Fixed Income Assignment 6

Nitish Ramkumar, Carlos Quicazan, Justin Ge, Yuying Wang

Question 1

For this question, we construct a binomial tree of rates by making sure the Dt for every time period matches the values provided in the question. Dt can be calculated by discounting the cashflows until time 0, using the predicted binomial rates.
We also use the fact that the second highest rate in every time period (t) =

2nd $highest_t = highest_t * \exp(-2\sigma_t\sqrt(dt))$

The final tree of the rates are as below:

Table 1: Binomial rates from time period 1 to 10

1	2	3	4	5	6	7	8	9	10
0.056605	0.0632699	0.0725723	0.0841249	0.0983264	0.1141517	0.1291161	0.1460415	0.1624982	0.1808178
0.000000	0.0549261	0.0612447	0.0695040	0.0795321	0.0910359	0.1026792	0.1158110	0.1292262	0.1442021
0.000000	0.0000000	0.0516852	0.0574241	0.0643301	0.0726011	0.0816553	0.0918382	0.1027667	0.1150011
0.000000	0.0000000	0.0000000	0.0474438	0.0520339	0.0578993	0.0649361	0.0728278	0.0817249	0.0917133
0.000000	0.0000000	0.0000000	0.0000000	0.0420881	0.0461747	0.0516403	0.0577525	0.0649915	0.0731413
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0368243	0.0410668	0.0457978	0.0516843	0.0583301
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0326582	0.0363177	0.0411018	0.0465182
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0287999	0.0326861	0.0370983
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0259935	0.0295859
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0235947
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

Table 2: Binomial rates from time period 11 to 20

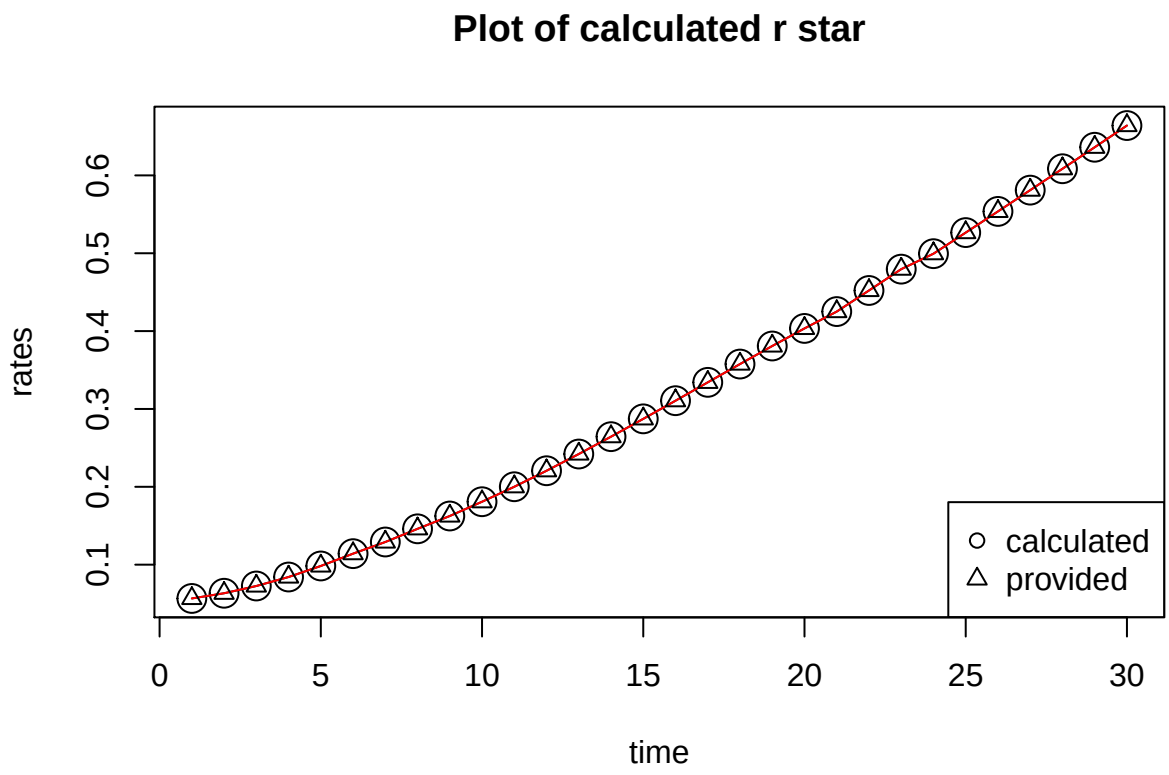
11	12	13	14	15	16	17	18	19	20
0.2000881	0.2206224	0.2421189	0.2644062	0.2872872	0.3106131	0.3341631	0.3576828	0.3808747	0.4034218
0.1602486	0.1774455	0.1955630	0.2144728	0.2340235	0.2541004	0.2745280	0.2950998	0.3155699	0.3356722
0.1283415	0.1427186	0.1579591	0.1739694	0.1906350	0.2078697	0.2255355	0.2434668	0.2614622	0.2793002
0.1027875	0.1147879	0.1275859	0.1411151	0.1552908	0.1700501	0.1852863	0.2008679	0.2166318	0.2323952
0.0823215	0.0923233	0.1030530	0.1144654	0.1264995	0.1391113	0.1522199	0.1657225	0.1794881	0.1933673
0.0659304	0.0742552	0.0832375	0.0928485	0.1030462	0.1138016	0.1250546	0.1367263	0.1487130	0.1608937
0.0528030	0.0597231	0.0672321	0.0753139	0.0839412	0.0930966	0.1027373	0.1128036	0.1232147	0.1338736
0.0422894	0.0480350	0.0543044	0.0610908	0.0683783	0.0761587	0.0844027	0.0930666	0.1020883	0.1113912
0.0338692	0.0386343	0.0438625	0.0495537	0.0557008	0.0623025	0.0693401	0.0767829	0.0845842	0.0926844
0.0271255	0.0310734	0.0354284	0.0401955	0.0453737	0.0509672	0.0569656	0.0633484	0.0700814	0.0771192
0.0217245	0.0249922	0.0286160	0.0326045	0.0369613	0.0416943	0.0467995	0.0522644	0.0580652	0.0641680
0.0000000	0.0201011	0.0231136	0.0264471	0.0301086	0.0341085	0.0384476	0.0431198	0.0481093	0.0533918
0.0000000	0.0000000	0.0186692	0.0214525	0.0245264	0.0279028	0.0315862	0.0355752	0.0398605	0.0444253
0.0000000	0.0000000	0.0000000	0.0174012	0.0199792	0.0228262	0.0259493	0.0293507	0.0330260	0.0369646
0.0000000	0.0000000	0.0000000	0.0000000	0.0162750	0.0186732	0.0213184	0.0242153	0.0273634	0.0307569
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0152758	0.0175139	0.0199784	0.0226716	0.0255917
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0143883	0.0164828	0.0187843	0.0212939
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0135988	0.0155636	0.0177178
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0128950	0.0147423
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0122665
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000

Table 3: Binomial rates from time period 21 to 30

21	22	23	24	25	26	27	28	29	30
0.4250638	0.4521561	0.4795586	0.4994297	0.5265159	0.5537277	0.5810568	0.6085253	0.6361532	0.6640090
0.3551833	0.3788918	0.4029924	0.4214753	0.4455923	0.4699491	0.4945400	0.5193856	0.5445043	0.5699568
0.2967912	0.3174988	0.3386508	0.3556886	0.3771063	0.3988461	0.4209053	0.4433035	0.4660591	0.4892265
0.2479988	0.2660535	0.2845820	0.3001703	0.3191464	0.3385009	0.3582344	0.3783663	0.3989152	0.4199310
0.2072278	0.2229440	0.2391457	0.2533177	0.2700947	0.2872859	0.3048949	0.3229414	0.3414446	0.3604508
0.1731596	0.1868197	0.2009638	0.2137781	0.2285821	0.2438197	0.2594974	0.2756354	0.2922536	0.3093955
0.1446922	0.1565487	0.1688780	0.1804102	0.1934499	0.2069300	0.2208594	0.2352590	0.2501494	0.2655718
0.1209048	0.1311826	0.1419150	0.1522505	0.1637173	0.1756216	0.1879744	0.2007971	0.2141111	0.2279554
0.1010281	0.1099267	0.1192569	0.1284862	0.1385545	0.1490502	0.1599859	0.1713834	0.1832647	0.1956672
0.0844191	0.0921149	0.1002164	0.1084312	0.1172592	0.1264990	0.1361647	0.1462783	0.1568622	0.1679523
0.0705406	0.0771893	0.0842159	0.0915066	0.0992369	0.1073598	0.1158904	0.1248508	0.1342635	0.1441631
0.0589437	0.0646821	0.0707700	0.0772236	0.0839845	0.0911163	0.0986348	0.1065620	0.1149205	0.1237434
0.0492534	0.0542014	0.0594709	0.0651700	0.0710764	0.0773305	0.0839486	0.0909523	0.0983643	0.1062160
0.0411561	0.0454190	0.0499758	0.0549978	0.0601522	0.0656304	0.0714490	0.0776292	0.0841932	0.0911713
0.0343901	0.0380596	0.0419967	0.0464134	0.0509070	0.0557006	0.0608106	0.0662577	0.0720637	0.0782575

21	22	23	24	25	26	27	28	29	30
0.0287363	0.0318927	0.0352915	0.0391689	0.0430828	0.0472731	0.0517561	0.0565520	0.0616817	0.0671729
0.0240121	0.0267250	0.0296569	0.0330551	0.0364611	0.0401207	0.0440499	0.0482680	0.0527954	0.0576583
0.0200645	0.0223947	0.0249219	0.0278957	0.0308572	0.0340505	0.0374911	0.0411974	0.0451893	0.0494914
0.0167659	0.0187660	0.0209428	0.0235415	0.0261145	0.0288987	0.0319088	0.0351626	0.0386790	0.0424813
0.0140096	0.0157253	0.0175991	0.0198670	0.0221008	0.0245263	0.0271577	0.0300118	0.0331066	0.0364641
0.0117064	0.0131773	0.0147893	0.0167660	0.0187040	0.0208155	0.0231141	0.0256156	0.0283370	0.0312993
0.0000000	0.0110421	0.0124280	0.0141491	0.0158293	0.0176661	0.0196725	0.0218633	0.0242546	0.0268659
0.0000000	0.0000000	0.0104438	0.0119406	0.0133964	0.0149932	0.0167433	0.0186606	0.0207603	0.0230606
0.0000000	0.0000000	0.0000000	0.0100768	0.0113374	0.0127248	0.0142503	0.0159271	0.0177694	0.0197942
0.0000000	0.0000000	0.0000000	0.0000000	0.0095949	0.0107995	0.0121285	0.0135941	0.0152094	0.0169905
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0091656	0.0103226	0.0116027	0.0130182	0.0145839
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0087856	0.0099031	0.0111427	0.0125182
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0084525	0.0095374	0.0107451
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0081634	0.0092231
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0079167

The plot of the solved r star between calculated and provided data is:



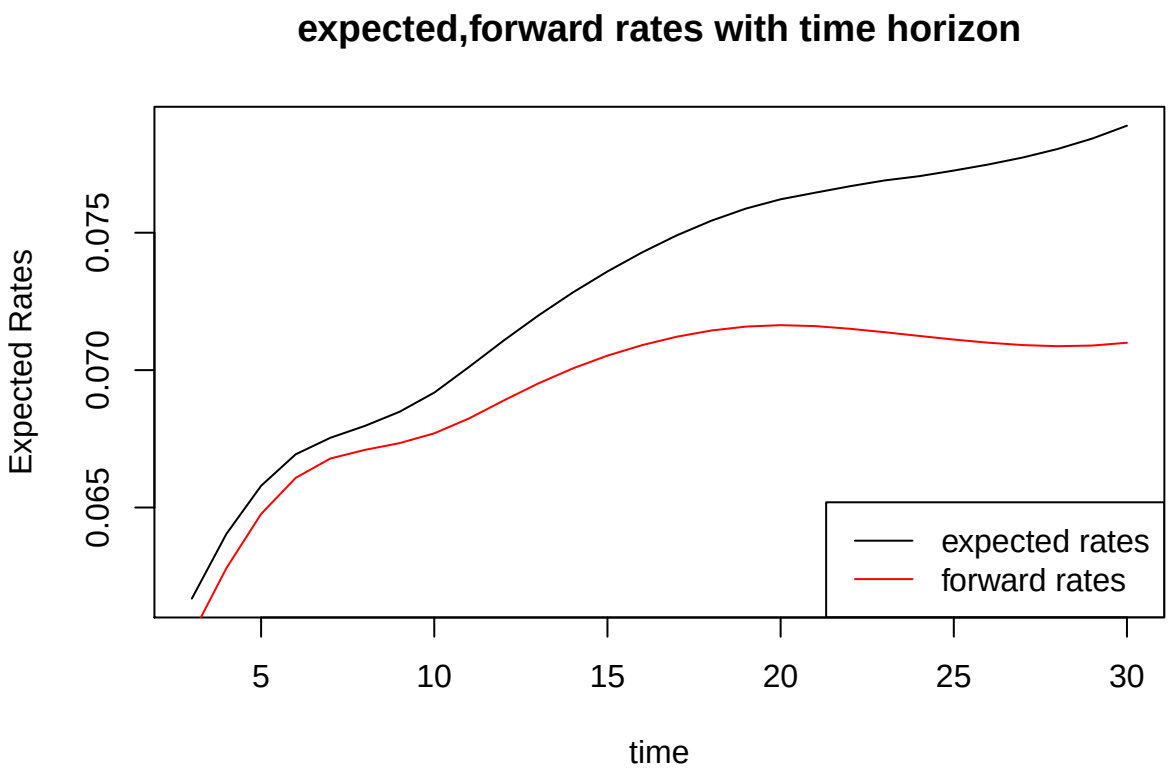
This shows that the calculated and provided data are exactly the same.

The correlation between the predicted and actual matrix is:

```
## [1] 1
```

Question 2

The expected rates are the weighted average of all the rates at each time step. The forward rates include the 0.5F1.5, 1F2 etc. Keeping this, We can plot the 2 graphs.



The difference might be because of the fact that the binomial method doesn't give the most accurate results due to the assumptions involved in the process.