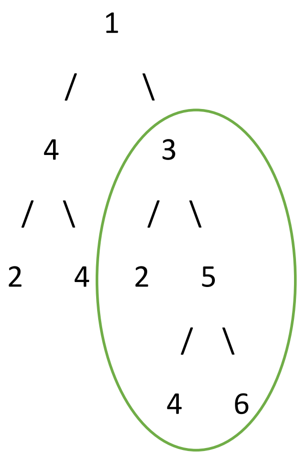
Calculate the Maximum Sum in the Valid Binary Search Tree Subtree LeetCode

Given a **binary tree** root, return *the maximum sum of all keys of* ***any*** *sub-tree which is also a Binary Search Tree (BST).*

Assume a BST is defined as follows:

- The left subtree of a node contains only nodes with keys **less than** the node's key.
- The right subtree of a node contains only nodes with keys **greater than** the node's key.
- Both the left and right subtrees must also be binary search trees.

Example:



```
          1
        /   \
       4     3
      / \   / \
     2   4 2   5
              / \
             4   6
```

Output: The Maximum of Valid BST Subtree: 20

Approach 1: Find the maximum sum of values in a Binary Search Tree (BST)

- **Function Purpose**:
  - The **maxSumBST** function finds the maximum sum of values in a BST.
- **Explanation**:
  - It checks if the entire tree is a BST within the full range of **INT_MIN** to **INT_MAX**.
  - If the tree is a valid BST, it calculates the sum of values.
  - Negative sums are set to 0.
  - It recursively finds the maximum sum in the left and right subtrees.
  - Returns the maximum of these three values.

- **Time Complexity**:
    - **O(N^2), where N is the number of nodes in the BST due to repeated isBST and getSum calls.**
- **Space Complexity:**
    - **O(H), where H is the height of the BST, for the call stack space.**

Approach 2: Find the maximum sum in a BST using an optimized approach

- **Function Purpose**:
    - The **maxSumBSTOptimized** function finds the maximum sum of values in a BST using an optimized approach.
- **Explanation**:
    - It uses a helper function **maxSumBSTHelper** to calculate information about a node and its subtrees.
    - The helper function checks if the node and its subtrees form a BST and calculates the sum.
    - The maximum sum is updated while calculating this information.
- **Time Complexity**:
    - **O(N), where N is the number of nodes in the BST, as it traverses all nodes once.**
- **Space Complexity:**
    - **O(H), where H is the height of the BST, for the call stack space.**

Conclusion:

- **Approach 2 is more efficient with a lower time complexity, making it the preferred approach.**