MODEL IDENTIFICATION OF COUPLED TWO TANKS

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Abstract: A commonly occurring control problem in the chemical process industries is the control of fluid levels in storage tanks, chemical blending and reaction vessels. In this paper we are addressing model identification of interacting coupled two tanks, which are coupled by an orifice. We have found practically the model by using MATLAB environment, which gives results almost identical to ideal model of plant.

Keywords: Coupled two tanks, orifice and model.

1. INTRODUCTION

There is a strong tendency to increase the complexity of the systems and the mathematical models. A system is defined as a collection of objects arranged in an ordered form, which is, in some sense, purpose or goal directed. A model may be defined as a representation of the essential aspects of a system, which presents knowledge of that system in a usable form (Sinha and Kuszta, 1983). A model to be useful must not be so complicated that it cannot be understood and thereby be unsuitable for predicting the behavior of the system, at the same time it must not be trivial to the extent that prediction of the behavior of the system based on the model are grossly inaccurate. On the other hand, a commonly occurring control problem in the chemical process industries is the control of fluid levels in storage tanks, chemical blending and reaction vessels. In this paper, system identification is presented for interacting coupled two tanks system.

The contents of this paper are complete plant description; model identification procedure and results obtained are given sequentially. Results of the model identification are shown and conclusions are discussed at the end.

2. THE SYSTEM: TWO INTERACTING COUPLED TANKS

A typical situation is one in which, it is required to supply fluid A to a chemical reactor at a constant rate q_A. To this end a reservoir or "holdup" tank as shown in Figure 1. may be used with the dual aim of filtering out any variations in the upstream supply flow (qi), and ensuring a temporary supply of reactant in case of process failure up stream of the "hold-up" tank. This may be achieved by a feedback control loop which maintains a constant level H of fluid in the tank by controlling the input flow rate qi, or the position of a valve in the outflow line q_A. A parallel situation may occur when the tank in Figure 1, is itself a reaction or blending vessel in which it is required that the level H (and hence the total volume of material in the vessel) is held constant. In this case the input to the tank will consist of a number of reactant flows and the out will be some desired chemical product.

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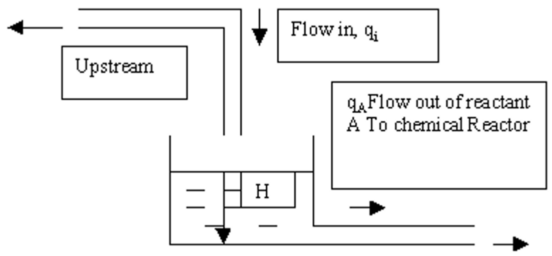

Fig.1. Basic Flow Control

The control of fluid level and flow is similarly important in power generation systems, where the control of fluid level in boilers is a central part of steam generation regulation. In the same vein but on a much larger scale, the distribution and planning of water recourses required the ideas of fluid level and flow regulation on a vast scale. Fluid level control is therefore a very basic and important problem in automatic control, while the dynamical properties of fluid level systems are relatively straightforward, they nonetheless involve challenging problems. In particular, the time scale of fluid level control is relatively long. For example, it is not unusual to encounter time constant of several hours or even days.

The basic experimental system consists of two hold-up tanks, which are coupled by an orifice. The input is supplied by variable speed pump, which supplies water to the first tank. The orifice allows this water to flow into the second tank and hence out to reservoir. The basic control problem is to control the level of fluid in one of the tank by varying the speed of the pump. The system is therefore a simple second order system with transfer function. Where see in Figure 2, $H_2(s)$ is the level in tank 2, q(s) is the flow rate of water into tank 1.

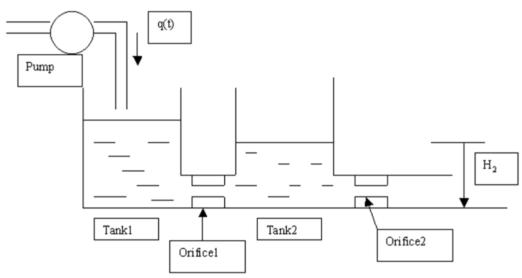

Fig. 2. Level control of second tank controlling flow in first tank

The system (D'azzo and Houpis, 1995) can be reduced to first order process by increasing the diameter of orifice 1. The aim of the coupled tanks apparatus is to determine the dynamical characteristics of a slow process system. Experiment deals with this aspect and illustrates how the time constants of fluid level system can be found by direct calculation and experimental observation. The coupled tanks apparatus consists of a transparent flexi-glass container measuring 20 cm long 10cm deep by 30cm high. A center partition is used to divide the container into two tanks. Flow between the two tanks is by means of a series of holes drilled at the base of the partition. Three holes, having diameters of 1.27cm, 0.95cm and 0.635cm are situated 3cm above the base of the tank. A smaller (bleed) hole, of 0.317cm diameter, is situated at a height of 1.5cm. These holes constitute "orifice 1" in Figure 2, the size of the orifice (and hence the degree of coupling between the tanks) is varied by plugging and unplugging the holes using the bungs provided. With all bungs removed the container can be considered as one big tank. On the other hand with the three largest holes plugged the remaining hole allows for a weak interaction between the fluid levels. Water is pumped from a reservoir into the first tank by a variable speed pump, which is driven by an electric motor. The actual flow rate is measured by flow meter, which is in the flow line between the pump and tank 1.

The depth of fluid is measured using parallel track depth sensors, which are stationed in tank1 and tank 2. This device performs as an electrical resistance, which varies with the water level. The changes in resistance are detected and provide an electrical signal, which is proportional to the height of water. The water, which flows into tank 2, is allowed to drain out via an adjustable tap, and the entire assembly is mounted in a large tray, which also forms the supply reservoir for the pump. Fully open the drain tap has a diameter of 0.70 cm. The motor drive and depth sensor signal processing is performed inside the instrumentation box such that the pump motor may be driven by a voltage between zero and ten volts applied to the pump drive socket. Corresponding the depth sensor outputs is provided as a voltage in the range 0 to -10 volts. The depth sensing system is prone to extraneous noise, and to remove this disturbance, filters are provided which may be switched in and out of circuit as required.

The pump motor drive is normally derived from an analogue or digital computer, however for demonstration purpose an interval drive is applied. This is marked on the instrumentation box front panel and should be switched in and out of circuit as required. The pump motor drive is normally derived from an analogue or digital computer, however for demonstration purpose an internal drive is supplied. This is marked on the instrumentation box front panel and should normally be switched out of circuit.

3. INSTRUMENTATION: DATA ACQUISITION

The coupled tanks instrumentation consists of:

1. A variable speed pump driven by an electric motor;
2. A direct reading flow meter;
3. Two parallel track fluid depth sensors.

The pump (Reference manual) is a three-diaphragm type mounted on a motor shaft. The motor drive is coupled to the pump drive shaft, which is fixed in the lower housing assembly. The drive shaft rotates freely in the front bearing. The drive bearing is inclined at 3° to the front bearing; this drive plate, which forms part of the diaphragm, prevents angular movement of its outer race. Subsequent rotation of the drive shaft causes drive plate to rotate. This imparts a reciprocating motion to the three diaphragms fixed to the drive plate, causing the fluid to be drawn in and then expelled from the upper housing. The valve assembly (Reference manual) ensures that the liquid is drawn in and then expelled from the other port. The pressure switch assembly senses backpressure in excess of 35 psi. The use of this pump is such that backpressure of this magnitude will never occur and so the micro switch has been removed.

The flow meter consists of a cylindrical bob weight inside a tapered tube. As fluid flows through the tube, the bob weight rises up until the pressure drop associated with flow round the bob is just balanced by the weight of the bob. The larger the flow rate, the higher the bob must rise to balance the pressure drop. Hence the weight of the bob in the tube gives a direct measurement of flow rate and the tube may be calibrated accordingly.

The depth sensor (Reference manual) consists of a pair of parallel tracks sited vertically in the tanks. A small alternating current is applied to the tracks, such that, as the water level changes, the voltage developed across the tracks varies correspondingly. This voltage is detected, filtered and amplified to from the output of the sensor.

4. MODELING: FROM THE FIRST PRINCIPLES

Consider the basic coupled tanks apparatus, as depicted in Figure 3. Taking flow balances about each tank may derive the dynamical equations of the system. For the first tank we have: Rate of change of fluid volume in tank1

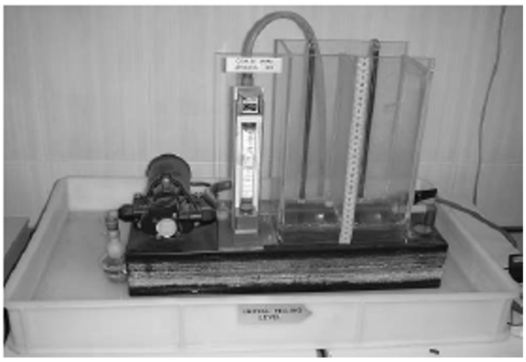

Fig. 3. Coupled two interacting tanks.

$$Q_i - Q_1 = \frac{dV_1}{dt} = A\frac{dH_1}{dt} \qquad (1)$$

Where V_1= the volume of fluid in tank 1, H_1 = height of fluid in tank1, Q_1 = flow rate of fluid from tank1 and tank2, Q_i = pump flow rate.

For the second tank: Rate of change of fluid volume in tank2

$$Q_i - Q_0 = \frac{dV_2}{dt} = A\frac{dH_2}{dt} \qquad (2)$$

Where V_2= the volume of fluid in tank2, H_2 = height of fluid in tank 1, Q_0 = flow rate of fluid out of tank2. If the inter tank holes and the drain tap are assumed to behave like orifices, then the following equations follows from the characteristic relation for orifices.

$$Q_1 = C_{d_1} a_1 \sqrt{2g(H_1 - H_2)} \qquad (3)$$

$$Q_0 = C_{d_2} a_2 \sqrt{2g(H_2 - H_3)} \qquad (4)$$

Where a_1 = cross sectional area of orifice 1, a_2 = cross sectional area of orifice 2, C_{d1}, C_{d2} = discharge coefficient ($\approx$ 0.6 for a sharp edged orifice given in reference manual), H_3 = height of drain tap, g = gravitational constant. Equation (1) to (4) describes the system dynamics in its true nonlinear form. For control systems studies it will be necessary to linearise the equations by considering small variations q_i in Q_i, q_1 in Q_1, q_0 in Q_0, h_1 in H_1 and h_2 in H_2. All measured about some mean level H_2. Assuming that in the steady state we have: $Q_i = Q_0 = Q_1$ then:

$$q_i - q_1 = A\frac{dh_1}{dt} \qquad (5)$$

$$q_1 - q_0 = A\frac{dh_2}{dt} \qquad (6)$$

$$q_1 = \frac{\delta Q_1}{\delta H_1} h_1 + \frac{\delta Q_1}{\delta H_2} h_2$$

$$= \frac{1}{2} C_{d_1} a_1 \sqrt{2g}\left[\frac{h_1 - h_2}{\sqrt{(H_1 - H_2)}}\right] \qquad (7)$$

$$q_0 = \frac{\delta Q_0}{\delta H_2} h_1 + \frac{\delta Q_1}{\delta H_2} h_2 = \frac{1}{2} C_{d_2} a_2 \sqrt{2g}\left[\frac{h_2}{\sqrt{(H_2 - H_3)}}\right] \qquad (8)$$

Substituting equation (7) and (8) into (5) and (6) gives

$$q_i - k_1 (h_1 - h_2) = A\frac{dh_1}{dt} \qquad (9)$$

$$k_1(h_1 - h_2) - k_2 h_2 = A\frac{dh_2}{dt} \qquad (10)$$

$$k_1 = \frac{C_{d1} a_1 \sqrt{2g}}{2\sqrt{(H_1 - H_2)}} \text{ and } k_2 = \frac{C_{d2} a_2 \sqrt{2g}}{2\sqrt{(H_2 - H_3)}} \qquad (11)$$

Rearranging equation sets (9) and (10) produces the following state space of the interacting coupled tanks:

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{pmatrix} \frac{-k_1}{A} & \frac{k_1}{A} \\ \frac{k_1}{A} & \frac{-(k_1+k_2)}{A} \end{pmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A} \\ 0 \end{bmatrix} \mathrm{qi} \quad (12)$$

By taking the Laplace transform of equation (12) the following transfer function is obtained:

$$\frac{h_2(s)}{q_i(s)} = \frac{\frac{1}{k_2}}{\left(\frac{A^2}{k_1 k_2}\right)s^2 + \left(\frac{A(2k_1+k_2)}{k_1 k_2}\right)s+1} \quad (13)$$

$$\frac{h_2(s)}{q_i(s)} = \frac{\frac{1}{k_2}}{(sT_1+1)+(sT_2+1)} \quad (14)$$

Where the time constants $T_1$and T_2 related to K_1, K_2 and A by:

$$T_1T_2 = \frac{A^2}{k_1k_2},\ T_1+T_2 = \frac{A(2k_1+k_2)}{k_1k_2} \quad (15)$$

The second order transfer function given by equation (13) describes the transfer function between small variations in the input flow $q_i(s)$ and small variations in the height $h_2(s)$ of fluid in tank 2. A similar, small signal transfer function can be found, which relates $q_i(s)$ to small variations in the height $h_2(s)$ of fluid in tank 2. A similar, small signal transfer function can be found, which relates $q_i(s)$ to small variations in the height $h_1(s)$ of fluid in tank1. Thus,

$$\frac{h_1(s)}{q_i(s)} = \frac{\frac{k_1+k_2}{k_1k_2}+\frac{A}{k_1k_2}s}{(sT_1+1)+(sT_2+1)} \quad (16)$$

The coupled tanks apparatus can be converted to a first order system by removing all the bungs from the inner-tank partition. This makes a1 in equation 11 relatively large, so that K_1 is approximately zero. Using this approximation in equation (13), gives the small signal transfer function for the tank with all bungs removed as:

$$\frac{h_1(s)}{q_i(s)} = \frac{\frac{1}{k_2}}{(sT+1)} \quad (17)$$

Where the time constant T is given by

$$T = \frac{2A}{k_2} \quad (18)$$

4.1 Measurement of System Dynamical Characteristics From physical Measurements.

The coupled tanks apparatus is an example of a system where the dynamical characteristics can be found from physical consideration. To be specific, the time constants, T_1 and T_2 in the coupled tanks transfer function equation (13) are determined by equation (15) the coefficients K_1, K_2 are a function of measurable parameters. Thus for a known operating point, the discharge coefficient C_{d1}, C_{d2} assume a value of 0.6 for a sharp orifice (Reference manual), and the gravitational constant g, to determine the coefficients of the transfer functions and state space model equation (12). The values for the constants a_1 = cross sectional area of orifice 1, a_2 = cross sectional area of orifice 2 are given in reference manual.

5. MODEL IDENTIFICATION OF TWO INTERACTING TANKS USING PRBS DATA

In this section system identification Using MATLAB is discussed. The identification is carried out in two parts. In the first part system input/output data is recorded which is subsequently used in the second part for fitting the model using MATLAB system identification toolbox.

5.1 Part 1: Procedure for system I/O data recordings.

The system input output data is gathered as explained below:
Following is the procedure used for the input output data collection form the system.
1. We are initially making system Power ON.
2. Then start compressor.
3. Initialize ADAM module. (a) Once the module is initialize run init_adam_rt.m
4. Then Global initialization required for noise free measurement. (a) Run global_initialization.m
5. Generate PRBS signal. As system is slow the steady state point is after 600sec. We are setting value near to the systems steady state has been reached. The range of PRBS signal in the PRBS_gen.m is 4±0.5 (reference set point value is 4). For initial 600sec we are not using the PRBS data. Once the system is settled after that we are applying 3000samples of PRBS signal. Frequency of PRBS signal is found from the actual time constant of the system. (b) Run the file.
6. Simulink model file: sys_id_level (a) In this model sysid_collectdata.m is used. (b) Do the required settings in this m-file. (c) Run the model. (d) Total 3700 samples are taken and saving this data for Ident. (e) Doing the identification using Ident tool box.

5.2 Part II: Model Identification using MATLAB

System Identification Input Output Plots:
For model identification we are using from 701 to 2200 (1500) samples for model identification and from 2201 to 3700 (1500) samples for system validation. A number of trials with various possible models available in the 'Ident' toolbox (MATLAB user Guide) were made. These included both parametric and process models. The best results were

obtained with armax 2121, which identifies the system maximum in all by 62% match. MATLAB command window is shown below for getting pole zero form of an identified model. Figure 4 is subdivided in two parts. Upper part gives relation between autocorrelation of residues obtained for different models. Proper model is that one whose response lies within the dotted lines. Lower part gives cross correlation for different models. These plots are generated by IDENT toolbox. For both the responses X-axis is samples.

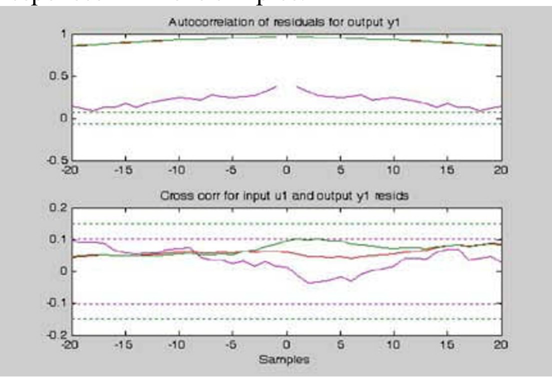

Fig. 4. Autocorrelation and cross correlation of the obtained model.

5.3 *Discussion on Results of System Identification*

Model transfer function is given as

$$\frac{0.00051976(s+2.017)}{(s+0.04551)(s+0.005906)} \tag{19}$$

We notice very important feature of the data. There is a high a high frequency (nearly periodic) noise in the data. This possibly comes by sampling the 50Hz power line pick up at 1Hz rate. Apart from following this disturbance, the model output variation is also compared the step response of the model with that of the process (obtained with a change 4 to 4.5V). The process response was normalized to 1 for comparison of the dynamic aspects. This is shown in Figure 5. This graph is automatically generated from IDENT toolbox for all types of models.

5.4 *Conclusion*

In the subsequent identification work we have ignored the zero in the transfer function of the model. The following are the reasons 1. From the physical experimentation of the process we obtain a transfer function with a gain and two poles. 2. The zero is too high in frequency compared to the region of interest. We find the step response of the identified model, which is almost identical to ideal one. So in our further work we are working with the following model whose step response is plotted in Figure 5.

$$\frac{0.00051976(2.017)}{(s+0.04551)(s+0.005906)} \tag{20}$$

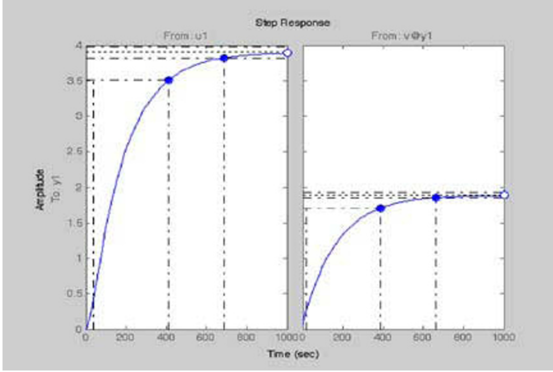

Fig. 5. Step response of the identified model

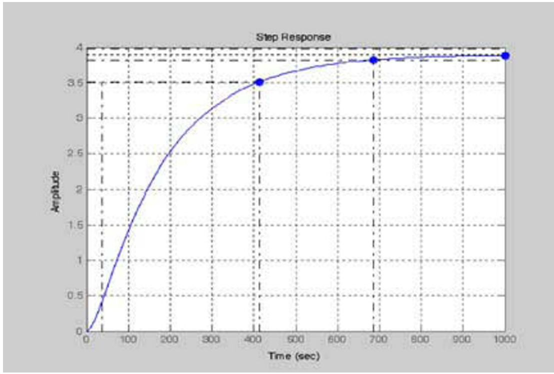

Fig. 6. Validation step response of the obtained model

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REFERENCES

D'azzo, J. J. and C. H. Houpis (1995), *Linear Control System Analysis and Design, Conventional and Modern,* Mc Graw-Hill Inc.,

Horowitz, M. (1993), *Synthesis of feedback systems*, Academic Press New York.

Mahmoud, M. S. and M. G. Singh (1981), *Large Scale System Modeling*, International Series on systems and control, Vol. 3, Pergamon Press, New York.

Sinha, N. K. and Kuszta (1983), *Modeling and identification of Dynamic Systems,* New York Van Nostrand Reinhold, Ch. 8 pp 133-164.

Reference manual by Tequipment Group of Companies Bonsall Street, LongEaton, Nottingham, NG10 2AN, England

MATLAB users guide (2001), The Mathworks Inc., MA, USA.