Definitions

Definition 1 (Similar matrices)

Let $A, B \in \mathbb{R}^{n \times n}$ be matrices. Then, A is *similar* to B, if and only if, there exists an invertible matrix $P \in \mathbb{R}^{n \times n}$, such that

$$A = PBP^{-1}.$$

Definition 2 (Diagonalisable matrix)

If $A \in \mathbb{R}^{n \times n}$ is similar to a diagonal $n \times n$ matrix, then A is said to be *diagonalisable*.

Exercise 1

Prove that similarity is an equivalence relation. That is, show the following:

1. **reflexivity**: for all $A \in \mathbb{R}^{n \times n}$, A is similar to itself;
2. **symmetry**: for all $A, B \in \mathbb{R}^{n \times n}$, if A is similar to B, then B is similar to A;
3. **transitivity**: for all $A, B, C \in \mathbb{R}^{n \times n}$, if A is similar to B, and B is similar to C, then A is similar to C.

Exercise 1: Solution

We show the properties of equivalence as follows:

1. **reflexivity**: Clearly $A = IAI^{-1}$, so A is similar to itself;
2. **symmetry**: Suppose $A = PBP^{-1}$, for some invertible $P \in \mathbb{R}^{n \times n}$. Then, $B = P^{-1}AP$, so B is similar to A;
3. **transitivity**: Suppose $A = PBP^{-1}$, and $B = QCQ^{-1}$. But then, $A = PQCQ^{-1}P^{-1} = (PQ)C(PQ)^{-1}$, and since $PQ \in \mathbb{R}^{n \times n}$ is invertible, A is similar to C.