Exercise Sheet 12d - Solutions
Propositional Logic – Natural Deduction

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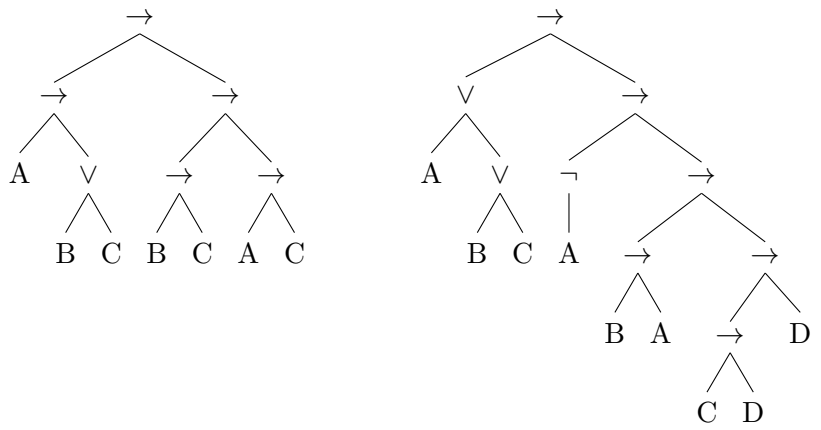
1. The parse trees of F and G are



2. Here is a constructive Natural Deduction proof of $(A \to (B \vee C)) \to (B \to C) \to A \to C$

$$\dfrac{\dfrac{\dfrac{\overline{A \to (B \vee C)}^{\,1} \quad \overline{A}^{\,3}}{B \vee C}\,[\to E] \quad \overline{B \to C}^{\,2} \quad \dfrac{\overline{C}^{\,4}}{C \to C}\,4\,[\to I]}{C}\,[\vee E]}{A \to C}\,3\,[\to I]}{(B \to C) \to A \to C}\,2\,[\to I]}{(A \to (B \vee C)) \to (B \to C) \to A \to C}\,1\,[\to I]$$

3. Here is a constructive Natural Deduction proof of $(A \vee B \vee C) \to \neg A \to (B \to A) \to (C \to D) \to D$

$$\dfrac{\dfrac{\dfrac{\dfrac{\overline{A \vee B \vee C}^{\,1} \quad \dfrac{\dfrac{\overline{A}^{\,5} \quad \overline{\neg A}^{\,2}}{\bot}\,[\neg E]}{D}\,[\bot E]}{A \to D}\,5\,[\to I] \quad \overline{B \vee C}^{\,6} \quad \dfrac{\dfrac{\dfrac{\overline{B \to A}^{\,3} \quad \overline{B}^{\,7}}{A}\,[\to E] \quad \overline{\neg A}^{\,2}}{\bot}\,[\neg E]}{D}\,[\bot E]}{B \to D}\,7\,[\to I] \quad \overline{C \to D}^{\,4}}{D}\,[\vee E]}{(B \vee C) \to D}\,6\,[\to I]}{D}\,[\vee E]}{(C \to D) \to D}\,4\,[\to I]}{(B \to A) \to (C \to D) \to D}\,3\,[\neg I]}{\neg A \to (B \to A) \to (C \to D) \to D}\,2\,[\to I]}{(A \vee B \vee C) \to \neg A \to (B \to A) \to (C \to D) \to D}\,1\,[\to I]$$

4. F and G are equivalent as they are both true and therefore are both equivalent to $\top$.

   Another explanation is that because both formulas are provable, by consistency they are both valid, which means that they are both true for all valuations, i.e., that they are equivalent.