1. **(2 marks)** Find out whether the following set is orthogonal or not: $\{\frac{1}{\sqrt{2}}(1,1)^T, \frac{1}{\sqrt{2}}(1,-1)^T\}$

   Solution:

   $U^T V = 0$, So orthogonal

2. **(8 marks)** A linear system is described by the following equations.

$$x1 - 3x2 + 4x3 = 3$$

$$2x1 - 5x2 + 6x3 = 6$$

$$-3x1 + 3x2 + 4x3 = 6$$

Does this system have any unique solution? Explain.

Solve the above linear system by Gaussian elimination method. Show your work (You have to show the row multipliers).

Solution:

A = $\begin{pmatrix} 1 & -3 & 4 \\ 2 & -5 & 6 \\ -3 & 3 & 4 \end{pmatrix}$, det(A) =4≠ 0. So, A is non-singular. This system has unique solution.

Augmented matrix = $\left(\begin{array}{ccc|c} 1 & -3 & 4 & 3 \\ 2 & -5 & 6 & 6 \\ -3 & 3 & 4 & 6 \end{array}\right) = \left(\begin{array}{ccc|c} 1 & -3 & 4 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & -6 & 16 & 15 \end{array}\right)$ [R2=R2-2R1, R3=R3+3R1] = $\left(\begin{array}{ccc|c} 1 & -3 & 4 & 3 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 4 & 15 \end{array}\right)$ [R3=R3+6R2] = U

Back Substitution: 4 x3 = 15 => x3=15/4, x2 = 15/2, x1 = 45/2-12=21/2

x1 = 21/2, x2 = 15/2, x3 = 15/4

1. **(2 marks)** Find out whether the following set is orthogonal or not: $\{(1,0)^T, (0,1)^T\}$

Solution:

$U^T V = 0$, So orthogonal

2. **(8 marks)** A linear system is described by the following equations.

x1 - 3x2 +4x3 = 3

2x1 - 5x2 +6x3 = 6

-3x1 + 3x2 +4x3 = 6

Solve the above linear system by LU decomposition method. Show your work (You have to show the Frobenius matrices).

F(1) = $\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix}$ A(2) = F(1)*A = $\begin{pmatrix} 1 & -3 & 4 \\ 0 & 1 & -2 \\ 0 & -6 & 16 \end{pmatrix}$, F(2) = $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 6 & 1 \end{pmatrix}$

b) L = $\begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -6 & 1 \end{pmatrix}$

c) A(3) = F(2)*A(2) = $\begin{pmatrix} 1 & -3 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 4 \end{pmatrix}$ = U,

Ly=b $\quad \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -3 & -6 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 6 \end{pmatrix}$ => $y_1 = 3, y_2 = 0, y_3 = 15$

Ux=y $\quad \begin{pmatrix} 1 & -3 & 4 \\ 0 & 1 & -2 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 15 \end{pmatrix}$ => $x_3 = 15/4, x_2 = 15/2, x_1 = 21/2$