Homework 8 Hints

237F

This assignment requires a reasonable amount of work but shouldn't be to hard conceptually. Basically, you are using the SMM model you developed in HW7 to value some fun and exciting exotic options. To do this you basically just value the terminal cash flow along a simulated path, discount back along that simulated path, then average across all the paths.

There may be two typos in the problem set: in problem 4, the 0.07 doesn't matter; in problem 5 you should use the 0.05 as the strike price, not 0.06.

General Comments

While the contracts that determine the cash flow differ, the basic approach to these problems is the same. Once you know the payoff formulae, then the price or value at time zero is:

$$p = E_t^Q \left[\sum_{i=1}^{N_x} x_{t_i} \exp\left(- \int_t^{t_i} r_s ds \right) \right] \tag{1}$$

where x_{t_i} is the cash flow associated with the security at time t_i. In the string model framework, this corresponds to the following formula

$$p = \frac{1}{N_{path}} \sum_{n=1}^{N_{path}} \sum_{i=1}^{N_x} \left(\prod_{j=0}^{i-1} D_n(j; j+1) \right) x_{ni} \tag{2}$$

Here n is the index over paths,i is the index over cash flows. So $D_n(j; j+1)$ is the 6-month 0-coupon bond price on path n from period j to period $j+1$. x_{ni} is the cash flow at time t_i on path n.

Speaking generally, if you have simulated paths for discount functions (what finance researchers frequently call "pricing kernels" or "state-price deflators") then you can effectively price almost any kind of financial derivative you can imagine. Of course, the models that are used to simulate various paths usually do not all agree, but that is a different issue.

As some of you have pointed out, there are sometimes analytical or closed-form solutions to the price of some of these derivatives, given a certain term-structure model. Frequently, however, these kinds of problems can only be solved through simulation. In any case, the derivation of these closed-form solutions is usually very difficult. As processing time is getting cheaper by the minute, simulations often offer a viable way to get numerical solutions to these kinds of problems without all the blood and guts commonly required to get closed-form solutions.

Problem Specific Comments

- Problem 1: Compute the CMS rates using the formula for par rates and the initial discount function.
- Problem 2: Note that you need to value the swap (i.e. the underlying) at the time of exercise as what the swap could be sold for, and then discount this back to $t = 0$. This means valuing the fixed and floating legs. The discount curve can be used at the time of exercise to value the fixed part. The floating part has a value of 1. Note that you can't take the realized cash flow streams as $(1/2)(s-L)$ along the remainder of the path and discount this with the paths as given. To do something like that, you would need to simulate paths starting from the exercise point on the particular path you were on. This is exactly analogous to a call option on stock. You don't calculate the value along a realized path based on having to hold the stock along that path past the exercise point. You calculate the payoff by taking how much the stock can be sold for minus the exercise price $(S-X)$. If you valued the payoffs from holding onto the stock over a large number of paths starting from the point where the option was exercised on a particular path, then you should get the same answer in both cases.
- Problem 3: For each spot rate maturity (by spot rates here I mean the actual spot rates at time 0, not the simulated spot rates, or futures rates) do the following for each spot rate maturity (i.e., $t = 0.5, 1, 1.5, \ldots, 10$):
  1. Shift the spot rate for that maturity UP by 1 basis point, leaving the rest of the term structure fixed.
  2. Generate a complete set of paths (however many simulations makes you feel complete, preferably more than one) for the discount rates with the single spot rate shifted UP.

3. Shift the spot rate for that maturity DOWN by 1 basis point, leaving the rest of the term structure fixed.

4. Generate a complete set of paths (however many simulations makes you feel complete, preferably more than one) for the discount rates with the single spot rate shifted DOWN.

Then, with these 'shifted' series of paths, recalculate the swaption price as in question 2. From this, you can calculate the DV01 as normal, but you will be getting the DV01 with respect to changes in each spot rate. Basically, instead of calculating a single DV01 for a parallel shift in the yield curve, you are simply calculating multiple DV01's for single spot rate shifts. Also, note that the spot rate with maturity t can be calculated from the initial zero-coupon bond price by rearranging the following (assuming semi-annual discrete compounding here):

$$D(t) = (1 + r/2)^{-2t}$$

Also, note that the forward swap rates are like the forward par rates.

- Problem 5: note that the CMS5 is a par rate at a particular time on a particular path, and is set in advance just like Libor.