MFE 237I: Financial Risk Management
Problem set 4

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due 5/8 before midnight

You should work with your assigned group but should write up your answer individually. Give the name of your group members in your writeup and **post your answer on CCLE** before Monday May 8 at midnight.

1 VaR for option on two underlying

1. We are interested in managing the risk of an option on two stocks with prices $S_{1,t}$ and $S_{2,t}$. Assume a short rate r and:

$$\begin{aligned}\frac{dS_{1,t}}{S_{1,t}} &= \mu_1 dt + \sigma_1 dW_{1,t} \\ \frac{dS_{1,t}}{S_{1,t}} &= \mu_2 dt + \sigma_2 dW_{2,t} \\ corr(dW_{1,t}, dW_{2,t}) &= \rho dt\end{aligned}$$

   where all the parameters are in daily units. Call $M(S_{1,t}, S_{2,t})$ the price of the option. *Derive a formula for the 99%-VaR for the option using the delta approach.*

2. *Derive a formula for the 99%-VaR for the option using the delta-gamma approach.*

3. Consider the case of a European option on the minimum of the two stock price, with maturity T and final payoff:

$$M_T = \max\left(\min\left(S_{1,T}, S_{2.T}\right) - K, 0\right)$$

   The price of the option when the time to maturity is τ is:

$$\begin{aligned}M(S_1, S_2) = {} & S_1 \mathcal{N}_2 \left(\gamma_1 + \sigma_1\sqrt{\tau}, \left(\ln\left(S_2/S_1\right) - \frac{1}{2}\sigma^2\sqrt{\tau}\right) / \left(\sigma\sqrt{\tau}\right), \left(\rho\sigma_2 - \sigma_1\right)/\sigma\right) \\ & + S_2 \mathcal{N}_2 \left(\gamma_2 + \sigma_2\sqrt{\tau}, \left(\ln\left(S_1/S_2\right) - \frac{1}{2}\sigma^2\sqrt{\tau}\right) / \left(\sigma\sqrt{\tau}\right), \left(\rho\sigma_1 - \sigma_2\right)/\sigma\right) \\ & - Ke^{-r\tau}\mathcal{N}_2\left(\gamma_1, \gamma_2, \rho\right)\end{aligned}$$

where

$$\gamma_1 = \frac{\ln(S_1/K) + \left(r - \frac{1}{2}\sigma_1^2\right)\tau}{\sigma_1\sqrt{\tau}}$$
$$\gamma_2 = \frac{\ln(S_2/K) + \left(r - \frac{1}{2}\sigma_2^2\right)\tau}{\sigma_2\sqrt{\tau}}$$
$$\sigma^2 = \sigma_1^2 + \sigma_2^2 - 2\rho\sigma_1\sigma_2$$

and $\mathcal{N}_2(\alpha, \beta, \theta)$ is the bivariate cumulative standard normal distribution with upper limits α and β and correlation θ. That is, if X_1 and $X_2$are standard normal with correlation θ:

$$\mathcal{N}_2(\alpha, \beta, \theta) = P(X_1 \leq \alpha, X_2 \leq \beta)$$

Assume: $r = 0.005\%, \sigma_1 = \sigma_2 = 2\%, \rho = 0.4, \mu_1 = \mu_2 = 0.03\%$. Further assume that at date 0, we have $T =$ 6months, and that $S_{1,0} = 99$, $S_{2,0} =$ 101 and $K = 100$. *Compute the price of the option at date* 0.

4. *Compute the VaR for the option using the two formulas you have derived before. Compare the results and explain the intuition behind this result.*

5. *Compute the VaR for the option using simulations. Compare to the results of the previous question and explain the intuition behind this result.*

6. *Now assume you are a trader in the real world and you do not know for sure that the model for the underlying is correct. What other types of risks would you worry about? If you had to worry about just one more risk, what would it be? Explain (quantitatively) how you get to this conclusion.*

2 Interview questions

1. Two options on the same underlying have the same Delta but different Gamma. Which one has the largest VaR?

2. If the EUR/USD exchange rate has drift $\mu$and volatility σ, what are the drift and volatility of the USD/EUR exchange rate?

3. You are long a call option on IBM stock. You have delta hedged your position. You hear on the radio that the CEO of IBMhas just been arrested for running a massive Ponzi scheme. The stock price plunges \$10. How do you adjust your hedge, i.e. do you sell stock and lend or buy stock and borrow?