Exercise Sheet 12c - Solutions
Propositional Logic – Natural Deduction

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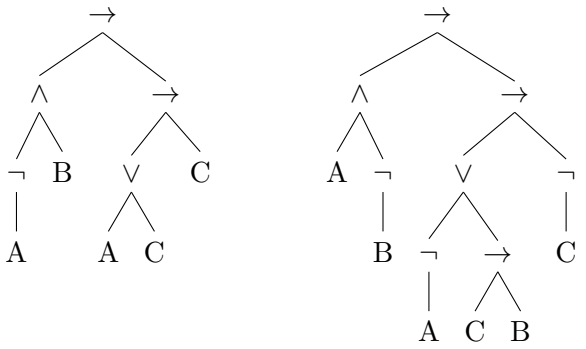
1. The parse trees of F and G are



2. Here is a constructive Natural Deduction proof of $(\neg A \wedge B) \to (A \vee C) \to C$

$$\dfrac{\dfrac{\overline{A\vee C}^{\,2} \qquad \dfrac{\dfrac{\dfrac{\overline{\neg A\wedge B}^{\,1}}{\neg A}\,[\wedge E_L] \quad \overline{A}^{\,3}}{\dfrac{\bot}{C}\,[\bot\, E]}\,[\neg E]}{A\to C}\,3\,[\to I] \qquad \dfrac{\overline{C}^{\,4}}{C\to C}\,4\,[\to I]}{C}\,[\vee E]}{(A\vee C)\to C}\,2\,[\to I]}{(\neg A\wedge B)\to(A\vee C)\to C}\,1\,[\to I]$$

3. Here is a constructive Natural Deduction proof of $(A \wedge \neg B) \to (\neg A \vee (C \to B)) \to \neg C$

$$\dfrac{\dfrac{\overline{\neg A\vee(C\to B)}^{\,2} \qquad \dfrac{\dfrac{\overline{A\wedge\neg B}^{\,1}}{A}\,[\wedge E_L] \quad \overline{\neg A}^{\,4}}{\bot}\,[\neg E] \qquad \dfrac{\dfrac{\overline{C\to B}^{\,5} \quad \overline{C}^{\,3}}{B}\,[\to E] \quad \dfrac{\overline{A\wedge\neg B}^{\,1}}{\neg B}\,[\wedge E_R]}{\bot}\,[\neg E]}{\bot}\,[\vee E]}{\neg C}\,3\,[\neg I]}{(\neg A\vee(C\to B))\to\neg C}\,2\,[\to I]}{(A\wedge\neg B)\to(\neg A\vee(C\to B))\to\neg C}\,1\,[\to I]$$

4. F and G are provable and therefore valid by soundness, so any valuation satisfies the formula such as $A = \mathbf{T}$, $B = \mathbf{T}$, and $C = \mathbf{T}$.