Exercise 1

In each of the following cases, find whether the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ form a basis in $\mathbb{R}^3$, and whether it is an orthogonal basis.

1. $\vec{u} = (3, 1, -4)$, $\vec{v} = (2, 5, 6)$, and $\vec{w} = (1, 4, 8)$;
2. $\vec{u} = (2, -3, 1)$, $\vec{v} = (4, 1, 1)$, and $\vec{w} = (0, -7, 1)$;
3. $\vec{u} = (2, 0, -1)$, $\vec{v} = (0, -4, 0)$ and $\vec{w} = (1, 0, 2)$.

Exercise 1: Solution

The vectors

1. form a basis but not orthogonal;
2. do not form a basis;
3. form an orthogonal basis.

Exercise 2

In each of the following cases, verify that the vectors in B form a basis of $\mathbb{Q}^2$ and find the coordinate vector of $\vec{v}$, relative to the basis B. This is written as $[\vec{v}]_B$.

1. $\vec{v} = (1, 1)$ and $B = \{(2, -4), (3, 8)\}$;
2. $\vec{v} = (4, 0)$ and $B = \{(1, 1), (1, -1)\}$;
3. $\vec{v} = (-2, 2)$ and $B = \{(-5, 1), (3, 0)\}$;
4. $\vec{v} = (0, 0)$ and $B = \{(10, 0), (8, -3)\}$.

Exercise 2: Solution

The coordinate vector is

1. $[\vec{v}]_B = (5/28, 3/14)$;
2. $[\vec{v}]_B = (2, 2)$;
3. $[\vec{v}]_B = (2, 8/3)$;
4. $[\vec{v}]_B = (0, 0)$.

Exercise 3

In each of the following cases, find $\vec{u} \cdot \vec{v}$, $\vec{u} \cdot \vec{u}$, and $\vec{v} \cdot \vec{v}$, and whether $\vec{u}$ and $\vec{v}$ are orthogonal.

1. $\vec{u} = (3, 1, 2)$ and $\vec{v} = (2, 2, -4)$;
2. $\vec{u} = (-5, 0, 2)$ and $\vec{v} = (1, 7, 3)$;
3. $\vec{u} = (0, 0, 0)$ and $\vec{v} = (-3, 8, -1)$.

Execise 3: Solution

We have

1. $\vec{u} \cdot \vec{v} = 0$ (orthogonal), $\vec{u} \cdot \vec{u} = 14$, and $\vec{v} \cdot \vec{v} = 24$;
2. $\vec{u} \cdot \vec{v} = 1$ (not orthogonal), $\vec{u} \cdot \vec{u} = 29$, and $\vec{v} \cdot \vec{v} = 59$;
3. $\vec{u} \cdot \vec{v} = 0$ (orthogonal), $\vec{u} \cdot \vec{u} = 0$, and $\vec{v} \cdot \vec{v} = 74$.