Computational Finance Problem Set 2

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QUESTION 1

The ρ calculated using simulation is **-0.709102**

QUESTION 2

The expected value using simulation is **1.56475**

QUESTION 3

a)

Expected value of $W_5^2 + \sin(W_5)$ = **5.08431**

Expected value of $e^{\left(\frac{0.5}{2}\right)} \cos(W_{0.5})$ = **0.996066**

Expected value of $e^{\left(\frac{3.2}{2}\right)} \cos(W_{3.2})$= **0.980965**

Expected value of $e^{\left(\frac{6.5}{2}\right)} \cos(W_{6.5})$= **1.02221**

b)

All the values for the last 3 integrals tend towards 1 as we increase the number of simulations. This matches the theoretical expectation of 1, as $\mathrm{E}(\cos(\mathrm{W_t})) = \mathrm{e}^{-\frac{t}{2}}$

c)

Variance Reduction:

For all 4, we can perform a **control variate** variance reduction strategy

This strategy involves

- Finding a Y, which has positive correlation with X (original value)
- Generating a T = X - gamma * (Y- E(Y)), where gamma is a constant

- Make sure that Var(T) < Var(X)

Let us take W_5^2 as the Y for 1st integral (Expected Value = 5), and $\cos(W_t)$ as the Y for last 3 integrals (Expected value = $e^{\left(\frac{-t}{2}\right)}$)

Expected values of all 4 remains almost the same with variance reduction

$W_5^2 + \sin(W_5)$ = **5.00689**, $e^{\left(\frac{0.5}{2}\right)} \cos(W_{0.5})$ = **0.99913,** $e^{\left(\frac{3.2}{2}\right)} \cos(W_{3.2})$= **0.984808**

$e^{\left(\frac{6.5}{2}\right)} \cos(W_{6.5})$= **1.02135**

There is a reduction in variance using this method for all 4 calculations.

Calculation	Standard Deviation (Normal)	Standard Deviation with Variance Reduction
$W_5^2 + \sin(W_5)$	7.20671	0.707259
$e^{\left(\frac{0.5}{2}\right)} \cos(W_{0.5})$	0.361974	0.0800684
$e^{\left(\frac{3.2}{2}\right)} \cos(W_{3.2})$	3.36572	2.6862
$e^{\left(\frac{6.5}{2}\right)} \cos(W_{6.5})$	18.2781	17.5694

As can be seen, the standard deviation reduces for all the 4 calculations using the control variate variance reduction technique

QUESTION 4

a)

Call Price using simulation = **18.4804**

b)

Call Price using Black Scholes = **18.2838**

Variance reduction - We use the antithetic method.

1) Calculate call price (c2) using -z (negative of random numbers)

2) Set (original price + c2)/2 as new set of prices and calculate mean and standard deviation

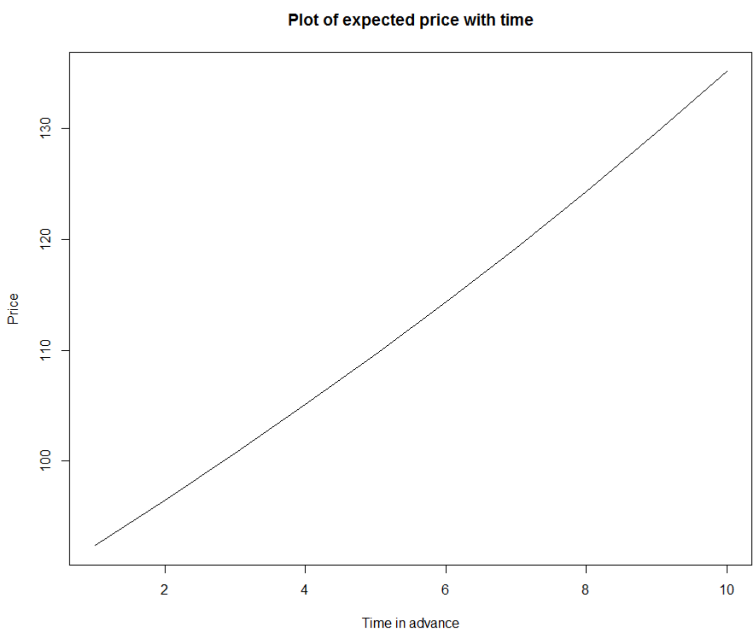
Call Price after antithetic variance reduction = 18.4173

Standard Deviation (Normal)	Standard Deviation after antithetic reduction
32.344	18.7054

As it can be seen, the standard deviation is reduced by this method. This is because the covariance between the variables is negative. This makes the average of the 2 variables have lesser covariance than just one of them.

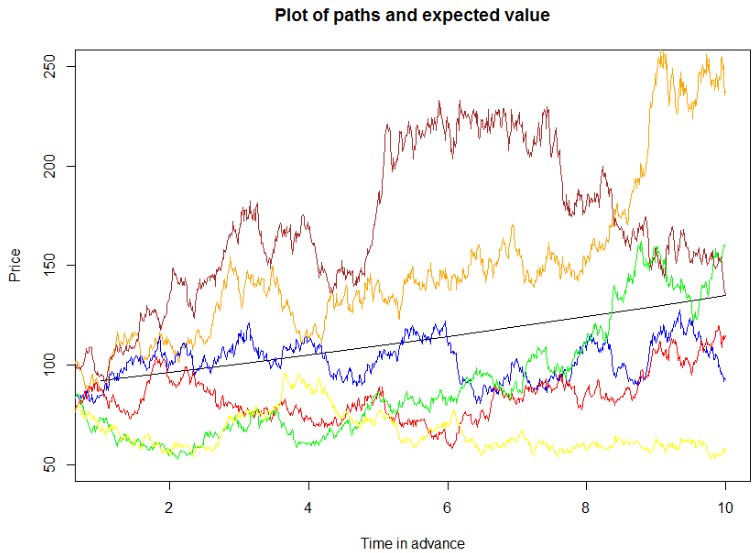
QUESTION 5

a) Plot of all $E(S_n)$ is as below



b) Perform simulation for 6 paths, with total time = 10, and each increment = 10/1000 = 0.01

c) Plot of all paths along with mean line from a) part

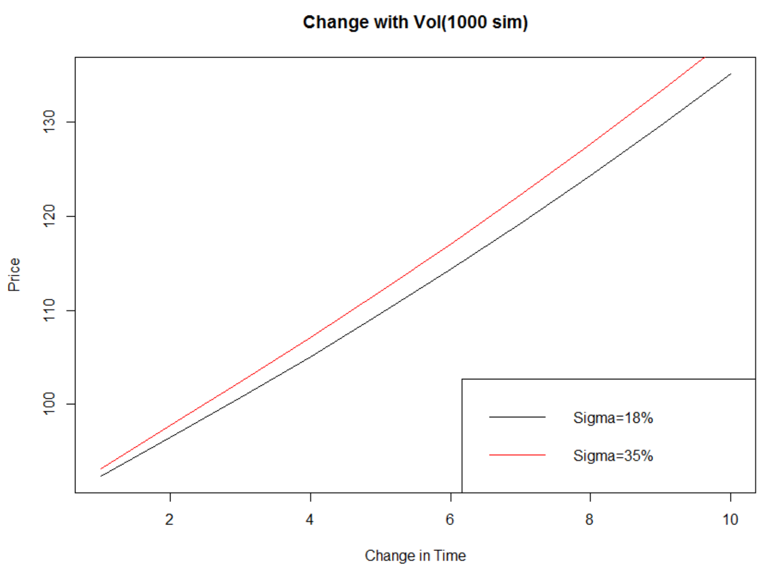


The black line is the mean from a) part. The colored lines are the random paths.

d)

Expected Values

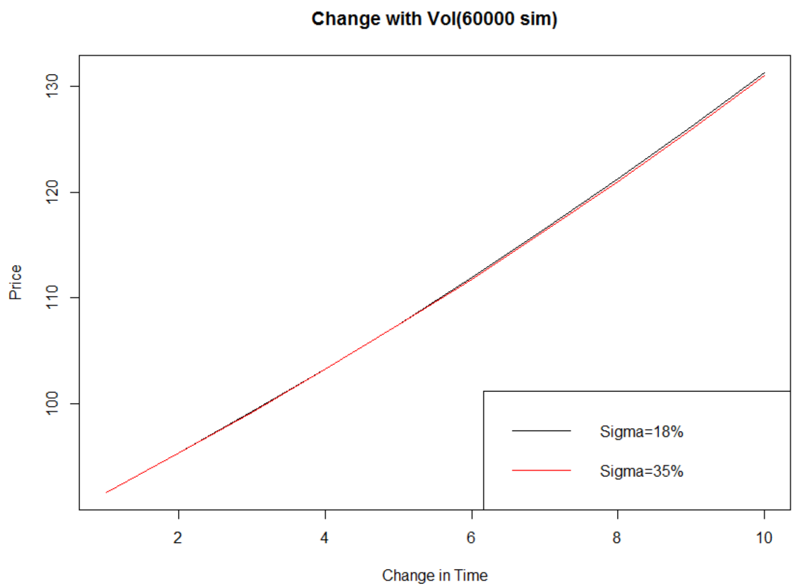
σ	1	2	3	4	5	6	7	8	9	10
0.18	92.34	96.47	100.71	105.09	109.63	114.35	119.25	124.35	129.65	135.17
0.35	93.16	97.74	102.38	107.13	112.03	117.08	122.3	127.7	133.27	139.03



For this sample, the prices have increased with increase in sigma.

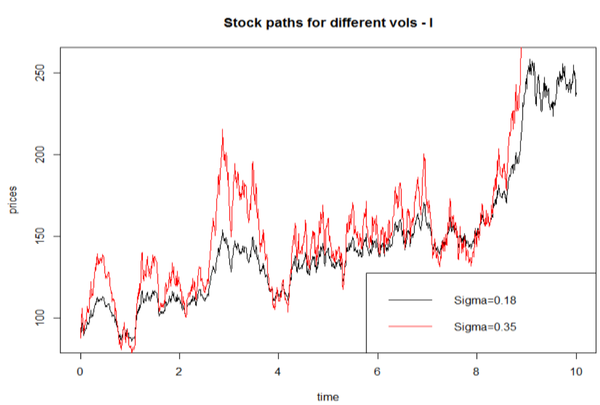
But when the number of simulations are increased, the expected value converges to the same value. This is as per the theoretical expected value e^{rt} , in which there is no volatility term

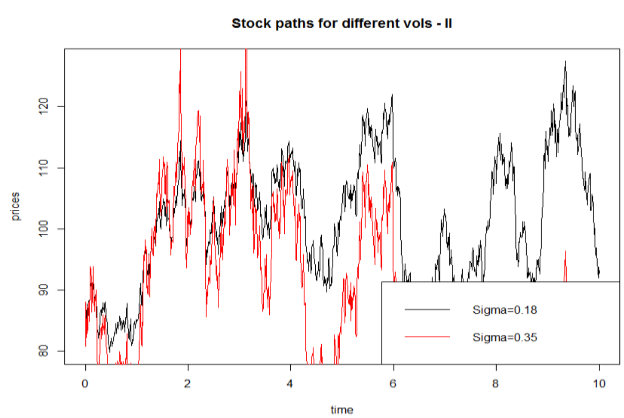
Here is the graph for 60000 iterations



Stock paths

The increase in sigma, increases the randomness of the paths and makes the swings more wild





QUESTION 6

The value of π from Euler Discretization is **3.14159**

The expected value of π from Monte Carlo Simulation is **3.14217**

The expected value of π from Importance Sampling is **3.13229**

Standard Deviation (Normal)	Standard Deviation (Importance Sampling)
0.896563	0.317536

As can be noticed, there is a reduction in the standard deviation after important sampling is applied.