

Provide Natural Deduction Proofs of
the following:

Logic Session - Mon 9th Oct - W3

Q1: Constructive: $\neg(A \wedge B) \rightarrow (A \rightarrow \neg B)$

Q2: Constructive: $(A \wedge B) \rightarrow ((A \rightarrow C) \rightarrow \neg(B \rightarrow \neg C))$

Q3: Constructive: $(A \vee B \vee C) \rightarrow \neg A \rightarrow (B \rightarrow A) \rightarrow (C \rightarrow D) \rightarrow D$

Q4: Constructive: $(P \rightarrow Q) \rightarrow (\neg Q \rightarrow \neg P)$

Q5: Constructive: $(A \rightarrow C) \wedge (B \rightarrow \neg C) \rightarrow \neg(A \wedge B)$

Q6: Constructive: Assuming Q , prove: $(Q \rightarrow R) \rightarrow R$

Q7: Constructive: $Q \rightarrow (Q \rightarrow R) \rightarrow R$

Q8: Constructive: $((P \wedge Q) \wedge R) \rightarrow (S \wedge T) \rightarrow Q \wedge S$

Logic Session - W4 & 5 - Mon 16th & 23rd

Q1: Constructive: $(A \vee (B \wedge A)) \rightarrow A$

Q2: Constructive: $((A \wedge B) \rightarrow C) \rightarrow A \rightarrow B \rightarrow C$

Q3: Classical: $(\neg Q \rightarrow P) \rightarrow (\neg P \rightarrow Q)$

Q4: Classical: $(\neg P \rightarrow Q \wedge R) \rightarrow P \vee Q$

Q5: Classical: $\neg(P \wedge Q) \rightarrow (\neg P \rightarrow R) \rightarrow (\neg Q \rightarrow R) \rightarrow R$

Q6: Classical: $\neg A \rightarrow (B \rightarrow C \vee A) \rightarrow (C \vee \neg B)$

Q7: Classical: $\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)$

Logic Session - Mon 30th Oct - W6

(Difficulty in yellow)

- Q1: Constructive: $((\neg A \vee B) \rightarrow (\neg D \vee C)) \rightarrow (\neg D \rightarrow C) \rightarrow A \rightarrow C$ - Moderate
- Q2: Classical: $(\neg A \rightarrow B \vee C) \rightarrow \neg B \rightarrow \neg C \rightarrow A$ - Moderate
- Q3: Constructive: $(A \wedge B \rightarrow \neg C) \rightarrow C \rightarrow A \rightarrow \neg B$ - Easy
- Q4: Classical: $((A \wedge \neg B) \rightarrow \neg C) \rightarrow C \rightarrow A \rightarrow B$ - Easy
- Q5: Constructive: $(A \rightarrow B \vee C) \rightarrow (B \rightarrow D) \rightarrow (C \rightarrow B \vee D) \rightarrow \neg D \rightarrow \neg A$ - Hard!
- Q6: Constructive: $(D \rightarrow A \vee C) \rightarrow (A \wedge B \rightarrow C) \rightarrow D \wedge B \rightarrow C$ - Moderate
- Q7: Constructive: $(C \rightarrow B \vee A) \rightarrow (B \rightarrow A) \rightarrow ((C \rightarrow A) \vee C)$ - Moderate
- Q8: Classical: $(C \rightarrow B) \rightarrow (B \rightarrow A) \rightarrow (\neg C \rightarrow A) \rightarrow A$ - Moderate
- Q9: Classical: $(C \rightarrow A) \rightarrow (A \rightarrow B) \rightarrow (\neg B \rightarrow C) \rightarrow B$ - Moderate
- Q10: Constructive: $A \rightarrow (B \rightarrow D) \rightarrow (\neg B \rightarrow C) \rightarrow (A \vee C)$ - Easy
- Q11: Constructive: $(D \rightarrow A \vee C) \rightarrow (A \wedge B \rightarrow C) \rightarrow (D \wedge B) \rightarrow C$ - Moderate

Logic Session - Mon 9th Oct - W3

Q1: Constructive: $\neg(A \wedge B) \rightarrow (A \rightarrow \neg B)$

Q2: Constructive: $(A \wedge B) \rightarrow ((A \rightarrow C) \rightarrow \neg(B \rightarrow \neg C))$

Q3: Constructive: $(A \vee B \vee C) \rightarrow \neg A \rightarrow (B \rightarrow A) \rightarrow (C \rightarrow D) \rightarrow D$

Q4: Constructive: $(P \rightarrow Q) \rightarrow (\neg Q \rightarrow \neg P)$

Q5: Constructive: $(A \rightarrow C) \wedge (B \rightarrow \neg C) \rightarrow \neg(A \wedge B)$

Q6: Constructive: Assuming Q , prove: $(Q \rightarrow R) \rightarrow R$

Q7: Constructive: $Q \rightarrow (Q \rightarrow R) \rightarrow R$

Q8: Constructive: $((P \wedge Q) \wedge R) \rightarrow (S \wedge T) \rightarrow Q \wedge S$

Q1: Constructive: $\neg(A \wedge B) \rightarrow (A \rightarrow \neg B)$

$$\frac{\frac{\frac{\neg(A \wedge B)}{1} \quad \frac{\frac{\bar{A}^2 \quad \bar{B}^3}{A \wedge B} [I\Gamma]}{[\neg E]}}{\perp} 3 [\neg I]}{\neg B} 2 [\rightarrow I]$$

$$\frac{(A \rightarrow \neg B)}{\neg(A \wedge B) \rightarrow (A \rightarrow \neg B)} 1 [\rightarrow I]$$

Q2: Constructive: $(A \wedge B) \rightarrow ((A \rightarrow C) \rightarrow \neg(B \rightarrow \neg C))$

$$\frac{\frac{\frac{\frac{B \rightarrow \neg C}{3} \quad \frac{\frac{A \wedge B}{1} [A E_R]}{B} [\rightarrow E]}{\frac{A \rightarrow C}{2}} 2 \quad \frac{\frac{A \wedge B}{1} [A E_L]}{A} [\rightarrow E]}{C} C [\neg E]}

$$\frac{\frac{\frac{\perp}{3 [\neg I]}}{\neg(B \rightarrow \neg C)} 2 [\rightarrow I]}{(A \rightarrow C) \rightarrow \neg(B \rightarrow \neg C)} 1 [\rightarrow I]$$

$$\frac{(A \wedge B) \rightarrow ((A \rightarrow C) \rightarrow \neg(B \rightarrow \neg C))}{(A \wedge B) \rightarrow ((A \rightarrow C) \rightarrow \neg(B \rightarrow \neg C))} 1 [\rightarrow I]$$$$

$\frac{\frac{\frac{\frac{\neg A}{\neg A}^2 \quad \frac{A}{A}^4 [\neg E]}{\perp C [\perp E]} \quad \frac{A \vee (B \vee C)}{A \rightarrow C}^1 \quad \frac{A \rightarrow C}{C}^2}{C}^3 \quad \frac{D}{(C \rightarrow D) \rightarrow D}^3 [\rightarrow I]}{(C \rightarrow D) \rightarrow D}^4 [\rightarrow I] \quad \frac{\neg A \rightarrow (B \rightarrow A) \rightarrow (C \rightarrow D) \rightarrow D}{(A \vee B \vee C) \rightarrow \neg A \rightarrow (B \rightarrow A) \rightarrow (C \rightarrow D) \rightarrow D}^1 [\rightarrow I]$	$\frac{\frac{\frac{\frac{\neg A}{\neg A}^2 \quad \frac{A}{A}^4 [\neg E]}{\perp C [\perp E]} \quad \frac{A \vee (B \vee C)}{A \rightarrow C}^1 \quad \frac{A \rightarrow C}{C}^2}{C}^3 \quad \frac{D}{(C \rightarrow D) \rightarrow D}^3 [\rightarrow I]}{(C \rightarrow D) \rightarrow D}^4 [\rightarrow I] \quad \frac{\neg A \rightarrow (B \rightarrow A) \rightarrow (C \rightarrow D) \rightarrow D}{(A \vee B \vee C) \rightarrow \neg A \rightarrow (B \rightarrow A) \rightarrow (C \rightarrow D) \rightarrow D}^1 [\rightarrow I]$
	$1. (A \vee B \vee C)$ $2. \neg A$ $3. (B \rightarrow A)$ $4. (C \rightarrow D)$

Q4: Constructive: $(P \rightarrow Q) \rightarrow (\neg Q \rightarrow \neg P)$ NB: This is called a contrapositive

$$\frac{\frac{\frac{\neg Q}{\neg Q \rightarrow \neg P} 2 \quad \frac{\overline{P \rightarrow Q}^1 \quad \overline{P}^3}{Q}}{Q} [\neg E]}{(\neg Q \rightarrow \neg P)} 2 [\neg I]$$

Q5: Constructive: $(A \rightarrow C) \wedge (B \rightarrow \neg C) \rightarrow \neg(A \wedge B)$

$$\frac{\frac{\frac{(A \rightarrow C) \wedge (B \rightarrow \neg C)}{B \rightarrow \neg C}^1 [AE_R] \quad \frac{\frac{(A \rightarrow C) \wedge (B \rightarrow \neg C)}{A \rightarrow C}^1 [AE_L]}{C} [\neg E]}{A} [\rightarrow E]}{(\neg(A \wedge B))} [\neg I]$$

Q6: Constructive: Assuming Q , prove: $(Q \rightarrow R) \rightarrow R$

$$\frac{\overline{Q \rightarrow R}^1 \quad Q}{\frac{R}{(Q \rightarrow R) \rightarrow R}} 1[\rightarrow I]$$

Q7: Constructive: $Q \rightarrow (Q \rightarrow R) \rightarrow R$

$$\frac{\overline{Q \rightarrow R}^1 \quad \overline{Q}^0}{\frac{R}{(Q \rightarrow R) \rightarrow R}} 1[\rightarrow I]$$
$$\frac{(Q \rightarrow R) \rightarrow R}{Q \rightarrow (Q \rightarrow R) \rightarrow R} 0[\rightarrow I]$$

Q8: Constructive: $((P \wedge Q) \wedge R) \rightarrow (S \wedge T) \rightarrow Q \wedge S$

$$\frac{\overline{(P \wedge Q) \wedge R}^1}{\frac{\overline{P \wedge Q}^2 [\wedge E] \quad \overline{R}^2}{\frac{P \wedge Q}{\frac{Q}{Q \wedge S}} 1[\wedge I]}} 2[\wedge I]$$
$$\frac{Q \wedge S}{(S \wedge T) \rightarrow Q \wedge S} 2[\rightarrow I]$$
$$\frac{(S \wedge T) \rightarrow Q \wedge S}{((P \wedge Q) \wedge R) \rightarrow (S \wedge T) \rightarrow Q \wedge S} 1[\rightarrow I]$$

Logic Session - W4&5 - Mon 16th & 23rd

Q1: Constructive: $(A \vee (B \wedge A)) \rightarrow A$

Q3: Classical: $(\neg Q \rightarrow P) \rightarrow (\neg P \rightarrow Q)$

Q4: Classical: $(\neg P \rightarrow Q \wedge R) \rightarrow P \vee Q$

Q5: Classical: $\neg(P \wedge Q) \rightarrow (\neg P \rightarrow R) \rightarrow (\neg Q \rightarrow R) \rightarrow R$

Q6: Classical: $\neg A \rightarrow (B \rightarrow C \vee A) \rightarrow (C \vee \neg B)$

Q7: Classical: $\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)$

Q1: Constructive: $(A \vee (B \wedge A)) \rightarrow A$

$$\frac{\frac{\frac{\overline{A \vee (B \wedge A)}}{1} \quad \frac{\frac{\overline{A}}{2} \quad \frac{\overline{B \wedge A}}{3} [\wedge E_R]}{A \rightarrow A} [\Rightarrow I]}{(B \wedge A) \rightarrow A} [\Rightarrow I]}{A} [\vee E]$$

Q2: Constructive: $((A \wedge B) \rightarrow C) \rightarrow A \rightarrow B \rightarrow C$

$$\frac{\frac{\frac{(A \wedge B) \rightarrow C}{\frac{C}{\frac{B \rightarrow C}{\frac{A \rightarrow B \rightarrow C}{((A \wedge B) \rightarrow C) \rightarrow A \rightarrow B \rightarrow C}}}}{1 [\rightarrow I]}}{2 [\rightarrow I]}}{3 [\rightarrow I]} \quad \begin{array}{l} 1. (A \wedge B) \rightarrow C \\ 2. A \\ 3. B \end{array}$$

Q3: Classical: $(\neg Q \rightarrow P) \rightarrow (\neg P \rightarrow Q)$

$$\frac{\cancel{-Q} \rightarrow P^1}{P} \frac{\cancel{-Q}^3}{[E]} \quad \frac{\cancel{-P}^2}{[E]}$$

DNE

$$\frac{1}{\gamma Q} \begin{cases} 3[\neg I] \\ [DNE] \end{cases}$$

$$\frac{Q}{P \rightarrow Q} 2 \{\rightarrow I\}$$

$$\frac{\neg P \rightarrow Q}{(\neg Q \rightarrow P) \rightarrow (\neg P \rightarrow Q)} \text{ [C-I]}$$

LFM:

$$\frac{\frac{\frac{\frac{Q \vee \neg Q}{[LEM]} \frac{\frac{\frac{\neg Q}{Q \rightarrow Q} \frac{Q \rightarrow I}{\neg Q \rightarrow Q} \frac{\frac{I}{Q}}{3[\neg I]}}{[vE]}}{[vE]}}{2[\neg I]}}{1[\neg \rightarrow I]}}{(\neg Q \rightarrow P) \rightarrow (\neg P \rightarrow Q)}$$

Q4: Classical: $(\neg P \rightarrow Q \wedge R) \rightarrow P \vee Q$

Q5: Classical: $\neg(P \wedge Q) \rightarrow (\neg P \rightarrow R) \rightarrow (\neg Q \rightarrow R) \rightarrow R$

$$1. \neg(P \wedge Q)$$

2. $\nabla P \rightarrow R$

3. $T \Omega \rightarrow R$

$\frac{\frac{\frac{\frac{\perp}{S(\neg I)}}{\neg Q}}{V_{\neg Q}}}{\neg P \vee \neg Q} 4 [\rightarrow I]}{P \rightarrow \neg P \vee \neg Q} [LEM]$	$\frac{4. P}{S.Q}$	$\frac{\frac{\frac{\neg P}{6}}{\neg P \vee Q} 6 [\rightarrow I]}{\neg P \rightarrow \neg P \vee \neg Q} 6 [V_E] 6. P$
$\neg P \vee \neg Q$	$\frac{\neg P \rightarrow R}{R} 2$	$\frac{\neg Q \rightarrow R}{R} 3$
$\frac{\frac{R}{(\neg Q \rightarrow R) \rightarrow R} 3 [\rightarrow I]}{(\neg P \rightarrow R) \rightarrow (\neg Q \rightarrow R) \rightarrow R} 2 [\rightarrow I]$		
$\frac{(\neg P \rightarrow R) \rightarrow (\neg Q \rightarrow R) \rightarrow R}{\neg(P \wedge Q) \rightarrow (\neg P \rightarrow R) \rightarrow (\neg Q \rightarrow R) \rightarrow R} 1 [\rightarrow I]$		

Q6: Classical: $\neg A \rightarrow (B \rightarrow C \vee A) \rightarrow ((C \vee \neg B)$

$$\begin{array}{c}
 \frac{\neg A^1 \quad \overline{A}^6}{\neg A \quad [\neg E]} \\
 \frac{B \rightarrow C \vee A^2 \quad \overline{B}^3}{\neg C \quad [\neg E]} \quad \frac{\neg C^5}{C \rightarrow C^5 \quad [\neg I]} \quad \frac{\perp^1}{C^1 \quad [\perp E]} \\
 \frac{\neg C \quad [\neg E]}{\neg A \rightarrow C^6 \quad [\neg I]} \\
 \frac{\perp^1}{C^1 \quad [\perp E]} \\
 \frac{C^1 \quad [\vee I_L]}{C \vee \neg B^7 \quad [3 \rightarrow I]} \quad \frac{\overline{B}^4}{B \vee \neg B^8 \quad [4 \rightarrow I]} \\
 \frac{C \vee \neg B^7 \quad [3 \rightarrow I] \quad B \vee \neg B^8 \quad [4 \rightarrow I]}{\neg B \rightarrow C \vee \neg B^9 \quad [\vee E]} \\
 \frac{\neg B \rightarrow C \vee \neg B^9 \quad [\vee E]}{C \vee \neg B^2 \quad [2 \rightarrow I]} \\
 \frac{(B \rightarrow C \vee A) \rightarrow (C \vee \neg B)^{10}}{\neg A \rightarrow (B \rightarrow C \vee A) \rightarrow (C \vee \neg B)^{11} \quad 1 \rightarrow I} \\
 \end{array}$$

1. $\neg A$
 2. $B \rightarrow C \vee A$
 3. B
 4. $\neg B$
 5. C
 6. A

Q7: Classical: $\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)$

$$\begin{array}{c}
 \text{LEM: } \frac{\neg(A \wedge B)^1 \quad \overline{A}^3 \overline{B}^4}{\overline{A \wedge B}^2 \quad [\wedge I]} \\
 \frac{\perp^1 \quad 4[\neg I]}{\neg B^3 \quad [\vee I_R]} \quad \frac{\overline{\neg A}^2}{\neg A \vee \neg B^3 \quad [\vee I_L]} \\
 \frac{\neg B^3 \quad 3 \rightarrow I \quad \neg A \vee \neg B^3 \quad 2 \rightarrow I}{\neg A \rightarrow (\neg A \vee \neg B)^4 \quad [\vee E]} \\
 \frac{\neg A \rightarrow (\neg A \vee \neg B)^4}{(\neg A \vee \neg B)^1 \quad 1 \rightarrow I} \\
 \frac{(\neg A \vee \neg B)^1}{\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)}
 \end{array}$$

$$\begin{array}{c}
 \text{DNE} \quad \frac{\neg(\neg A \vee \neg B)^2 \quad \overline{\neg A}^3}{\overline{\neg(\neg A \vee \neg B)}^1 \quad [\neg E]} \quad \frac{\perp^5}{\neg(\neg A \vee \neg B)^2 \quad [\neg E]} \\
 \frac{\perp^3 \quad 3[\neg I]}{\overline{\neg A}^4 \quad [\text{DNE}]} \quad \frac{\perp^5 \quad [\neg I]}{\overline{\neg B}^4 \quad [\text{DNE}]} \\
 \frac{\overline{\neg A}^4 \quad [\text{DNE}]}{A^1} \quad \frac{\overline{\neg B}^4 \quad [\text{DNE}]}{B^1 \quad [\wedge I]} \\
 \frac{\perp^1}{\neg(A \wedge B)} \\
 \frac{A \wedge B \quad [\neg E]}{\perp^2 \quad 2[\neg I]} \\
 \frac{\perp^2 \quad 2[\neg I] \quad \neg(A \wedge B)^1}{\neg(\neg A \vee \neg B)^3 \quad [\text{DNE}]} \\
 \frac{\neg(\neg A \vee \neg B)^3}{(\neg A \vee \neg B)^1 \quad 1 \rightarrow I} \\
 \frac{(\neg A \vee \neg B)^1}{\neg(A \wedge B) \rightarrow (\neg A \vee \neg B)}
 \end{array}$$

Logic Session - Mon 30th Oct - W6

(Difficulty in yellow)

- Q1: Constructive: $((\neg A \vee B) \rightarrow (\neg D \vee C)) \rightarrow (\neg D \rightarrow C) \rightarrow A \rightarrow C$ - Moderate
- Q2: Classical: $(\neg A \rightarrow B \vee C) \rightarrow \neg B \rightarrow \neg C \rightarrow A$ - Moderate
- Q3: Constructive: $(A \wedge B \rightarrow \neg C) \rightarrow C \rightarrow A \rightarrow \neg B$ - Easy
- Q4: Classical: $((A \wedge \neg B) \rightarrow \neg C) \rightarrow C \rightarrow A \rightarrow B$ - Easy
- Q5: Constructive: $(A \rightarrow B \vee C) \rightarrow (B \rightarrow D) \rightarrow (C \rightarrow B \vee D) \rightarrow \neg D \rightarrow \neg A$ - Hard!
- Q6: Constructive: $(D \rightarrow A \vee C) \rightarrow (A \wedge B \rightarrow C) \rightarrow D \wedge B \rightarrow C$ - Moderate
- Q7: Constructive: $(C \rightarrow B \vee A) \rightarrow (B \rightarrow A) \rightarrow ((C \rightarrow A) \vee C)$ - Moderate
- Q8: Classical: $(C \rightarrow B) \rightarrow (B \rightarrow A) \rightarrow (\neg C \rightarrow A) \rightarrow A$ - Moderate
- Q9: Classical: $(C \rightarrow A) \rightarrow (A \rightarrow B) \rightarrow (\neg B \rightarrow C) \rightarrow B$ - Moderate
- Q10: Constructive: $A \rightarrow (B \rightarrow D) \rightarrow (\neg B \rightarrow C) \rightarrow (A \vee C)$ - Easy
- Q11: Constructive: $(D \rightarrow A \vee C) \rightarrow (A \wedge B \rightarrow C) \rightarrow (D \wedge B) \rightarrow C$ - Moderate

- Q1: Constructive: $((\neg A \vee B) \rightarrow (\neg D \vee C)) \rightarrow (\neg D \rightarrow C) \rightarrow A \rightarrow C$ - Moderate

$$\begin{array}{c}
 \frac{\overline{\neg A}^5 \quad \overline{A}^3 [\neg E]}{\frac{1}{(\neg A \vee B) \rightarrow (\neg D \vee C)} \quad \frac{\frac{\perp}{\neg \neg A}^5 [\neg \neg I]}{\frac{\neg \neg A \vee B}{[\neg E]}}^1} \\
 \frac{2}{\frac{\neg D \rightarrow C}{\frac{3}{\frac{C}{A \rightarrow C}^3 [\neg \neg I]}^2}}^2 \\
 \frac{4}{\frac{C \rightarrow C}{[\neg E]}^4 [\neg \neg I]} \\
 \frac{5}{\frac{1 [\neg \neg I]}{((\neg A \vee B) \rightarrow (\neg D \vee C)) \rightarrow (\neg D \rightarrow C) \rightarrow A \rightarrow C}}^1
 \end{array}$$

1. $(\neg A \vee B) \rightarrow (\neg D \vee C)$
2. $(\neg D \rightarrow C)$
3. A
4. C
5. $\neg A$

Q2: Classical: $(\neg A \rightarrow B \vee C) \rightarrow \neg B \rightarrow \neg C \rightarrow A$ - Moderate

$$\begin{array}{c}
 \frac{\neg A \rightarrow B \vee C}{\neg A} \text{ 1} \quad \frac{\neg A}{\neg A} \text{ 4} \\
 \hline
 \frac{}{B \vee C} \quad \frac{\neg B^2 \quad B^5_{[\neg E]}}{\perp} \text{ 5 } [\rightarrow I] \quad \frac{\neg C^3 \quad C^6_{[\neg E]}}{\perp} \text{ 6 } [\rightarrow I] \\
 \hline
 \frac{\perp}{\neg A} \text{ 4 } [\neg I] \\
 \frac{\neg A}{A} \text{ DNE} \\
 \frac{A}{\neg C \rightarrow A} \text{ 3 } [\rightarrow I] \\
 \frac{\neg C \rightarrow A}{\neg B \rightarrow \neg C \rightarrow A} \text{ 2 } [\rightarrow I] \\
 \hline
 (\neg A \rightarrow B \vee C) \rightarrow \neg B \rightarrow \neg C \rightarrow A \text{ 1 } [\rightarrow I]
 \end{array}$$

Q3: Constructive: $(A \wedge B \rightarrow \neg C) \rightarrow C \rightarrow A \rightarrow \neg B$ - Easy

$$\begin{array}{c}
 \frac{\neg C^2}{\frac{A \wedge B \rightarrow \neg C}{\neg C} \text{ 1 } [\neg E]} \\
 \frac{\perp}{\neg B} \text{ 4 } [\neg I] \\
 \frac{\neg B}{\frac{A \rightarrow \neg B}{C \rightarrow A \rightarrow \neg B} \text{ 3 } [\rightarrow I]} \\
 \hline
 (A \wedge B \rightarrow \neg C) \rightarrow C \rightarrow A \rightarrow \neg B \text{ 1 } [\rightarrow I]
 \end{array}$$

Q4: Classical: $((A \wedge \neg B) \rightarrow \neg C) \rightarrow C \rightarrow A \rightarrow B$ - Easy

$$\begin{array}{c}
 \frac{\frac{(A \wedge \neg B) \rightarrow \neg C}{\neg C} \text{ 1 } \frac{\frac{A^3 \neg B^4}{A \wedge \neg B} \text{ [\wedge I]} \text{ 2 } [\rightarrow E]}{\neg C} \text{ 2 } \\
 \hline
 \frac{\perp}{\neg \neg B} \text{ 4 } [\neg I] \\
 \frac{\neg \neg B}{B} \text{ DNE} \\
 \frac{B}{\frac{A \rightarrow B}{C \rightarrow A \rightarrow B} \text{ 3 } [\rightarrow I]} \\
 \hline
 ((A \wedge \neg B) \rightarrow \neg C) \rightarrow C \rightarrow A \rightarrow B \text{ 1 } [\rightarrow I]
 \end{array}$$

Q5: Constructive: $(A \rightarrow B \vee C) \rightarrow (B \rightarrow D) \rightarrow (C \rightarrow B \vee D) \rightarrow \neg D \rightarrow \neg A$ - Hard!

$$\begin{array}{c}
 \frac{\overline{\neg D}^4 \quad \frac{\overline{B \rightarrow D}^2 \overline{B}^9}{D}}{D} \\
 \frac{\overline{A \rightarrow B \vee C}^1 \overline{A}^5 \quad \frac{\perp}{C} \quad \frac{\overline{C}^8 \quad 8[\rightarrow I]}{C \rightarrow C} \quad \frac{\overline{B \rightarrow D}^2 \overline{B}^6 \quad [\rightarrow E]}{D \quad [G E]}}{B \rightarrow C \quad 9[\rightarrow I] \quad \frac{\perp}{\neg D}^4 \quad \frac{\perp}{D \rightarrow \perp} \quad [\rightarrow I]} \\
 \frac{C}{B \vee D} \quad \frac{B \rightarrow \perp \quad 6[\rightarrow I]}{B \rightarrow \perp} \quad \frac{\perp}{D \rightarrow \perp \quad 7[\rightarrow I]} \\
 \hline
 \frac{\frac{\perp}{\neg A} \quad 5[\neg I]}{\neg D \rightarrow \neg A} \quad 4[\rightarrow I] \\
 \frac{}{(C \rightarrow B \vee D) \rightarrow \neg D \rightarrow \neg A} \\
 \frac{(B \rightarrow D) \rightarrow (C \rightarrow B \vee D) \rightarrow \neg D \rightarrow \neg A}{2[\rightarrow I]} \\
 \frac{(A \rightarrow B \vee C) \rightarrow (B \rightarrow D) \rightarrow (C \rightarrow B \vee D) \rightarrow \neg D \rightarrow \neg A}{1[\rightarrow I]}
 \end{array}$$

Alternatively:

$$\begin{array}{c}
 \frac{\overline{A \rightarrow B \vee C}^1 \overline{A}^5}{B \vee C} \quad \frac{\overline{C \rightarrow B \vee D}^3 \overline{C}^6 \quad \frac{\overline{B \rightarrow D}^2 \quad \frac{\overline{D}^7 \quad 7[\rightarrow I]}{D \rightarrow D \quad [G E]}}{B \vee D}}{B \rightarrow D} \quad \frac{D}{C \rightarrow D \quad 6[\rightarrow I]} \\
 \frac{}{B \rightarrow D} \quad \frac{}{C \rightarrow D} \quad \frac{}{D} \\
 \hline
 \frac{D}{\neg D}^4 \quad [G E]
 \end{array}$$

$$\begin{array}{c}
 \frac{\perp}{\neg A} \quad 5[\neg I] \\
 \frac{\neg D \rightarrow \neg A}{\neg D \rightarrow \neg A} \quad 4[\rightarrow I] \\
 \frac{}{(C \rightarrow B \vee D) \rightarrow \neg D \rightarrow \neg A} \\
 \frac{(B \rightarrow D) \rightarrow (C \rightarrow B \vee D) \rightarrow \neg D \rightarrow \neg A}{2[\rightarrow I]} \\
 \frac{(A \rightarrow B \vee C) \rightarrow (B \rightarrow D) \rightarrow (C \rightarrow B \vee D) \rightarrow \neg D \rightarrow \neg A}{1[\rightarrow I]}
 \end{array}$$

(Credit to Emil for this proof)

Q6: Constructive: $(D \rightarrow A \vee C) \rightarrow (A \wedge B \rightarrow C) \rightarrow D \wedge B \rightarrow C$ - Moderate

$$\begin{array}{c}
 \frac{\frac{\frac{\frac{D \rightarrow A \vee C}{D \wedge B}^1 \quad \frac{D \wedge B}{D}^2 [E] \quad \frac{A \wedge B \rightarrow C}{A \wedge B}^3 [E_R]}{A \vee C}^4 [E]}{A \rightarrow C}^5 [\rightarrow I] \quad \frac{\overline{C}^6}{\overline{C}^7}^6 [\rightarrow I]}{C}^7 [\rightarrow I] \\
 \frac{\frac{C}{A \rightarrow C}^8 [\rightarrow I] \quad \frac{\overline{C}^9}{\overline{C}^10}^9 [\rightarrow I]}{C \rightarrow C}^{10} [\vee E] \\
 \frac{\frac{C}{D \wedge B \rightarrow C}^3 [\rightarrow I]}{D \wedge B}^2 [\rightarrow I] \\
 \frac{(A \wedge B \rightarrow C) \rightarrow D \wedge B \rightarrow C}{(D \rightarrow A \vee C) \rightarrow (A \wedge B \rightarrow C) \rightarrow D \wedge B \rightarrow C}^1 [\rightarrow I]
 \end{array}$$

Q7: Constructive: $(C \rightarrow B \vee A) \rightarrow (B \rightarrow A) \rightarrow ((C \rightarrow A) \vee C)$ - Moderate

$$\begin{array}{c}
 \frac{\frac{C \rightarrow B \vee A}{C}^1 \quad \frac{C}{C \rightarrow A}^2 [\rightarrow E]}{B \vee A}^3 \quad \frac{\overline{A}^4}{A \rightarrow A}^4 [\rightarrow I] \\
 \frac{\frac{A}{C \rightarrow A}^5 [\rightarrow I]}{C \rightarrow A}^6 [\vee I_L] \\
 \frac{(C \rightarrow A) \vee C}{(B \rightarrow A) \rightarrow ((C \rightarrow A) \vee C)}^7 [\rightarrow I] \\
 \frac{(B \rightarrow A) \rightarrow ((C \rightarrow A) \vee C)}{(C \rightarrow B \vee A) \rightarrow (B \rightarrow A) \rightarrow ((C \rightarrow A) \vee C)}^1 [\rightarrow I]
 \end{array}$$

Q8: Classical: $(C \rightarrow B) \rightarrow (B \rightarrow A) \rightarrow (\neg C \rightarrow A) \rightarrow A$ - Moderate

$$\begin{array}{c}
 \frac{\frac{\frac{C \rightarrow B}{C}^1 \quad \frac{C}{C \rightarrow A}^2 [\rightarrow E]}{B \vee \neg C}^3 [\vee I_L] \quad \frac{\frac{\neg C}{\neg C \rightarrow A}^4 [\rightarrow I]}{B \vee \neg C}^5 [\rightarrow I]}{C \vee \neg C}^6 [\text{LEM}] \\
 \frac{\frac{B \vee \neg C}{C \rightarrow B \vee \neg C}^7 [\rightarrow I] \quad \frac{\frac{\neg C \rightarrow B \vee \neg C}{\neg C \rightarrow A}^8 [\vee E]}{\neg C \rightarrow A}^9 [\rightarrow I]}{B \vee \neg C}^2 \\
 \frac{\frac{A}{(\neg C \rightarrow A) \rightarrow A}^1 [\rightarrow I]}{(\neg C \rightarrow A) \rightarrow A}^2 \\
 \frac{(\neg C \rightarrow A) \rightarrow A}{(B \rightarrow A) \rightarrow (\neg C \rightarrow A) \rightarrow A}^3 \\
 \frac{(B \rightarrow A) \rightarrow (\neg C \rightarrow A) \rightarrow A}{(C \rightarrow B) \rightarrow (B \rightarrow A) \rightarrow (\neg C \rightarrow A) \rightarrow A}^1
 \end{array}$$

Alternatively

$$\begin{array}{c}
 \frac{\frac{\frac{B \rightarrow A}{C \rightarrow B}^1 \quad \frac{C}{C \rightarrow A}^2 [\rightarrow E]}{B \vee \neg C}^3 [\vee I_R]}{C \vee \neg C}^4 [\text{LEM}] \\
 \frac{\frac{\neg C}{\neg C \rightarrow A}^5 [\rightarrow I]}{C \vee \neg C}^6 [\rightarrow I] \\
 \frac{A}{(\neg C \rightarrow A) \rightarrow A}^7 [\rightarrow I] \\
 \frac{(\neg C \rightarrow A) \rightarrow A}{(B \rightarrow A) \rightarrow (\neg C \rightarrow A) \rightarrow A}^8 \\
 \frac{(B \rightarrow A) \rightarrow (\neg C \rightarrow A) \rightarrow A}{(C \rightarrow B) \rightarrow (B \rightarrow A) \rightarrow (\neg C \rightarrow A) \rightarrow A}^1
 \end{array}$$

Q9: Classical: $(C \rightarrow A) \rightarrow (A \rightarrow B) \rightarrow (\neg B \rightarrow C) \rightarrow B$ - Moderate

$$\begin{array}{c}
 \frac{}{A \rightarrow B} 2 \\
 \frac{\frac{\frac{C \rightarrow A}{C} 1 \quad \frac{\neg B \rightarrow C}{} 3 \quad \frac{\neg B}{} 4}{C \rightarrow E} [\rightarrow E]}{A} [\rightarrow E] \\
 \frac{}{B} \\
 \frac{\frac{\frac{1}{\neg B} 4 [\neg I]}{B} [DNE]}{B} 3 [\rightarrow I] \\
 \frac{\frac{(A \rightarrow B) \rightarrow (\neg B \rightarrow C) \rightarrow B}{(A \rightarrow B) \rightarrow (\neg B \rightarrow C) \rightarrow B} 2 [\rightarrow I]}{1 [\rightarrow I]} \\
 (C \rightarrow A) \rightarrow (A \rightarrow B) \rightarrow (\neg B \rightarrow C) \rightarrow B
 \end{array}$$

Q10: Constructive: $A \rightarrow (B \rightarrow D) \rightarrow (\neg B \rightarrow C) \rightarrow (A \vee C)$ - Easy

$$\begin{array}{c}
 \frac{\neg A}{} 1 \\
 \frac{(\neg A) \vee C}{} [\vee I L] \\
 \frac{}{(A \vee C)} 3 [\rightarrow I] \\
 \frac{(\neg B \rightarrow C) \rightarrow (A \vee C)}{(\neg B \rightarrow C) \rightarrow (A \vee C)} 2 [\rightarrow I] \\
 \frac{(B \rightarrow D) \rightarrow (\neg B \rightarrow C) \rightarrow (A \vee C)}{A \rightarrow (B \rightarrow D) \rightarrow (\neg B \rightarrow C) \rightarrow (A \vee C)} 1 [\rightarrow I]
 \end{array}$$

Q11: Constructive: $(D \rightarrow A \vee C) \rightarrow (A \wedge B \rightarrow C) \rightarrow (D \wedge B) \rightarrow C$ - Moderate

$$\begin{array}{c}
 \frac{D \rightarrow A \vee C}{} 1 \\
 \frac{\frac{\frac{D \wedge B}{} 3 [\wedge E L]}{D} [\rightarrow E]}{A \vee C} \\
 \frac{\frac{\frac{A \wedge B \rightarrow C}{} 2 \frac{\frac{D \wedge B}{} 3 [\wedge E R]}{B} [\wedge I]}{A \wedge B} [\rightarrow E]}{A \rightarrow C} 4 [\rightarrow I] \\
 \frac{\frac{\frac{C}{C \rightarrow C} 7 [\neg I]}{C} [\neg E]}{C \rightarrow C} 7 [\rightarrow I] \\
 \frac{\frac{C}{(D \wedge B) \rightarrow C} 3 [\rightarrow I]}{(D \wedge B) \rightarrow C} \\
 \frac{\frac{(A \wedge B \rightarrow C) \rightarrow (D \wedge B) \rightarrow C}{(A \wedge B \rightarrow C) \rightarrow (D \wedge B) \rightarrow C} 2 [\rightarrow I]}{(D \rightarrow A \vee C) \rightarrow (A \wedge B \rightarrow C) \rightarrow (D \wedge B) \rightarrow C} 1 [\rightarrow I]
 \end{array}$$