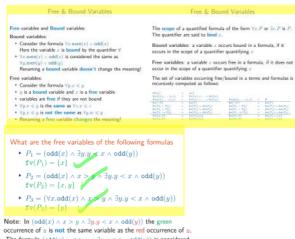
```
Substitution is defined recursively on terms and formulas:
  P[x \mid t] substitute all the free occurrences of x in P with t.
                                                             Substitution
Substitution is defined recursively on terms and formulas:
 P[x \mid t] substitute all the free occurrences of x in P with t.
1st attempt (WRONG)
                                 \begin{aligned} x_1x_{ij} &= t \\ x_1y_{ij} &= x \\ f(t_{i1}, t_n)|_{x^2 \in \mathbb{R}} &= f(t_{1}|_{x^2}), \dots, t_n|_{x^2}) \\ (f(t_{i1}, \dots t_n)|_{x^2}) &= p(t_{1}|_{x^2}), \dots, t_n|_{x^2}) \\ &= p(t_{1}|_{x^2}), \dots, t_n|_{x^2}) \\ (-P)|_{x^2}|_{x^2} &= P|_{x^2}|_{x^2} \\ (P_1 \wedge P_2)|_{x^2}|_{x^2} &= P|_{x^2}|_{x^2} \\ (P_2 P_2)|_{x^2}|_{x^2} &= 3x, P \\ (3y, P)|_{x^2}|_{x^2} &= 3y, P|_{x^2}|_{x^2} \end{aligned}
 Why is this wrong? (\forall y.y > x)[x \setminus y] would return \forall y.y > y, where
the free y is now bound! The free y got captured! The red occurrences of y stand for different variables than the green ones.
 Substitution is defined recursively on terms and formulas:
  P[x \mid t] substitute all the free occurrences of x in P with t.
2nd attempt (CORRECT)
                                \begin{array}{lll} x\|y|t| & = & t \\ (f(t_1,\dots,t_n))[x]t| & = & x \\ (f(t_1,\dots,t_n))[x]t| & = & f(t_1[x]t),\dots,t_n[x]t] \\ (\neg P)[x]t| & = & \neg P[x]t| \\ (\neg P)[x]t| & = & \neg P[x]t| \\ (P_1 \wedge P_2)[x]t| & = & P_1[x]t| \\ (P_2 \wedge P_2)[x]t| & = & P_1[x]t| \\ (P_1 - P_2)[x]t| & = & P_1[x]t| \\ (P_2 - P_2)[x]t| & = & P_1[x]t| \\ (Vx_tP)[x]t| & = & Vx_tP \\ (Vx_tP)[x]t| & = & Vx_tP \\ (Vx_tP)[x]t| & = & Vx_tP \\ (\partial y_tP)[x]t| & = & yy_tP \\ (\partial y_tP)[x]t| & = & yy_tP \\ (\partial y_tP)[x]t| & = & yy_tP(x]t|, \ if \ y \notin tv(t) \\ (\partial y_tP)[x]t| & = & yy_tP(x]t|, \ if \ y \notin tv(t) \\ \end{array}
 The additional conditions ensure that free variables do not get
 These conditions can always be met by silently renaming
  bound variables before substituting
   ules for Substitution:
           Example: If P=\forall y.(y< x \wedge odd(y)) and you want to substitute x with 2, it becomes \forall y.(y<2 \wedge odd(y)).
            When substituting a bound variable, ensure the substitution do meaning of the expression.
            Example: If P = \forall x.(x < y \land odd(x)) and you want to substitute x with z, it becomes \forall x.(x < y \land odd(x)).
  Similarly for (\exists y.P)[x \setminus t]:
 If P=x>y and we substitute t=z+1 for x, the result x\in \exists y.(z+1>y), as long as y\not\in \mathrm{fv}(t). Again, this avoids any accidental binding or capture.
  By following the rule y \notin fv(t), we ensure no unintended interactions between the free variables in t.

What if y \in fv(t)?
```



The formula $(\mathrm{odd}(x) \wedge x > y \wedge \exists y.y < x \wedge \mathrm{odd}(y))$ is considered the same as $(\mathrm{odd}(x) \wedge x > y \wedge \exists z.z < x \wedge \mathrm{odd}(z))$ Here $\operatorname{odd}(y)$ is bound by \exists , which is why changing the variable name

If you don't get this (^) you're a fucking idiot

Inference rules for ∀ and ∃?

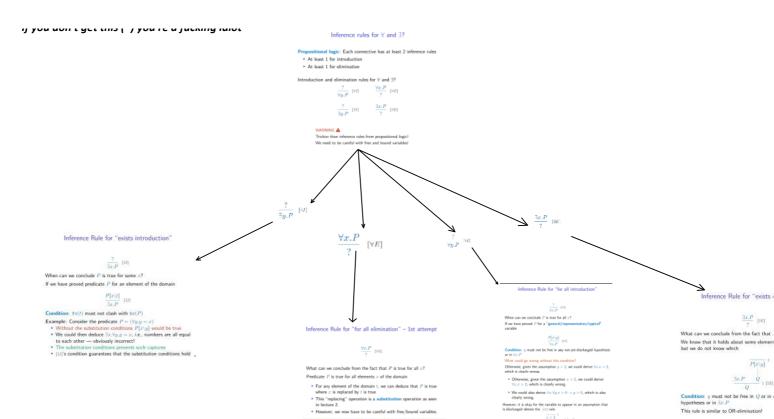
Propositional logic: Each connective has at least 2 inference rules

At least 1 for introduction

At least 1 for elimination

Introduction and elimination rules for V and 3?

-



Inference Rule for "for all elimination" — 2nd attempt $\frac{\forall x.P}{P[x|t]} \ |rt|$

Condition: V(t) must not clash with any bound variables of P Example: consider the formally V(t, p) = x.

* True over domain of natural numbers.

* P Big by y = x.

* In the formal V(t, p) = x.

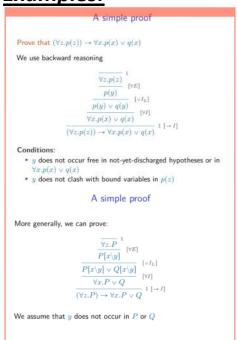
* This condition guarantees that we can do the substitution.

* Substituting x with y without example bound variables would give this wong answer (one prescous table).

* Therefore, we first remain bound variables that Clash with P(t)(t, t, x) with T(t, t, t, x) with T(t, t, t, x).

* Then, we substitute: T(t, t, t, x).

Examples:



Summary of all 4 inference rules:

elimination"

P is true for some x? t of the domain,

not-yet-discharged

Descriptor logic Des

All four inference rules in one slide

$$\frac{P[x \backslash y]}{\forall x.P} \quad [\forall I]$$

 ${\color{red}\textbf{Condition:}} \ y \ \text{must not be free in any not-yet-discharged hypothesis or in } \ \forall x.P$

$$\frac{\forall x.P}{P[x\backslash t]} \quad [\forall E]$$

 ${\bf Condition} \colon {\tt fv}(t) \ {\tt must not clash with} \ {\tt bv}(P) \\$

$$\frac{P[x \backslash t]}{\exists x.P} \quad [\exists I]$$

Condition: fv(t) must not clash with bv(P)

$$\frac{}{P[x \setminus y]} \begin{array}{c} 1 \\ \vdots \\ \hline \frac{\exists x. P \quad Q}{Q} \end{array} 1 \ [\exists E]$$

Condition: y must not be free in Q or in not-yet-discharged hypotheses or in $\exists x.P$