SIMPLEX METHOD (Case NO Alt. Soln)

General form : Max $c_1x_1 + c_2x_2 + \cdots + c_nx_n$

s.to $a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le b_m$

Assume: $\begin{cases} x_j \ge 0 \;, j = 1, 2, \cdots, n \\ b_m \ge 0 \;, \; m \le n \end{cases}$

Using slack variables convert this to the following standard form:

Max $c_1x_1 + c_2x_2 + \cdots + c_nx_n + 0\,x_{n+1} + 0\,x_{n+2} + \cdots + 0\,x_{n+m}$

s.to

$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + x_{n+1} = b_1$

$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + x_{n+2} = b_2$

$\vdots$

$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + x_{n+m} = b_m$

Notations :

Basis Vector $= X_B = (X_{B1}, X_{B2}, \cdots, X_{Bm})^T$

Initial basis vector $= (X_{n+1}, X_{n+2}, \cdots, X_{n+m})^T$

$= (b_1, b_2, \cdots, b_m)^T$

$$X_j = \begin{pmatrix} a_{1j} \\ a_{2j} \\ \vdots \\ a_{mj} \end{pmatrix}, \quad j = 1, 2, \cdots, n+m$$

$C_B = (C_{B1}, C_{B2}, \cdots, C_{Bm})^T$,

C_{Bi} = Cost coffecient of X_{Bi} in the objective function.

Objective Value $= Z = C_B^T X_B$

$$\Delta_j = C_B^T X_j - C_j$$

Simplex Table

$i\downarrow$ $j\rightarrow$			C_1	C_2	...	C_r	...	C_n	...	C_{n+m}	Min Ratio
	C_B	X_B	X_1	X_2	...	X_r	...	X_n	...	X_{n+m}	
1	C_{B1}	X_{B1} : b_1	a_{11}	a_{12}	..	a_{1r}	..	a_{1n}	....	$a_{1\,n+m}$	
2	C_{B2}	X_{B2} : b_2	a_{21}	a_{22}	..	a_{2r}	..	a_{2n}	...	$a_{2\,n+m}$	
⋮		⋮ : ⋮	⋮								
s	C_{Bs}	X_{Bs} : b_s	a_{s1}	a_{s2}	..	$\boxed{a_{sr}}$		a_{sn}	...	$a_{s\,n+m}$	
⋮		⋮ : ⋮	⋮			⋮					
m	C_{Bm}	X_{Bm} : b_m	a_{m1}	a_{m2}	..	a_{mr}	..	a_{mn}	..	$a_{m\,n+m}$	
		$Z = C_B^T X_B$	Δ_1	...	Δ_2	...	$\Delta_r\uparrow$	..	Δ_n	..	$\Delta_{m\,n+1}$

How to change the BASIS and move from feasibility to optimality ? :

Find $\Delta_j = C_B^T X_j - C_j$

Let $\Delta_r = \underset{j}{\text{Min}} \{\Delta_j \mid \Delta_j < 0\}$

$\Rightarrow$ r^{th} column will enter into basis

Minimum Ratio :

$$\frac{X_{Bs}}{a_{sr}} = \underset{i}{\text{Min}} \left\{ \frac{X_{Bi}}{a_{ir}} \;\middle|\; a_{ir} > 0 \right\}$$

$$= \underset{i}{\text{Min}} \left\{ \frac{b_i}{a_{ir}} \;\middle|\; a_{ir} > 0 \right\}$$

$\Rightarrow$ s^{th} vector will leave basis.

i.e. X_{Bs} is replaced by X_r

Pivot element is a_{sr}

Transform the table:

$$a_{ij}^{New} = \begin{cases} \dfrac{a_{ij}}{a_{sr}} & \text{for } i = s,\ j = 1,2,\ldots,m+n \\ a_{ij} - \left(\dfrac{a_{sj}}{a_{sr}}\right) a_{ir} & \text{for } i \neq s \\ & j = 1,2,\ldots m+n \end{cases}$$

$$b_i^{New} = \begin{cases} \dfrac{b_i}{a_{sr}} & \text{for } i = s \\ b_i - \left(\dfrac{b_s}{a_{sr}}\right) a_{ir} & \text{for } i \neq s \end{cases}$$

- STOP if all Δ_j in tranformed table are ≥ 0.
  Optimal soln is X_B
  Optimal value is $C_B^T X_B$.

Ex Max $-x_1+3x_2-2x_3$

s.to $3x_1-x_2+3x_3 \le 7$

$-2x_1+4x_2 \le 12$

$-4x_1+3x_2 \le 10$, $x_j \ge 0$.

Standard From : Slack variables $x_4, x_5, x_6 \ge 0$.

Maximize $-x_1+3x_2-2x_3$

s.t $3x_1-x_2+3x_3+x_4 = 7$

$-2x_1+4x_2+x_5 = 12$

$-4x_1+3x_2+x_6 = 10$

$x_j \ge 0$.

Initial solution : $x_B = (x_4, x_5, x_6) = (7, 12, 10)$

Initial Basis : $(X_4 \quad X_5 \quad X_6) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

	$C_j \rightarrow$		-1	3	-2	0	0	0	
C_B	X_B	b	X_1	X_2	X_3	X_4	X_5	X_6	Min Ratio
0	X_4	7	3	-1	3	1	0	0	-
0	X_5	12	-2	[4]	0	0	1	0	12/4 ✓
0	X_6	10	-4	3	0	0	0	1	10/3
	$Z = 0$		1	-3 ↑	2	0	0	0	
0	X_4	10	[5/2]	0	3	1	1/4	0	10 × 2/5
3	X_2	3	-1/2	1	0	0	1/4	0	-
0	X_6	1	-5/2	0	0	0	-3/4	1	-
		9	-1/2 ↑	0	0	0	3/4	0	
-1	X_1	4	1	0	6/5	2/5	1/10	0	
3	X_2	5	0	1	3/5	1/5	6/20	0	
0	X_6	11	0	0	3	1	-1/4	1	
		11	0	0	13/5	1/5	16/20	0	

All $Z_j \geq 0$, optimal soln : $x_1 = 4$, $x_2 = 5$, $x_3 = 0$ Basic
$x_4 = 0$, $x_5 = 0$, $x_6 = 11$
Non basic.