C.K. Raju. ***Cultural Foundations of Mathematics: The Nature of Mathematical Proof and the Transmission of the Calculus from India to Europe in the 16th c. CE.*** *History of Science, Philosophy and Culture in Indian Civilization.* Vol. X, Pt 4. New Delhi: Centre for Studies in Civilizations/Delhi: Pearson Education, 2007. ISBN 81-317-0871-3. Pp. xlv + 477.

Reviewed by José Ferreirós*

This book is part of a major project undertaken by the Centre for Studies in Civilizations (New Delhi), being one of a total of ninety-six planned volumes. The author is a statistician and computer scientist by training, who has concentrated on historical matters for the last ten years or so. The book has very ambitious aims, proposing an alternative philosophy of mathematics and a deviant history of the calculus. Throughout, there is an emphasis on the need to combine history and philosophy of mathematics, especially in order to evaluate properly the history of mathematics in India, in particular the history of the calculus.

The pivotal goals of the book are (i) to oppose the Eurocentric account of the history of science in general and mathematics in particular; (ii) to avoid the usual philosophical idea of the centrality of proof for mathematical knowledge, in favour of the traditional Indian notion of *pramāṇa* [validation] encompassing empirical elements and emphasizing calculation; (iii) to analyze the thousand-year-long development of infinite series in India, starting in the fifth century; and (iv) to show 'how and why the calculus was imported into Europe' from about 1500. The result is a picture in which inputs from the Indian subcontinent and epistemology are the driving forces of the history of mathematics, as people in the European subcontinent struggle to adopt new calculation techniques from the East in spite of Western philosophico-religious biases (pp. xxxix–xli). Thus, a 'first math war' involved the *algorismus de numero indorum*, adopted for practical reasons, which forced Europeans to modify their epistemology of number and quantities. A 'second math war' revolved around infinite series and the calculus, since the background of Western epistemology created 'spurious difficulties' about infinities and infinitesimals, partially resolved with theories of real numbers. And a 'third math war' is under way, with computers forcing a new epistemological strife between math-as-calculation and math-as-proof.

The book is stimulating to read and poses interesting questions, although the author would have done better, I believe, to avoid opening so many fronts and to concentrate instead on dealing adequately with some of

* Departamento de Filosofia y Logica, Universidad de Sevilla, 41018 Sevilla, Spain. josef@us.es

them. Often the tone is more that of a scientist writing popularization, than an academic doing serious history or philosophy, and one regrets that the editorial board did not work to temper Raju's enthusiasm. The result is unbalanced and presents some serious shortcomings. The following remarks relate to part I, 'The nature of mathematical proof', which is divided into two chapters, 'Euclid and Hilbert' and 'Proof *vs. pramāṇa*', corresponding roughly to aims (i) and (ii) above. Given the nature of this journal, we shall not discuss the book's central part, 210 pages (out of a total of 477) devoted to a study of the origins of the calculus in India.

In his interest to revise traditional historiography and oppose proof-centred mathematics, Raju devotes a lot of effort to questioning the existence of Euclid and insisting that the text of the *Elements* originates at the earliest in 370 CE (with Theon) or perhaps even in the tenth century. In my opinion, this is useless and does not help advance the author's main theses. For historical purposes, what is relevant is that *Elements* represents a systematisation of a large portion of geometrical knowledge in the Greek-speaking world before the common era. ('Euclid' is simply the name of its otherwise unknown author, whose dates—it is true—are dubious; incidentally, an interesting question would be whether philologists find reason to think that the text of *Elements* was written by different authors.) Raju insists on the idea that Proclus's views represent the original philosophy of mathematics in the *Elements* (p. 25), and he overemphasizes the connections between geometrical proof, Platonism, and Christian religion (see below).

The remarks on Hilbert and his ideas about proof are astonishingly weak, suggesting that the author does not know Hilbert's work, but attributes to him assorted ideas about mathematical proof that Raju must have known during university studies and academic life. The following sentence of Hilbert will come as a big surprise to him:

> There is no talk of arbitrariness here. Mathematics is not like a game in which the problems are determined by rules invented arbitrarily—it is a conceptual system [endowed] with inner necessity, that can only be this and not any other way.[1]

Even more surprised, however, would Hilbert be if he could learn of Raju's assertions, such as when he says that Hilbert's view of mathematics 'was entirely mechanical—where Proclus sought to persuade human beings,

[1] *Natur und Mathematisches Erkennen*. David E. Rowe, ed. Basel: Birkhäuser, 1992, p. 14 (text of a lecture course delivered at Göttingen in 1919–1920). Contrast with words that are often attributed to Hilbert, such as: 'Mathematics is a game played according to certain simple rules with meaningless marks on paper', a sentence that cannot be found anywhere in his papers, books or lectures, and severely misrepresents his thought.

Hilbert sought to persuade machines!' (p. 69; this assumes, wrongly, that Hilbert's proof theory represents his whole conception of mathematics); or the following astonishing remark:

> Hilbert's notion of proof is derived from the Procluvian notion of proof [*i.e.*, that of Proclus] by eliminating all empirical, political, and human significance in the latter, and bringing it in line with later-day Christian theological beliefs. (p. 9; notice that Hilbert had little sympathy for religion.)

Absurd as these assertions are, however, I would not like readers to discard Raju's text on the basis of them.

In previous work of the 1990s, Raju dealt with the philosophy of time in connection with quantum mechanics and quasi-truth-functional logic. His chapter 2 emphasizes that bivalent logic is not 'universal', meaning not shared by all cultures or philosophical schools, and that, because formalistic philosophy of mathematics has rejected the empirical, the choice of classical logic can only be 'based on social and cultural authority' (pp. 88–90, 99). Looking for the historical background of the Western adoption of classical logic, he points to the connections between geometry, math-as-proof, and Christian religion. Hence also his concern to link Proclus with the *Elements*, and to explain the transformation of the original philosophy of the *Elements* at the hands of Christians. There is an interesting parallel between this reconstruction and the situation with traditional logical schools in India, as they were linked with religious ideas and, according to Raju, also with different understandings of time. Unfortunately, the corresponding discussion is not a good introduction to the history of logical ideas in India, making scattered comments about them in a quick, less than clear, way. It is to be expected that this topic is treated in some other volume of the series, but the reader finds no cross-reference.

Raju thus proposes to deviate from classical logic, taking into account the empirical, in search of 'the logic of the empirical world' (p. 89). Although we cannot enter into the question in any detail, let me sketch an argument that Raju does not seem to consider. As usually understood, logic is not concerned with the world, but with assertions about the world—or more generally, with representations of phenomena. Logic is not a reflection on ontological matters, but on language and representations. And when it comes to representing, it seems most natural to consider just two options: a representation can be either adequate (to some degree of accuracy) or inadequate, *tertium exclusum*. This way of grounding bivalent logic, by the way, has little to do with culturally charged conceptions of God or religion or the mind. (That, however, is not to say that there have been no historical connections between Western mathematics and religion; but in my view the topic should be pursued along a line different from Raju's insistence on the alleged theological basis of Western notions of logic and proof.)

The author asserts that abolishing the separation between mathematics and empirical science 'is fatal to the present-day (Western) notion of mathematics' (p. xxxix). This suggests that he is not well acquainted with relevant philosophical literature, such as Quine or Putnam, or relevant historical figures like Riemann, Poincaré, Weyl, or even Hilbert. The assertion is linked with his insistence throughout on considering formalism and a formal notion of proof as the quintessence of modern mathematics, but this view, perhaps natural in a computer scientist, is highly dubious and ignores too many other aspects of the discipline.

To conclude my partial summary of the relevant parts of this book, on the basis of his twofold criticism of the ideal of proof (based on its theological underpinnings and its reliance on bivalent logic), Raju concludes that mathematics is best conceived as calculation, not proof. This leads him, *e.g.*, to question the usual theories of real numbers (which he again qualifies as 'formal', in my opinion wrongly) and to argue for a flexible computational approach to numbers, more in line with the tradition of Indian mathematics. Since in Raju's view the choice of logic must depend on empirical considerations, and logic in turn determines inference and proof, he finds reason to believe that the Western separation of proof from the empirical is fundamentally wrong. Hence his preference for the traditional Indian notion of *pramāṇa*, and also his insistence throughout this work that 'deduction will forever remain more fallible than induction' (*e.g.*, p. 99). In this reviewer's opinion, and for the reasons sketched above, the argument remains far from convincing.

In conclusion, the book under review addresses many questions in an intriguing but often deficient way. The volume will best be approached as a thought-provoking program for future work that has still to be carried out in a thorough and scholarly way.

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