

Problem Set 1

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Question 1

Construct the equal weighted market return, value weighted market return and lagged total market capitalization using CRSP Bond data. Your output should be from January 1926 to December 2015, at a month frequency.

Data Clean up

Before calculating the required return information, I cleaned up the bond data go out of CRSP database (CRSPA.TFZ_MTH table). This involved changing bond returns, which are equal to -99 to NA.

Calculation Process

- 1) Firstly sort the data based on Month and Year of each monthly entry. 2) Calculate the lag of the Bond market capitalization.
- 3) Now calculate the sum of the lag of market capitalization, value-weighted and equal-weighted returns of the various month and years.
- 4) The output should be constrained between Jan 1926 and Dec 2015.

Sample of the calculated output is as below:

| Year | Month | Bond_lag_MV | Bond_Ew_Ret | Bond_Vw_Ret |
|------|-------|-------------|-------------|-------------|
| 1931 | 1 | 809 | -0.0010403 | -0.0109272 |
| 1931 | 2 | 809 | 0.0022608 | 0.0080689 |
| 1931 | 3 | 809 | 0.0030875 | 0.0098680 |
| 2015 | 10 | 12064584 | -0.0028533 | -0.0030831 |
| 2015 | 11 | 12195107 | -0.0026195 | -0.0027881 |
| 2015 | 12 | 12296771 | -0.0014463 | -0.0015439 |

Question 2

Aggregate stock, bond and riskless datatables. For each year-month, calculate the lagged market value and excess value-weighted returns for both stocks and bonds. Your output should be from Jan 1926 to Dec 2015, at a monthly frequency.

Data

- 1) For this question, we use the stock returns based on the method used in the first problem set. The range of data is from Jan 1926 to Dec 2015.
- 2) The risk free information is retrieved from CRSP using the Treasury and inflation (CRSPA.MCTI) table in CRSP.

Calculation Process

- 1) Merge all the data table after sorting them based on month and year. 2) Subtract the 30-day T-bill returns (risk free for 30 days) from the stock and bond value weighted return. 3) The output is finally constrained between Jan 1926 to Dec 2015.

Sample output out of this function is as below:

| Year | Month | Stock_lag_MV | Stock_Excess_Vw_Ret | Bond_lag_MV | Bond_Excess_Vw_Ret |
|------|-------|--------------|---------------------|-------------|--------------------|
| 1931 | 1 | 41988.56 | 0.0628242 | 809 | -0.0114822 |
| 1931 | 2 | 44577.75 | 0.1087491 | 809 | 0.0076619 |
| 1931 | 3 | 49250.31 | -0.0632994 | 809 | 0.0079810 |
| 2015 | 10 | 27455716.59 | 0.0757357 | 12064584 | -0.0030561 |
| 2015 | 11 | 29465271.61 | 0.0027610 | 12195107 | -0.0028081 |
| 2015 | 12 | 29461661.85 | -0.0229034 | 12296771 | -0.0016469 |

Question 3

Calculate the monthly unlevered and levered risk-parity portfolio returns as defined by Asness, Frazzini and Pedersen (2012). For the levered risk-parity portfolio, match the value-weighted portfolio's σ over the longest matched holding period of both

This question has these calculations.

- 1) Calculate value weighted return of a portfolio using the stock and bond excess returns along with the stock and bond lagged market cap returns as weights.
- 2) 60-40 Portfolio return can be calculated by weighing the stock with 60% and bond with 40%.
- 3) σ^{-1} for both stock and bond on each day is the standard deviation of the asset excess returns over the previous 36 months, ignoring the current month.(perform shift operator on the standard deviation rollapply for 36 months). Let's keep the stock's value as σ_s^{-1} and bond's value as σ_b^{-1} .
- 4) The unlevered K can be calculated as $\frac{1}{\sigma_s^{-1} + \sigma_b^{-1}}$
- 5) For the levered K, we need to match the portfolio standard deviation to the value weighted market portfolio standard deviation.

$K_{lev} * sd(\Sigma \sigma_i^{-1} R_{it}) = \text{Volatility of market portfolio}$

$K_{lev} = \text{Market portfolio volatility} / sd(\sigma_s^{-1} \mathbf{R}_{st} + \sigma_b^{-1} \mathbf{R}_{bt})$

- 6) Using the levered K, we can then find the weight on both the assets ($k\sigma^{-1}$), and then we can calculate the return on the levered RP portfolio.

Sample of the results out of this question:

| Year | Month | Stock_Excess_Vw_Ret | Bond_Excess_Vw_Ret | Excess_Vw_Ret | Excess_60_40_Ret |
|------|-------|---------------------|--------------------|---------------|------------------|
| 1931 | 1 | 0.0628242 | -0.0114822 | 0.0614196 | 0.0331017 |
| 1931 | 2 | 0.1087491 | 0.0076619 | 0.1069472 | 0.0683142 |
| 1931 | 3 | -0.0632994 | 0.0079810 | -0.0621474 | -0.0347872 |
| 2015 | 10 | 0.0757357 | -0.0030561 | 0.0516825 | 0.0442190 |
| 2015 | 11 | 0.0027610 | -0.0028081 | 0.0011307 | 0.0005333 |
| 2015 | 12 | -0.0229034 | -0.0016469 | -0.0166439 | -0.0144008 |

| Year | Month | Stock_inverse_sigma_hat | Bond_inverse_sigma_hat | Unlevered_k | Excess_Unlevered_RP_Ret |
|------|-------|-------------------------|------------------------|-------------|-------------------------|
| 1931 | 1 | 13.86384 | 86.38830 | 0.0099748 | -0.0012063 |
| 1931 | 2 | 13.69634 | 85.50535 | 0.0100805 | 0.0216185 |

| Year | Month | Stock_inverse_sigma_hat | Bond_inverse_sigma_hat | Unlevered_k | Excess_Unlevered_RP_Ret |
|------|-------|-------------------------|------------------------|-------------|-------------------------|
| 1931 | 3 | 13.28032 | 84.97742 | 0.0101773 | -0.0016531 |
| 2015 | 10 | 35.47001 | 142.80446 | 0.0056093 | 0.0126205 |
| 2015 | 11 | 33.28532 | 142.41370 | 0.0056916 | -0.0017531 |
| 2015 | 12 | 33.26018 | 142.23957 | 0.0056980 | -0.0056754 |

| Year | Month | Levered_k | Excess_Levered_RP_Ret |
|------|-------|-----------|-----------------------|
| 1931 | 1 | 0.0261428 | -0.0031617 |
| 1931 | 2 | 0.0261428 | 0.0560658 |
| 1931 | 3 | 0.0261428 | -0.0042464 |
| 2015 | 10 | 0.0261428 | 0.0588193 |
| 2015 | 11 | 0.0261428 | -0.0080524 |
| 2015 | 12 | 0.0261428 | -0.0260390 |

Question 4

Replicate and report annualized excess returns, t-statistics of average excess returns, annualized volatility, annualized Sharpe Ratio, skewness and excess kurtosis. Your sample should be from January 1926 to June 2010 at monthly frequency. Discuss the difference between your table and the table reported in the paper

Given monthly returns on the stocks, bonds, value weighted, 60-40 weighted, unlevered risk parity and levered risk parity portfolios, we can calculate the annualized mean, standard deviation, sharpe ratio (arithmetic), along with t test value for the mean, skewness and excess kurtosis.

The data is restricted between January 1929 and June 2010 for these calculations.

Mean : Means of the monthly returns for all the portfolios were calculated and annualized on an arithmetic basis (i.e. $\text{Mean Return}_{\text{annual}} = \text{Mean Return}_{\text{monthly}} * 12$)

Standard Deviation : Standard deviations of the monthly returns of all the portfolios were calculated and annualized on an arithmetic basis (i.e. $SD_{\text{annual}} = SD_{\text{monthly}} * \sqrt{12}$)

Sharpe Ratio : Sharpe ratio was calculated on a monthly basis by dividing the annualized mean and annualized standard deviation.

t-test : T stat of the the means was calculated by using the `t.test()` R function on the returns distribution.

Skewness, Excess Kurtosis : Skewness and Excess kurtosis was calculated using the R functions for the entire data set. No form of annualization was done for these metrics.

| | Excess Return | t-stat | Volatility | Sharpe Ratio | Skewness | Excess Kurtosis |
|--------------------------|---------------|--------|------------|--------------|----------|-----------------|
| CRSP Stocks | 6.78 | 3.21 | 19.07 | 0.36 | 0.21 | 7.85 |
| CRSP Bonds | 1.50 | 4.16 | 3.26 | 0.46 | -0.02 | 4.61 |
| Value-weighted portfolio | 3.95 | 2.35 | 15.18 | 0.26 | 0.46 | 13.87 |
| 60/40 portfolio | 4.67 | 3.60 | 11.69 | 0.40 | 0.23 | 7.78 |
| unlevered RP | 2.17 | 4.63 | 4.24 | 0.51 | 0.05 | 4.69 |
| levered RP | 7.91 | 4.72 | 15.12 | 0.52 | -0.36 | 1.97 |

The difference might be due to the

- 1) **Difference in date ranges :** The question asks us to restrict the data from Jan 1929 to June 2015. But the research paper uses Jan 1926 to June 2015. When I change to the range in the research paper,

the bond results match more accurately, but the stock results don't match the paper.

- 2) **Difference in data retrieval and manipulation** : The data for calculation of stocks might be different between what was used in paper compared to what was used in this assignment. Detail steps on how each data set was created is not provided in the paper. As all the combined portfolios depend on the stock data out of CRSP, their values are affected by this as well.