

Logic Session - W4 & 5 - Mon 16th & 23rd

Q1: Constructive: $(A \vee (B \wedge A)) \rightarrow A$

Q2: Constructive: $((A \wedge B) \rightarrow C) \rightarrow A \rightarrow B \rightarrow C$

Q3: Classical: $(\neg Q \rightarrow P) \rightarrow (\neg P \rightarrow Q)$

Q4: Classical: $(\neg P \rightarrow Q \wedge R) \rightarrow P \vee Q$

Q5: Classical: $\neg(P \wedge Q) \rightarrow (\neg P \rightarrow R) \rightarrow (\neg Q \rightarrow R) \rightarrow R$

Q6: Classical: $\neg A \rightarrow (B \rightarrow C \vee A) \rightarrow (C \vee \neg B)$

Q7: Classical: $\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)$

Q1: Constructive: $(A \vee (B \wedge A)) \rightarrow A$

$$\frac{\frac{\frac{}{A \vee (B \wedge A)}}{A} 1 \quad \frac{\frac{\frac{}{A}}{A \rightarrow A} 2 [\rightarrow I] \quad \frac{\frac{\frac{}{B \wedge A}}{A} 3 [\wedge E]}{(B \wedge A) \rightarrow A} 3 [\rightarrow I]}{A} [\vee E]}{A \vee (B \wedge A) \rightarrow A} 1 [\rightarrow I]$$

Q2: Constructive: $((A \wedge B) \rightarrow C) \rightarrow A \rightarrow B \rightarrow C$

$$\frac{\frac{\frac{\frac{}{(A \wedge B) \rightarrow C}}{C} 3 [\rightarrow I] \quad \frac{\frac{\frac{}{A \wedge B}}{A} 2 [\wedge I] \quad \frac{}{B} 3}{A \wedge B} [\wedge I]}{B \rightarrow C} 3 [\rightarrow I]}{A \rightarrow B \rightarrow C} 2 [\rightarrow I]}{((A \wedge B) \rightarrow C) \rightarrow A \rightarrow B \rightarrow C} 1 [\rightarrow I]$$

1. $(A \wedge B) \rightarrow C$
2. A
3. B

Q3: Classical: $(\neg Q \rightarrow P) \rightarrow (\neg P \rightarrow Q)$

$$\frac{\frac{\frac{}{\neg Q \rightarrow P} 1 \quad \frac{}{\neg Q} 3}{P} \quad \frac{}{\neg P} 2}{\neg P} [\neg E]$$

DNE

$$\frac{\frac{\frac{}{Q}}{\neg \neg Q} 3 [\neg I] \quad \frac{}{Q} [\text{DNE}]}{Q} 2 [\rightarrow I]}{\neg P \rightarrow Q} 1 [\rightarrow I]}{(\neg Q \rightarrow P) \rightarrow (\neg P \rightarrow Q)} 1 [\rightarrow I]$$

LEM:

$$\frac{\frac{\frac{\frac{}{\neg Q \rightarrow P} 1 \quad \frac{}{\neg Q} 3}{P} \quad \frac{}{\neg P} 2}{\neg P} [\neg E] \quad \frac{\frac{}{Q}}{Q \rightarrow Q} 4 [\rightarrow I] \quad \frac{\frac{}{Q}}{\neg Q \rightarrow Q} 3 [\rightarrow I]}{Q \vee \neg Q} [\text{LEM}]}{Q} 2 [\rightarrow I]}{\neg P \rightarrow Q} 1 [\rightarrow I]}{(\neg Q \rightarrow P) \rightarrow (\neg P \rightarrow Q)} 1 [\rightarrow I]$$

Q4: Classical: $(\neg P \rightarrow Q \wedge R) \rightarrow P \vee Q$

$$\begin{array}{c}
 \frac{\frac{P \vee \neg P}{P \vee \neg P} \text{ [LEM]} \quad \frac{\frac{\frac{P}{P \vee Q} \text{ [VIL]} \quad \frac{P \rightarrow P \vee Q}{P \rightarrow P \vee Q} \text{ 2 [}\rightarrow\text{I]}}{P \rightarrow P \vee Q} \text{ 2 [}\rightarrow\text{I]}}{P \vee Q} \text{ 1 [}\rightarrow\text{I]} \\
 \frac{P \vee Q}{(\neg P \rightarrow Q \wedge R) \rightarrow P \vee Q} \text{ 1 [}\rightarrow\text{I]}
 \end{array}$$

Q5: Classical: $\neg(P \wedge Q) \rightarrow (\neg P \rightarrow R) \rightarrow (\neg Q \rightarrow R) \rightarrow R$

[illegible]

Q6: Classical: $\neg A \rightarrow (B \rightarrow C \vee A) \rightarrow (C \vee \neg B)$

[illegible]

Q7: Classical: $\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)$

$$\begin{array}{c} \text{LEM: } \frac{\frac{\overline{A^3 B^4}}{A \wedge B} [\neg I] \quad \frac{\frac{\frac{1}{\neg B} [\neg I]}{\neg A \vee \neg B} [\vee I_R] \quad \frac{\frac{\overline{\neg A^2}}{\neg A \vee \neg B} [\vee I_L]}{A \rightarrow (\neg A \vee \neg B)} [\rightarrow I] \quad \frac{}{A \vee \neg A} [\text{LEM}] \quad \frac{}{A \rightarrow (\neg A \vee \neg B)} [\rightarrow I]}{A \vee \neg A} [\text{LEM}] \quad \frac{A \rightarrow (\neg A \vee \neg B) \quad A \rightarrow (\neg A \vee \neg B)}{\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)} [\vee E] \\ \frac{}{\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)} [\rightarrow I] \end{array}$$

DNE

$$\begin{array}{c}
 \text{VE} \\
 \frac{\frac{\frac{}{1} \neg(A \wedge B)}{\neg(\neg A \vee \neg B)} 2}{\frac{}{\neg A} 3} \frac{\frac{}{\neg B} 5}[\vee I_R]}{\frac{}{\neg(\neg A \vee \neg B)} [\neg E]} \\
 \frac{\frac{\frac{}{1} \neg(A \wedge B)}{\neg A} [\neg E]}{\frac{}{A} [\neg E]} \\
 \frac{\frac{}{1} \neg(A \wedge B)}{A \wedge B} [\neg E] \\
 \frac{\frac{\frac{}{1} \neg(A \wedge B)}{\neg(\neg A \vee \neg B)} 2}{\frac{}{\neg A \vee \neg B} [\neg E]} \\
 \frac{}{\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)} 1[\rightarrow I]
 \end{array}$$