Exercise Sheet for Maths Material Covered in Week 7 (Gaussian Elimination) + (Introduction to Vector Spaces)

Note that you will be given feedback for question (2).

(1) Solve the following system of four linear equations in four variables x_1, x_2, x_3, x_4 over the set of rational numbers:

$$x_1 + 5x_2 - 2x_3 + 3x_4 = -11$$
$$3x_1 - 2x_2 + 7x_3 + x_4 = 5$$
$$-2x_1 - x_2 - x_3 - 2x_4 = 0$$
$$5x_1 + 3x_2 + 4x_3 - x_4 = 13$$

Show all the intermediate steps (not just the final values obtained for x_1, x_2, x_3, x_4).

Comment: For practice, solve first using the "standard" elimination method and then also using the compact way (using a matrix). Once you obtain values for x_1, x_2, x_3, x_4 plug them back into each of the four equations to verify that your answer is correct.

(2) **(feedback)** On Friday I bought two candy bars, a croissant and three coffees for 13.5 pounds. On Saturday, I bought three candy bars, five croissants and two coffees for 21.5 pounds. On Sunday (when I was very sleepy the whole day) I bought a candy bar, three croissants and six coffees for 26.5 pounds. Tomorrow, I wish to purchase four candy bars, two croissants and one coffee. How much will this cost me?
Comment: Show your working, i.e., don't just write down the final answer.

(3) Let V be a vector space over a field F. Show each of the following:

- Additive identity $\vec{0}$ is unique
- $0\vec{v} = \vec{0}$
- $s\vec{0} = \vec{0}$
- $(-1)\vec{v} = \overrightarrow{-v}$
- $s\vec{v} = \vec{0}$ implies $s = 0$ or $\vec{v} = \vec{0}$

Comment: To prove the above, you can use the following:

- Any of the 8 conditions available due to V being a vector space over F
- Any of the conditions available due to F being a field

(4) Verify that $\mathbb{Q}^2$ is a vector space over the field $\mathbb{Q}$ with the following definition of vector addition and scalar multiplication:

(a) Vector addition: If $\vec{v_1} = \binom{a_1}{b_1}$ and $\vec{v_2} = \binom{a_2}{b_2}$ then $\vec{v_1} + \vec{v_2}$ is defined to be $\binom{a_1+a_2}{b_1+b_2}$

(b) Scalar multiplication: $\vec{v} = \binom{a}{b} \in \mathbb{Q}^2$ and $r \in \mathbb{Q}$ then $r\vec{v} = \binom{ra}{rb}$

Comment: Each of the 8 conditions in the definition of vector space over a field needs to be verified. You can use any of the conditions available due to $\mathbb{Q}$ being a field.