Assignment - 2

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Problem:

Solving BVP using Block Tridiagonal Method

$$Y''' + 4Y'' + Y' - 6Y = 1$$
$$Y(0) = 0$$
$$Y'(0) = 0$$
$$Y'(1) = 1$$

Code: (Python Script)

```
import sys
import numpy as np
import matplotlib.pyplot as plt

def block_diagonal(a, b, c, d):
    n = len(d)
    b_ = np.zeros(a.shape)
    c_ = np.zeros(a.shape)
    d_ = np.zeros((n,2,1))

    c_[0] = np.linalg.inv(b[0]).dot(c[0])
    d_[0] = np.linalg.inv(b[0]).dot(d[0])

    for i in range(1,n):
        b_[i] = b[i] - a[i].dot(c_[i-1])
        c_[i] = np.linalg.inv(b_[i]).dot(c[i])
        d_[i] = np.linalg.inv(b_[i]).dot((d[i] - a[i].dot(d_[i-1])))

    for i in range(n-2, -1, -1):
        d_[i] = d_[i] - c_[i].dot(d_[i+1])
```

```
    return d_

def bvp(l, r, h):
    n = int ((r-l) / (1.0*h))
    a = np.zeros((n-1,2,2))
    b = np.zeros((n-1,2,2))
    c = np.zeros((n-1,2,2))
    d = np.zeros((n-1,2,1))

    for i in range(n-1):
        # As i starts from 0, we define x = l + (i+1)*h
        x = l+(i+1)*h
        a[i] = np.array(([[1.0 / h**2 - 2/h, 0.0],[-h/2, -1.0]]))
        #b[i] = np.array(([[- 2.0 / h**2 + 1.0, 1/h**2 + 2.0/h],[-h/2,
1.0]]))
        b[i] = np.array(([[- 2.0 / h**2 + 1.0, -6],[-h/2, 1.0]]))
        c[i] = np.array(([[1.0 / h**2 + 2/h, 0],[0, 0]]))
        d[i] = np.array(([[1],[0]]))

    d[0] = d[0] - a[0].dot(np.array(([[0.0], [0.0]])))
    #d[n-2] = d[n-2] -  c[n-2].dot(np.array(([[a2], [a4]])))
    d[n-2] = d[n-2] - np.array([[1/h**2 + 2/h], [0.0]])
    w = block_diagonal(a, b, c, d)
    a1 = 0.0
    a3 = 0.0
    a4 = d[n-2][1] + h/2 + h/2*d[n-2][0]
    a2 = 1.0

    w = np.vstack(([np.array(([[a1], [a3]]))], w))
    return np.vstack((w, [np.array(([[a2], a4]))]))

def main():

    a = 0
    b = 1

    h1 = 0.1
    h2 = 0.05
    h3 = 0.02
    '''
    x1 = np.linspace(a,b,(b - a)/h1+1)
    n = (b - a)/h1 + 1
    #x1 = x1[0:(b - a)/h1]
```

```
    w1 = bvp(a, b, h1)
    #print w1
    z1 = w1[[range(w1.shape[0])],[0],[0]]
    y1 = w1[[range(w1.shape[0])],[1],[0]]
    y1 = y1[0]
    z1 = z1[0]
    #print y1, n
    y1[n-1] = h1*(z1[n-2]+1) / 2 + y1[n-2]
    print y1

    x2 = np.linspace(a,b,(b - a)/h2 +1)
    #x2 = x2[0:(b - a)/h2]
    w2 = bvp(a, b, h2)
    y2 = w2[[range(w2.shape[0])],[1],[0]]
    y2 = y2[0]


    x3 = np.linspace(a,b,(b - a)/h3 +1)
    #x3 = x3[0:(b - a)/h3]
    w3 = bvp(a, b, h3)
    y3 = w3[[range(w3.shape[0])],[1],[0]]
    y3 = y3[0]
    #print len(x3), len(y3)

    plt.ylabel('y')
    plt.xlabel('x')

    p1, p2, p3 = plt.plot(x3, np.interp(x3, x1, y1),
                          x3, np.interp(x3, x2, y2), x3, y3)

    plt.legend([p1, (p1, p2), (p1,p2,p3)], ["h = 0.1", "h =0.05", "h =
0.005"], loc =4)

    plt.show()
    '''

    h = [0.5, 0.1, 0.05, 0.01]
    for step in h:
      n = int((b - a)/step) + 1
      file = open("Resut_h_" + str(step) + ".txt", 'w')
      x = np.linspace(a,b,(b - a)/step+1)
      w = bvp(a, b, step)
      y= w[[range(w.shape[0])],[1],[0]]
```

```
	z = w[[range(w.shape[0])],[0],[0]]
	y = y[0]
	z = z[0]
	y[n-1] = step*(z[n-2]+1) / 2 + y[n-2]
	file.write("Value of y(x) with respect to x:\n\n\tx\ty(x)\n\n")
	for i in xrange(n):
		file.write("\t" + str(x[i]) + "\t" + str(y[i]) + "\n")
		plt.plot(x, y,  label="h={0}".format(step))
		plt.xlabel('x')
		plt.ylabel('y(x)')
		plt.legend(bbox_to_anchor=(2,2),
bbox_transform=plt.gcf().transFigure)
		plt.legend(loc="upper left", bbox_to_anchor=[0, 1],
		ncol=2, shadow=True, title="Legend", fancybox=True)
		plt.savefig('Plot_h_' + str(step) + '.png')
		plt.clf()
		#file.close()


	plt.legend(bbox_to_anchor=(2,2),
bbox_transform=plt.gcf().transFigure)
	plt.legend(loc="upper left", bbox_to_anchor=[0, 1],
		ncol=2, shadow=True, title="Legend", fancybox=True)
    #plt.legend(bbox_to_anchor=(1.05, 1), loc=2,
bbox_transform=plt.gcf().transFigure)
	plt.savefig('Plot_h.png')
	plt.show()

main()
```

Output:

<u>For h = 0.5:</u>

Value of y(x) with respect to x:

x	y(x)
0.0	0.0
0.5	0.205882352941
1.0	0.661764705882

For h = 0.1:

Value of y(x) with respect to x:

x	y(x)
0.0	0.0
0.1	0.0122488896478
0.2	0.0452578101855
0.3	0.0928087770617
0.4	0.150836683241
0.5	0.216808096722
0.6	0.289277865241
0.7	0.367574503952
0.8	0.45157871506
0.9	0.54156915572
1.0	0.638116698752

For h = 0.05:

Value of y(x) with respect to x:

x	y(x)
0.0	0.0
0.05	0.00330831896801
0.1	0.0126821904435
0.15	0.0271165701181
0.2	0.0457857643067
0.25	0.0680158661792
0.3	0.0932613473902
0.35	0.121085198289
0.4	0.151142097412
0.45	0.183164165981
0.5	0.216948927438
0.55	0.252349147174
0.6	0.289264274801
0.65	0.327633251806

```
0.7   0.367428482056
0.75  0.408650792302
0.8   0.451325235228
0.85  0.495497609351
0.9   0.541231588662
0.95  0.588606370856
1.0   0.637714766571
```

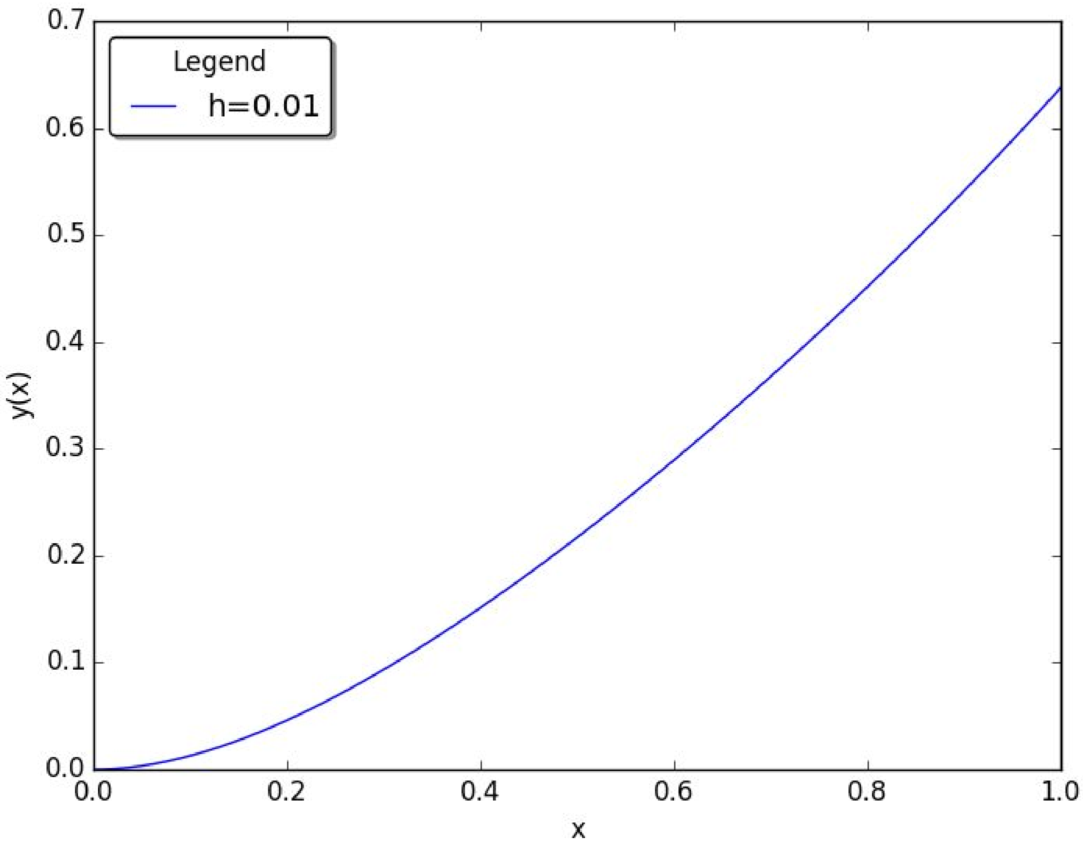
For h = 0.01:

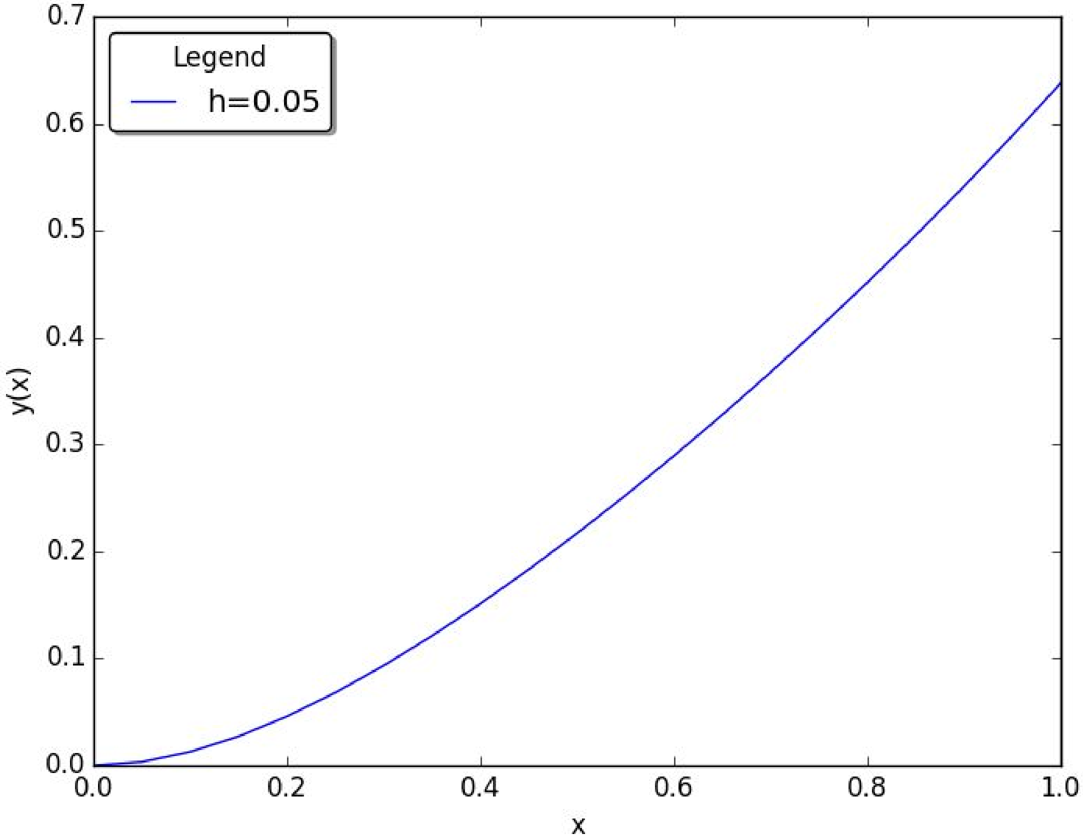
Value of y(x) with respect to x:

x	y(x)
0.0	0.0
0.01	0.000141678206357
0.02	0.000561633540095
0.03	0.00124989446132
0.04	0.0021968574259
0.05	0.00339327494583
0.06	0.00483024402741
0.07	0.00649919497572
0.08	0.00839188055405
0.09	0.0105003654874
0.1	0.0128170162992
0.11	0.015334491472
0.12	0.0180457319198
0.13	0.0209439517659
0.14	0.0240226294134
0.15	0.0272754989014
0.16	0.0306965415375
0.17	0.0342799777982
0.18	0.0380202594883
0.19	0.0419120621523
0.2	0.0459502777292
.	
.	
.	
.	
0.9	0.541124883994
0.91	0.550463496659

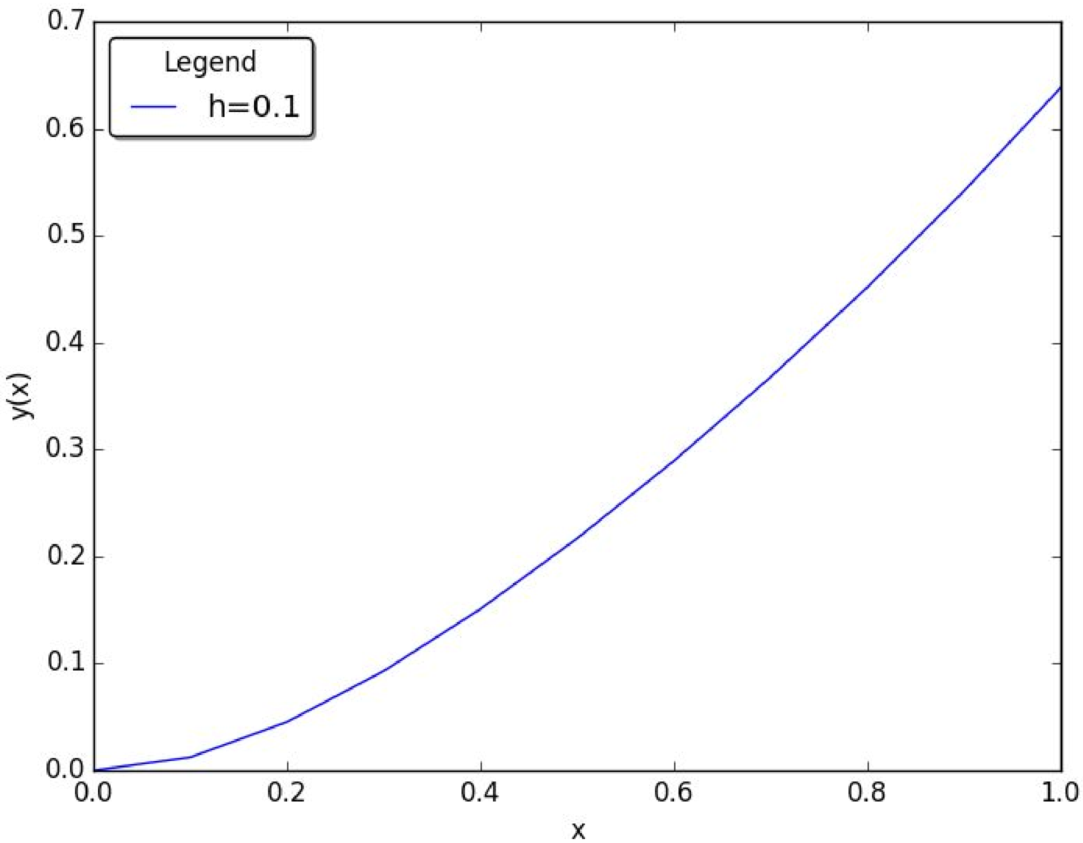
0.92	0.559868460963
0.93	0.5693404989
0.94	0.578880353095
0.95	0.588488786321
0.96	0.59816658103
0.97	0.607914538916
0.98	0.617733480489
0.99	0.627624244674
1.0	0.637587688423

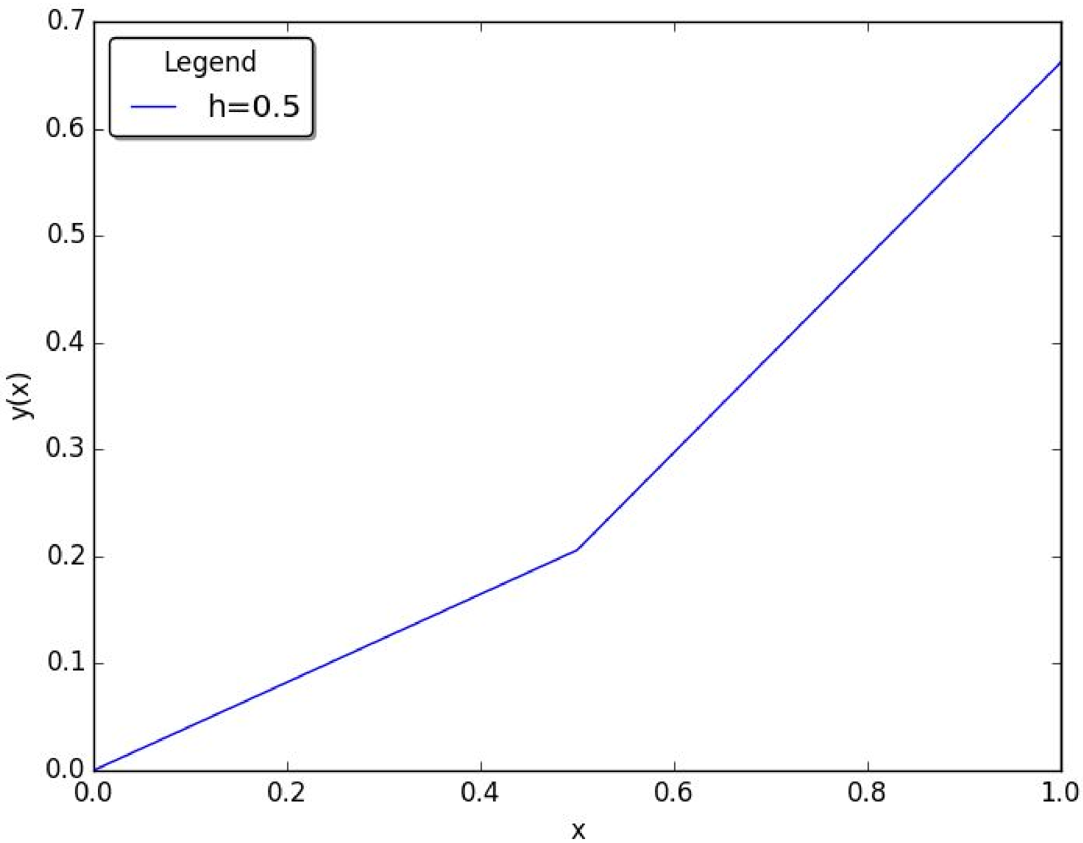
Plots:





Legend
h=0.05
y(x)
x
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.0
0.2
0.4
0.6
0.8
1.0


Legend
h=0.1
y(x)
x
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.0
0.2
0.4
0.6
0.8
1.0



Legend
h=0.5
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
y(x)
0.0
0.2
0.4
0.6
0.8
1.0
x