

Exercise Sheet 11b

Predicate Logic – Natural Deduction & Semantics

Consider the following signature:

- Function symbols: **zero** (arity 0); **succ** (arity 1)
- Predicate symbols: $<$ (arity 2); \leq (arity 2)

We will use infix notation for the binary symbols $<$ and \leq . For simplicity we write 0 for **zero**, 1 for **succ(zero)**, 2 for **succ(succ(zero))**, etc. Consider the following formulas that capture properties of the above symbols:

- let S_1 be $\neg\exists x.0 \leq x$
- let S_2 be $\forall x.\forall y.\text{succ}(x) < y \rightarrow x \leq \text{succ}(y)$
- let S_3 be $\neg\exists x.1 < x$

1. Provide a constructive Natural Deduction proof of $(S_1) \rightarrow \forall x.\neg 0 \leq \text{succ}(x)$
2. Provide a Constructive Natural Deduction proof of $(S_1) \rightarrow (S_2) \rightarrow S_3$
3. Provide a model M_1 such that $\models_{M_1} \exists x.\exists y.x \leq y \wedge \neg y < x$.
4. Provide a model M_2 such that $\models_{M_2} \neg\exists x.\exists y.x \leq y \wedge \neg y < x$.