

## Exercise Sheet 12b

### Predicate Logic – Natural Deduction & Semantics

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Consider the following signature:

- Function symbols: **zero** (arity 0); **succ** (arity 1)
- Predicate symbols:  $<$  (arity 2);  $\leq$  (arity 2)

We will use infix notation for the binary symbols  $<$  and  $\leq$ . For simplicity we write 0 for **zero**, 1 for **succ(zero)**, 2 for **succ(succ(zero))**, etc. Consider the following formulas that capture properties of the above symbols:

- let  $S_1$  be  $\exists x.\forall y.x < y$
- let  $S_2$  be  $\forall x.\neg x < x$
- let  $S_3$  be  $\forall x.\forall y.x < y \vee y \leq x$

1. Provide a constructive Natural Deduction proof of  $(S_1) \rightarrow (S_2) \rightarrow \perp$
2. Provide a Constructive Natural Deduction proof of  $(S_2) \rightarrow (S_3) \rightarrow \forall x.x \leq x$
3. Provide a model  $M_1$  such that  $\models_{M_1} S_3$ .
4. Provide a model  $M_2$  such that  $\models_{M_2} \neg S_3$ .