Fixed Income Markets: Homework 2 Hints

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Here are some detailed hints for HW2:

Question 1: You should start by computing/estimating the D(T) function. You have been provided with raw data from the Strips market. Each of these bonds is a zero coupon bond. For example, Lines 1 and 2 of the Strips data are:

Maturity	Price
0.0833	99.73
0.3333	98.79

The first column gives you the maturity time (T) of the bond, and the second column gives you its price. Recall that strips are zero coupon bonds. You are to assume that each of these bonds has a face value of \$100. Thus, the price of a bond maturing in 0.0833 years (= 1/12 years = 1 month) is given by:

$$P = \$100 \times D\,(0.0833)$$

You can use the above equation to solve for $D(0.0833)$ as:

$$D(0.0833) = \frac{99.73}{100} = 0.9973$$

Similarly:

$$D(0.3333) = \frac{98.79}{100} = 0.987$$

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Given the $D(T)$ function, you can now solve for spot rates and 3-month forward rates (m = 0.25 in the formula given in class) as follows:

$$D(0.0833) = 0.9973 = \left(\frac{1}{1+\frac{r}{2}}\right)^{2*0.0833}$$

And:

$$1+\frac{r}{2} = \left(\frac{1}{0.9973}\right)^{\frac{1}{2*0.0833}}$$

This gives you the 1-month spot rate:

$$r = 0.0327$$

Similarly you can solve for the 3-month spot rate:

$$r = 0.03685$$

Given the values of the discount function $D(T)$, you can also solve for the 3-month forward rates using the formula:

$$1 + m *_n f_m = \frac{D(n)}{D(n+m)}$$

Here n = dates in first column in years, and m = 0.25. Hence we can solve for the rate $(1/12)f\left(\frac{3}{12}\right)$ or the forward rate for a loan/investment that begins in 1- month from now and lasts for 3-months as:

$$1 + 0.25 *_{0.0833} f_{0.25} = \frac{D(0.0833)}{D(0.0833+0.25)} = \frac{D(0.0833)}{D(0.3333)} = \frac{0.9973}{0.9879}$$

$${}_{0.0833}f_{0.25} = 0.03806$$

Question 2: You have to add extra columns to the table given in Excel:

Maturity(T)	Price	D(T)	T	T^2	T^3	T^4	T^5	$\ln D(T)$
0.0833	99.73	0.9973	0.0833	6.94×10^{-3}	5.79×10^{-4}	4.82×10^{-5}	4.02×10^{-6}	-0.0027
0.3333	98.79	0.9879	0.3333	1.11×10^{-1}	3.70×10^{-2}	1.23×10^{-2}	4.12×10^{-3}	-0.0122

Now run a regression in Excel with various powers of T as your independent variables, and ln D(T) as the dependent variable. Remember to check the box "No intercept/constant" while running the regression. You should end up with coefficient values similar to the following values:

Coefficient	Value
a	-3.263×10^{-2}
b	-1.075×10^{-3}
c	-1.981×10^{-5}
d	$+2.824\times10^{-6}$
e	-4.682×10^{-8}

Questions 3-5: Now, given these coefficients, make a new table in Excel as follows:

Maturity(T)	$a\times T$	$b\times T^2$	$c\times T^3$	$d\times T^4$	$e\times T^5$	$\ln D(T)$
0.5	-0.01631	-2.686×10^{-4}	-2.477×10^{-6}	1.765×10^{-7}	-1.460×10^{-9}	-0.0165
1.0	-0.03263	-1.074×10^{-3}	-1.981×10^{-5}	2.824×10^{-6}	-4.682×10^{-8}	-0.0337
1.5	-0.04894	-2.418×10^{-3}	-6.687×10^{-5}	1.429×10^{-5}	-3.555×10^{-7}	-0.0514

Complete this table out to 25 years, and with the new values of D(T) from this table, compute the spot rates again as in Question 1. Also solve for the par rates and the 6-month forward rates. The initial entries in your table should be:

T	D(T)	Spot	Par	Forward
0.5	0.9836	3.3447	3.3447	3.4565
1	0.9668	3.4006	3.4001	3.5701

You can answer questions 3-5 simultaneously by plotting all three curves (spot, forward and par) in the same graph. You should get a hump-shaped forward curve.

Questions 6 and 7: You have to use the data on T-notes and Bonds. You also have to do a regression similar to the one described above. Here your dependent variable or Y for the regression will be the "Yield" as defined in column D and the dependent variables will be the various powers of T (T being the maturity given in Column B). Again, run the regression, estimate the coefficients, a - f and plug these back in for T = 0.5, 1, 1.5... to estimate Y(0.5), Y(1), Y(1.5)...

As a hint, the first entries in your table should be:

Price	Maturity	Coupon	Yield	T	T2	T3	T4	T5
101.03	0.0833333	6.75	2.34	0.0833	0.0069	0.0006	0.0000	0.0000
101.84	0.3333333	6.5	2.71	0.3333	0.1111	0.0370	0.0123	0.0041

And your estimated coefficients should be close to:

	Coefficient
a	2.5944
b	0.5127
c	-0.0794
d	0.0065
e	-0.0003
f	0.0000

The Y(T) that you have estimated above is the par rate for different maturities. Given these par rates you have to back out the discount function for different maturities. In order to do this, **assume that each of the bonds trades at par or at a value of $100. Do not use the price in column 1 to bootstrap the discount function.** As an example, once you have run the above regression and plugged back the values of the coefficients, you will find that:

$$Y(0.5) = 2.831630$$
$$Y(1.0) = 3.033882$$

You can use these values to determine the discount functions D(0.5) and D(1) as follows. Assume that the price of the bond is $100 (these are par bonds).

$$100 = \frac{2.831630}{2} * D(0.5) + 100 * D(0.5)$$
$$D(0.5) = 0.9860$$

Similarly, for the 1 year bond:

$$100 = \frac{3.033882}{2} * D(0.5) + \frac{3.033882}{2} * D(1) + 100 * D(1)$$

$$D(1.0) = 0.9703$$

Note that to determine the value of D(1.0), you had to use the value of D(0.5) determined in the previous step. This is what we mean by bootstrapping. Complete this procedure for all the bonds, to determine the values of D(0.5) - D(25). Once you have the values of the discount functions, you can compute the spot and forward rates, using the procedure described for question 1.