Computational Finance Project 7 Nitish Ramkumar

1)

a)

The values for European Option put using the different methods and different dt order for S0 = 10 are as follows:

Method	dtOrder	Price of European Put
Explicit	1	0.464541
Explicit	3	0.463417
Explicit	4	0.463142
Implicit	1	0.463406
Implicit	3	0.462855
Implicit	4	0.462579
Crank-Nicolson	1	0.463686
Crank-Nicolson	3	0.463136
Crank-Nicolson	4	0.462861
Black Scholes		0.464695

b) The prices for the various types and the different orders of dX are as below

SO	EFD _dt	EFD _3dt	EFD_ 4dt	IFD_ dt	IFD_ 3dt	IFD_ 4dt	CNF _dt	CNF_ 3dt	CNF_ 4dt	BlackS choles
4	5.787	5.794	5.787	4.639	5.632	5.721	4.608	5.630	5.721	5.802
5	4.781	4.745	4.735	4.759	4.744	4.735	4.758	4.744	4.735	4.802
6	3.797	3.712	3.743	3.797	3.712	3.743	3.797	3.712	3.743	3.802
7	2.751	2.804	2.689	2.752	2.804	2.689	2.752	2.804	2.689	2.805
8	1.783	1.799	1.782	1.783	1.799	1.783	1.783	1.799	1.783	1.844
9	0.980	0.946	0.925	0.980	0.946	0.925	0.980	0.946	0.925	1.024
10	0.465	0.463	0.463	0.463	0.463	0.463	0.464	0.463	0.463	0.465
11	0.164	0.145	0.147	0.164	0.145	0.147	0.164	0.145	0.147	0.172
12	0.048	0.049	0.041	0.048	0.049	0.041	0.048	0.049	0.041	0.052
13	0.012	0.013	0.012	0.012	0.013	0.012	0.012	0.013	0.012	0.014
14	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003	0.003
15	0.001	0.000	0.001	0.001	0.000	0.001	0.001	0.000	0.001	0.001
16	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

The error percentage for all these prices by comparing it with black scholes is as follows:

	EFD_d	EFD_3d	EFD_4d	IFD_d	IFD_3d	IFD_4d	CNF_d	CNF_3d	CNF_4
S0	t	t	t	t	t	t	t	t	dt
				-			-		
4	-0.26%	-0.14%	-0.26%	20.05%	-2.93%	-1.40%	20.57%	-2.96%	-1.40%
5	-0.45%	-1.19%	-1.39%	-0.90%	-1.20%	-1.39%	-0.91%	-1.20%	-1.39%
6	-0.14%	-2.38%	-1.56%	-0.14%	-2.38%	-1.56%	-0.14%	-2.38%	-1.56%
7	-1.92%	-0.06%	-4.15%	-1.91%	-0.06%	-4.14%	-1.92%	-0.06%	-4.15%
8	-3.34%	-2.47%	-3.35%	-3.33%	-2.45%	-3.33%	-3.34%	-2.46%	-3.34%
9	-4.36%	-7.62%	-9.73%	-4.33%	-7.64%	-9.75%	-4.32%	-7.63%	-9.74%
10	-0.03%	-0.27%	-0.33%	-0.28%	-0.40%	-0.46%	-0.22%	-0.34%	-0.39%
					1	-			-
11	-4.30%	-15.47%	-14.38%	-4.23%	15.62%	14.53%	-4.14%	-15.54%	14.46%
						-			-
12	-9.37%	-6.41%	-21.15%	-8.81%	-6.27%	20.95%	-8.88%	-6.34%	21.05%
	-			-		-	-		-
13	11.48%	-3.53%	-12.05%	10.89%	-2.32%	10.86%	11.49%	-2.92%	11.45%
14	-9.14%	-11.24%	-9.10%	-6.62%	-7.91%	-5.75%	-8.30%	-9.57%	-7.42%
	-			-	-		-		-
15	17.09%	-29.71%	-15.75%	11.40%	23.68%	-9.06%	14.76%	-26.69%	12.41%
	-				-	-	-		-
16	18.70%	-27.84%	-31.85%	-5.23%	17.03%	21.58%	11.34%	-22.47%	26.75%

- 1) The error percentage reduces as we decrease dX for the Implicit and Crank Nicolson methods (except for S0 = 4). So, there is unconditional convergence for both the methods.
- 2) The OTM put option errors are higher because they are very small numbers
- 3) In general, the prices are very similar for ITM put options for all types and dx.
- 4) External with $dx = \sigma \sqrt{dt}$ case, has errors moving around a lot (in one case it is very low error and in the next case it is very high). This shows that this case isn't stable.

2)

The American call and American put prices using dS = 0.5 are as below

Explicit_Call	0.652503
Explicit_Put	0.472490
Implicit_Call	0.651897
Implicit_Put	0.471615
Crank-Nicolson_Call	0.652200
Crank-Nicolson Put	0.472051

These values match expected values out of binomial and trinomial models from previous homeworks.

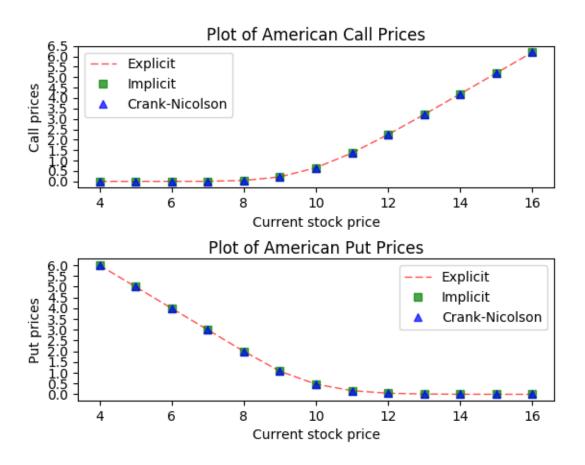
The American call and American put prices using dS = 1 are as below

Explicit_Call	0.621424
Explicit_Put	0.439066
Implicit_Call	0.620664
Implicit_Put	0.438166
Crank-Nicolson_Call	0.621044
Crank-Nicolson_Put	0.438616

The value becomes less accurate with increase in dS.

b)

We can plot values from Explicit, Implicit and Crank-Nicolson for American call and puts for current stock prices from 4 to 16.



It can be noticed that American call and put prices are very similar for 3 methods.