Logic Session - W4&5 - Mon 16th & 23rd

Q1: Constructive: $(A \vee (B \wedge A)) \to A$

Q2: Constructive: $((A \wedge B) \to C) \to A \to B \to C$

Q3: Classical: $(\neg Q \to P) \to (\neg P \to Q)$

Q4: Classical: $(\neg P \to Q \wedge R) \to P \vee Q$

Q5: Classical: $\neg(P \wedge Q) \to (\neg P \to R) \to (\neg Q \to R) \to R$

Q6: Classical: $\neg A \to (B \to C \vee A) \to (C \vee \neg B)$

Q7: Classical: $\neg(A \wedge B) \to (\neg A \vee \neg B)$

Q1: Constructive: $(A \vee (B \wedge A)) \to A$

$$\dfrac{\dfrac{}{A \vee (B \wedge A)}\,1 \quad \dfrac{\dfrac{}{A}\,2}{A \to A}\,[\to I]_2 \quad \dfrac{\dfrac{\dfrac{}{B \wedge A}\,3}{A}\,[\wedge E_R]}{(B \wedge A) \to A}\,3\,[\to I]}{A}\,[\vee E]$$

$$\dfrac{A}{A \vee (B \wedge A) \to A}\,1\,[\to I]$$

Q2: Constructive: $((A \wedge B) \to C) \to A \to B \to C$

$$\dfrac{\dfrac{\dfrac{}{(A \wedge B) \to C}\,1 \quad \dfrac{\dfrac{}{A}\,2 \quad \dfrac{}{B}\,3}{A \wedge B}\,[\wedge I]}{C}\,[\to E]}{B \to C}\,3\,[\to I]$$

$$\dfrac{A \to B \to C}{((A \wedge B) \to C) \to A \to B \to C}\,2\,[\to I]\;1\,[\to I]$$

1. $(A \wedge B) \to C$
2. A
3. B

Q3: Classical: $(\neg Q \to P) \to (\neg P \to Q)$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\neg Q \to P}\,1 \quad \dfrac{}{\neg Q}\,3}{P} \quad \dfrac{}{\neg P}\,2}{\bot}\,[\neg E]}{\neg\neg Q}\,3\,[\neg I]$$

DNE

$$\dfrac{Q}{\neg P \to Q}\,2\,[\to I]$$

$$\dfrac{}{(\neg Q \to P) \to (\neg P \to Q)}\,1\,[\to I]$$

LEM:

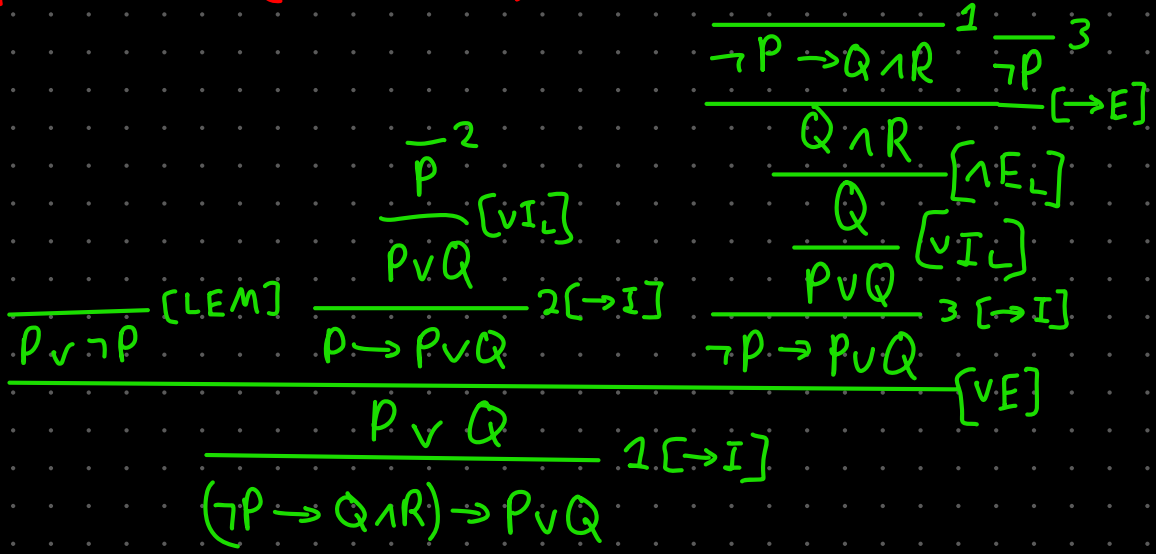
$$\dfrac{\dfrac{\dfrac{\dfrac{}{\neg Q \to P}\,1 \quad \dfrac{}{\neg Q}\,3}{P} \quad \dfrac{}{\neg P}\,2}{\bot}\,[\neg E]}{Q}\,[\bot E]$$

$$\dfrac{\dfrac{}{Q \vee \neg Q}\,[LEM] \quad \dfrac{\dfrac{}{Q}\,4}{Q \to Q}\,4\,[\to I] \quad \dfrac{Q}{\neg Q \to Q}\,3\,[\to I]}{Q}\,[\vee E]$$

$$\dfrac{Q}{\neg P \to Q}\,2\,[\to I]$$

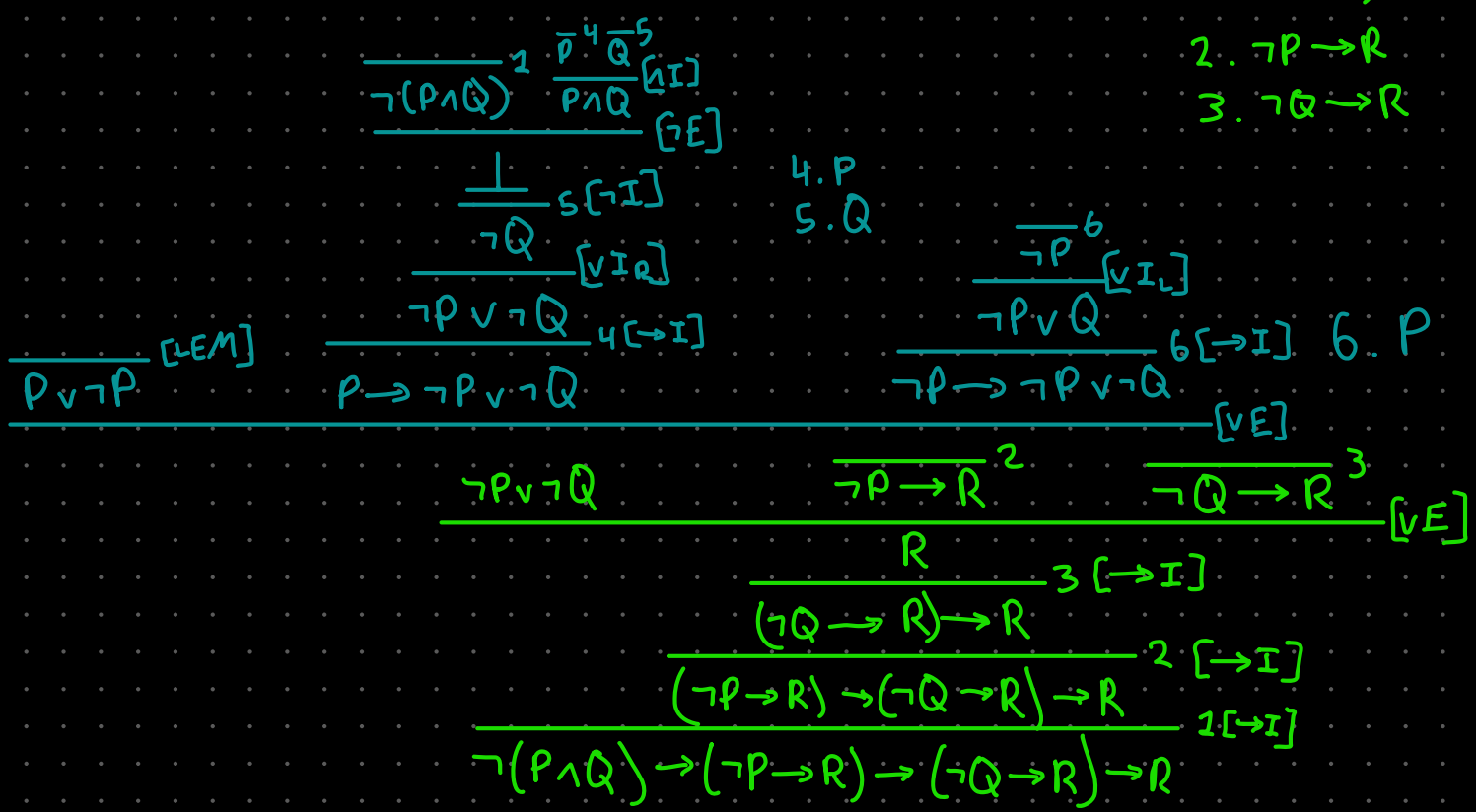
$$\dfrac{}{(\neg Q \to P) \to (\neg P \to Q)}\,1\,[\to I]$$

Q4: Classical: $(\neg P \to Q \wedge R) \to P \vee Q$

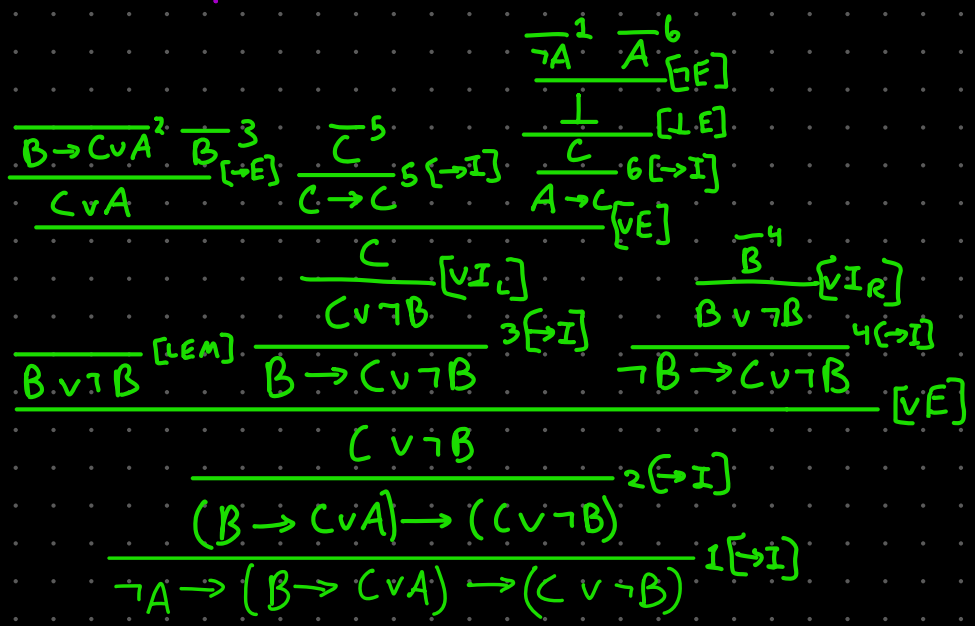


Q5: Classical: $\neg(P \wedge Q) \to (\neg P \to R) \to (\neg Q \to R) \to R$

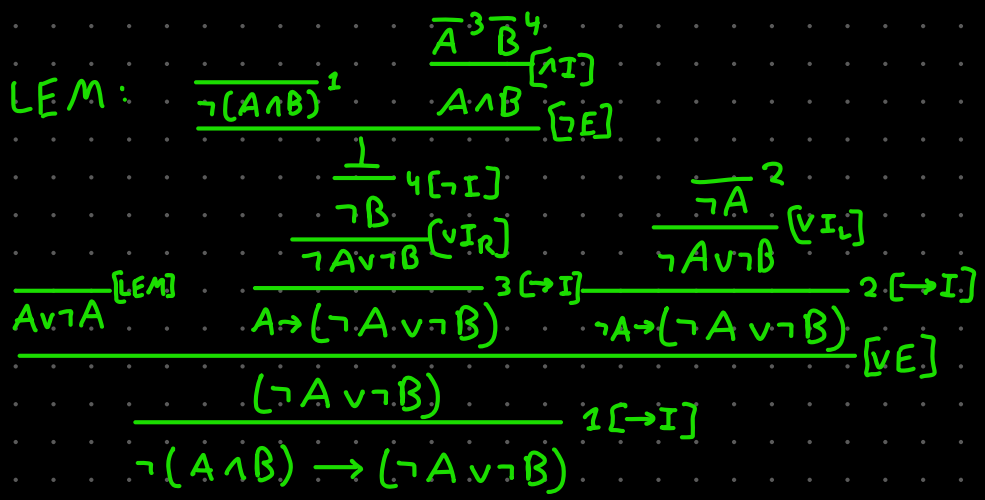
1. $\neg(P \wedge Q)$
2. $\neg P \to R$
3. $\neg Q \to R$



Q6: Classical: $\neg A \to (B \to C \vee A) \to (C \vee \neg B)$



Q7: Classical: $\neg(A \wedge B) \to (\neg A \vee \neg B)$



DNE