MFE 237I: Financial Risk Management
Problem set 8

Valentin Haddad

due 6/7 before midnight

You should work with your assigned group but should write up your answer individually. Give the name of your group members in your writeup and **post your answer on CCLE** before Wednesday 6/7 at midnight.

1 CVA and DVA

Consider a European call option on a non-dividend-paying stock where the stock price is \$52, the strike price \$50, the risk-free rate is 5%, the volatility is 30%, and the time to maturity is one year. Answer the following questions assuming no recovery in the event of default, that the probability of default is independent of the option valuation, no collateral is posted, and no other transactions between the parties are outstanding.

1. What is the value of the option assuming no possibility of a default?
2. What is the value of the option to the buyer if there is a 2% chance that the option seller will default at maturity?
3. Suppose that, instead of paying the option price up front, the option buyer agrees to pay the option price (with accumulated interest) at the end of option's life. By how much does this reduce the cost of defaults to the option buyer in the case where there is a 2% chance of the option seller defaulting?
4. If in case (3) the option buyer has a 1% chance of defaulting at the end of the life of the option, what is the default risk to the option seller? Discuss the two-sided nature of default risk in the case and the value of the option to each side.

2 CDOs

Consider the following stylized model of CDOs. We assume no difference between historical and risk-neutral default probabilities.

- Two $1 bonds
- Each bond has a 5% probability of paying nothing
- Against this pool of loans, we issue two claims:
  - A junior tranche that pays $1 if both securities deliver and $0 if either one defaults.
  - A senior tranche that pays $1 if either security delivers and $0 if both default.
- We assume independent default events.

For each of the following questions, show your calculations.

1. Compute the price of the two tranches. Assuming that bond defaults are perfectly correlated, compute again the price of the two tranches.
2. Assume that the joint distribution of time to default of the two bonds can be represented by a normal bivariate copula with correlation ρ. Plot the default probability of the two tranches as a function of ρ. Comment on the role of the correlation for the default probability of the tranches.
3. Go back to the uncorrelated case. Now assume we have a portfolio of three bonds. We form three tranches: Senior (defaults if all underlying bond defaults), Mezzanine (defaults if 2 or more bonds default) and Junior (defaults if any underlying bond defaults). Comment on the impact of adding more uncorrelated assets for the default probability of the senior-most tranche.
4. Now imagine you have three identical CDOs like in question 2. Assume you create a pool with the three mezzanine tranches of the asset and form a new CDOs. This new asset is called a CDO^2 or CDO-squared. Comment on the riskiness of the two senior-most tranches of the CDO-squared.
5. Redo the calculations of questions 3 and 4, now assuming a default probability of 10%. Express for each tranche the ratio of the default probability with 10% default risk vesus with 4% default risk. Comment on the sensitivity of the various tranches to the default probability.