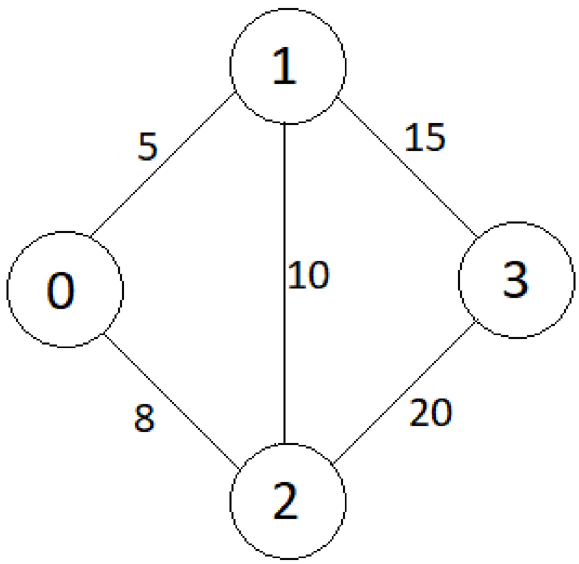
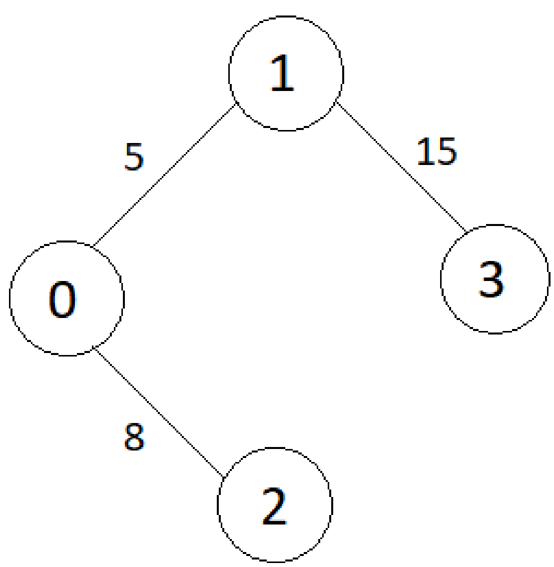
Minimum Spanning Tree in Undirected Graph using Prim’s Algorithm [CodeStudio]

You are given an undirected connected weighted graph having ‘N’ nodes numbered from 1 to 'N'. A matrix ‘E’ of size M x 2 is given which represents the ‘M’ edges such that there is an edge directed from node E[i][0] to node E[i][1]. You are supposed to return the minimum spanning tree where you need to return weight for each edge in the MST.



Output:



addEdge Function:

- **Purpose:**
    - Adds edges to the undirected graph, considering the weights.
- **Explanation:**
    - Iterates through each edge in the **edges** vector.
    - Extracts the source vertex **u**, destination vertex **v**, and weight **w**.
    - Adds edges from both **u** to **v** and **v** to **u** in the adjacency list.
- **Time Complexity:**
    - **O(E), where E is the number of edges in the input vector.**
- **Space Complexity:**
    - **O(E), where E is the number of edges. Each edge results in the creation of entries in the adjacency list for both vertices.**

Approach 1: Function to find the Minimum Spanning Tree using Prim's Algorithm

- **Purpose:**
    - Finds the MST using Prim's algorithm.
- **Explanation:**
    - Initializes vectors to store key values, MST set, and parent vertices.
    - Starts from the first vertex.
    - Iteratively selects the vertex with the minimum key value not yet in the MST.
    - Includes the selected vertex in the MST and updates key values and parent vertices for neighboring vertices.
    - Constructs the result in the form of edges and their weights.
- **Time Complexity:**
    - **O(V^2), where V is the number of vertices.**
- **Space Complexity:**
    - **O(V + E), where V is the number of vertices and E is the number of edges.**

Approach 2: Function to find the Minimum Spanning Tree using Optimized Prim's Algorithm with a Priority Queue

- **Purpose:**
    - Finds the MST using an optimized version of Prim's algorithm with a priority queue.
- **Explanation:**
    - Initializes vectors to store key values, MST set, and parent vertices.
    - Uses a priority queue to efficiently find the minimum key value.
    - Iteratively selects the vertex with the minimum key value not yet in the MST.
    - Includes the selected vertex in the MST and updates key values and parent vertices for neighboring vertices.
    - Constructs the result in the form of edges and their weights.
- **Time Complexity:**
    - **O((V + E) * log(V)), where V is the number of vertices and E is the number of edges.**
- **Space Complexity:**
    - **O(V + E), where V is the number of vertices and E is the number of edges.**

Conclusion:

- Prim's algorithm efficiently finds the Minimum Spanning Tree in an undirected weighted graph.
- The optimized version using a priority queue further improves the algorithm's efficiency.