Problem-2

Given that,

$$f(z) = e^{-\frac{z}{2}}$$

where $z = g(y)$, $g(y) = y^T S^{-1} y$, $y = h(x)$, $h(x) = x - u$

Here,

$$\frac{d}{dx} f(z) = \frac{d}{dx} e^{-\frac{z}{2}}$$

$$= \frac{d}{dx}\left(e^{-\frac{(x-u)^T S^{-1}(x-u)}{2}} \right)$$

$$= -e^{-\frac{(x-u)^T S^{-1}(x-u)}{2}} \frac{d}{dx}\left(-\frac{(x-u)^T S^{-1}(x-u)}{2} \right)$$

$$= \frac{1}{2} e^{-\frac{(x-u)^T S^{-1}(x-u)}{2}} \frac{d}{dx}\left(\frac{(x-u)^T S^{-1}(x-u)}{\cancel{2}} \right)$$

$$= \frac{1}{2} e^{(x-u)^T S^{-1}(x-u)} \{$$

$$= \frac{1}{2} e^{-\frac{(x-u)^T S^{-1}(x-u)}{2}} \left\{ S^{-1} \frac{d}{dx}\left((x-u)^T (x-u)\right) \right\}$$

$$= \frac{1}{2} e^{-\frac{(x-u)^T S^{-1}(x-u)}{2}} \cdot 2 \cdot S^{-1}(x-u)^T$$

$$= e^{-\frac{(x-u)^T S^{-1}(x-u)}{2}} (x-u)^T S^{-1}$$

(Ans)