Exercise Sheet for Maths Material Covered in Week 10
Matrix Multiplication, Identity, Determinant and Inverse

(1) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(a) Find a 2×2 matrix B (if it exists) such that $BA = I$?

(b) Find a 2×2 matrix C (if it exists) such that $AC = I$?

Comment: After answering part (a), is there an easy way to answer part (b)?

(2) Let $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(a) Find a 3×3 matrix B (if it exists) such that $BA = I$?

(b) Find a 3×3 matrix C (if it exists) such that $AC = I$?

(3) Let $X = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 4 \\ 1 & 0 & 5 \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

(a) Find a 3×3 matrix Y (if it exists) such that $XY = I$?

(b) Find a 3×3 matrix Z (if it exists) such that $ZX = I$?

(4) Let $a, b, c, d \in \mathbb{Q}$ be such that none of them are zero. Let $X = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. If $ad - bc = 0$, then show that there is no 2×2 matrix Y whose entries are rational numbers and which satisfies $XY = I$.

(5) Let $A = \begin{pmatrix} 1 & 2 & 0 \\ 4 & 5 & 1 \\ 1 & -1 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}$

(a) Calculate the determinant of both matrices.

(b) For both matrices, calculate the inverse, if it exists. If the inverse does not exist, explain why.

(6) Alice and Bob in the fields: Around Mathsdale village, there are three fields named $\mathbb{C}, \mathbb{Q}$, and $\mathbb{R}$. Bob wants to meet Alice each afternoon, after the Maths tutorials. He gives her the following hint: "Let $A = \begin{pmatrix} 1 & a \\ 1 & 1 \end{pmatrix}$ be a 2×2 matrix over a field. On Monday, let $a = 0$, on Tuesday $a = 2$ and on Wednesday $a = -1$. Caculate the eigenvalues of A, and find me in the respective field. "

(a) Can you help Alice, for each day, to find out where to search for Bob?

(b) On which days can Alice locate Bob more quickly?

(c) Can you find values for a that could direct Alice immediately to the other field(s)?