

Introduction of Circuit Elements

Charge + charge is The electrical quantity is a
property of the atomic particles of which all matter
consists. we know that the atom may be pictured
as composed of a nucleus with a positive charge
surrounded by negatively charged particles called
electrons. If the number of electrons in the atom
equals to the number of protons in the nucleus then
the atom is said to be in neutral, if one electron
is removed then the atom is said to be positively
charged & gains negative charge.

The unit of charge is the Coulomb (C)

It is the charge contained in 6.24×10^{18} electrons.

It is the charge contained in 6.28×10^{-18} coulombs.

law of conservation of charge.

Current - The drift & movement & flow of electrons
in a conductor in one direction is known as the
electric current. The flow of electric current
takes place by the movement of the electrons in
a conductor. since the electrons are negatively
charged so the direction of their motion is opposite
to the direction of conventional current which is
from high potential to low potential.

Electric current = Rate of flow of electrons

= Quantity of electricity passed during a given time

$$I \rightarrow \frac{Q}{T} \text{ coulombs/sec or Ampere}$$

The ampere is the current which, if maintained in two straight parallel conductors of infinite length, in each of negligible circular cross-section, and placed one meter apart in a vacuum, would produce between these conductors a force of 2×10^{-7} N/meter length.

Voltage = The energy required to bring a unit

(positive) charge from infinite distance to the point in the electrical field is called as potential.

The ratio of work to charge is given the name voltage.

Power = power is the rate of doing work and

is expressed in Joules per second. When one coulomb of electrical charge moves through a potential difference

of one volt in one second the work done is one Joule/second and in electrical engineering it is expressed as one watt and is denoted by symbol P.

$$P = V \times I \text{ watts. } P = IVR \text{ & } \frac{VR}{R}$$

Energy :- The capacity of to do work & is the total amount of work done and is expressed in Joules & in watt-seconds in electrical engineering. It is denoted by W

$$W = Pxt = I^2 R t = \frac{V^2}{R} t = VIt \text{ watt sec}$$

The electrical power utility companies measure energy in watt-hours (Wh) $1 \text{ Wh} = 3600 \text{ Joule}$

Circuit Element :- circuit element refers to the mathematical model of a physical device. Resistors, Inductors and capacitors are the circuit elements.

Types of circuit elements :-

(i) Simple circuit elements.

(ii) General circuit elements.

(i) **Simple circuit elements** :- A simple circuit element is the mathematical model of a two terminal electrical device, and it can be completely characterised by the voltage current relationship, but cannot be subdivided into another two terminal devices.

ex:- Resistors, inductors and capacitors

(ii) **General circuit elements** :- A general circuit element may be composed of more than one simple circuit elements. ex:- Transformer.

Electric Network: The inter connection of two or more circuit elements called as electric network.

Electric Circuit: The inter connection of two or more circuit element with at least one closed path is called as electric circuit.

Resistance: Resistance is the property of a material by virtue of which it opposes the flow of electrons (current) through it. Thus, resistance restricts the flow of electric current through it. The unit of resistance is ohms (Ω).

Ohm's Law: Ohm's law may be defined as the temperature remaining constant, the potential difference across the ends of the conductor is directly proportional to the current flowing through it.

$$V \propto I \Rightarrow V = IR \quad J = \frac{V}{R}$$

where R is Resistance and is a proportional constant.

$$\boxed{1\Omega = \frac{1V}{1A}}$$

Limitations of Ohm's Law:

Ohm's law cannot be applied to circuits consisting of electronic valves & transistors because these elements are not bilateral.

Ohm's law cannot be applied to circuits consisting of non-linear elements such as powdered carbon, thyrite, electric arc, etc.

Laws of Resistance

The resistance of the wire depends upon its length, area of cross-section, type of material, purity and hardness of material of which it is made of and working temperature.

Resistance of wires is

(i) directly proportional to its length, ie $R \propto l$.

(ii) Inversely proportional to its area of cross section

$$R \propto \frac{1}{a}$$

combining above two facts $R \propto \frac{l}{a}$

$$R = \rho \frac{l}{a} \quad \Omega$$

where ρ is the proportionality constant and is denoted by Specific resistance or Resistivity

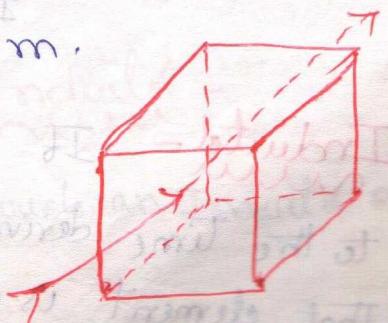
Specific resistance (ρ) = specific resistance of conductivity
is defined as the resistance between opposite faces of a unit cube of that material.

(d)

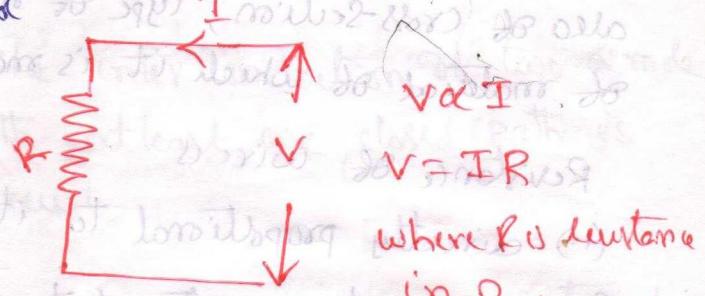
The resistance of a material of unit length (ie $l=1$)

having unit cross-sectional area ($a=1$) is defined as The Resistivity and specific resistance and is denoted by letter ρ .

' ρ ' and the unit is $\Omega\text{-m}$.



Resistor :- If the voltage across the element is direct proportional to the current through it, then that element is a Resistor.



Power absorbed by a resistor is given by

$$P = I^2 R = VI = \frac{V^2}{R} \text{ watts}$$

Inductance :- (L) Inductance is the property of a material

by virtue of which it oppose any change of current through it. Thus the current through inductance cannot change instantaneously.

Inductance is denoted by 'L' and is given by

$$I L = \frac{\Psi}{I} \quad \Psi = \text{flux linkages total}$$

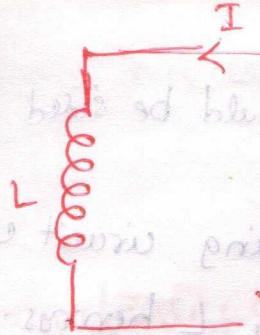
$$\Psi = N\Phi$$

$$\frac{N\Psi}{I} \text{ henrys}$$

Inductance :- If the terminal voltage is proportional

to the time derivative of current through it then that element is an inductor.

$$V \propto \frac{di}{dt} \Rightarrow V = L \frac{di}{dt}$$



$$V = L \frac{dI}{dt}$$

Characteristics of Inducto:

(i) There is no voltage across an inductor if the current through it is not changing with time i.e. an inductor acts as a short circuit to DC.

$$V = L \frac{di}{dt}$$

(ii) A finite amount of energy can be stored in an inductor even if the voltage across inductor is zero

$$E = \frac{1}{2} L I^2 \text{ Joules.}$$

(iii) An Inductor opposes the change in current through it.

(iv) An Inductor stores energy in magnetic form and is given by $E(t) = \frac{1}{2} L i^2(t)$

(v) The average power absorbed by it is zero.

Energy stored in an Inducto

When a current flows through an Inductor a magnetic field is established.

Work needs to be done in establishing the magnetic field, since the inductor opposes the setting up of the field due to its inherent property of electrical

inertia i.e. inductance

The energy supplied to the inductor would be stored in its magnetic field.

Thus an inductor is a energy storing circuit element consider an inductor of inductance L henrys.

let a current I A is forced through it. The current increases from zero to a finite value I . At any time

the emf developed across inductance is $e = L \frac{di}{dt}$

$$e = -L \frac{di}{dt} \text{ volt}$$

$$\text{Instantaneous work done / sec} = P = V I = \left(L \frac{di}{dt} \right) i dt$$

Total work done in establishing current varying from

0 to I is given by $W = \int_0^I L i di = \frac{1}{2} L I^2 \text{ joules}$

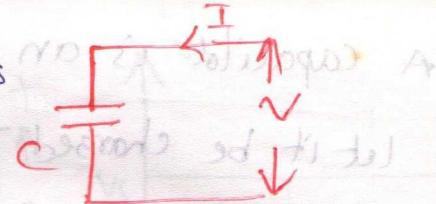
$$\text{Energy stored in the inductor } W = \frac{1}{2} L I^2 \text{ joules}$$

Capacitance - Capacitance is the property of a circuit element to store electric charge. It opposes any change of voltage across it. Thus, the voltage across the capacitor cannot change instantaneously. It is denoted by 'C' and measured in Farads (F).

Capacitor - If the terminal voltage is proportional to the integral of current through it then that element is a capacitor $V \propto \int i dt$

$$V = \frac{1}{C} \int i dt$$

Where C is capacitance in Farads



Characteristics of capacitor :-

- (i) The current through capacitor is zero, if the voltage across it is not changing with time i.e. a capacitor acts as an open circuit to D.C under steady state.

$$\therefore I = C \frac{dV}{dt}$$

- (ii) A finite amount of energy can be stored in a capacitor even if the current through the capacitor is zero.

$$\text{ie } E = \frac{1}{2} CV^2 \text{ Joules}$$

- (iii) It is impossible to change the voltage across a capacitor by a finite amount in zero time, for this it requires an infinite current through the capacitor i.e. a capacitor resists abrupt change in voltage across it.

- (iv) The capacitor stores energy in an electric field and given by $E(t) = \frac{1}{2} CV^2(t)$ Joules

- (v) The average power absorbed by it is zero.

Energy stored in a capacitor :-

The capacitor opposes the changes in voltage,

Therefore when a capacitor is to be charged, work needs to be done in overcoming this opposition, and the energy supplied by the charging source is stored in the electrostatic field of the dielectric.

A capacitor is an energy storing circuit element.

Let it be charged to a final voltage V . Let the instantaneous charging current be ' i '

$$i = C \frac{dv}{dt} \quad (\text{if } i \text{ and } v \text{ are instantaneous values})$$

Total work done in establishing voltage V across the capacitor is

$$W = \int_0^V V idt = \int_0^V [C \frac{dv}{dt}] dt$$

$$= C \int_0^V v dv$$

$$\boxed{\text{Energy stored in the capacitor, } = \frac{1}{2} C V^2 \text{ joules}}$$

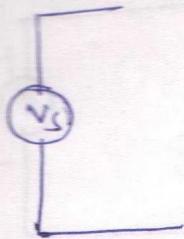
Types of Sources +

There are two types of sources.

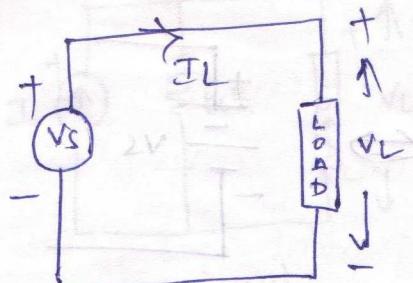
(1) voltage source (2) current source

They are classified as Ideal source and practical source

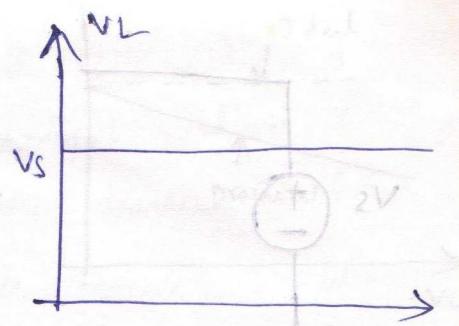
(a) Ideal voltage source - An Ideal voltage source is a device that provides a constant voltage across its terminals ($V = E$) irrespective of current flowing in it ie the internal resistance is zero or negligible



Symbol



circuit



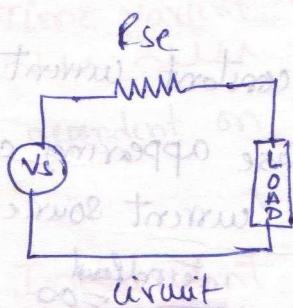
characteristics. IL

(b) practical voltage source

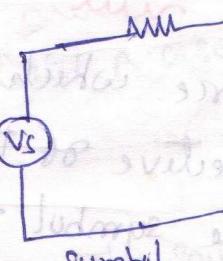
practically, every voltage source has small internal resistance shown in series with the voltage and is represented by R_{se} as shown in fig. Because of the R_{se} , voltage across the terminals decreases slightly with increase in current and is given by.

$$V_L = V_S - I_L R_{se}$$

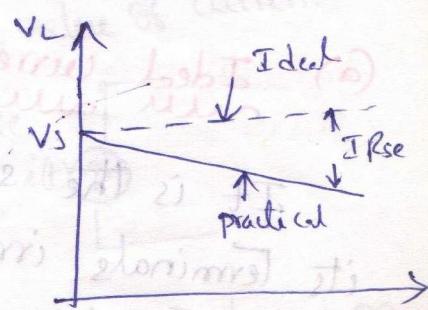
for ideal voltage source $R_{se} = 0$



circuit



symbol



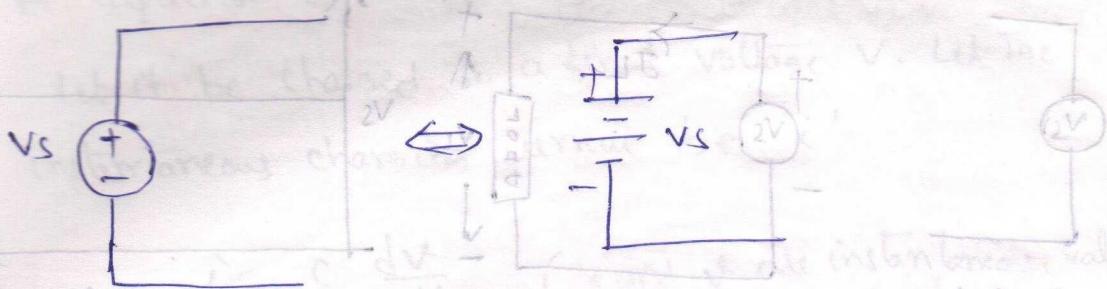
characteristics. IL

voltage sources are further classified into

- (i) Time invariant voltage source (d) DC source.
- (ii) Time variant voltage source (a) AC source

(i) Time invariant voltage source :-

The source in which voltage is not varying with time, are known as time invariant voltage source



(1) Time variant voltage source :-

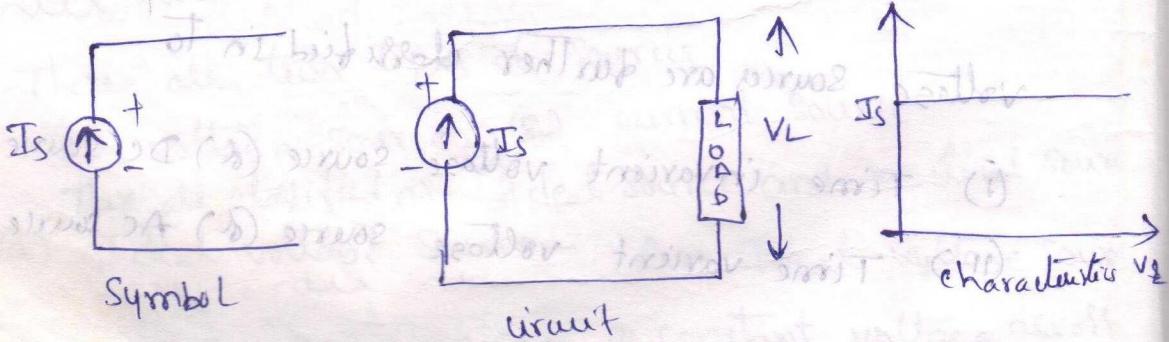
The source in which voltage is varying with respect to time is called A.C. source.



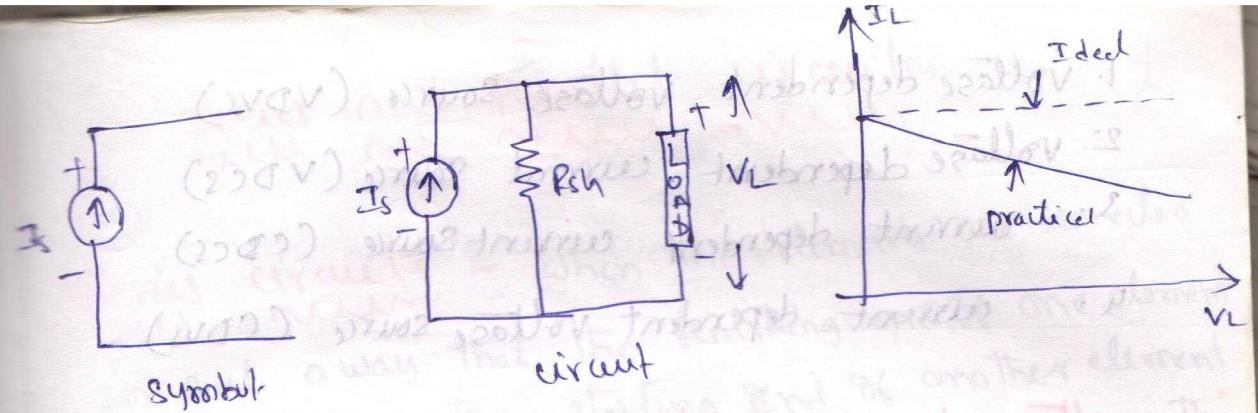
(2) Current Source :-

(a) Ideal current source :-

It is the source which gives constant current as its terminals irrespective of the voltage appearing across its terminals. The symbol for ideal current source and its characteristics are shown in fig.



(b) practical current source - practically every current source has high internal resistance shown in parallel with current source and it is represented by Rsh.

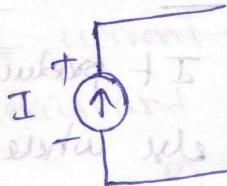


symbol

$$I_L = I_s - I_{sh}$$

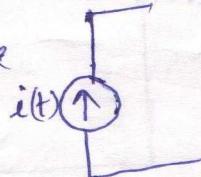
~~Time invariant current source (d) DC source~~

The source in which current is not varying with time
are known as Time invariant current source.



~~Time variant current source = The value of current source~~

~~dependent on time is called A.C. source~~



Independent sources : The sources does not depend on other voltages or currents in the network & Their values are represented by circle with a polarity of voltage & direction of current indicated inside.

Dependent Sources : Dependent sources are those whose value of source depends on voltage & current in the circuit. Such sources are indicated by diamond and they are further classified as

• Transistor
• Diode
• Inductor
• Capacitor

1. voltage dependent voltage source (VDVS)

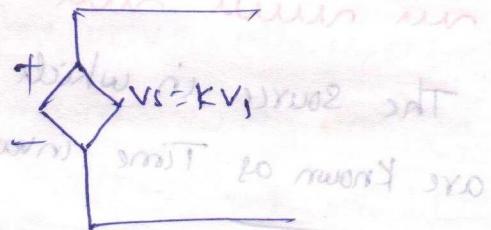
2. voltage dependent current source (VDCS)

3. current dependent current source (CDCS)

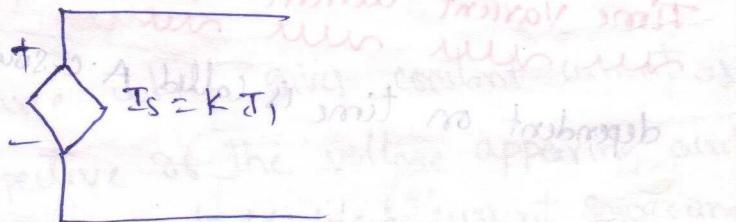
4. current dependent voltage source (CDVS)

1. voltage dependent voltage source :-

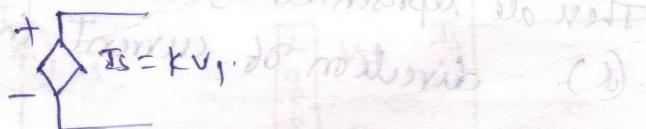
It produces a voltage as a function of voltages elsewhere in the given circuit. This is called voltage dependent voltage source.



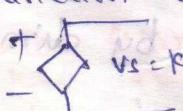
2. current dependent current source :- It produces a current as a function of currents elsewhere in the given circuit. This is called current dependent current source.



3. voltage dependent current source :- It produces a current as a function of voltage elsewhere in the given circuit



4. current dependent voltage source :- It produces a voltage as a function of current elsewhere in the given circuit

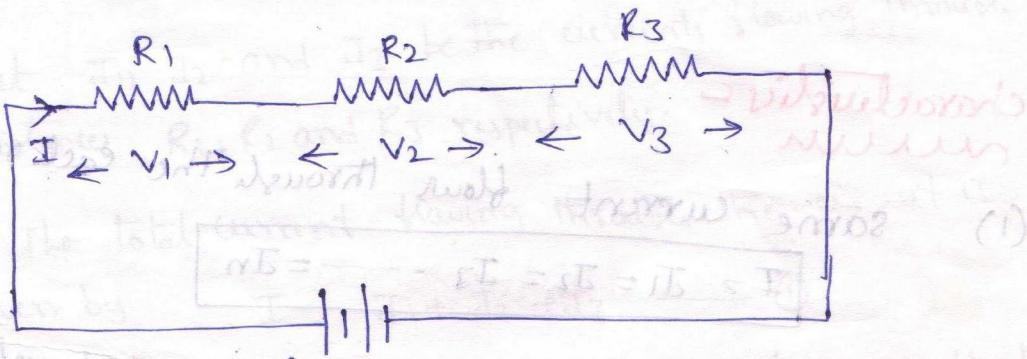


where k is the constant. V_i, I_i are the voltage and current value in the given circuit. Dependent sources are also called controlled sources.

Series and parallel circuits

Series circuits - When the elements are connected in such a way that the finishing end of one element is connected to the starting end of another element and so on and the current having only one path then that circuit is called series circuit.

Resistors in series - when the resistors are connected end to end so that they form only one path for the flow of current. Then resistors are said to be connected in series and such circuits are known as series circuits.



Voltage distribution -

Let V_1, V_2 and V_3 be the voltages across the terminals of resistances R_1, R_2 and R_3 respectively.

$$V = V_1 + V_2 + V_3 \quad \text{--- (1)}$$

Now according to Ohm's law

$$\begin{aligned} V_1 &= I_1 R_1 \\ V_2 &= I_2 R_2 \\ V_3 &= I_3 R_3 \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} - (2)$$

For overall circuit

$$V = I R_{\text{eq}} \quad \text{--- (3)}$$

We know that for series circuit the current having

$$\text{only one path} \quad I = I_1 = I_2 = I_3 \quad \text{--- (4)}$$

Substitute the equations (2), (3) and (4) in (1)

$$I R_{\text{eq}} = I R_1 + I R_2 + I R_3$$

$$R_{\text{eq}} = R_1 + R_2 + R_3$$

For n resistors connected in series, the equivalent

$$R_{\text{eq}} = R_1 + R_2 + R_3 + \dots + R_n$$

characteristics -

(1) same current flows through each resistance

$$I = I_1 = I_2 = I_3 = \dots = I_n$$

(2) supply voltage 'V' is sum of individual voltage drop across the resistors i.e. $V = V_1 + V_2 + V_3 + \dots + V_n$

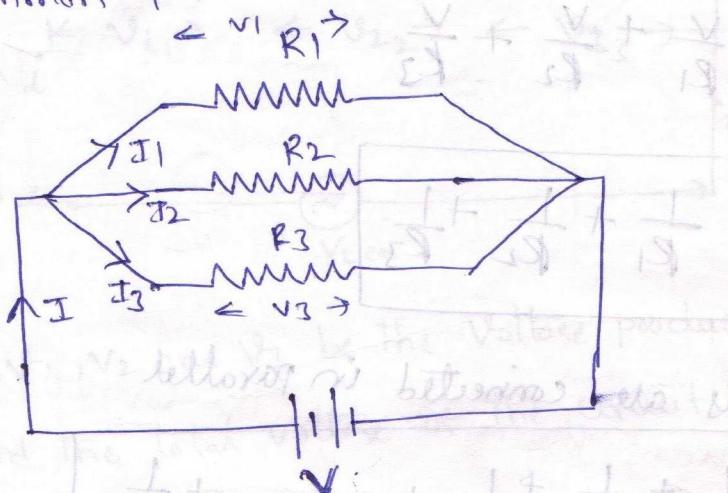
(3) The equivalent resistance $R_{\text{eq}} = R_1 + R_2 + R_3 + \dots + R_n$

(4) R_{eq} is greater than $R_1, R_{\text{eq}} > R_2, \dots$

$$R_{\text{eq}} > R_n$$

Resistors in parallel - all in one word

When number of resistors are connected in such away that one end of each of them is joined to a common point and the other ends being joined to another common point, then resistors are said to be in parallel.



Let I_1 , I_2 and I_3 be the currents flowing through the resistors R_1 , R_2 and R_3 respectively.

The total current flowing through the circuit is given by $I = I_1 + I_2 + I_3$ — ①

In parallel circuits, the potential drops across all the elements equal to the source voltage

$$V = V_1 = V_2 = V_3 \quad \text{--- ②}$$

According to Ohms law, the current flowing through each resistor is given by

$$\left. \begin{aligned} I_1 &= \frac{V_1}{R_1} \\ I_2 &= \frac{V_2}{R_2} \\ I_3 &= \frac{V_3}{R_3} \end{aligned} \right\} \quad \text{--- ③}$$

Total current flowing through equivalent resistance

$$\text{Resistors in parallel} \quad I_{\text{eq}} = \frac{V}{R_{\text{eq}}} \quad (1)$$

Substitute each (2), (3) & (4) in (1)

$$\frac{V}{R_{\text{eq}}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

If n resistors are connected in parallel

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

Characteristics :-

1. The same potential difference gets across all the resistances in parallel. $V = V_1 = V_2 = V_3 = \dots = V_n$

2. The total current I is given by

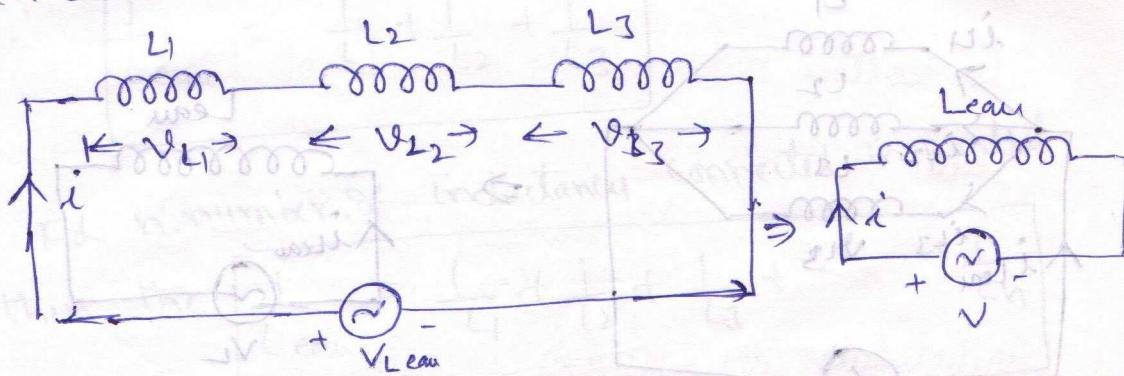
$$I = I_1 + I_2 + I_3 + \dots + I_n$$

$$3. \frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_n}$$

$$4. R_{\text{eq}} < R_1 < R_2 < R_3 < \dots < R_n$$

Inductances in series :-

Let L_1, L_2 and L_3 are the three inductances connected in series as shown in fig. below



v_1, v_2 and v_3 be the voltage produced by each inductl
and the total voltage of the circuit is given by

$$v_{\text{Leak}} = v_{L1} + v_{L2} + v_{L3} \quad \textcircled{1}$$

Voltage produced by each inductl is given by

$$v_{L1} = L_1 \frac{di}{dt}, \quad v_{L2} = L_2 \frac{di}{dt}, \quad v_{L3} = L_3 \frac{di}{dt} \quad \textcircled{2}$$

The voltage produced by equivalent inductl is given

$$\text{by } v_{\text{Leak}} = \text{Leak} \frac{di}{dt} \quad \textcircled{3}$$

Substitute $\textcircled{3}$ & $\textcircled{2}$ in $\textcircled{1}$

$$\text{Leak} \frac{di}{dt} = L_1 \frac{di}{dt} + L_2 \frac{di}{dt} + L_3 \frac{di}{dt}$$

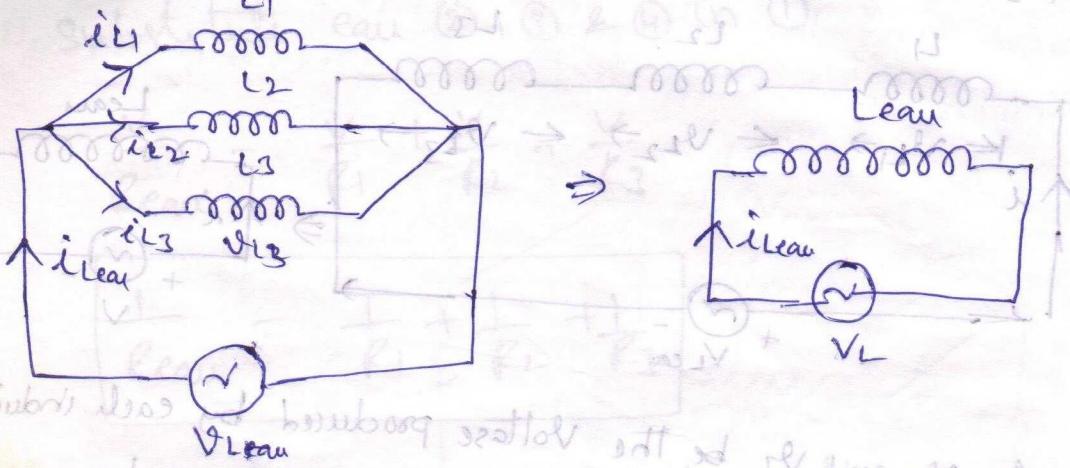
$$\therefore \boxed{\text{Leak} = L_1 + L_2 + L_3}$$

For n number of inductances connected in series

$$\boxed{\text{Leak} = L_1 + L_2 + L_3 + \dots + L_n}$$

Inductance in parallel \rightarrow reciprocal sum

Let L_1, L_2 and L_3 be the three inductances connected in parallel shown in fig.



The total current flowing through the circuit is given by

$$i_{Lean} = i_{L1} + i_{L2} + i_{L3} \quad \text{--- (1)}$$

In parallel circuits, the voltage produced across each inductor is equal to the total voltage, i.e.

$$V_{L1} = V_{L2} = V_{L3} = V_{Lean} \quad \text{--- (2)}$$

The current flowing through the each inductor is given by

$$i_{L1} = \frac{1}{L_1} \int V_{L1} dt$$

$$i_{L2} = \frac{1}{L_2} \int V_{L2} dt$$

$$i_{L3} = \frac{1}{L_3} \int V_{L3} dt$$

$$\left. \right\} \quad \text{--- (3)}$$

The current flowing in the equivalent inductance $Lean$ is given by

$$i_{Lean} = \frac{1}{Lean} \int V_L dt \quad \text{--- (4)}$$

Substitute equation ④, ③ and ② in ①

$$\frac{1}{L_{\text{equ}}} \int v_L dt = \frac{1}{L_1} \int v_L dt + \frac{1}{L_2} \int v_L dt + \frac{1}{L_3} \int v_L dt$$

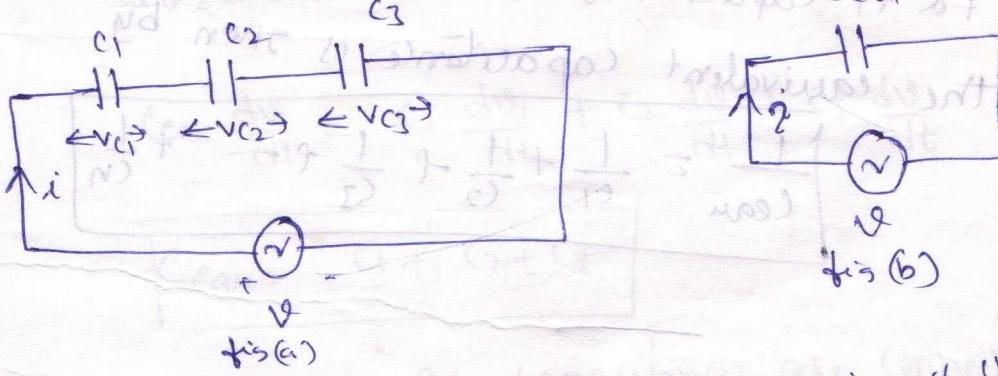
$$\therefore \frac{1}{L_{\text{equ}}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}$$

For n number of inductances connected in parallel

$$\text{then } \frac{1}{L_{\text{equ}}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \dots + \frac{1}{L_n}$$

Capacitance in series =
reciprocal sum of all

Let C_1, C_2 and C_3 be the capacitances connected in series shown in fig (a) and its equivalent capacitance shown in fig (b).



The total voltage across the series circuit is given by

by the sum of voltage produced by individual capacitance

$$V_C = V_{C1} + V_{C2} + V_{C3} \quad \rightarrow ①$$

According to series circuit concept the current having only one path i.e. $i = i_1 = i_2 = i_3 \dots \rightarrow ②$

We know that the voltage produced by individual capacitance are given by

$$V_{c1} = \frac{1}{C_1} \int i_1 dt$$

$$V_{c2} = \frac{1}{C_2} \int i_2 dt$$

$$V_{c3} = \frac{1}{C_3} \int i_3 dt$$

The voltage produced by equivalent capacitance

C_{equ} is given by

$$V_{\text{equ}} = \frac{1}{C_{\text{equ}}} \int i dt \quad \text{--- (4)}$$

Substitute (2) & (3) in (4)

$$\frac{1}{C_{\text{equ}}} \int i dt = \frac{1}{C_1} \int i_1 dt + \frac{1}{C_2} \int i_2 dt + \frac{1}{C_3} \int i_3 dt$$

$$\frac{1}{C_{\text{equ}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

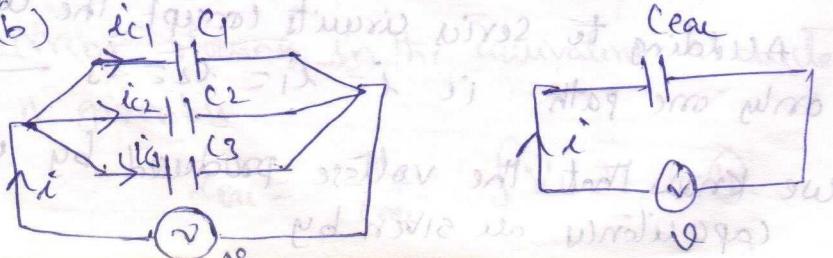
If n capacitors are connected in series then

the equivalent capacitance is given by

$$\frac{1}{C_{\text{equ}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots + \frac{1}{C_n}$$

Capacitance in parallel

Let C_1, C_2 and C_3 be the three capacitors connected in parallel shown in fig (a) its equivalent circuit shown in fig (b)



The total current flowing through the parallel circuit is the sum of individual currents flowing through the each capacity $i = i_1 + i_2 + i_3$ — ①

In parallel circuit the total voltage is equal to the sum of individual voltage produced by each capacity

$$V = V_1 = V_2 = V_3 \quad \text{— ②}$$

The current flowing through the individual capacity are given by

$$\begin{aligned} i_1 &= C_1 \frac{dV}{dt} \\ i_2 &= C_2 \frac{dV}{dt} \\ i_3 &= C_3 \frac{dV}{dt} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \quad \text{— ③}$$

The total current is given by $i = C_{\text{eq}} \frac{dV}{dt}$ — ④

Substitute equation ①, ③ & ④ in ④

$$C_{\text{eq}} \frac{dV}{dt} = C_1 \frac{dV}{dt} + C_2 \frac{dV}{dt} + C_3 \frac{dV}{dt}$$

$$C_{\text{eq}} = C_1 + C_2 + C_3$$

For n number of capacitors connected in parallel series their equivalent capacitance is given by

$$C_{\text{eq}} = C_1 + C_2 + C_3 + \dots + C_n$$