Antras laboratorinis darbas

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1 Uždavinys

Išspręsti pirmos eilės diferencialinę lygtį su Koši pradine sąlyga naudojanti Rungės-Kuto 3-pakopį ir 4-pakopį skaitinius modelius.

$$u' = x + 2x^2 \sin(u) \tag{1}$$

$$u(0) = -1 \tag{2}$$

2 Skaitiniai modeliai

Skaitiniai modeliai įgyvendinti naudojant python programavimo kalbą, numpy ir scipy paketus.

2.1 Rungė-Kuto 3-pakopis modelis

$$\begin{cases} k_1 &= f(x_n, y_n) \\ k_2 &= f(x_n + \tau, y_n + \tau k_1) \\ k_3 &= f(x_n + \frac{\tau}{2}, y_n + \frac{\tau}{2}\frac{k_1+k_2}{2}) \end{cases} \tag{3}$$

$$y_{n+1} = y_n + \frac{\tau}{6}(k_1 + k_2 + 4k_3). \tag{4}$$

2.2 Rungė-Kuto 4-pakopis modelis

$$\begin{cases} k_1 &= f(x_n, y_n) \\ k_2 &= f(x_n + \frac{\tau}{2}, y_n + \frac{\tau}{2}k_1) \\ k_3 &= f(x_n + \frac{\tau}{2}, y_n + \frac{\tau}{2}k_2) \\ k_4 &= f(x_n + \tau, y_n + \tau k_3) \end{cases} \tag{5}$$

$$y_{n+1} = y_n + \frac{\tau}{6}(k_1 + 2k_2 + 2k_3 + k_4). \tag{6}$$

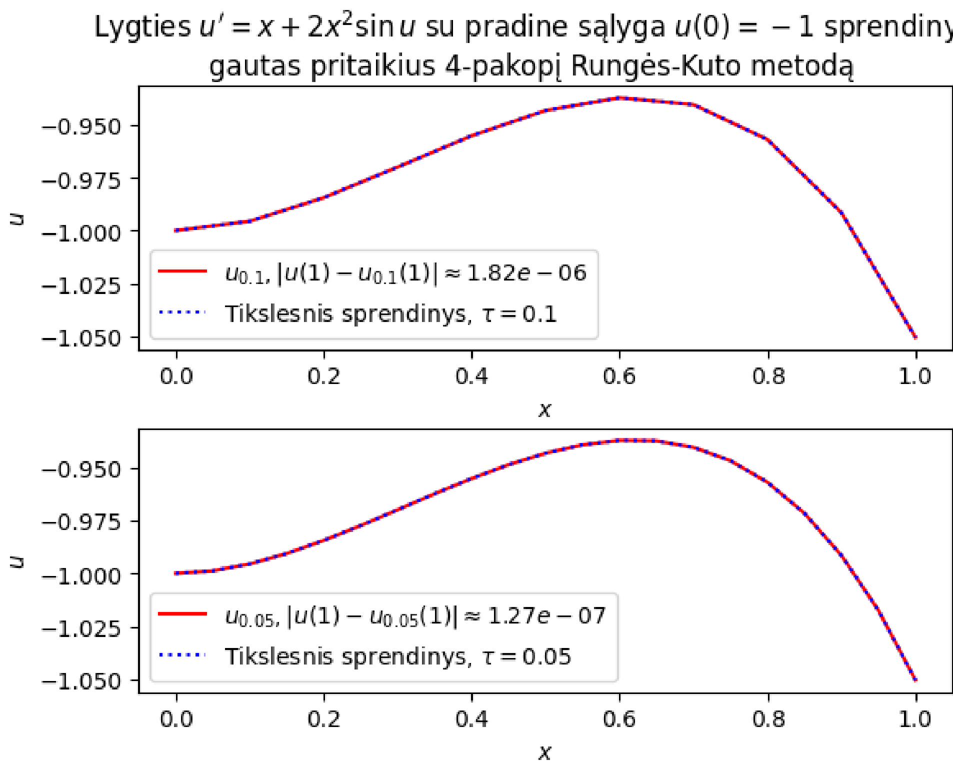
3 Rezultatai

3.1 Žymėjimas

x - laisvas kintamasis, u_τ - skaitinis sprendinys su žingsniu τ, $u_\tau(x)$ - skaitinio sprendinio su žingsniu τ reikšmė koordinatėje x, $u(x)$ - analitinio sprendinio reikšmė koordinatėje x.

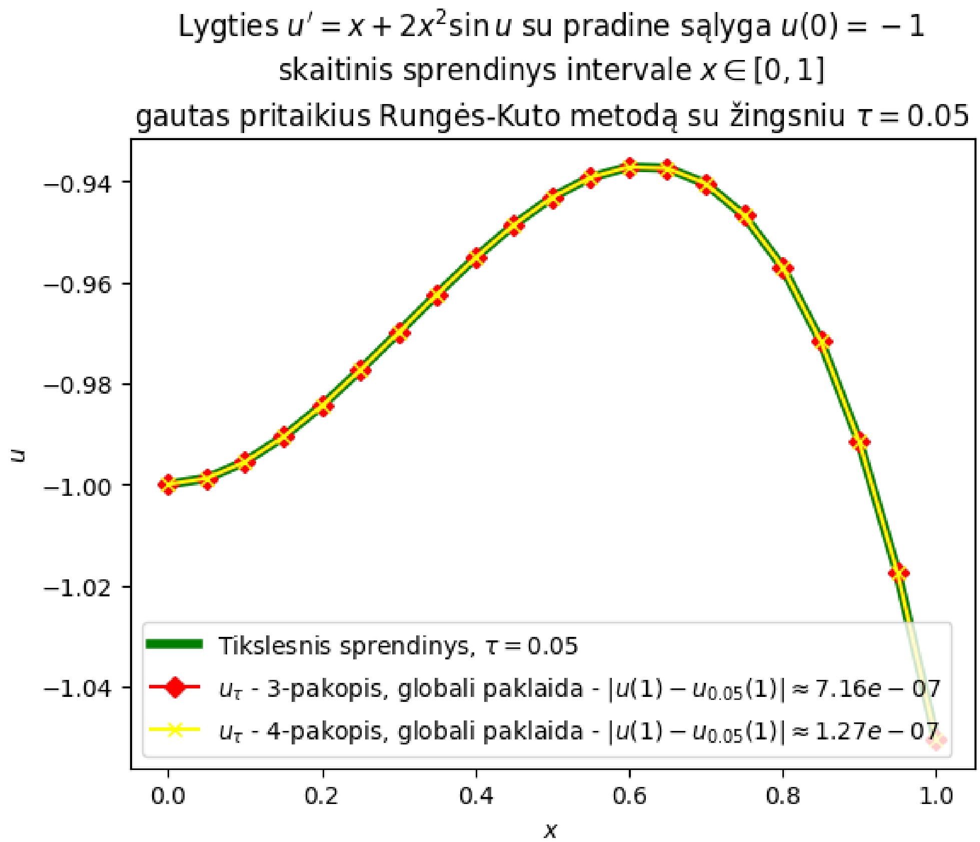
3.2 Sprendinių grafikai

3.2.1 (a) dalis



1 pav.: Skaitiniai lygties sprendiniai, kai $\tau = 0.1, 0.05$ lyginami su tikslesniu sprendiniu gautu naudojant scipy metodą odeint.

3.2.2 (b) dalis

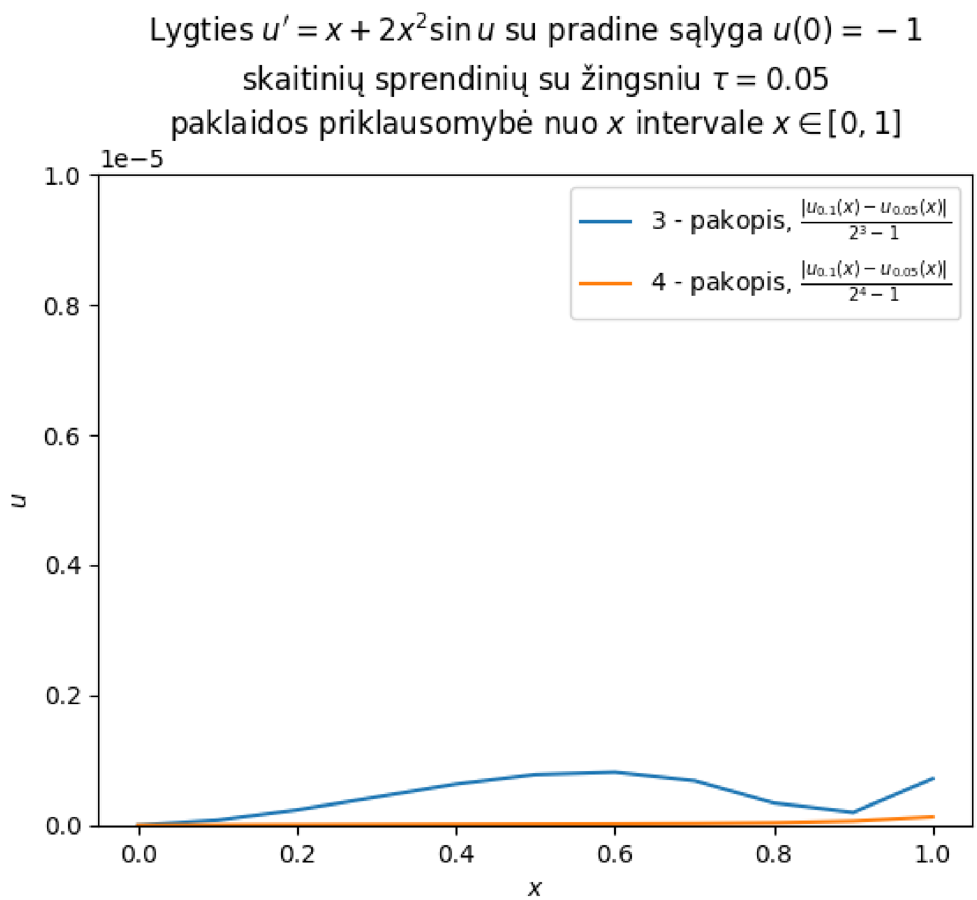


2 pav.: Skaitiniai lygties sprendiniai gauti skirtingais metodais lyginami su tikslesniu sprendiniu gautu naudojant scipy metodą odeint.

3.3 Paklaidos vertinimas

Rungės metodas paklaidai įvertinti

$$|u(x) - u_\tau(x)| \approx \frac{|u_{2\tau}(x) - u_\tau(x)|}{2^p - 1} \tag{7}$$



3 pav.: Skaitinių metodų paklaidos.

4 Priedai

Programos kodas naudotas sugeneruoti visus šiame dokumente esančius grafikus.

4.1 solver.py

```
import numpy as np

def rk3(f, t_0, y_0, tau, N):
    ts = np.linspace(t_0, t_0 + (N - 1) * tau, N)
    ys = np.zeros(N)
    ys[0] = y_0
    for i in range(N - 1):
        k1 = f(ts[i], ys[i])
        k2 = f(ts[i] + tau, ys[i] + tau * k1)
        k3 = f(ts[i] + tau/2, ys[i] + tau/2 * (k1 + k2)/2 )
        ys[i + 1] = ys[i] + tau/6 * (k1 + k2 + 4*k3)
    return ts, ys

def rk4(f, t_0, y_0, tau, N):
    ts = np.linspace(t_0, t_0 + (N - 1) * tau, N)
    ys = np.zeros(N)
    ys[0] = y_0
    for i in range(N - 1):
        k1 = f(ts[i], ys[i])
        k2 = f(ts[i] + tau/2, ys[i] + tau/2 * k1)
        k3 = f(ts[i] + tau/2, ys[i] + tau/2 * k2)
        k4 = f(ts[i] + tau, ys[i] + tau * k3)
        ys[i + 1] = ys[i] + tau/6 * (k1 + 2*k2 + 2*k3 + k4)
    return ts, ys
```

4.2 different-steps.py

```
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt
from solver import rk3, rk4

def f(t, u):
    return t + 2 * t ** 2 * np.sin(u)

u_0 = -1
x_0 = 0
x_end = 1

fig, axes = plt.subplots(2, sharey=True)

fig.suptitle("")

plt.xlabel("")
plt.ylabel("")

for index, tau in enumerate([0.1, 0.05]):

    N = int((x_end - x_0) / tau) + 1
    M = int((x_end - x_0) / (2 * tau)) + 1

    xs, us = rk4(f, x_0, u_0, tau, N)

    _, us_2tau = rk4(f, x_0, u_0, 2 * tau, M)
    error = np.abs(us_2tau[-1] - us[-1]) / (2**4 - 1)
    axes[index].plot(xs, us, label="", color='red')

    us_real = odeint(f, u_0, xs, tfirst=True)
    axes[index].plot(xs, us_real,label="", linestyle='dotted', color='blue')

    axes[index].set_xlim((-0.05, x_end+0.05))

    axes[index].legend()

fig.subplots_adjust(wspace=0.2)

plt.show()
```

4.3 different-method.py

```
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt
from solver import rk3, rk4

def f(t, u):
    return t + 2 * t ** 2 * np.sin(u)

u_0 = -1
x_0 = 0
x_end = 1
tau = 0.05

plt.title("")

plt.xlabel("")
plt.ylabel("")

N = int((x_end - x_0) / tau) + 1
M = int((x_end - x_0) / (2 * tau)) + 1

xs, us_3 = rk3(f, x_0, u_0, tau, N)

us_real = odeint(f, u_0, xs, tfirst=True)
plt.plot(xs, us_real,label="", linewidth=4, color='green')

_, us_3_2tau = rk3(f, x_0, u_0, 2 * tau, M)
error_3_global = np.abs(us_3_2tau[-1] - us_3[-1]) / (2**3 - 1)
plt.plot(xs, us_3, label="", marker='D', color='red')

_, us_4 = rk4(f, x_0, u_0, tau, N)
_, us_4_2tau = rk4(f, x_0, u_0, 2 * tau, M)
error_4_global = np.abs(us_4_2tau[-1] - us_4[-1]) / (2**4 - 1)
plt.plot(xs, us_4, label="", marker='x', color='yellow')

plt.gca().set_xlim((-0.05, x_end+0.05))

plt.legend()
plt.show()
```

4.4 error.py

```
import numpy as np
from scipy.integrate import odeint
import matplotlib.pyplot as plt
from solver import rk3, rk4

def f(t, u):
    return t + 2 * t ** 2 * np.sin(u)

u_0 = -1
x_0 = 0
x_end = 1
tau = 0.05

N = int((x_end - x_0) / tau) + 1
M = int((x_end - x_0) / (2 * tau)) + 1

_, us_3 = rk3(f, x_0, u_0, tau, N)
xs, us_3_2tau = rk3(f, x_0, u_0, 2 * tau, M)

_, us_4 = rk4(f, x_0, u_0, tau, N)
_, us_4_2tau = rk4(f, x_0, u_0, 2 * tau, M)

error_rk3 = np.abs(us_3_2tau - us_3[::2]) / (2**3 - 1)
error_rk4 = np.abs(us_4_2tau - us_4[::2]) / (2**4 - 1)

plt.title("")

plt.plot(xs, error_rk3, label="")
plt.plot(xs, error_rk4, label="")
plt.legend()
plt.xlabel("")
plt.ylabel("")

plt.gca().set_ylim((0, 0.00001))
plt.gca().set_xlim((-0.05, x_end+0.05))

plt.show()
```