Exercise Sheet 12g - Solutions
Propositional Logic – Natural Deduction

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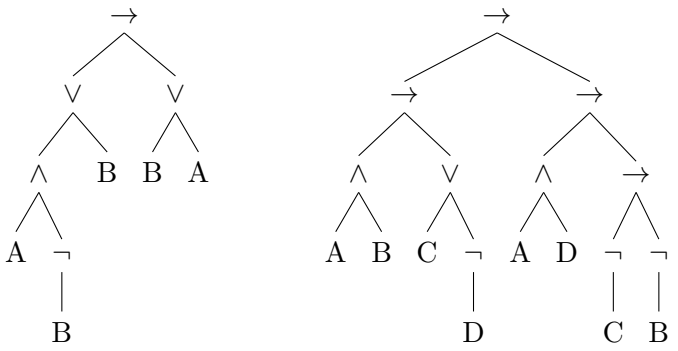
1. The parse trees of F and G are



2. Here is a constructive Natural Deduction proof of $(A \wedge \neg B) \vee B \to B \vee A$

$$\dfrac{\dfrac{\overline{(A \wedge \neg B) \vee B}^{\,1} \quad \dfrac{\dfrac{\dfrac{\overline{A \wedge \neg B}^{\,2}}{A}\,[\wedge E_L]}{B \vee A}\,[\vee I_R]}{A \wedge \neg B \to B \vee A}^{\,2}\,[\to I] \quad \dfrac{\dfrac{\overline{B}^{\,3}}{B \vee A}\,[\vee I_L]}{B \to B \vee A}^{\,3}\,[\to I]}{B \vee A}\,[\vee E]}{(A \wedge \neg B) \vee B \to B \vee A}^{\,1}\,[\to I]$$

3. Here is a constructive Natural Deduction proof of $(A \wedge B \to C \vee \neg D) \to A \wedge D \to \neg C \to \neg B$

$$\dfrac{\dfrac{\dfrac{\overline{A \wedge B \to C \vee \neg D}^{\,1} \quad \dfrac{\dfrac{\overline{A \wedge D}^{\,2}}{A}\,[\wedge E_L] \quad \overline{B}^{\,4}}{A \wedge B}\,[\wedge I]}{C \vee \neg D}\,[\to E] \quad \dfrac{\dfrac{\overline{C}^{\,5} \quad \overline{\neg C}^{\,3}}{\bot}\,[\neg E]}{C \to \bot}^{\,5}\,[\to I] \quad \dfrac{\dfrac{\overline{A \wedge D}^{\,2}}{D}\,[\wedge E_R] \quad \overline{\neg D}^{\,6}}{\bot}\,[\neg E]}{\neg D \to \bot}^{\,6}\,[\to I]}{\bot}\,[\vee E]}{\neg B}^{\,4}\,[\neg I]}{\neg C \to \neg B}^{\,3}\,[\to I]}{A \wedge D \to \neg C \to \neg B}^{\,2}\,[\to I]}{(A \wedge B \to C \vee \neg D) \to A \wedge D \to \neg C \to \neg B}^{\,1}\,[\to I]$$

4. F is provable and therefore valid by soundness, and so any valuation satisfies the formula such as $A = \mathbf{T}$, $B = \mathbf{T}$.
   G is provable and therefore valid by soundness, and therefore not falsifiable.