

Signature : • functions : zero, succ  
• predicates :  $\leq^2$ ;  $<^2$

$S_3 \stackrel{df}{=} \forall x. \exists y. x \leq y$

$$(2) \models_{M_2} \neg S_3 \Leftrightarrow \neg \models_{M_2} S_3$$

$$\in \underbrace{D^0 \rightarrow D}_D \quad \in D^1 \rightarrow D \quad \in D^2 \quad \in D^2 \stackrel{d^2}{=} D \times D$$

- $M_1 = \langle N, \langle 0, (n) \mapsto n \rangle, \langle \frac{1}{2} \langle x, y \rangle \mid x \leq y \rangle, \emptyset \rangle$

$$\bullet M_2 = \langle \mathbb{N}, \langle 0, \langle n \rangle \mapsto n \rangle, \{ \langle x, y \rangle \mid x < y \}, \emptyset \rangle$$

- $M_2' = \langle \mathbb{N}, \langle 0, \langle n \rangle \mapsto n \rangle, \{ \langle x, y \rangle \mid \text{True} \}, \emptyset \rangle$

$\models_{M_1} \Sigma_3 \Leftrightarrow$  For all  $d \in \mathbb{N}$ , there exists  $e \in \mathbb{N}$ , True  $\Leftrightarrow$  True

②  $\models_{M_2} \neg S_3 \Leftrightarrow \neg \models_{M_2} S_3$

$$\bullet M_2 = \langle N, \langle 0, \langle n \rangle \mapsto n \rangle, \langle \{ \langle x, y \rangle \mid y \in x \}, \emptyset \rangle \rangle$$

$$\bullet \mu'_2 = \langle N, \langle \alpha, \langle n \rangle \vdash n \rangle, \langle \emptyset, \emptyset \rangle \rangle$$

$\neg \text{FA}_2 S_3 \Leftrightarrow \neg \text{for all } d \in \mathbb{N}, \text{ there exists } e \in \mathbb{N}, \langle d, e \rangle \in \varnothing \Leftrightarrow \text{True}$   
False  
False  
False