Exercise Sheet 7
Predicate Logic – Syntax

Note that you can submit question 5 for feedback.

1. Consider the following example from lecture 11. Consider the database composed of the 3 following tables (we only show some of the rows here):

Student	
sid	**name**
0	Alice
1	Bob

Module	
mid	**name**
0	Math
1	OOP

Enroll	
sid	**mid**
0	0
1	1

   These 3 tables can be seen as 3 relations:

   - $\mathit{Student}(\mathit{sid}, \mathit{name})$: predicate *Student* relates student ids and names
   - $\mathit{Module}(\mathit{mid}, \mathit{name})$: predicate *Module* relates module ids and names
   - $\mathit{Enroll}(\mathit{sid}, \mathit{mid})$: predicate *Enroll* relates student and module ids

   Write the following queries as predicate logic formulas:

   - find the student names x enrolled either in the Math module or the OOP module
   - find all the pairs of student names x and y that are enrolled in the same module

2. Consider the domain of Boolean expressions. Write a signature that allows to at least state the two de Morgan's laws, and state predicate logic formulas that capture those laws.

3. Provide an example of a domain and a signature that includes an equivalence relation, and state predicate logic formulas that capture the fact that it is an equivalence relation.

4. Define the ring laws in predicate logic. Indicate what signature you are using.

5. [**feedback**] Consider the following domain and signature:

   - Domain: $\mathbb{N}$
   - Functions: $0, 1, 2, \ldots$ (arity 0); $+, \times$ (arity 2)
   - Predicates: `prime`, `even`, `odd` (arity 1); $=, >, \geq$ (arity 2)

   Express the following sentences in predicate logic:

   - If a number is strictly greater than another number then it must be greater than or equal to the successor of that number
   - A prime number is a number greater than 1 that cannot be expressed as the multiplication of two numbers greater than 1

6. Provide a constructive Natural Deduction proof of (let's keep practicing)

$$(A \wedge \neg B) \to (A \to C) \to (D \to B) \to (\neg D \wedge C)$$