Supplementary Material to: Direct Sparse Visual-Inertial Odometry with Stereo Cameras

Ziqiang Wang

March, 2019

Ziqiang Wang is an engineer of UISEE Technologies Shanghai Co., Ltd and was a postgraduate student with the college of Electronics and Information Engineering of Tongji Universty, Shanghai, China.(e-mail: ziqiang.wang@ uisee.com).

Content

Chapter1 Visual-inertial Preliminaries

In our main paper [Ⅳ], The term $\mathrm{J}_r(\boldsymbol{\xi})$ is the right Jacobian of $SE(3)$ can be calculated by (1.1).

$$\begin{aligned}
&\mathtt{Exp}(\boldsymbol{\xi}^\wedge)=\mathbf{T}=\begin{pmatrix}\mathbf{R} & \mathbf{t}\\ \mathbf{0}^T & 0\end{pmatrix}_{4\times4},\mathtt{Exp}(\delta\boldsymbol{\xi}^\wedge)=\begin{pmatrix}\delta\boldsymbol{\phi}^\wedge & \delta\boldsymbol{\rho}\\ \mathbf{0}^T & 0\end{pmatrix}_{4\times4},\mathbf{p}\in\mathbb{R}^3\\
&\frac{\partial(\mathbf{Tp})}{\partial\delta\boldsymbol{\xi}}=\lim_{\delta\boldsymbol{\xi}\to\mathbf{0}}\frac{\mathtt{Exp}(\boldsymbol{\xi}^\wedge)\mathtt{Exp}(\delta\boldsymbol{\xi}^\wedge)\mathbf{p}-\mathtt{Exp}(\boldsymbol{\xi}^\wedge)\mathbf{p}}{\delta\boldsymbol{\xi}}\\
&\approx\lim_{\delta\boldsymbol{\xi}\to\mathbf{0}}\frac{\mathtt{Exp}(\boldsymbol{\xi}^\wedge)(\mathbf{I}-\delta\boldsymbol{\xi}^\wedge)\mathbf{p}-\mathtt{Exp}(\boldsymbol{\xi}^\wedge)\mathbf{p}}{\delta\boldsymbol{\xi}}=\lim_{\delta\boldsymbol{\xi}\to\mathbf{0}}-\frac{\mathtt{Exp}(\boldsymbol{\xi}^\wedge)\delta\boldsymbol{\xi}^\wedge\mathbf{p}}{\delta\boldsymbol{\xi}}\\
&=\lim_{\delta\boldsymbol{\xi}\to\mathbf{0}}-\frac{\begin{pmatrix}\mathbf{R} & \mathbf{t}\\ \mathbf{0}^T & 0\end{pmatrix}\begin{pmatrix}\delta\boldsymbol{\phi}^\wedge\mathbf{p}+\delta\boldsymbol{\rho}\\ 1\end{pmatrix}}{\delta\boldsymbol{\xi}}=\lim_{\delta\boldsymbol{\xi}\to\mathbf{0}}-\frac{\begin{pmatrix}\mathbf{R}\delta\boldsymbol{\phi}^\wedge\mathbf{p}+\mathbf{R}\delta\boldsymbol{\rho}+\mathbf{t}\\ \mathbf{0}^T\end{pmatrix}}{\delta\boldsymbol{\xi}}\\
&=\lim_{\delta\boldsymbol{\xi}\to\mathbf{0}}-\frac{\begin{pmatrix}-\mathbf{R}\mathbf{p}^\wedge\delta\boldsymbol{\phi}+\mathbf{R}\delta\boldsymbol{\rho}+\mathbf{t}\\ \mathbf{0}^T\end{pmatrix}}{\begin{pmatrix}\delta\boldsymbol{\rho}\\ \delta\boldsymbol{\phi}=\end{pmatrix}}=\begin{pmatrix}-\mathbf{R} & \mathbf{R}\mathbf{p}^\wedge\\ \mathbf{0}^T & \mathbf{0}^T\end{pmatrix}_{4\times6}
\end{aligned}\tag{1.1}$$

Homogeneous camera calibration matrices are denoted by $\mathbf{K}$ as (1.2.1). and homogeneous 2D image coordinate point $\mathbf{p}$ is represented by its image coordinate and inverse depth as (1.2.3) relative to its host keyframe i^L. Corresponding homogeneous 3D camera coordinate point $\mathbf{p}_\mathtt{C}$ is denoted as (1.2.4). $\Pi_\mathbf{K}$ are used to denote camera projection functions. The jacobian of $\mathtt{I}_i^L$, $\Pi_\mathbf{K}$ is denoted as (1.5)

$$\begin{aligned}
&\mathbf{K}=\begin{pmatrix}f_x & 0 & c_x & 0\\ 0 & f_y & c_y & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix},\mathbf{K}^{-1}=\begin{pmatrix}f_x^{-1} & 0 & -f_x^{-1}c_x & 0\\ 0 & f_y^{-1} & -f_y^{-1}c_y & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix},\\
&\mathbf{p}=\begin{pmatrix}u^i\\ v^i\\ 1\\ d_\mathbf{p}\end{pmatrix},\mathbf{p}_\mathtt{C}=\begin{pmatrix}x\\ y\\ z\\ 1\end{pmatrix},d_\mathbf{p}=z^{-1},\mathbf{p}=d_\mathbf{p}\mathbf{K}\mathbf{p}_\mathtt{C}=\Pi_\mathbf{K}(\mathbf{p}_\mathtt{C})\\
&\frac{\partial(\mathtt{I}_i^L(\mathbf{p}))}{\partial\mathbf{p}}=(g_x,g_y,0,0),\frac{\partial\mathbf{p}}{\partial\mathbf{p}_\mathtt{C}}=\frac{\partial\Pi_\mathbf{K}(\mathbf{p}_\mathtt{C})}{\partial\mathbf{p}_\mathtt{C}}=\begin{pmatrix}f_xz^{-1} & 0 & -xf_xz^{-2} & 0\\ 0 & f_yz^{-1} & -yf_yz^{-2} & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & z^{-2} & 0\end{pmatrix}
\end{aligned}\tag{1.2}$$

Chapter2 IMU Error Factors

1.1 Time-closest measurements selection strategy

Dynamic multi-view stereo residuals $E_{ij}^{\mathbf{p}}$ are defined as

$$E_{ij}^{\mathbf{p}} = w_{\mathbf{p}}||(r_{\mathbf{p}}^{d})_{ij}||_{\gamma}, \quad (r_{\mathbf{p}}^{d})_{ij} := I_j^L(\mathbf{p}^{'}) - b_j^L - \frac{e^{a_j^L}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L) \tag{2.2}$$

γ is Huber norm. a_i^L, b_i^L is affine brightness parameters to frame iL . w_{p} is a gradient-dependent weighting parameters, $\mathbf{p}$ in frame I_i^L projected to I_j^L is $\mathrm{p}^{'}$ as:

$$w_{\mathbf{p}} := \frac{c^2}{c^2 + ||\nabla I_i(\mathbf{p})||_2^2}, \quad \mathbf{p}^{'} = d_{\mathbf{p}}^{jL}\mathbf{K}(\mathbf{T}_j\mathbf{T}_i^{-1}((d_{\mathbf{p}}^{iL})^{-1}\mathbf{K}^{-1}\mathbf{p})) \tag{2.3}$$

Static one-view stereo residuals $E_{is}^{\mathbf{p}}$ are modified to

$$E_{is}^{\mathbf{p}} = w_{\mathbf{p}}||r_{\mathbf{p}}^{s}||_{\gamma}, \quad r_{\mathbf{p}}^{s} := I_i^R(\mathbf{p}^{'}) - b_i^R - \frac{e^{a_i^R}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L) \tag{2.2}$$

Hostframe of $\mathbf{p}$ is I_i^L. a_i^R, b_i^R is affine brightness parameters to frame iR. $\mathbf{p}$ in frame I_i^L projected to I_i^R is $\mathrm{p}^{'}$ as :

$$\mathbf{p}^{'} = d_{\mathbf{p}}^{iR}\mathbf{K}(\mathbf{T}_{RL}((d_{\mathbf{p}}^{iL})^{-1}\mathbf{K}^{-1}\mathbf{p})) \tag{2.2}$$

Total residuals

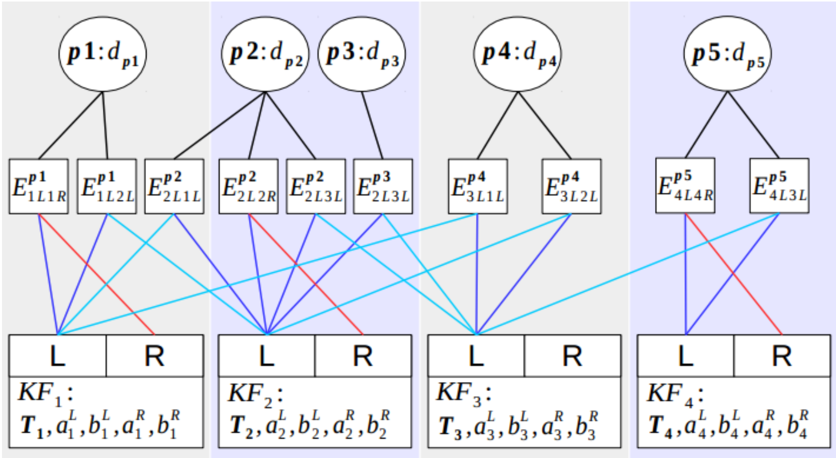
$$E(\delta) = \sum_{i\in\mathcal{F}}\sum_{\mathbf{p}\in\mathcal{P}_i}(\sum_{j\in obs^t(\mathbf{P})} E_{ij}^{\mathbf{p}} + \lambda E_{is}^{\mathbf{p}}) \tag{2.2}$$

$$\boldsymbol{\delta} = \begin{pmatrix} (\boldsymbol{\xi}_1^T, \dots, \boldsymbol{\xi}_{N_f}^T)^T \\ (d_{\mathbf{p}_1}, \dots, d_{\mathbf{p}_{N_p}})^T \\ (a_1^L, a_1^R, b_1^L, b_1^R)^T \\ \vdots \\ (a_{N_f}^L, a_{N_f}^R, b_{N_f}^L, b_{N_f}^R)^T \\ (f_x, f_y, c_x, c_y)^T \end{pmatrix} \in \mathbb{R}^{10N_f+N_p+4}, \boldsymbol{\xi}_i = (ln\mathbf{T}_i)^{\vee} \in \mathbb{R}^6 \tag{2.1}$$

To balance the relative weights of temporal multi-view and static stereo, we introduce a coupling factor λ to weight the constraints from static stereo differently. $\mathcal{P}_i$ is a set of all image point host by frame iL. $obs^t(\mathrm{p})$ are the observations of $\mathbf{p}$ from temporal multi-view stereo. If there are N_p image point and N_f keyframes in $\boldsymbol{\mathcal{F}}$, optimization variable $\boldsymbol{\delta}$ is

In this example, there are 7 dynamic residuals and 3 static residuals, Factor graph of the residuals function is

Total residuals is



Iteration δ^* can be calculated by

$$
\begin{aligned}
E(\delta) &= E^{\mathbf{p}_1}_{1L2L} + E^{\mathbf{p}_2}_{2L1L} + E^{\mathbf{p}_2}_{2L3L} + E^{\mathbf{p}_3}_{2L3L} + E^{\mathbf{p}_4}_{3L1L} + E^{\mathbf{p}_4}_{3L2L} + E^{\mathbf{p}_5}_{4L3L} \\
&\quad + E^{\mathbf{p}_1}_{1L1R} + E^{\mathbf{p}_2}_{2L2R} + E^{\mathbf{p}_5}_{4L4R} \\
&= E_d(\delta) + E_s(\delta)
\end{aligned}
$$

$$
E_s(\delta) = \begin{pmatrix} r^s_{\mathbf{p}_1} \\ r^s_{\mathbf{p}_2} \\ r^s_{\mathbf{p}_5} \end{pmatrix}^T \begin{pmatrix} \lambda w_{\mathbf{p}_1} & 0 & 0 \\ 0 & \lambda w_{\mathbf{p}_2} & 0 \\ 0 & 0 & \lambda w_{\mathbf{p}_5} \end{pmatrix} \begin{pmatrix} r^s_{\mathbf{p}_1} \\ r^s_{\mathbf{p}_2} \\ r^s_{\mathbf{p}_5} \end{pmatrix} = (\mathbf{r}^s)^T \mathbf{W}^s \mathbf{r}^s
$$

$$
\mathbf{J}_s = \begin{pmatrix}
\frac{\partial r^s_{\mathbf{p}_1}}{\partial \boldsymbol{\xi}_1} \cdots & \frac{\partial r^s_{\mathbf{p}_1}}{\partial \boldsymbol{\xi}_4} & \frac{\partial r^s_{\mathbf{p}_1}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial r^s_{\mathbf{p}_1}}{\partial d_{\mathbf{p}_5}} & \frac{\partial r^s_{\mathbf{p}_1}}{\partial a^L_1} \cdots & \frac{\partial r^s_{\mathbf{p}_1}}{\partial b^R_4} & \frac{\partial r^s_{\mathbf{p}_1}}{\partial f_x} \cdots & \frac{\partial r^s_{\mathbf{p}_1}}{\partial c_y} \\
\frac{\partial r^s_{\mathbf{p}_2}}{\partial \boldsymbol{\xi}_1} \cdots & \frac{\partial r^s_{\mathbf{p}_2}}{\partial \boldsymbol{\xi}_4} & \frac{\partial r^s_{\mathbf{p}_2}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial r^s_{\mathbf{p}_2}}{\partial d_{\mathbf{p}_5}} & \frac{\partial r^s_{\mathbf{p}_2}}{\partial a^L_1} \cdots & \frac{\partial r^s_{\mathbf{p}_2}}{\partial b^R_4} & \frac{\partial r^s_{\mathbf{p}_2}}{\partial f_x} \cdots & \frac{\partial r^s_{\mathbf{p}_2}}{\partial c_y} \\
\frac{\partial r^s_{\mathbf{p}_5}}{\partial \boldsymbol{\xi}_1} \cdots & \frac{\partial r^s_{\mathbf{p}_5}}{\partial \boldsymbol{\xi}_4} & \frac{\partial r^s_{\mathbf{p}_5}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial r^s_{\mathbf{p}_5}}{\partial d_{\mathbf{p}_5}} & \frac{\partial r^s_{\mathbf{p}_5}}{\partial a^L_1} \cdots & \frac{\partial r^s_{\mathbf{p}_5}}{\partial b^R_4} & \frac{\partial r^s_{\mathbf{p}_5}}{\partial f_x} \cdots & \frac{\partial r^s_{\mathbf{p}_5}}{\partial c_y}
\end{pmatrix}_{3\times 49} \tag{2.2}
$$

$$
E_d(\delta) = \begin{pmatrix} (r^d_{\mathbf{p}_1})_{12} \\ (r^d_{\mathbf{p}_1})_{21} \\ \vdots \\ (r^d_{\mathbf{p}_5})_{43} \end{pmatrix}^T \begin{pmatrix} w_{\mathbf{p}_1} & 0 & \dots & 0 \\ 0 & w_{\mathbf{p}_1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & w_{\mathbf{p}_5} \end{pmatrix} \begin{pmatrix} (r^d_{\mathbf{p}_1})_{12} \\ (r^d_{\mathbf{p}_1})_{21} \\ \vdots \\ (r^d_{\mathbf{p}_5})_{43} \end{pmatrix} = (\mathbf{r}^d)^T \mathbf{W}^d \mathbf{r}^d
$$

$$
(\mathbf{J}_s^T \lambda \mathbf{W}^s \mathbf{J}_s + \mathbf{J}_d^T \mathbf{W}^d \mathbf{J}_d)\delta^* = -(\mathbf{J}_s^T \lambda \mathbf{W}^s \mathbf{r}^s + \mathbf{J}_d^T \mathbf{W}^d \mathbf{r}^d) \tag{2.2}
$$

$$
\mathbf{J}_s \in \mathbb{R}^{3\times 49}, \mathbf{W}^s \in \mathbb{R}^{3\times 3}, \mathbf{J}_s \in \mathbb{R}^{7\times 49}, \mathbf{W}^s \in \mathbb{R}^{7\times 7},
$$

$$
\mathbf{J}_d = \begin{pmatrix}
\frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial \boldsymbol{\xi}_1} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial \boldsymbol{\xi}_4} & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial d_{\mathbf{p}_5}} & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial a^L_1} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial b^R_4} & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial f_x} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial c_y} \\
\frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial \boldsymbol{\xi}_1} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial \boldsymbol{\xi}_4} & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial d_{\mathbf{p}_5}} & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial a^L_1} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial b^R_4} & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial f_x} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial c_y} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial \boldsymbol{\xi}_1} \cdots & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial \boldsymbol{\xi}_4} & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial d_{\mathbf{p}_5}} & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial a^L_1} \cdots & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial b^R_4} & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial f_x} \cdots & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial c_y}
\end{pmatrix}_{7\times 49} \tag{2.2}
$$

We construct residuals and its formulation.

1.2 Errors and covariance calculation pseudo code

We know for a Lie algebra $\boldsymbol{\rho} \in \mathbb{R}^3, \boldsymbol{\phi} \in \mathbb{R}^3, \boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{\rho} \\ \boldsymbol{\phi} \end{pmatrix} \in \mathbb{R}^6$ and $\mathbf{p}_w$:

$$
\begin{aligned}
&\boldsymbol{\xi}^{\wedge} = \begin{pmatrix} \boldsymbol{\rho} \\ \boldsymbol{\phi} \end{pmatrix}^{\wedge} = \begin{pmatrix} \boldsymbol{\phi}^{\wedge} & \boldsymbol{\rho} \\ \mathbf{0}^T & \mathbf{0}^T \end{pmatrix} \in \mathbb{R}^{4\times 4} \\
&\boldsymbol{\epsilon} \in \mathbb{R}^3, \begin{pmatrix} \boldsymbol{\epsilon} \\ 1 \end{pmatrix}^{\odot} = \begin{pmatrix} \mathbf{E} & -\boldsymbol{\epsilon}^{\wedge} \\ \mathbf{0}^T & \mathbf{0}^T \end{pmatrix} \in \mathbb{R}^{4\times 6} \\
&\frac{\partial(exp(\xi^{\wedge})\mathbf{p}_w)}{\partial\boldsymbol{\xi}} = \frac{\partial(\mathbf{T}\mathbf{p}_w)}{\partial\boldsymbol{\xi}} = (\mathbf{T}\mathbf{p}_w)^{\odot} \\
&\mathbf{T}\mathbf{p}_w = exp(\xi^{\wedge})\mathbf{p}_w \approx (\mathbf{E} + \xi^{\wedge})\mathbf{p}_w \\
&\frac{\partial(exp(\xi^{\wedge})\mathbf{p}_w)}{\partial\boldsymbol{\xi}} \approx \frac{\partial(\mathbf{E} + \xi^{\wedge})}{\partial\boldsymbol{\xi}} = \mathbf{0} + \frac{\partial(\xi^{\wedge}\mathbf{p}_w)}{\partial\boldsymbol{\xi}} \approx (\mathbf{T}\mathbf{p}_w)^{\odot} \\
&since, \frac{\partial(\mathbf{T}\mathbf{p}_w)}{\partial\boldsymbol{\xi}} = (\mathbf{T}^{-1}\mathbf{p}_w)^{\odot} = \frac{\partial(exp(-\xi^{\wedge})\mathbf{p}_w)}{\partial\boldsymbol{\xi}} \\
&= \frac{\partial(\mathbf{E} - \xi^{\wedge})}{\partial\boldsymbol{\xi}} = -(\mathbf{T}\mathbf{p}_w)^{\odot}
\end{aligned}
\tag{2.2}
$$

1.3 Jacobian derivation

Chapter3 Photo Error Factors

3.1 Construction residual errors

Dynamic multi-view stereo residuals $E_{ij}^{\mathbf{p}}$ are defined as

$$E_{ij}^{\mathbf{p}} = w_{\mathbf{p}}||(r_{\mathbf{p}}^{d})_{ij}||_{\gamma}, \quad (r_{\mathbf{p}}^{d})_{ij} := I_j^L(\mathbf{p}') - b_j^L - \frac{e^{a_j^L}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L) \tag{2.2}$$

γ is Huber norm. a_i^L, b_i^L is affine brightness parameters to frame iL . $w_{\mathbf{p}}$ is a gradient-dependent weighting parameters, $\mathbf{p}$ in frame I_i^L projected to I_j^L is p' as:

$$w_{\mathbf{p}} := \frac{c^2}{c^2 + ||\nabla I_i(\mathbf{p})||_2^2}, \quad \mathbf{p}' = d_{\mathbf{p}}^{jL}\mathbf{K}(\mathbf{T}_j\mathbf{T}_i^{-1}((d_{\mathbf{p}}^{iL})^{-1}\mathbf{K}^{-1}\mathbf{p})) \tag{2.3}$$

Static one-view stereo residuals $E_{is}^{\mathbf{p}}$ are modified to

$$E_{is}^{\mathbf{p}} = w_{\mathbf{p}}||r_{\mathbf{p}}^{s}||_{\gamma}, \quad r_{\mathbf{p}}^{s} := I_i^R(\mathbf{p}') - b_i^R - \frac{e^{a_i^R}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L) \tag{2.2}$$

Hostframe of $\mathbf{p}$ is I_i^L. a_i^R, b_i^R is affine brightness parameters to frame iR. $\mathbf{p}$ in frame I_i^L projected to I_i^R is p' as :

$$\mathbf{p}' = d_{\mathbf{p}}^{iR}\mathbf{K}(\mathbf{T}_{RL}((d_{\mathbf{p}}^{iL})^{-1}\mathbf{K}^{-1}\mathbf{p})) \tag{2.2}$$

Total residuals

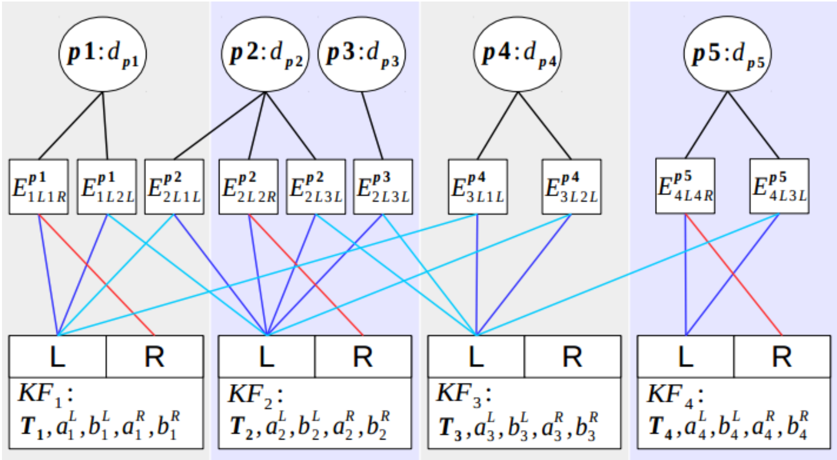
$$E(\delta) = \sum_{i\in\mathcal{F}}\sum_{\mathbf{p}\in\mathcal{P}_i}\Big(\sum_{j\in obs^t(\mathbf{P})} E_{ij}^{\mathbf{p}} + \lambda E_{is}^{\mathbf{p}}\Big) \tag{2.2}$$

$$\boldsymbol{\delta} = \begin{pmatrix} (\boldsymbol{\xi}_1^T, \ldots, \boldsymbol{\xi}_{N_f}^T)^T \\ (d_{\mathbf{p}_1}, \ldots, d_{\mathbf{p}_{N_p}})^T \\ (a_1^L, a_1^R, b_1^L, b_1^R)^T \\ \vdots \\ (a_{N_f}^L, a_{N_f}^R, b_{N_f}^L, b_{N_f}^R)^T \\ (f_x, f_y, c_x, c_y)^T \end{pmatrix} \in \mathbb{R}^{10N_f+N_p+4}, \boldsymbol{\xi}_i = (ln\mathbf{T}_i)^{\vee} \in \mathbb{R}^6 \tag{2.1}$$

To balance the relative weights of temporal multi-view and static stereo, we introduce a coupling factor λ to weight the constraints from static stereo differently. $\mathcal{P}_i$ is a set of all image point host by frame iL. $obs^t(\mathrm{p})$ are the observations of $\mathbf{p}$ from temporal multi-view stereo. If there are N_p image point and N_f keyframes in $\boldsymbol{\mathcal{F}}$, optimization variable $\boldsymbol{\delta}$ is

In this example, there are 7 dynamic residuals and 3 static residuals, Factor graph of the residuals function is

Total residuals is



Iteration δ^* can be calculated by

$$
\begin{aligned}
E(\delta) &= E^{\mathbf{p}_1}_{1L2L} + E^{\mathbf{p}_2}_{2L1L} + E^{\mathbf{p}_2}_{2L3L} + E^{\mathbf{p}_3}_{2L3L} + E^{\mathbf{p}_4}_{3L1L} + E^{\mathbf{p}_4}_{3L2L} + E^{\mathbf{p}_5}_{4L3L} \\
&\quad + E^{\mathbf{p}_1}_{1L1R} + E^{\mathbf{p}_2}_{2L2R} + E^{\mathbf{p}_5}_{4L4R} \\
&= E_d(\delta) + E_s(\delta)
\end{aligned}
$$

$$
E_s(\delta) = \begin{pmatrix} r^s_{\mathbf{p}_1} \\ r^s_{\mathbf{p}_2} \\ r^s_{\mathbf{p}_5} \end{pmatrix}^T \begin{pmatrix} \lambda w_{\mathbf{p}_1} & 0 & 0 \\ 0 & \lambda w_{\mathbf{p}_2} & 0 \\ 0 & 0 & \lambda w_{\mathbf{p}_5} \end{pmatrix} \begin{pmatrix} r^s_{\mathbf{p}_1} \\ r^s_{\mathbf{p}_2} \\ r^s_{\mathbf{p}_5} \end{pmatrix} = (\mathbf{r}^s)^T \mathbf{W}^s \mathbf{r}^s
$$

$$
\mathbf{J}_s = \begin{pmatrix}
\frac{\partial r^s_{\mathbf{p}_1}}{\partial \xi_1} \cdots & \frac{\partial r^s_{\mathbf{p}_1}}{\partial \xi_4} & \frac{\partial r^s_{\mathbf{p}_1}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial r^s_{\mathbf{p}_1}}{\partial d_{\mathbf{p}_5}} & \frac{\partial r^s_{\mathbf{p}_1}}{\partial a^L_1} \cdots & \frac{\partial r^s_{\mathbf{p}_1}}{\partial b^R_4} & \frac{\partial r^s_{\mathbf{p}_1}}{\partial f_x} \cdots & \frac{\partial r^s_{\mathbf{p}_1}}{\partial c_y} \\
\frac{\partial r^s_{\mathbf{p}_2}}{\partial \xi_1} \cdots & \frac{\partial r^s_{\mathbf{p}_2}}{\partial \xi_4} & \frac{\partial r^s_{\mathbf{p}_2}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial r^s_{\mathbf{p}_2}}{\partial d_{\mathbf{p}_5}} & \frac{\partial r^s_{\mathbf{p}_2}}{\partial a^L_1} \cdots & \frac{\partial r^s_{\mathbf{p}_2}}{\partial b^R_4} & \frac{\partial r^s_{\mathbf{p}_2}}{\partial f_x} \cdots & \frac{\partial r^s_{\mathbf{p}_2}}{\partial c_y} \\
\frac{\partial r^s_{\mathbf{p}_5}}{\partial \xi_1} \cdots & \frac{\partial r^s_{\mathbf{p}_5}}{\partial \xi_4} & \frac{\partial r^s_{\mathbf{p}_5}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial r^s_{\mathbf{p}_5}}{\partial d_{\mathbf{p}_5}} & \frac{\partial r^s_{\mathbf{p}_5}}{\partial a^L_1} \cdots & \frac{\partial r^s_{\mathbf{p}_5}}{\partial b^R_4} & \frac{\partial r^s_{\mathbf{p}_5}}{\partial f_x} \cdots & \frac{\partial r^s_{\mathbf{p}_5}}{\partial c_y}
\end{pmatrix}_{3\times 49} \tag{2.2}
$$

$$
E_d(\delta) = \begin{pmatrix} (r^d_{\mathbf{p}_1})_{12} \\ (r^d_{\mathbf{p}_1})_{21} \\ \vdots \\ (r^d_{\mathbf{p}_5})_{43} \end{pmatrix}^T \begin{pmatrix} w_{\mathbf{p}_1} & 0 & \dots & 0 \\ 0 & w_{\mathbf{p}_1} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & w_{\mathbf{p}_5} \end{pmatrix} \begin{pmatrix} (r^d_{\mathbf{p}_1})_{12} \\ (r^d_{\mathbf{p}_1})_{21} \\ \vdots \\ (r^d_{\mathbf{p}_5})_{43} \end{pmatrix} = (\mathbf{r}^d)^T \mathbf{W}^d \mathbf{r}^d
$$

$$
(\mathbf{J}_s^T \lambda \mathbf{W}^s \mathbf{J}_s + \mathbf{J}_d^T \mathbf{W}^d \mathbf{J}_d)\delta^* = -(\mathbf{J}_s^T \lambda \mathbf{W}^s \mathbf{r}^s + \mathbf{J}_d^T \mathbf{W}^d \mathbf{r}^d) \tag{2.2}
$$

$$
\mathbf{J}_s \in \mathbb{R}^{3\times 49}, \mathbf{W}^s \in \mathbb{R}^{3\times 3}, \mathbf{J}_s \in \mathbb{R}^{7\times 49}, \mathbf{W}^s \in \mathbb{R}^{7\times 7},
$$

$$
\mathbf{J}_d = \begin{pmatrix}
\frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial \xi_1} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial \xi_4} & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial d_{\mathbf{p}_5}} & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial a^L_1} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial b^R_4} & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial f_x} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{12}}{\partial c_y} \\
\frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial \xi_1} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial \xi_4} & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial d_{\mathbf{p}_5}} & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial a^L_1} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial b^R_4} & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial f_x} \cdots & \frac{\partial (r^d_{\mathbf{p}_1})_{21}}{\partial c_y} \\
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\
\frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial \xi_1} \cdots & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial \xi_4} & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial d_{\mathbf{p}_1}} \cdots & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial d_{\mathbf{p}_5}} & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial a^L_1} \cdots & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial b^R_4} & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial f_x} \cdots & \frac{\partial (r^d_{\mathbf{p}_5})_{43}}{\partial c_y}
\end{pmatrix}_{7\times 49} \tag{2.2}
$$

We construct residuals and its formulation.

3.2 Jacobian citation

We know for a Lie algebra $\boldsymbol{\rho} \in \mathbb{R}^3, \boldsymbol{\phi} \in \mathbb{R}^3, \boldsymbol{\xi} = \begin{pmatrix} \boldsymbol{\rho} \\ \boldsymbol{\phi} \end{pmatrix} \in \mathbb{R}^6$ and $\mathbf{p}_w$:

$$\begin{aligned}
&\boldsymbol{\xi}^\wedge = \begin{pmatrix} \boldsymbol{\rho} \\ \boldsymbol{\phi} \end{pmatrix}^\wedge = \begin{pmatrix} \boldsymbol{\phi}^\wedge & \boldsymbol{\rho} \\ \mathbf{0}^T & \mathbf{0}^T \end{pmatrix} \in \mathbb{R}^{4\times4} \\
&\boldsymbol{\epsilon} \in \mathbb{R}^3, \begin{pmatrix} \boldsymbol{\epsilon} \\ 1 \end{pmatrix}^\odot = \begin{pmatrix} \mathbf{E} & -\boldsymbol{\epsilon}^\wedge \\ \mathbf{0}^T & \mathbf{0}^T \end{pmatrix} \in \mathbb{R}^{4\times6} \\
&\frac{\partial(exp(\xi^\wedge)\mathbf{p}_w)}{\partial\boldsymbol{\xi}} = \frac{\partial(\mathbf{T}\mathbf{p}_w)}{\partial\boldsymbol{\xi}} = (\mathbf{T}\mathbf{p}_w)^\odot \\
&\mathbf{T}\mathbf{p}_w = exp(\xi^\wedge)\mathbf{p}_w \approx (\mathbf{E}+\xi^\wedge)\mathbf{p}_w \\
&\frac{\partial(exp(\xi^\wedge)\mathbf{p}_w)}{\partial\boldsymbol{\xi}} \approx \frac{\partial(\mathbf{E}+\xi^\wedge)}{\partial\boldsymbol{\xi}} = \mathbf{0} + \frac{\partial(\xi^\wedge\mathbf{p}_w)}{\partial\boldsymbol{\xi}} \approx (\mathbf{T}\mathbf{p}_w)^\odot \\
&since, \frac{\partial(\mathbf{T}\mathbf{p}_w)}{\partial\boldsymbol{\xi}} = (\mathbf{T}^{-1}\mathbf{p}_w)^\odot = \frac{\partial(exp(-\xi^\wedge)\mathbf{p}_w)}{\partial\boldsymbol{\xi}} \\
&= \frac{\partial(\mathbf{E}-\xi^\wedge)}{\partial\boldsymbol{\xi}} = -(\mathbf{T}\mathbf{p}_w)^\odot
\end{aligned} \tag{2.2}$$

1.3 Jacobian derivation

1.3.1 Dynamic Parameter

Firstly, if $\mathbf{p}$ is neither observed by frame mL, mR nor hosted by nL, nR:

$$\frac{\partial(r^d_{\mathbf{p}})_{ij}}{\partial\boldsymbol{\xi_m}} = \frac{\partial(r^d_{\mathbf{p}})_{ij}}{\partial\boldsymbol{\xi_n}} = \mathbf{0}^T, so \quad \frac{\partial(r^d_{\mathbf{p}_1})_{12}}{\partial\boldsymbol{\xi}_3} = \frac{\partial r^d_{(\mathbf{p}_1)}{}_{12}}{\partial\boldsymbol{\xi}_4} = \ldots = \mathbf{0}^T, \tag{2.2}$$

$$\frac{\partial(r^d_{\mathbf{p}})_{ij}}{\partial\boldsymbol{\xi_i}} = \frac{\partial(I^L_j(\mathbf{p}'))}{\partial\boldsymbol{\xi_i}} = \frac{\partial(I^L_j(\mathbf{p}'))}{\partial\mathbf{p}'}\frac{\partial\mathbf{p}'}{\partial\mathbf{p}'_w}\frac{\partial\mathbf{p}'_w}{\partial\boldsymbol{\xi_i}} \tag{2.2}$$

$$\mathbf{p}'_w = \mathbf{T}_j\mathbf{T}_i^{-1}\mathbf{p}_w = \mathbf{T}_j\mathbf{T}_i^{-1}((d^{iL}_{\mathbf{p}})^{-1}\mathbf{K}^{-1}\mathbf{p})$$

otherwise, we follow

For one frame iL, we have $\mathbf{p}$ and $\mathbf{K}$, then we can get

Secondly, according to

$$\left\{ \mathbf{p}_w = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} = \begin{pmatrix} f_x^{-1}(d_{\mathbf{p}}^{iL})^{-1}(u^i - c_x) \\ f_y^{-1}(d_{\mathbf{p}}^{iL})^{-1}(v^i - c_y) \\ (d_{\mathbf{p}}^{iL})^{-1} \\ 1 \end{pmatrix} \right. \tag{2.2}$$

$$\begin{aligned}
&\frac{\partial \mathbf{p}_w^{'}}{\partial \boldsymbol{\xi_i}} = \mathbf{T}_j \frac{\partial (\mathbf{T}_i^{-1}\mathbf{p}_w^{'})}{\partial \boldsymbol{\xi_i}} = -\mathbf{T}_j(\mathbf{T}_i\mathbf{p}_w)^{\odot} \\
&\frac{\partial \mathbf{p}_w^{'}}{\partial \boldsymbol{\xi_j}} = \frac{\partial (\mathbf{T}_j\mathbf{T}_i^{-1}\mathbf{p}_w)}{\partial \boldsymbol{\xi_i}} = (\mathbf{T}_j\mathbf{T}_i^{-1}\mathbf{p}_w)^{\odot} \\
&= \begin{pmatrix} x^{'} \\ y^{'} \\ z^{'} \\ 1 \end{pmatrix}^{\odot} = \begin{pmatrix} 1 & 0 & 0 & 0 & z^{'} & -y^{'} \\ 0 & 1 & 0 & -z^{'} & 0 & x^{'} \\ 0 & 1 & 0 & y^{'} & -x^{'} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \\
&\Rightarrow \frac{\partial (r_{\mathbf{p}}^d)_{ij}}{\partial \boldsymbol{\xi_j}} = \frac{\partial (I_j^L(\mathbf{p}^{'}))}{\partial \mathbf{p}^{'}} \frac{\partial \mathbf{p}^{'}}{\partial \mathbf{p}_w^{'}} \frac{\partial \mathbf{p}_w^{'}}{\partial \boldsymbol{\xi_j}} \\
&= (g_x^{'}, g_y^{'}, 0, 0) \begin{pmatrix} f_x(z^{'})^{-1} & 0 & -x^{'}f_x(z^{'})^{-2} & 0 \\ 0 & f_y(z^{'})^{-1} & -y^{'}f_y(z^{'})^{-2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & (z^{'})^{-2} & 0 \end{pmatrix}
\end{aligned} \tag{2.2}$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 & z^{'} & -y^{'} \\ 0 & 1 & 0 & -z^{'} & 0 & x^{'} \\ 0 & 0 & 1 & y^{'} & -x^{'} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} g_x^{'}f_x(z^{'})^{-1} \\ g_y^{'}f_y(z^{'})^{-1} \\ -(g_x^{'}x^{'}f_x + g_y^{'}y^{'}f_y)(z^{'})^{-2} \\ -g_y^{'}f_y - (g_x^{'}x^{'}y^{'}f_x + g_y^{'}(y^{'})^2 f_y)(z^{'})^{-2} \\ g_x^{'}f_x + (g_x^{'}(x^{'})^2 f_x + g_y^{'}x^{'}y^{'}f_y)(z^{'})^{-2} \\ -g_x^{'}f_x y^{'}(z^{'})^{-1} + g_y^{'}f_y x^{'}(z^{'})^{-1} \end{pmatrix}^T$$

$$(r_{\mathbf{p}}^d)_{ij} := I_j^L(\mathbf{p}^{'}) - b_j^L - \frac{e^{a_j^L}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L) \tag{2.2}$$

$$\begin{aligned}
&\frac{\partial (r_{\mathbf{p}}^d)_{ij}}{\partial a_i} = \frac{e^{a_j^L}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L), \frac{\partial (r_{\mathbf{p}}^d)_{ij}}{\partial a_j} = -\frac{e^{a_j^L}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L) \\
&\frac{\partial (r_{\mathbf{p}}^d)_{ij}}{\partial b_i} = \frac{e^{a_j^L}}{e^{a_i^L}}, \qquad \frac{\partial (r_{\mathbf{p}}^d)_{ij}}{\partial b_j} = -1
\end{aligned} \tag{2.2}$$

We have:

add detail Calibration derivation……

$$
\begin{aligned}
&\mathbf{p}^{'} = d_{\mathbf{p}}^{jL}\mathbf{K}(\mathbf{T}_j\mathbf{T}_i^{-1}\mathbf{p}_w)\\
&assume: \mathbf{T}_j\mathbf{T}_i^{-1} = \begin{pmatrix} r_{11}^{ji} & r_{12}^{ji} & r_{13}^{ji} & t_1^{ji}\\ r_{21}^{ji} & r_{22}^{ji} & r_{23}^{ji} & t_2^{ji}\\ r_{31}^{ji} & r_{32}^{ji} & r_{33}^{ji} & t_3^{ji}\\ 0 & 0 & 0 & 1 \end{pmatrix}\\
&\mathbf{p}_w^{'} = \begin{pmatrix} r_{11}^{ji}f_x^{-1}(d_{\mathbf{p}}^{iL})^{-1}(u^i - c_x) + r_{12}^{ji}f_y^{-1}(d_{\mathbf{p}}^{iL})^{-1}(v^i - c_y) + r_{13}^{ji}(d_{\mathbf{p}}^{iL})^{-1} + t_1^{ji}\\ r_{21}^{ji}f_x^{-1}(d_{\mathbf{p}}^{iL})^{-1}(u^i - c_x) + r_{22}^{ji}f_y^{-1}(d_{\mathbf{p}}^{iL})^{-1}(v^i - c_y) + r_{23}^{ji}(d_{\mathbf{p}}^{iL})^{-1} + t_2^{ji}\\ r_{31}^{ji}f_x^{-1}(d_{\mathbf{p}}^{iL})^{-1}(u^i - c_x) + r_{32}^{ji}f_y^{-1}(d_{\mathbf{p}}^{iL})^{-1}(v^i - c_y) + r_{33}^{ji}(d_{\mathbf{p}}^{iL})^{-1} + t_3^{ji}\\ 1 \end{pmatrix}\\
&= \begin{pmatrix} \frac{a}{d_{\mathbf{p}}^{iL}} + t_1^{ji}\\ \frac{b}{d_{\mathbf{p}}^{iL}} + t_2^{ji}\\ \frac{c}{d_{\mathbf{p}}^{iL}} + t_3^{ji}\\ 1 \end{pmatrix} = \begin{pmatrix} x^{'}\\ y^{'}\\ z^{'}\\ 1 \end{pmatrix} \Rightarrow d_{\mathbf{p}}^{jL} = (z^{'})^{-1}, \mathbf{p}^{'} = \begin{pmatrix} f_x x^{'} d_{\mathbf{p}}^{jL} + c_x\\ f_y y^{'} d_{\mathbf{p}}^{jL} + c_y\\ 1\\ d_{\mathbf{p}}^{jL} \end{pmatrix}\\
&\frac{\partial(I_j^L(\mathbf{p}^{'}))}{\partial\mathbf{p}^{'}} = (g_x^{'}, g_y^{'}, 0, 0)\\
&\frac{\partial\mathbf{p}^{'}}{\partial\mathbf{p}_w^{'}} = \begin{pmatrix} f_x(z^{'})^{-1} & 0 & -x^{'}f_x(z^{'})^{-2} & 0\\ 0 & f_y(z^{'})^{-1} & -y^{'}f_y(z^{'})^{-2} & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & (z^{'})^{-2} & 0 \end{pmatrix}, \frac{\partial\mathbf{p}_w^{'}}{\partial d_{\mathbf{p}}^{iL}} = \begin{pmatrix} -\frac{a}{(d_{\mathbf{p}}^{iL})^2}\\ -\frac{b}{(d_{\mathbf{p}}^{iL})^2}\\ -\frac{c}{(d_{\mathbf{p}}^{iL})^2}\\ 0 \end{pmatrix}\\
&\Rightarrow \frac{\partial(r_{\mathbf{p}}^d)_{ij}}{\partial d_{\mathbf{p}}^{iL}} = \frac{\partial(I_j^L(\mathbf{p}^{'}))}{\partial d_{\mathbf{p}}^{iL}} = \frac{\partial(I_j^L(\mathbf{p}^{'}))}{\partial\mathbf{p}^{'}}\frac{\partial\mathbf{p}^{'}}{\partial\mathbf{p}_w^{'}}\frac{\partial\mathbf{p}_w^{'}}{\partial d_{\mathbf{p}}^{iL}}\\
&= -\frac{g_x^{'}f_x a}{z^{'}(d_{\mathbf{p}}^{iL})^2} - \frac{g_y^{'}f_y b}{z^{'}(d_{\mathbf{p}}^{iL})^2} + \frac{c(g_x^{'}x^{'}f_x + g_y^{'}y^{'}f_y)}{(z^{'}d_{\mathbf{p}}^{iL})^2}\\
&= \frac{c(g_x^{'}x^{'}f_x + g_y^{'}y^{'}f_y) - g_x^{'}f_x a z^{'} - g_y^{'}f_y b z^{'}}{(z^{'}d_{\mathbf{p}}^{iL})^2}
\end{aligned}
\tag{2.2}
$$

1.3.2 Static Parameter

Firstly, For a stereo frame i: inverse depth $d_{\mathbf{p}}^{iL} = d_{\mathbf{p}}^{iR}$, a left frame iL pixel $\mathbf{p}$ is projected to right frame iR with $\mathbf{p}'$:

$$
\begin{aligned}
&\mathbf{p} = \begin{pmatrix} u^i \\ v^i \\ 1 \\ d_{\mathbf{p}}^{iL} \end{pmatrix}, \mathbf{p}_w = \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}, d_{\mathbf{p}}^{iL} = z^{-1}, \mathbf{p}_w = (d_{\mathbf{p}}^{iL})^{-1}\mathbf{K}^{-1}\mathbf{p} \\
&= \begin{pmatrix} f_x^{-1}(d_{\mathbf{p}}^{iL})^{-1}(u^i - c_x) \\ f_y^{-1}(d_{\mathbf{p}}^{iL})^{-1}(v^i - c_y) \\ (d_{\mathbf{p}}^{iL})^{-1} \\ 1 \end{pmatrix}, \mathbf{T}_{RL} = \begin{pmatrix} 1 & 0 & 0 & t_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \\
&\mathbf{p}' = d_{\mathbf{p}}^{iR}\mathbf{K}(\mathbf{T}_{RL}\mathbf{p}_w) \\
&= d_{\mathbf{p}}^{iL} \begin{pmatrix} f_x & 0 & c_x & 0 \\ 0 & f_y & c_y & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} f_x^{-1}(d_{\mathbf{p}}^{iL})^{-1}(u^i - c_x) + t_1 \\ f_y^{-1}(d_{\mathbf{p}}^{iL})^{-1}(v^i - c_y) \\ (d_{\mathbf{p}}^{iL})^{-1} \\ 1 \end{pmatrix} = \begin{pmatrix} u^i + t_1 f_x d_{\mathbf{p}}^{iL} \\ v^i \\ 1 \\ d_{\mathbf{p}}^{iL} \end{pmatrix} \\
&\frac{\partial r_{\mathbf{p}}^s}{\partial d_{\mathbf{p}}^{iL}} = \frac{\partial (I_i^R(\mathbf{p}')) - \frac{e^{a_i^R}}{e^{a_i^L}}(I_i^L(\mathbf{p}))}{\partial d_{\mathbf{p}}^{iL}} = (\frac{\partial (I_i^R(\mathbf{p}'))}{\partial \mathbf{p}'} - \frac{e^{a_i^R}}{e^{a_i^L}} \frac{\partial (I_i^L(\mathbf{p}))}{\partial \mathbf{p}'}) \frac{\partial \mathbf{p}'}{\partial d_{\mathbf{p}}^{iL}} \\
&= [(g_x^{iR}, g_y^{iR}, 0, 0) - \mathbf{0}^T] \begin{pmatrix} t_1 f_x \\ 0 \\ 0 \\ 1 \end{pmatrix} = g_x^{iR} t_1 f_x
\end{aligned} \tag{1}
$$

Secondly, according to:

$$
r_{\mathbf{p}}^s := I_i^R(\mathbf{p}') - b_i^R - \frac{e^{a_i^R}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L) \tag{2.2}
$$

We have:

$$
\begin{aligned}
\frac{\partial (r_{\mathbf{p}}^s)_{ij}}{\partial a_i} &= \frac{e^{a_j^L}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L), \frac{\partial (r_{\mathbf{p}}^s)_{ij}}{\partial a_j} = -\frac{e^{a_j^L}}{e^{a_i^L}}(I_i^L(\mathbf{p}) - b_i^L) \\
\frac{\partial (r_{\mathbf{p}}^s)_{ij}}{\partial b_i} &= \frac{e^{a_j^L}}{e^{a_i^L}}, \qquad \frac{\partial (r_{\mathbf{p}}^s)_{ij}}{\partial b_j} = -1
\end{aligned} \tag{2.2}
$$

add detail Calibration derivation……