Risk Management Homework 4

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Question 1

The change in value of the option can be shown as a change in price of stock multiplied by its delta (using taylor's series)

M_{t+1} - M_t = $\Delta_1 dS_1$ + $\Delta_2 dS_2$

$= \Delta_1(\mu_1 S_1 dt + \sigma_1 S_1 dW_{t1})$ + $\Delta_2(\mu_2 S_2 dt + \sigma_2 S_2 dW_{t2})$
$= (\Delta_1 \mu_1 S_1 + \Delta_2 \mu_2 S_2)dt + (\Delta_1 \sigma_1 S_1 dW_{t1} + \Delta_2 \sigma_2 S_2 dW_{t2})$

So the dt term is the μ_c and the σ_c is the combined standard deviation of the two dW_t terms.

VaR = -μ_c + 2.326 σ_c

$= -(\mathbf{\Delta_1}\mu_\mathbf{1}\mathbf{S_{t,1}} + \mathbf{\Delta_2}\mu_\mathbf{2}\mathbf{S_{t,2}}) + \mathbf{2.326}(\sqrt{\mathbf{\Delta^2{}_1 S^2{}_{t,1}}\sigma^\mathbf{2}{}_\mathbf{1} + \mathbf{\Delta^2{}_2 S^2{}_{t,2}}\sigma_\mathbf{1}{}^\mathbf{2} + \mathbf{2\Delta_1\Delta_2}\sigma_\mathbf{1}\sigma_\mathbf{2}\mathbf{S_1 S_2}\rho})$

Question 2

For the delta gamma method, we need to introduce more terms from the taylor's expansion. These terms will include the gamma of the stocks

M_{t+1} - M_t = $\Delta_1 dS_1$ + $\Delta_2 dS_2$ + $\frac{1}{2}\gamma_1 dS_1{}^2$ + $\frac{1}{2}\gamma_2 dS_2{}^2$ + $\gamma_{1,2} dS_1 dS_2$

$= (\Delta_1 \mu_1 S_1 + \Delta_2 \mu_2 S_2)dt + (\Delta_1 \sigma_1 S_1 dW_{t1} + \Delta_2 \sigma_2 S_2 dW_{t2})$ + $\frac{1}{2}\gamma_1 \sigma_1{}^2 S_1{}^2 dt$ + $\frac{1}{2}\gamma_2 \sigma_2{}^2 S_2{}^2 dt$ + $\gamma_{1,2} S_1 S_2 \sigma_1 \sigma_2 \rho dt$

$= (\Delta_1 \mu_1 S_1 + \Delta_2 \mu_2 S_2 + \frac{1}{2}\gamma_1 \sigma_1{}^2 S_1{}^2 + \frac{1}{2}\gamma_2 \sigma_2{}^2 S_2{}^2 + \gamma_{1,2} S_1 S_2 \sigma_1 \sigma_2 \rho)dt + (\Delta_1 \sigma_1 S_1 dW_{t1} + \Delta_2 \sigma_2 S_2 dW_{t2})$

So the dt term is the μ_c and the σ_c is the combined standard deviation of the two dW_t terms.

VaR = -μ_c + 2.326 σ_c

$= -(\mathbf{\Delta_1}\mu_\mathbf{1}\mathbf{S_1} + \mathbf{\Delta_2}\mu_\mathbf{2}\mathbf{S_2} + \mathbf{\frac{1}{2}}\gamma_\mathbf{1}\sigma_\mathbf{1}{}^\mathbf{2}\mathbf{S_1}{}^\mathbf{2} + \mathbf{\frac{1}{2}}\gamma_\mathbf{2}\sigma_\mathbf{2}{}^\mathbf{2}\mathbf{S_2}{}^\mathbf{2} + \gamma_\mathbf{1,2}\mathbf{S_1 S_2}\sigma_\mathbf{1}\sigma_\mathbf{2}\rho)+$

$\mathbf{2.326}(\sqrt{\mathbf{\Delta^2{}_1 S^2{}_{t,1}}\sigma^\mathbf{2}{}_\mathbf{1} + \mathbf{\Delta^2{}_2 S^2{}_{t,2}}\sigma_\mathbf{1}{}^\mathbf{2} + \mathbf{2\Delta_1\Delta_2}\sigma_\mathbf{1}\sigma_\mathbf{2}\mathbf{S_1 S_2}\rho})$

Question 3

The price of the option can be calculated by using the closed form solution provided in the question. All the values are annualized while using the closed form solution. The price of the option at t=0 is

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## [1] 1.728485
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Question 4

The formula from question 1 and question 2 contain delta and gamma terms. this can be calculated by checking how the price of the option and the delta changes by increasing the stock price by 0.01. The cross gamma is calculated by increasing both the stock prices by 0.01 simultaneously.

The option price from both the methods are as follows:
Delta Approach

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## [1] 0.472288
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So the VaR using the delta approach is 47.23%.

Delta Gamma approach

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## [1] 0.460004
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The VaR using the delta gamma approach is 46%. As can be seen, the VaR due to the Delta gamma approach has reduced compared to the Delta approach. This is because convexity is always helpful and has increased the returns. The delta approach makes a big approximation which can be very wrong as the convexity increases.

Question 5

The VaR from simulation can be calculated by finding the price of the option at t=0 and t=1 and then finding the return based on that. For calculating the price of the option at t=1, we need to simulate the price of S1 and S2 at t=1 using the provided stochastic equation. The value of VaR is

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## [1] 0.4833693
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This value of VaR is 48.33%, which is higher than that produced out of both the Gamma and Delta approaches. This is because simulation considers other types of risks as well (like vega, rho etc.). Due to this the VaR value is greater than others.

Question 6

We would need to worry about the other greeks, for which we have assumed the values are the same. This involves the greeks of Rho (wrt interest rate), theta (wrt time decay) and Vega (wrt sigma). The value of these greeks are as below:

Vega1	Vega2	Theta	Rho
2.135479	1.614589	-8.555906	9.777822

In Option trading, one of the most important greek is the vega. Volatility is an important input into the black scholes equation and how option price changes with change in volatility is a key risk which needs to be considered.

Interview Questions

Question 1

Higher Gamma leads to a higher VaR. This is because higher gamma means higher returns and higher risk. This will in turn increase the VaR.

Question 2

Assume the exchange process follows a GBM dX = μX_t dt + $\sigma X_t dW_t$
d($\frac{1}{X_t}$) = $\frac{-1}{{X_t}^2} dS_t + \frac{1}{{X_t}^3} d{X_t}^2$

$= \mathbf{\frac{1}{X_t}(\sigma^2 - r)dt - \frac{1}{X_t}\sigma dW_t}$

So it drifts with mean $(\sigma^2 - r)$ and volatility $-\sigma$

Question 3

Initially you are long a call option. To Delta hedge this, we need to short the stocks. Assume the delta of the call option = 0.7. In this case, we are short 0.7 units of stock

When the stock price falls, the delta of the call option will fall (as gamma is positive). Assume it falls to 0.1. So we need to reduce the short position of the stock. For that, we need to **buy stock and borrow**.