Fixed Income Assignment 7

Question 1

We know dynamics of D(t,T) as

dD(t,T_i) = r D(t,T_i) + σ (T-t) dZ_i

σ is based on the time to maturity of the bond. We have different σ for various maturities, but is constant across bonds

Table 1: first 7 time steps

0.5	1	1.5	2	2.5	3	3.5
0.9724765	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.9445693	0.9723657	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.9163238	0.9435355	0.9708587	0.0000000	0.0000000	0.0000000	0.0000000
0.8879337	0.9142475	0.9406621	0.9677334	0.0000000	0.0000000	0.0000000
0.8597406	0.8855595	0.9109536	0.9368460	0.9683264	0.0000000	0.0000000
0.8320438	0.8571382	0.8820332	0.9072176	0.9376270	0.9683244	0.0000000
0.8050770	0.8296667	0.8538201	0.8780369	0.9075755	0.9370586	0.9679040
0.7789041	0.8026401	0.8260199	0.8491699	0.8777865	0.9061445	0.9361123
0.7534803	0.7764934	0.7994135	0.8215914	0.8493520	0.8766115	0.9053303
0.7287341	0.7511464	0.7733641	0.7949949	0.8220544	0.8487157	0.8767530
0.7045800	0.7262850	0.7476484	0.7684480	0.7951800	0.8210503	0.8481326
0.6810105	0.7017554	0.7223042	0.7424687	0.7682648	0.7932320	0.8194281
0.6580423	0.6780183	0.6976780	0.7173177	0.7421341	0.7661582	0.7917086
0.6356925	0.6549809	0.6739035	0.6930749	0.7167338	0.7397877	0.7643233
0.6139770	0.6325046	0.6506960	0.6693540	0.6921952	0.7146030	0.7382641
0.5929047	0.6108696	0.6287321	0.6470309	0.6691655	0.6908931	0.7136021
0.5724819	0.5895735	0.6066935	0.6243235	0.6457930	0.6668074	0.6886179
0.5527129	0.5692104	0.5857967	0.6027740	0.6234834	0.6436880	0.6646593
0.5336010	0.5495704	0.5655750	0.5817017	0.6018362	0.6210012	0.6412235
0.5151482	0.5306037	0.5460119	0.5618043	0.5813671	0.5998049	0.6191701

Table 2: next 7 time steps

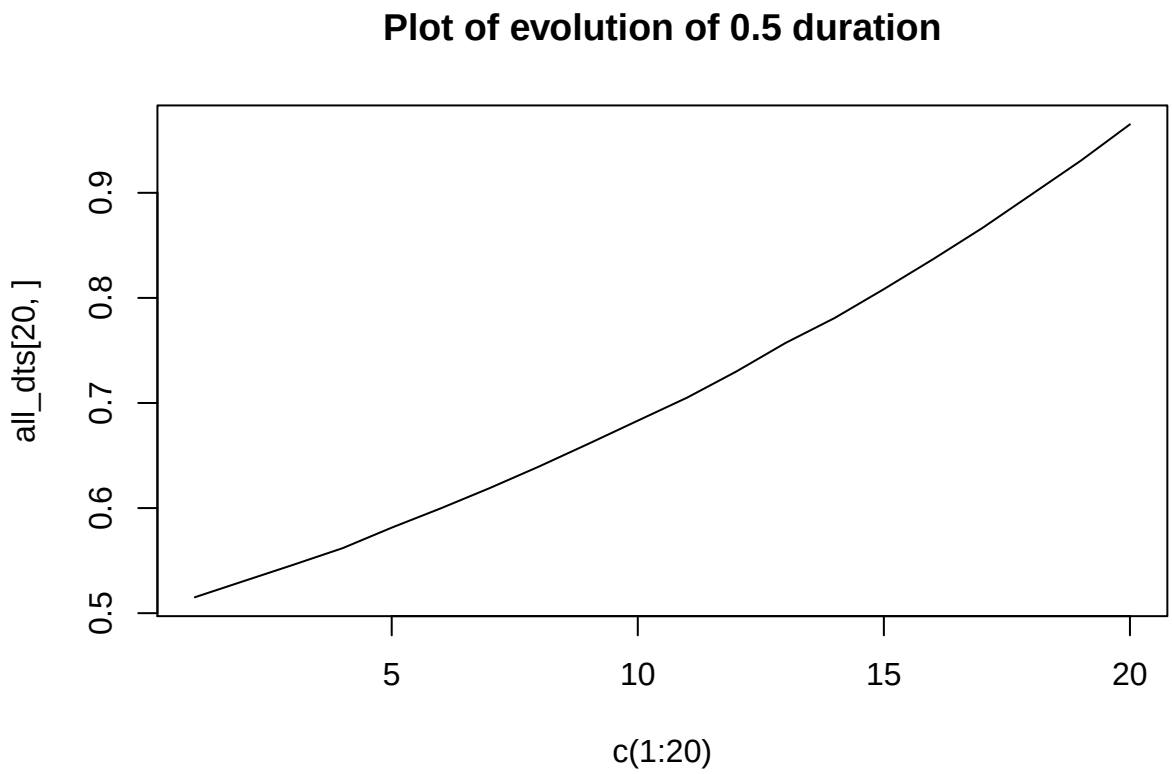
4	4.5	5	5.5	6	6.5	7
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.9675182	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.9355685	0.9672961	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.9060309	0.9370124	0.9691399	0.0000000	0.0000000	0.0000000	0.0000000
0.8763246	0.9060206	0.9372016	0.9673712	0.0000000	0.0000000	0.0000000
0.8465206	0.8752283	0.9053098	0.9346066	0.9663336	0.0000000	0.0000000
0.8179870	0.8457955	0.8748238	0.9033377	0.9341447	0.9679676	0.0000000

4	4.5	5	5.5	6	6.5	7
0.7895554	0.8165085	0.8446216	0.8721078	0.9020164	0.9350451	0.9655032
0.7623328	0.7883126	0.8149730	0.8412376	0.8703259	0.9019671	0.9312953
0.7367915	0.7619373	0.7875096	0.8127663	0.8411811	0.8717739	0.8999589
0.7110256	0.7350124	0.7598903	0.7840457	0.8111579	0.8408969	0.8680037
0.6863665	0.7096457	0.7335633	0.7568561	0.7830858	0.8117470	0.8379214
0.6623029	0.6843503	0.7074249	0.7299130	0.7555602	0.7834978	0.8087869
0.6396777	0.6612341	0.6831849	0.7050666	0.7299124	0.7570728	0.7808717

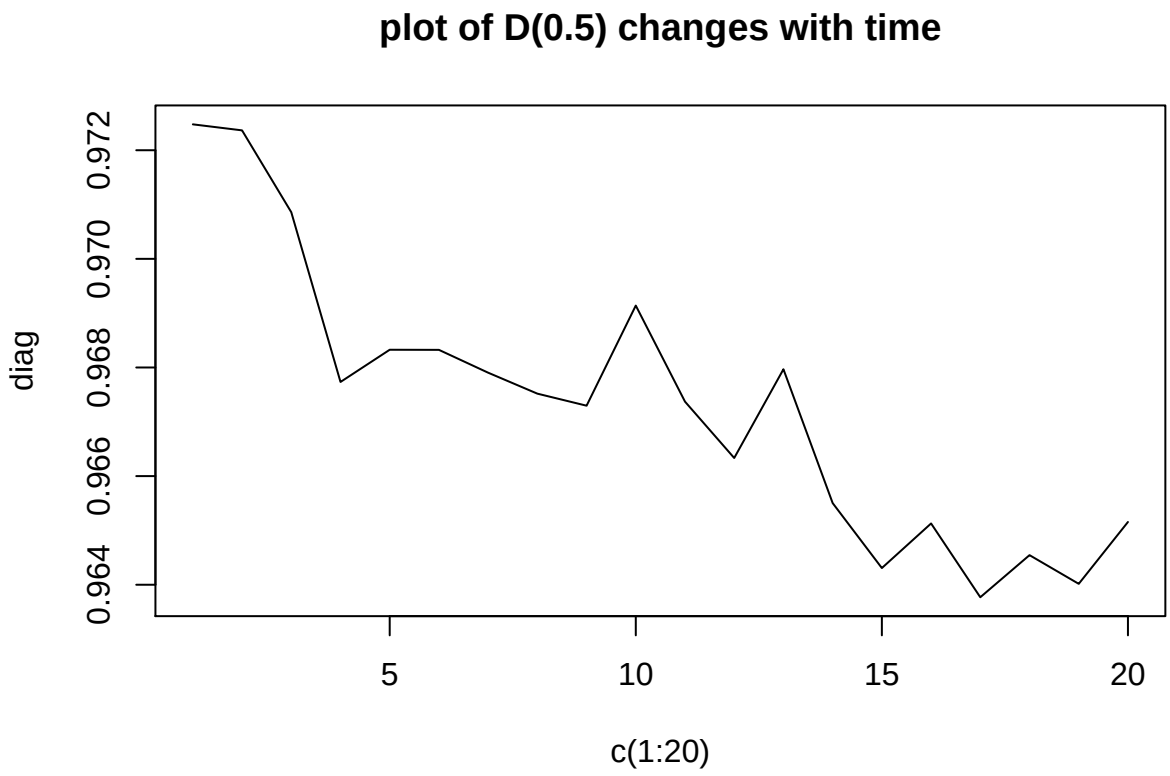
Table 3: final 6 time steps

7.5	8	8.5	9	9.5	10
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.0000000	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.9643075	0.0000000	0.0000000	0.0000000	0.0000000	0.0000000
0.9316483	0.9651280	0.0000000	0.0000000	0.0000000	0.0000000
0.8986338	0.9308045	0.9637698	0.0000000	0.0000000	0.0000000
0.8673837	0.8983661	0.9301866	0.9645448	0.0000000	0.0000000
0.8371371	0.8668183	0.8973901	0.9305527	0.9640165	0.0000000
0.8083432	0.8368408	0.8664630	0.8984772	0.9305278	0.9651553

The plot of how maturity 10 bond changes with time is as below



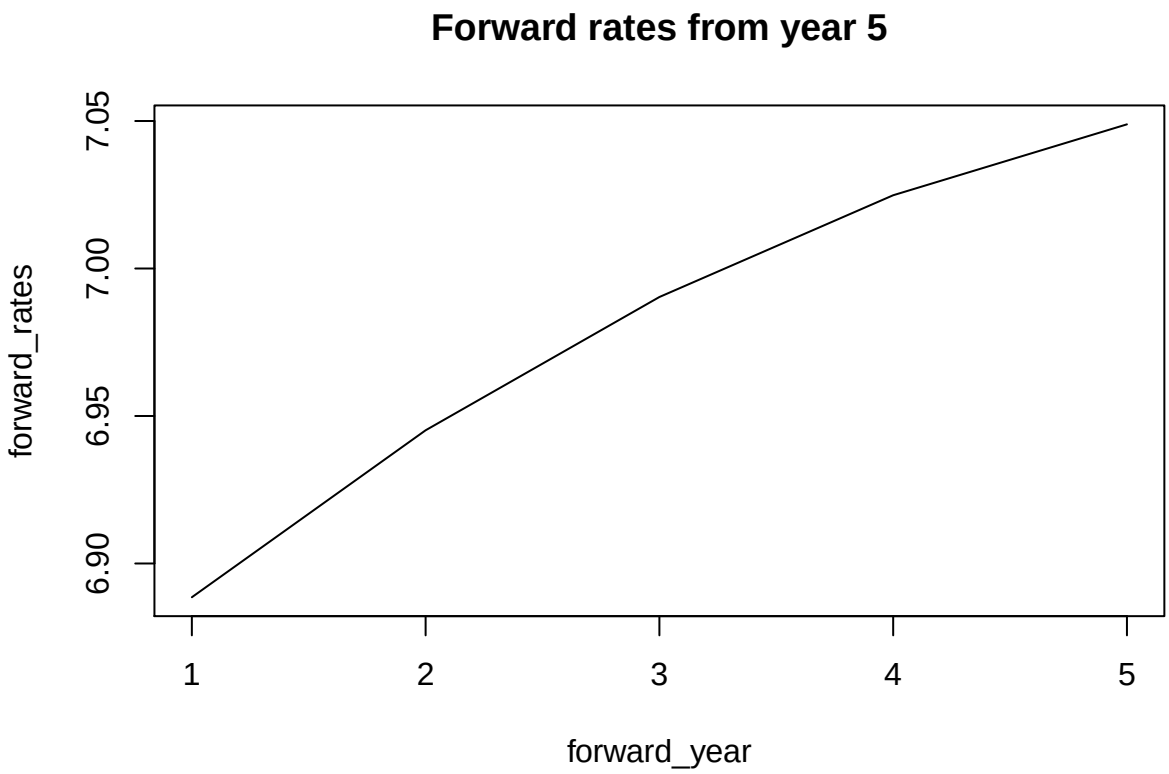
The plot of the diagonal elements (which is evolution of D(0.5))



Question 2

The forward par rates can be calculated by using the initial dt values. The forward par rates starts at 5 years, and has maturity of 1,2,3,4,5 years.

1	2	3	4	5
6.888558	6.945151	6.990333	7.024829	7.048864



Question 3

For this question, for every simulation, we will calculate the dt for all time periods and use that to calculate the price of the coupon bond (coupon based on the prar rate from previous question). Using the price, we can get the minimum price and compare it with price of no cheapest to deliver option (in this case we assume the 5 year forward bond 5 years from now). Once we repeat this for all simulations, we can get the price of the option.

The price of the option at time = 5 is the difference between price with no cheapest to deliver option (a 1,2,3,4, 5 year bond after time 5), and option with a cheapest to deliver option. The values when we choose different bonds for the non-cheapest to delivery comparison is:

1	2	3	4	5
2.640042	8.760511	14.84127	20.4941	25.50591

The price of the 5 bonds are

Table 6: Price of all bonds available

1	2	3	4	5
103.0491	109.1695	115.2503	120.9031	125.9149

The price of the CTD is:

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## [1] 100.409
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