Problem-2

1 $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 4 & 16 \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

$u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $u_2 = \begin{pmatrix} 0 \\ 4 \\ -1 \\ 1 \end{pmatrix}$ $u_3 = \begin{pmatrix} 0 \\ 16 \\ 1 \\ 1 \end{pmatrix}$

2 $P_1 = u_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$ $|P_1| = 2$

$$q_1 = \frac{P_1}{|P_1|} = \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

$$P_2 = u_2 - (u_2^T q_1)\, q_1$$

$$(u_2^T q_1) = \frac{1}{2}\begin{pmatrix} 0 & 4 & -1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = 2$$

$$P_2 = \begin{pmatrix} 0 \\ 4 \\ -1 \\ 1 \end{pmatrix} - 2 \cdot \frac{1}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix}$$

$$|P_2| = \sqrt{1+9+4} = \sqrt{14}$$

$$q_2 = \frac{P_2}{|P_2|} = \frac{1}{\sqrt{14}}\begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix}$$

$$P_3 = u_3 - \left((u_3^T q_1)\, q_1 + (u_3^T q_2)\, q_2 \right)$$

$$(u_3^T q_1) = \frac{1}{2}\begin{pmatrix} 0 & 16 & 1 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} = \frac{18}{2} = 9$$

$$u_3^T q_2 = \frac{1}{\sqrt{14}}\begin{pmatrix} 0 & 16 & 1 & 1 \end{pmatrix}\begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix} = \frac{46}{\sqrt{14}}$$

$$(u_3^T q_1)\, q_1 = \frac{9}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} \qquad (u_3^T q_2)\, q_2 = \frac{46}{14}\begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix}$$

$$P_3 = \begin{pmatrix} 0 \\ 16 \\ 1 \\ 1 \end{pmatrix} - \frac{9}{2}\begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} - \frac{23}{7}\begin{pmatrix} -1 \\ 3 \\ -2 \\ 0 \end{pmatrix} = \frac{1}{14}\begin{pmatrix} -17 \\ 23 \\ 43 \\ -49 \end{pmatrix}$$

$$q_3 = \frac{P_3}{|P_3|} \qquad |P_3| = 5.08$$

$$q_3 = \begin{pmatrix} -0.239 \\ 0.323 \\ 0.604 \\ -0.688 \end{pmatrix}$$

3.

$$R = \begin{pmatrix} u_1^T q_1 & u_2^T q_1 & u_3^T q_1 \\ 0 & u_2^T q_2 & u_3^T q_2 \\ 0 & 0 & u_3^T q_3 \end{pmatrix}$$

$$= \begin{pmatrix} 2 & 2 & 9 \\ 0 & 3.74 & 12.3 \\ 0 & 0 & 5.08 \end{pmatrix}$$

4.

$$Rx = \begin{pmatrix} 2 & 2 & 9 \\ 0 & 3.74 & 12.3 \\ 0 & 0 & 5.08 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$$

$$= \begin{pmatrix} 2a_0 + 2a_1 + 9a_2 \\ 3.74a_1 + 12.3a_2 \\ 5.08a_2 \end{pmatrix}$$

$$Q^T b = \begin{pmatrix} 2 \\ -3.475 \\ -0.843 \end{pmatrix}$$

$\therefore$

5.

$a_0 = 2.129$

$a_p = -0.3833$ $\qquad P_2(x) = 2.129 - 0.3833x - 0.1659x^2$

$a_2 = -0.1659$

1)

$$f(x) = 0.2 + 25x + 3x^2$$

$$f(0) = 0.2 \qquad f(2) = 0.2 + 50 + 12 = 62.2$$

(a) $$I_1(f) = \frac{(b-a)}{2}\left[f(a) + f(b)\right] = \frac{2-0}{2} \times \frac{0.2 + 62.2}{2} = 62.4$$

(b) $$\int_0^2 f(x)\,dx = \left(0.2x + 12.5x^2 + x^3\right) = \left(0.2 \times 2 + 12.5 \times 2^2 + 2^3\right) - 0 = 58.4$$

(c) $\therefore$ error $$= \left|\frac{62.4 - 58.4}{58.4}\right| \times 100\% = 6.85\%$$

2) $f(x) = \int_e^{e+1} \frac{1}{x(\ln x)^2} dx$

$u = \ln x$

$du = \ln x \, dx$

$\frac{du}{dx} = \frac{1}{x}$

(a) $f(x) = \int_e^{e+1} \frac{1}{u^2} dx$

$= \int_e^{e+1} u^{-2} dx$

$= \frac{u^{-2+1}}{-1}$

$= \left[-\frac{1}{u}\right]_e^{e+1}$

$= \left[\frac{-1}{\ln x}\right]_e^{e+1} = 0.23853$

(b) here, $m = 4$ $\therefore h = \frac{b-a}{m} = \frac{e+1-e}{4} = \frac{1}{4}$

$x_0 = e$; $x_1 = x_0 + h = e + \frac{1}{4}$; $x_2 = e + \frac{1}{2}$; $x_3 = e + \frac{3}{4}$; $x_4 = e + 1$

Using Composite - Newton Cotes formula,

$C_{1,4}(f) = \frac{h}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$

$= \frac{1}{8}\left[0.36787 + 0.56922 + 0.45487 + 0.37283 + 0.15594\right]$

$= 0.24076$

(c) Error : $\left|\frac{0.24076 - 0.23853}{0.23853}\right| \times 100\% = 0.93489\%$