Exercise Sheet 12f - Solutions
Propositional Logic – Natural Deduction

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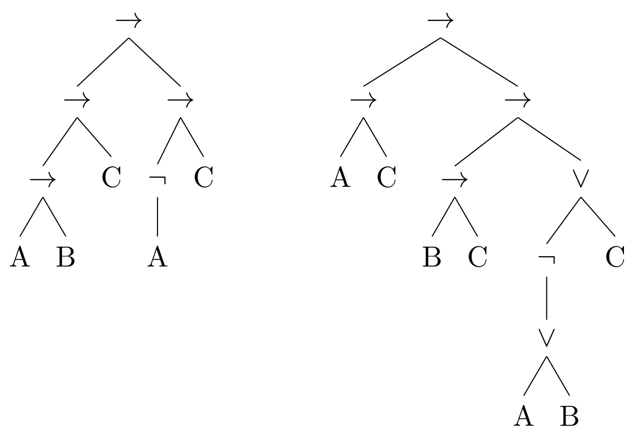
1. The parse trees of F and G are



2. Here is a constructive Natural Deduction proof of $((A \to B) \to C) \to \neg A \to C$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(A \to B) \to C}\,1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{A}\,3 \quad \dfrac{}{\neg A}\,2}{\bot}\,[\neg E]}{B}\,[\bot E]}{A \to B}\,3\,[\to I]}{C}\,[\to E]}{\neg A \to C}\,2\,[\to I]}{((A \to B) \to C) \to \neg A \to C}\,1\,[\to I]$$

3. Here is a classical Natural Deduction proof of $(A \to C) \to (B \to C) \to \neg(A \vee B) \vee C$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{C \vee \neg C}\,[LEM] \quad \dfrac{\dfrac{\dfrac{}{C}\,3}{\neg(A \vee B) \vee C}\,[\vee I_R]}{C \to \neg(A \vee B) \vee C}\,3\,[\to I] \quad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{A \vee B}\,5 \quad \dfrac{\dfrac{\dfrac{\dfrac{}{A \to C}\,1 \quad \dfrac{}{A}\,6}{C}\,[\to E] \quad \dfrac{}{\neg C}\,4}{\bot}\,[\neg E]}{A \to \bot}\,6\,[\to I] \quad \dfrac{\dfrac{\dfrac{\dfrac{}{B \to C}\,2 \quad \dfrac{}{B}\,7}{C}\,[\to E] \quad \dfrac{}{\neg C}\,4}{\bot}\,[\neg E]}{B \to \bot}\,7\,[\to I]}{\bot}\,[\vee E]}{\neg(A \vee B)}\,5\,[\neg I]}{\neg(A \vee B) \vee C}\,[\vee I_L]}{\neg C \to \neg(A \vee B) \vee C}\,4\,[\to I]}{\neg(A \vee B) \vee C}\,[\vee E]}{(B \to C) \to \neg(A \vee B) \vee C}\,2\,[\to I]}{(A \to C) \to (B \to C) \to \neg(A \vee B) \vee C}\,1\,[\to I]$$

4. F and G are equivalent as they are both true and therefore are both equivalent to $\top$.

   Another explanation is that because both formulas are provable, by consistency they are both valid, which means that they are both true for all valuations, i.e., that they are equivalent.