Heimadæmi – heimadæmi 3

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1.

GR 3

1. ⓐ $T(n) = 8T(n/2) + n^3$ Dýpt $= \log_2 n$

Tré → n^3 $\qquad$ n^3

$n/2 \; \ldots\ldots$ $\qquad$ $8 \cdot \frac{n}{2} = 4n$

$8^2 \cdot \frac{n}{4} = 16n$

$T(n) = \sum_{i=0}^{\log n} 8^i \cdot \frac{n}{2^i} = 4^i n \Rightarrow T(n) = n^{\log_2 8} = n^3$ //

ⓑ $T(n) = 10T(n/3) + n$ Dýpt $= \log_3 n$

Tré → $\sum_{i=0}^{\log n} 10^i \frac{n}{3^i}$

n

$10 \cdot n/3 = \frac{10}{3} n = 3{,}33n$

$10^2 \cdot n/9 = \frac{100}{9} n = 11{,}111n$

$10^3 \cdot n/27 = \frac{1000}{27} n = 37{,}037\ldots n$

vaxandi kvótaruna

$\Rightarrow n^{\log_3 10} \approx n^{2{,}095\ldots}$ //

ⓒ $T(n) = 3T(n-1) + 1$

$T(0) = 0$

$T(1) = 3T(0) + 1 = 1$

$T(2) = 3 \cdot 1 + 1 = 4$

$T(3) = 3 \cdot 4 + 1 = 13$

$T(4) = 3 \cdot 13 + 1 = 40$

virðist vera: $\frac{3^n - 1}{2}$

venjuleg þrepun

Gr: $T(0) = \frac{3^0 - 1}{2} = 0$

Þrepunar tilvik: Grf. að gildi fyrir n-1

$T(n) = 3 \cdot (n-1) + 1$

$= 3 \cdot \left(\frac{3^{n-1} - 1}{2}\right) + 1$

$= \frac{3^n - 3}{2} + \frac{2}{2} = \frac{3^n - 1}{2}$ OK

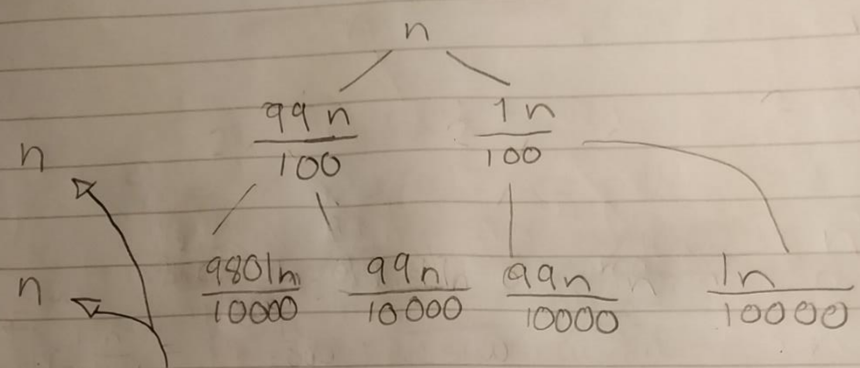
svo $O(3^n)$ //

• Þá er reiknirit B með $O(n^{2{,}095\ldots})$ best af þessum 3.

2.

99% 1%

2. $T(n) = T(0{,}99n) + T(0{,}01n) + n$

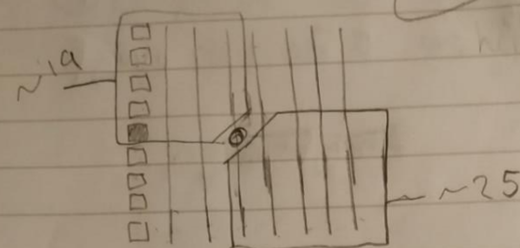


allir liðir jafnir, svo þetta er ennþá nlogn tímaflækja. það þarf enn verri skiptingu til þess að þetta verði $O(n^2)$

3.

GR 3

3. (a) $T(n) = T\left(\frac{n}{9}\right) + T\left(\frac{13n}{18}\right) + O(n)$



$T\left(\frac{n}{9}\right)$: Finna MoM í fyllkinu
$T\left(\frac{13n}{18}\right)$: Versta mögulega tilfelli í endurkvæmninni

$5n : 9 = \frac{5}{9}n$
$5n : 9 \cdot \frac{1}{2} = 5/18\, n$
svo amk. $5/18\, n$ öðrum megin svo $13/18\, n$ í versta falli.

$O(n)$: kostnaður við skiptingu

(b)

	n			n
$\frac{2}{18}n$		$\frac{13}{18}n$		$\frac{5}{6}n$
$\frac{1}{81}n$	$\frac{13}{162}n$	$\frac{13}{162}n$	$\frac{169}{324}n$	$\frac{25}{36}n$

Lækkandi kvótaröð, svo $O(n)$

4.

4. (a) $n^{\log_2 3} \approx n^{1,58} \to$ þegar $n/2 \to n^{\log_2}$ (r)

$n^{\log_3 6} \approx n^{1,6309} \to$ þegar $n/3 \to n^{\log_3}$ (r)

$n^{\log_3 5} \approx n^{1,465}$ svo 5 eða færri margf.

n $\quad$ n $\quad$ 3margf = $n \log n$

$n/3 \quad n/3 \quad n/3$ $\quad$ n

$n/3 \quad n/3 \quad n/3$ $\quad$ $9 \cdot \frac{n}{3^2} = n$

$= n$

n $\quad$ n $\quad$ 5margf = $n^{1,465}$

$n/3 \quad n/3 \quad n/3 \quad n/3 \quad n/3$ $\quad$ $5/3 n$

$25/9 n$

n $\quad$ n $\quad$ 4margf = $n^{1,26}$

$n/3 \quad n/3 \quad n/3 \quad n/3$ $\quad$ $4/3 n$

$16/9 n$

svo 5 eða færri

(b) $\log_4 9$ er sama og $\log_2 3$ eða 1,5849..
svo færri en 9 margf. væru með betri tímaflækju

5. a) $(a + b)(c + d) - ac - bd = ac + ad + bc + bd - ac - bd = ad + bc = bc + ad$

b)

```python
import math

# Úr bókinni
def FastMultiply(x, y, n):
    if n == 1:
        return x * y
    else:
        m = math.ceil(n/2)
        a = math.floor(x/math.pow(10, m))
        b = x % pow(10, m)
        c = math.floor(y/pow(10, m))
        d = y % pow(10, m)
        e = FastMultiply(a, c, m)
        f = FastMultiply(b, d, m)
        g = FastMultiply(a-b, c-d, m)
        return (pow(10, 2*m) * e + (pow(10, m) * (e + f - g) + f))


# Úr upphaflegu greininni
def FastMultiplyOld(x, y, n):
    if n == 1:
        return x * y
    else:
        m = math.ceil(n/2)
        a = math.floor(x/math.pow(10, m))
        b = x % pow(10, m)
        c = math.floor(y/pow(10, m))
        d = y % pow(10, m)
        e = FastMultiply(a, c, m)
        f = FastMultiply(b, d, m)
        g = FastMultiply(a+b, c+d, m)
        return (pow(10, 2*m) * e + (pow(10, m) * (- e - f + g) + f))

print(FastMultiply(153, 263, 3))
print(FastMultiplyOld(153, 263, 3))
```

```
C:\Users\addi\Desktop\Háskóli\onn 4\GreiningReiknirita\vika3>python daemi5.py
40239
40239
```

c) a, b og c eru með m-fjölda tölustafa, en a+b og c+d geta verið með m+1 fjölda tölustafa