Computational Finance Homework 5

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Question 1

The American Put price is calculated for all combinations of initial stock price (S0,time K and polynomial type). This involves following the Longstaff-Schwartz Least Square Monte Carlo process for 100,000 paths (50000 with normal random numbers and 50000 with their antithetic version).

Part a

Laguerre Method

Table 1: Various combinations of Laguerre method

K	S0	Time	Price
2	36	0.5	3.9898083
3	36	0.5	4.0820664
4	36	0.5	4.2311124
2	40	0.5	1.4691544
3	40	0.5	1.7934998
4	40	0.5	1.9010330
2	44	0.5	0.5193598
3	44	0.5	0.6579726
4	44	0.5	0.6652625
2	36	1.0	3.9824003
3	36	1.0	4.1003357
4	36	1.0	4.3203969
2	40	1.0	1.5428079
3	40	1.0	1.9573960
4	40	1.0	2.2868338
2	44	1.0	0.6588360
3	44	1.0	0.9082778
4	44	1.0	1.1219611
2	36	2.0	3.9690197
3	36	2.0	4.1225006
4	36	2.0	4.3778332
2	40	2.0	1.6241695
3	40	2.0	2.0509575
4	40	2.0	2.4651989
2	44	2.0	0.7730722
3	44	2.0	1.0828167
4	44	2.0	1.3711712

Part b

Hermite Method

Table 2: Various combinations of Hermite method

	K	S0	Time	Price
28	2	36	0.5	4.2278499
29	3	36	0.5	4.2481401
30	4	36	0.5	4.2651242
31	2	40	0.5	1.8524034
32	3	40	0.5	1.9214519
33	4	40	0.5	1.9191456
34	2	44	0.5	0.6458856
35	3	44	0.5	0.6742260
36	4	44	0.5	0.6779837
37	2	36	1.0	4.5411902
38	3	36	1.0	4.5847754
39	4	36	1.0	4.6021074
40	2	40	1.0	2.4054154
41	3	40	1.0	2.4972551
42	4	40	1.0	2.4929797
43	2	44	1.0	1.1487614
44	3	44	1.0	1.2035667
45	4	44	1.0	1.2084382
46	2	36	2.0	4.9545767
47	3	36	2.0	5.0416918
48	4	36	2.0	5.0384275
49	2	40	2.0	3.0229030
50	3	40	2.0	3.1339812
51	4	40	2.0	3.1301712
52	2	44	2.0	1.7606173
53	3	44	2.0	1.8397176
54	4	44	2.0	1.8481431

Part c

Monomial Method

Table 3: Various combinations of Monomial method

	K	S0	Time	Price
55	2	36	0.5	4.2278499
56	3	36	0.5	4.2481401
57	4	36	0.5	4.2651242
58	2	40	0.5	1.8524034
59	3	40	0.5	1.9214519
60	4	40	0.5	1.9191456
61	2	44	0.5	0.6458856
62	3	44	0.5	0.6742260
63	4	44	0.5	0.6779837
64	2	36	1.0	4.5411902
65	3	36	1.0	4.5847754
66	4	36	1.0	4.6021074
67	2	40	1.0	2.4054154
68	3	40	1.0	2.4972551
69	4	40	1.0	2.4929797
70	2	44	1.0	1.1487614
71	3	44	1.0	1.2035667
72	4	44	1.0	1.2084382
73	2	36	2.0	4.9545767
74	3	36	2.0	5.0416918
75	4	36	2.0	5.0384275
76	2	40	2.0	3.0229030
77	3	40	2.0	3.1339812
78	4	40	2.0	3.1301712
79	2	44	2.0	1.7606173
80	3	44	2.0	1.8397176
81	4	44	2.0	1.8481431

Observations
1) The price converges towards expected price from binomial pricing as we increase k
2) The Laguerre polynomial doesn't give a good estimate for k=2
3) Values of Hermite and Monomial are consistent with changes in k. Both the values are very similar because both have a similar structure (sum of monomials of the form x, x^2, x^3)

Time taken

Laguerre	Hermite	Monomial
6.0685	4.9605	5.207

As can be seen, the time taken for Hermite and Monomial are better than Laguerre. This shows that the ordinary monomials take lesser time as compared to the orthogonal basis functions like Laguerre polynomials.

On the whole, it is slightly better to use Monomials or Hermite over the other methods.

Question 2

a

European price can be calculated by 1) identifying strike as stock price at T=t
2) calculating payoff at T=T with previously identified strike
3) Discount back to t=0 to get price

```
## [1] 3.172378
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We get a price of **3.1724**

b

For American prices, we can use the Least Square Monte Carlo approach. It needs to be noted that for every simulation, we will have different strikes. So while running a regression at ever step, we will have more variation in the continuation value for each simulation. For this problem, we can assume that this won't cause an issue.

The following steps can be used
1) Identify strike as stock price at T=t
2) Now, trim the stock paths from T=t to T=T and input into the least Square Monte carlo function. Use type as monomial and k = 4
3) Discount the price coming out of the Least Square Monte Carlo function back to T=0.

```
## [1] 3.350598
```

We get a price of **3.3506**