Cosmology 2

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Abstract

All derivations not shown may or may not be trivial but are left as exercises anyways

Table 1: **Useful Constants**

Rest Energy of Neutron	=	939.57 MeV
Rest Energy of Proton	=	938.27 MeV
Rest Energy of Electron	=	0.511 MeV
Typical Nuclear Binding Energy	≈	1 MeV
Ionization Energy of Hydrogen Atom	=	13.6 eV
$n \to \bar{\nu}_e + p + e^-$		
$n + \nu_e \rightleftharpoons p + e^-$		
$n + e^+ \rightleftharpoons p + \bar{\nu}_e$		

1 TST Problem 2022 CMB Origins Decoupling

In this problem, we will explore the origins of the Cosmic Microwave Background (CMB). First we need some prerequisites. The total radiation energy density of a blackbody at temperature T is given by,

$$\epsilon_{rad} = \alpha T^4$$

where $\alpha = 7.565 \times 10^{-16} Jm^{-3}K^{-4}$. You also know that the typical energy of a photon in a thermal distribution is

$$E = 3k_BT$$

a. At the current temperature of the universe, what is the number density of photons?

From Planck results, it is also known that the current energy density of baryonic matter is $\epsilon_{bar} = 3.9 \times 10^{-11} Jm^{-3}$. And the rest energies of a neutron and proton are 939.57 MeV and 938.27 MeV respectively.

b. What is the current number density of baryons?

c. Find the photon to baryon ratio, η.

Now, we're in a position to estimate the epoch of the origin of the CMB. One can naively estimate that the first H atoms formed when the mean energy of the photons was equal to the first ionisation energy of Hydrogen, $Q = 13.6eV$

d. Using this assumption, what was the temperature of the universe when the formation of H atoms was first possible?

However from the solution you found in 'c', you should realize that there is a long tail of photons with energies much higher than 13.6 eV. These prevent the formation of H atoms, and we have to consider that. The fraction of photons with energy exceeding I is given approximately by

$$\eta \approx exp(-I/k_BT)$$

e. What is the revised temperature now? This is approximately the temperature when 'decoupling' occurs.

f. Integration of the Planck function (which you can try yourself if you have time on your hands) shows that if $I >> k_BT$ the fraction of photons of energy greater than I is

$$\eta \approx (I/k_BT)^2 exp(-I/k_BT)$$

Find the temperature such that there is one ionizing photon per baryon.

f. How does the value of the baryon-to-photon ratio, η, affect the recombination temperature in the early universe. Express the fractional ionization X as a function of temperature. First assuming $\eta = 4 \times 10^{-10}$, then assuming $\eta = 8 \times 10^{-10}$, if we define T_{rec} as the temperature at which $X = 1/2$

g. Assuming a baryon-to-photon ratio $\eta = 6.1 \times 10^{-10}$, at what temperature T will there be one ionizing photon, with $hf > Q = 13.6$ eV, per baryon? Is the temperature you calculate greater than or less than $T_{rec} = 3760$ K?

2 TST Problem 2023

Problem inspiration - Barbara Ryden Section 8.3

2.1 What if the CMB had a different origin story?

We know the origin story of the CMB, photons didn't have enough energy to ionize Hydrogen atoms and thus 'decoupled' from electrons. But what if the Hydrogen stayed ionized? Photons would still decouple from the electrons eventually, but by a different mechanism. To understand that we need some primary concepts.

The likelihood of interactions between two particles is given by a quantity called the cross-section. The cross-section for Thomson scattering (photon-electron interaction) is $\sigma_e = 6.65 \times 10^{-29} m^2$. This means, if you have a photon within a gas of electrons, they will act like particles with a cross sectional area of σ_e

a. If the number density of such a gas of electrons is n_e, what is the mean free path, λ of the photon? [1]

b. Given the current electron number density is $n_{e,0} = 0.25 m^{-3}$, what is the scattering rate of the photon, Γ at any scale factor a? [2]

When the photon scattering rate drops below the cosmic expansion rate aka the Hubble parameter, H, then the electrons are being diluted by expansion more rapidly than the photons can interact with them. The photons then decouple from the electrons and the universe becomes transparent. We assume the decoupling occurred during the matter dominated era.

c. Find an expression for H in the matter dominated era. Take $\Omega_{m,0} = 0.31$ and $H_0 = 2.2 \times 10^{-18} s^{-1}$.

d. At what redshift, z and temperature, T do photons decouple from electrons?

3 Big Bang Nucleosynthesis

3.1 Neutron And Proton - USAAAO 2021 Second Exam Q4

In the very early universe, everything is in thermodynamic equilibrium and particles are freely created, destroyed, and converted between each other due to the high temperature. In one such process, the reaction converting between neutrons and protons happens at a very high rate. In thermal equilibrium, the relative number density of particle species is given approximately by the Boltzmann factor:

$$n_i \propto m_i^{3/2} exp\left[-\frac{m_i c^2}{k_B T}\right]$$

where i could represent any particle species.

Find an expression for the ratio of number densities of neutron to proton?

Thus, the temperature dictates the ratio of neutrons to protons when they are in thermal equilibrium.This thermal equilibrium is maintained through the interactions between baryons and neutrinos. At one point in time, the neutrinos decouple from the neutrons and protons, and the ratio of neutrons to protons is "frozen". This happens at a temperature where $k_B T \approx 0.8 MeV$ known as the freeze-out temperature

What was the ratio of number densities of neutrons to protons at this temperature?

After the freezing out, ^{4}He and other lighter elements started forming. This process is called the Big Bang Nucleosynthesis. However, $0.8MeV$ is still close to the binding energy of a nucleus, so the temperature had to fall just a little more to allow all neutrons to fuse. You can consider this lower energy to be reached 200s later (or 400s). Also, free neutrons are unstable and they'll decay into protons with a half life of about 614s. By the time all the neutrons combined with a proton, some of them decayed.

Thus find the new number density of neutron to proton after the BBN.

If $f = n_n/n_p$, find Y_p, the fraction of baryonic mass in the form of ^{4}He, in terms of f. Assume all neutrons go into forming ^{4}He.

The total luminosity of the stars in our galaxy is L = 3×10^{10} $L_\odot$. Suppose that the luminosity of our galaxy has been constant for the past 10 Gyr. How much energy has our galaxy emitted in the form of starlight during that time?

Most stars are powered by the fusion of H into 4He, with the release of 28.4 MeV for every helium nucleus formed. How many helium nuclei have been created within stars in our galaxy over the course of the past 10 Gyr, assuming that the fusion of H into 4He is the only significant energy source?

If the baryonic mass of our galaxy is $M \approx 10^{11} M_\odot$, by what amount has the helium fraction Y of our galaxy been increased over its primordial value $Y_p = 0.24$?