Basic logic

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What is logic?

What sort of reasoning can logic help us with?

A puzzle:

- There are 4 cards, each with a letter on one side and a number on the other
- Rule: "every card with a vowel has an even number on the other side"

$\boxed{Q}$ $\boxed{E}$ $\boxed{6}$ $\boxed{3}$

- Which card(s) must you turn over in order to check this rule?
- E and 3
- Why do we not need to turn over Q and 6?

Another puzzle:

- There are 4 cards, each with name of a drink on one side and an age on the other
- Rule: "if the age is under 18, then the drink on the other side of the card is non-alcoholic"

$\boxed{\text{Juice}}$ $\boxed{35}$ $\boxed{\text{Beer}}$ $\boxed{16}$

- Which card(s) must you turn over in order to check this rule?
- Beer and 16
- Why do we not need to turn over Juice and 35?

A **proposition** is a sentence which states a fact

i.e. a statement that can (in principle) be true or false

Example sentences:

- Birmingham is north of London
  proposition, and true
- $8 \times 7 = 42$
  proposition, and false
- Please mind the gap
  not a proposition!
- Every even natural number > 2 is the sum of two primes
  proposition
  Goldbach Conjecture: unknown whether it is true or false!
- Is black the opposite of white?
  not a proposition!

An **argument** is a list of propositions

- the last of which is called the **conclusion**
- and the others are called **premises**

Example: 2 premises and 1 conclusion

1. Premise 1: **If** there is smoke, **then** there is a fire
2. Premise 2: There is no fire
3. Conclusion: **Therefore**, there is no smoke

An argument is **valid** if (and only if), whenever the premises are true, then so is the conclusion

Is the argument from the previous slide valid?

1. Premise 1: **If** there is smoke, **then** there is a fire
2. Premise 2: There is no fire
3. Conclusion: **Therefore**, there is no smoke

Yes, it is valid!

If an argument is not valid, then it is invalid

Is this valid?

1. If John is at home, then his television is on

Is this valid?

1. If John is at home, then his television is on.
2. His television is not on.
3. Therefore, John is not at home.

Valid

Is this valid?

1. You can eat a burger or pasta.
2. You ate a burger.
3. Therefore, you did not eat pasta.

Invalid

Is this valid? Invalid

1. If the control software crashes, then the car's brakes will fail.
2. The car's brakes failed.
3. Therefore, the control software crashed.

Is this valid? Invalid (for the same reason as above)

1. If (2+2=5) then (3+3=6).
2. 3+3=6.
3. Therefore, 2+2=5.

More generally (with **symbols**) this argument is not valid (we saw 2 counterexamples):

1. If P then Q.
2. Q.
3. Therefore, P.

Basic concepts: More Example Arguments

Is this valid? Invalid

1. If the control software crashes, then the car's brakes will fail.
2. The control software did not crash.
3. Therefore, the car's brakes did not fail.

Is this valid? Invalid (for the same reason as above)

1. If (2+2=5) then (3+3=6).
2. 2+2 is not 5.
3. Therefore, 3+3 is not 6.

More generally (with **symbols**) this argument is not valid (we saw 2 counterexamples):

1. If P then Q.
2. $\neg P$.
3. Therefore, $\neg Q$.