MFE 237I: Financial Risk Management
Problem set 7

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due 5/31 before midnight

You should work with your assigned group but should write up your answer individually. Give the name of your group members in your writeup and **post your answer on CCLE** before Wednesday 5/31 at midnight.

1 Bootstrapping CDS curve

1. Recover the hazard rate curve from slide 15 of the notes.
2. Use this hazard rate curve to price a 7-year bond on the same company which pays 2.5% coupon every 6 month and has face value $100.

2 Dynamic credit model

Consider 8 categories: AAA, AA, BBB, BB, B, CCC and default. We are interested in constructing a stochastic dynamic model of rating and default in continuous time. For this we will use the information in slide 7 of the notes.

1. Let us call $P(t)$ the 8×8 matrix of transition probability after time t. This means that $P_{ij}(t)$ is the probability of being in category j at date t if the firm is in category i at date 0.
   (a) What is $P(0)$?
   (b) What is $P(1)$?
2. Just like we defined the hazard rate has the instantaneous probability of default, we can consider instantaneous transition probability λ_{ij} such that $\lambda_{ij} dt$ is the probability of going from rating i to rating j during an interval dt if $i \neq j$. When $i = j$, we define λ_{ii} as the opposite of the intensity of leaving state i: $\lambda_{ii} = -\sum_{j \neq i} \lambda_{ij}$. We can put all these in a matrix Λ. Express Λ as a function of P and its first derivative.
3. Assuming that Λ is constant over time, derive an expression relating $P(1)$ and Λ.

4. Compute Λ for the values of slide 7.
5. Use this matrix Λ to compute the probabilities of default at horizon 1, 2, 3, 4, 5, 7, and 10 years given each initial rating.
6. Compare your results to slide 6 of the notes. What can explain the similarities and differences?
7. Use this model to price a 7-year bond on a BBB company which pays 2.5% coupon every 6 month and has face value \$100. Assume that the risk-free interest rate is 0% and recovery is 60%
8. Compute the 3, 5, and 10-year CDS spreads for the same company.

3 CDS-Bond Basis

1. Explain how to combine a 1-year zero-coupon bond and 1-year CDS on the same firm to form a risk free asset. Derive a relation between CDS price, bond price, and risk-free bond price. To simplify, assume the following:
   - The bond has face value F, the interest rate is i, the default probability d and the recovery is a random number ρ between 0 and 1.
   - The CDS terms are the following: in case of a default you receive $(1-\rho)F$, in case of no default you receive nothing.
2. If this no arbitragre relation does not hold, the difference is called the CDS-Bond basis. Find some evidence on the behavior of the CDS-Bond basis, over roughly the last 15 years. Discuss the evidence: when are CDS cheaper than bonds, when do we see the opposite, what can explain those variations?