Quiz #4

1. Consider a set of three data points, f(500) = 120, f(700) = 180 and f(990) = 200.
   - (2 marks) Represent the above information of the overdetermined system using matrices.
   - (8 marks) Use the QR-decomposition method to find the equation of a straight line that gives the least error while fitting the data above.

5 (a) Eq^n of straight line = $a_0 + a_1 x$

$$\underbrace{\begin{bmatrix} 1 & 500 \\ 1 & 700 \\ 1 & 990 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 120 \\ 180 \\ 200 \end{bmatrix}}_{b}$$

(b) $\begin{bmatrix} 1 & 500 \\ 1 & 700 \\ 1 & 990 \end{bmatrix}$, u_1 = first column, u_2 = second column

$$P_1 = u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$P_2 = u_2 - \frac{u_2 \cdot P_1}{P_1 \cdot P_1} P_1$$

$$= \begin{bmatrix} 500 \\ 700 \\ 990 \end{bmatrix} - \frac{(500\times1)+(700\times1)+(990\times1)}{(1\times1)+(1\times1)+(1\times1)} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 500 \\ 700 \\ 990 \end{bmatrix} - 730 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} -230 \\ -30 \\ 260 \end{bmatrix}$$

$$q_1 = \frac{P_1}{|P_1|} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

$$q_2 = \frac{P_2}{|P_2|} = \frac{1}{348.425} \begin{bmatrix} -230 \\ -30 \\ 260 \end{bmatrix} = \begin{bmatrix} -0.660 \\ -0.086 \\ 0.746 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1/\sqrt{3} & -0.660 \\ 1/\sqrt{3} & -0.086 \\ 1/\sqrt{3} & 0.746 \end{bmatrix} = \begin{bmatrix} 0.577 & -0.660 \\ 0.577 & -0.086 \\ 0.577 & 0.746 \end{bmatrix}$$

$R = Q^T A$

$$= \begin{bmatrix} 0.577 & 0.577 & 0.577 \\ -0.660 & -0.086 & 0.746 \end{bmatrix} \begin{bmatrix} 1 & 500 \\ 1 & 700 \\ 1 & 990 \end{bmatrix}$$

$$R = \begin{bmatrix} 1.731 & 1263.63 \\ 0 & 348.34 \end{bmatrix}$$

$Rx = Q^T b$

$$\begin{bmatrix} 1.731 & 1263.63 \\ 0 & 348.34 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 0.577 & 0.577 & 0.577 \\ -0.660 & -0.086 & 0.746 \end{bmatrix} \begin{bmatrix} 120 \\ 180 \\ 200 \end{bmatrix}$$

$$\begin{bmatrix} 1.731 & 1263.63 \\ 0 & 348.34 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 288.5 \\ 54.52 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1.731 & 1263.63 \\ 0 & 348.34 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 288.5 \\ 54.52 \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 52.412 \\ 0.156 \end{bmatrix}$$

$\therefore$ Eq^n of straight line = $52.412 + 0.156x$

1. Consider a set of three data points, f(100) = 70, f(220) = 180 and f(430) = 300.
   - (2 marks) Represent the above information of the overdetermined system using matrices.
   - (8 marks) Use the QR-decomposition method to find the equation of a straight line that gives the least error while fitting the data above.

⑤(a) Eqn of straight line = $a_0 + a_1 x$

$$\underbrace{\begin{bmatrix} 1 & 100 \\ 1 & 220 \\ 1 & 430 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} a_0 \\ a_1 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 70 \\ 180 \\ 300 \end{bmatrix}}_{b}$$

(b) $\begin{bmatrix} 1 & 100 \\ 1 & 220 \\ 1 & 430 \end{bmatrix}$ $\to u_1, u_2$

$$P_1 = u_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$P_2 = u_2 - \frac{u_2 \cdot P_1}{P_1 \cdot P_1} P_1$$

$$= \begin{bmatrix} 100 \\ 220 \\ 430 \end{bmatrix} - \frac{(100\times1)+(220\times1)+(430\times1)}{(1\times1)+(1\times1)+(1\times1)} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 100 \\ 220 \\ 430 \end{bmatrix} - 250 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$P_2 = \begin{bmatrix} -150 \\ -30 \\ 180 \end{bmatrix}$$

$$q_1 = \frac{P_1}{|P_1|} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{bmatrix}$$

$$q_2 = \frac{P_2}{|P_2|} = \frac{1}{30\sqrt{62}} \begin{bmatrix} -150 \\ -30 \\ 180 \end{bmatrix} = \begin{bmatrix} -5\sqrt{62}/62 \\ -\sqrt{62}/62 \\ 3\sqrt{62}/31 \end{bmatrix}$$

$$\therefore Q = \begin{bmatrix} 1/\sqrt{3} & -5\sqrt{62}/62 \\ 1/\sqrt{3} & -\sqrt{62}/62 \\ 1/\sqrt{3} & 3\sqrt{62}/31 \end{bmatrix} = \begin{bmatrix} 0.577 & -0.635 \\ 0.577 & -0.127 \\ 0.577 & 0.762 \end{bmatrix}$$

$$R = Q^T A$$

$$= \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -5\sqrt{62}/62 & -\sqrt{62}/62 & 3\sqrt{62}/31 \end{bmatrix} \begin{bmatrix} 1 & 100 \\ 1 & 220 \\ 1 & 430 \end{bmatrix}$$

$$R = \begin{bmatrix} \sqrt{3} & 250\sqrt{3} \\ 0 & 30\sqrt{62} \end{bmatrix} = \begin{bmatrix} 1.732 & 433.013 \\ 0 & 236.220 \end{bmatrix}$$

$$Rx = Q^T b$$

$$\begin{bmatrix} \sqrt{3} & 250\sqrt{3} \\ 0 & 30\sqrt{62} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ -5\sqrt{62}/62 & -\sqrt{62}/62 & 3\sqrt{62}/31 \end{bmatrix} \begin{bmatrix} 70 \\ 180 \\ 300 \end{bmatrix}$$

$$\begin{bmatrix} \sqrt{3} & 250\sqrt{3} \\ 0 & 30\sqrt{62} \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 550/\sqrt{3} \\ 635\sqrt{2}/\sqrt{31} \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} \sqrt{3} & 250\sqrt{3} \\ 0 & 30\sqrt{62} \end{bmatrix}^{-1} \cdot \begin{bmatrix} 550/\sqrt{3} \\ 635\sqrt{2}/\sqrt{31} \end{bmatrix}$$

$$\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 1175/93 \\ 127/186 \end{bmatrix} = \begin{bmatrix} 12.634 \\ 0.683 \end{bmatrix}$$

$\therefore$ Eq^n of straight line = $12.634 + 0.683x$