = -1
\end{aligned} \tag{2.2}
$$