# Antras laboratorinis darbas

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# 1 Uždavinys

Išspręsti pirmos eilės diferencialinę lygtį su Koši pradine sąlyga naudojanti Rungės-Kuto 3-pakopį ir 4-pakopį skaitinius modelius.

$$u' = x + 2x^2 \sin(u) \tag{1}$$

$$u(0) = -1 \tag{2}$$

#### 2 Skaitiniai modeliai

Skaitiniai modeliai įgyvendinti naudojant python programavimo kalbą, numpy ir scipy paketus.

### 2.1 Rungė-Kuto 3-pakopis modelis

$$\begin{cases} k_1 &= f(x_n, y_n) \\ k_2 &= f(x_n + \tau, y_n + \tau k_1) \\ k_3 &= f(x_n + \frac{\tau}{2}, y_n + \frac{\tau}{2} \frac{k_1 + k_2}{2}) \end{cases}$$
(3)

$$y_{n+1} = y_n + \frac{\tau}{6}(k_1 + k_2 + 4k_3). \tag{4}$$

#### 2.2 Rungė-Kuto 4-pakopis modelis

$$\begin{cases}
k_1 &= f(x_n, y_n) \\
k_2 &= f(x_n + \frac{\tau}{2}, y_n + \frac{\tau}{2}k_1) \\
k_3 &= f(x_n + \frac{\tau}{2}, y_n + \frac{\tau}{2}k_2) \\
k_4 &= f(x_n + \tau, y_n + \tau k_3)
\end{cases}$$
(5)

$$y_{n+1} = y_n + \frac{\tau}{6}(k_1 + 2k_2 + 2k_3 + k_4). \tag{6}$$

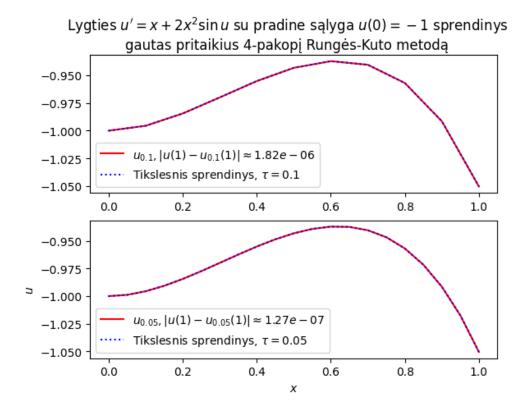
# 3 Rezultatai

# 3.1 Žymėjimas

x - laisvas kintamasis,  $u_{\tau}$  - skaitinis sprendinys su žingsniu  $\tau$ ,  $u_{\tau}(x)$  - skaitinio sprendinio su žingsniu  $\tau$  reikšmė koordinatėje x, u(x) - analitinio sprendinio reikšmė koordinatėje x.

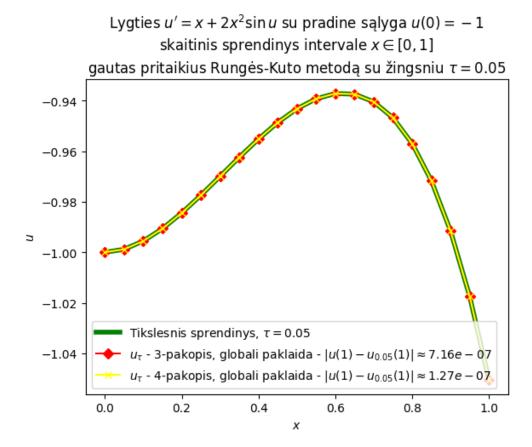
#### 3.2 Sprendinių grafikai

#### 3.2.1 (a) dalis



1 pav.: Skaitiniai lygties sprendiniai, kai  $\tau=0.1,0.05$  lyginami su tikslesniu sprendiniu gautu naudojant scipy metodą odeint.

### 3.2.2 (b) dalis



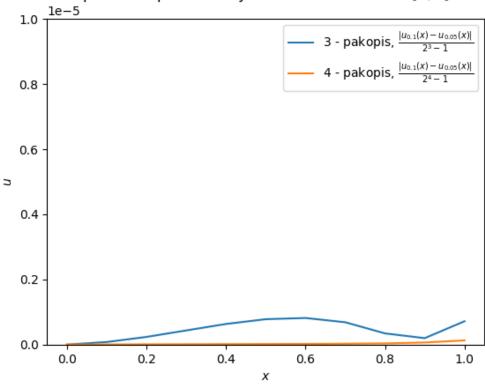
2 pav.: Skaitiniai lygties sprendiniai gauti skirtingais metodais lyginami su tikslesniu sprendiniu gautu naudojant scipy metodą odeint.

# 3.3 Paklaidos vertinimas

Rungės metodas paklaidai įvertinti

$$|u(x) - u_{\tau}(x)| \approx \frac{|u_{2\tau}(x) - u_{\tau}(x)|}{2^p - 1}$$
 (7)

Lygties  $u' = x + 2x^2 \sin u$  su pradine sąlyga u(0) = -1 skaitinių sprendinių su žingsniu  $\tau = 0.05$  paklaidos priklausomybė nuo x intervale  $x \in [0, 1]$ 



3 pav.: Skaitinių metodų paklaidos.

# 4 Priedai

Programos kodas naudotas sugeneruoti visus šiame dokumente esančius grafikus.

#### 4.1 solver.py

```
import numpy as np
def rk3 (f, t 0, y 0, tau, N):
    ts = np.linspace(t_0, t_0 + (N - 1) * tau, N)
    ys = np.zeros(N)
    ys[0] = y 0
    for i in range (N-1):
        k1 = f(ts[i], ys[i])
        k2 = f(ts[i] + tau, ys[i] + tau * k1)
        k3 = f(ts[i] + tau/2, ys[i] + tau/2 * (k1 + k2)/2)
        ys[i + 1] = ys[i] + tau/6 * (k1 + k2 + 4*k3)
    return ts, ys
def rk4(f, t_0, y_0, tau, N):
    ts = np.linspace(t_0, t_0 + (N - 1) * tau, N)
    ys = np.zeros(N)
    ys[0] = y 0
    for i in range (N-1):
        k1 = f(ts[i], ys[i])
        k2 = f(ts[i] + tau/2, ys[i] + tau/2 * k1)
        k3 = f(ts[i] + tau/2, ys[i] + tau/2 * k2)
        k4 = f(ts[i] + tau, ys[i] + tau * k3)
        ys[i + 1] = ys[i] + tau/6 * (k1 + 2*k2 + 2*k3 + k4)
    return ts, ys
```

#### 4.2 different-steps.py

```
import numpy as np
from scipy integrate import odeint
import matplotlib.pyplot as plt
from solver import rk3, rk4
def f(t, u):
    return t + 2 * t ** 2 * np.sin(u)
u\ 0\ =\ -1
x 0 = 0
x \text{ end} = 1
fig, axes = plt.subplots(2, sharey=True)
fig.suptitle("")
plt.xlabel("")
plt.ylabel("")
for index, tau in enumerate ([0.1, 0.05]):
    N = int((x_end - x_0) / tau) + 1
    M = int((x_end - x_0) / (2 * tau)) + 1
    xs, us = rk4(f, x 0, u 0, tau, N)
    _{\rm ,} us_{\rm 2}tau = rk4(f, x_{\rm 0}, u_{\rm 0}, 2 * tau, M)
    error = np.abs(us_2tau[-1] - us[-1]) / (2**4 - 1)
    axes[index].plot(xs, us, label="", color='red')
    us real = odeint(f, u 0, xs, tfirst=True)
    axes[index].plot(xs, us_real, label="", linestyle='dotted', color='blue')
    axes [index].set xlim ((-0.05, x \text{ end} + 0.05))
    axes [index].legend()
fig.subplots\_adjust(wspace=0.2)
plt.show()
```

#### 4.3 different-method.py

```
import numpy as np
from scipy integrate import odeint
import matplotlib.pyplot as plt
from solver import rk3, rk4
def f(t, u):
    return t + 2 * t ** 2 * np.sin(u)
u \ 0 = -1
x 0 = 0
x end = 1
tau = 0.05
plt.title("")
plt.xlabel("")
plt.ylabel("")
N = int((x_end - x_0) / tau) + 1
M = int((x_end - x_0) / (2 * tau)) + 1
xs, us 3 = rk3(f, x 0, u 0, tau, N)
us real = odeint(f, u 0, xs, tfirst=True)
plt.plot(xs, us real, label="", linewidth=4, color='green')
_{,} us_3_2tau = rk3(f, x_0, u_0, 2 * tau, M)
error_3_global = np.abs(us_3_2tau[-1] - us_3[-1]) / (2**3 - 1)
plt.plot(xs, us_3, label="", marker='D', color='red')
_{-}, us_{-}4 = rk4(f, x_{-}0, u_{-}0, tau, N)
_{,} us_{,}4_{,}2tau = rk4(f, x_{,}0, u_{,}0, 2 * tau, M)
error_4_global = np.abs(us_4_2tau[-1] - us_4[-1]) / (2**4 - 1)
plt.plot(xs, us_4, label="", marker='x', color='yellow')
plt.gca().set_xlim((-0.05, x end+0.05))
plt.legend()
plt.show()
```

#### 4.4 error.py

```
import numpy as np
from scipy integrate import odeint
import matplotlib.pyplot as plt
from solver import rk3, rk4
def f(t, u):
     return t + 2 * t ** 2 * np.sin(u)
u\ 0\,=\,-1
x 0 = 0
x \text{ end} = 1
tau = 0.05
N = int((x_end - x_0) / tau) + 1
M = int((x end - x 0) / (2 * tau)) + 1
\_, \ us\_3 \ = \ rk3 \, (\, f \; , \ x\_0 \, , \ u\_0 \, , \ tau \; , \ N)
xs, us 3 2tau = rk3 (f, x 0, u 0, 2 * tau, M)
_{,} us _{,} 4 = rk4 (f, x_{,} 0, u_{,} 0, tau, N)
_{,} us _{,} 4 _{,} 2 tau = rk4 (f, x_0, u_0, 2 * tau, M)
error rk3 = np.abs (us 3 2tau - us 3 [::2]) / (2**3 - 1)
error rk4 = np. abs (us 4 2tau - us 4[::2]) / (2**4 - 1)
plt.title("")
plt.plot(xs, error rk3, label="")
plt.plot(xs, error_rk4, label="")
plt.legend()
plt.xlabel("")
plt.ylabel("")
plt.gca().set_ylim((0, 0.00001))
plt.gca().set_xlim((-0.05, x_end+0.05))
plt.show()
```