Hausaufgaben zum 7. 12. 2012

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25. Dezember 2012

1.

a)

$f(x) = (x_3 \vee \overline{x_2}) \wedge (x_2 \vee \overline{x_1})$

x_1	x_2	x_3	$f(x)$
0	0	0	1
0	0	1	1
0	1	0	0
1	0	0	0
0	1	1	1
1	1	0	0
1	0	1	0
1	1	1	1

$\Rightarrow$ **KNF:** $(x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee x_3) \wedge (\overline{x_1} \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3})$
$\Rightarrow$ **DNF:** $(\overline{x_1} \wedge \overline{x_2} \wedge \overline{x_3}) \vee (\overline{x_1} \wedge \overline{x_2} \wedge x_3) \vee (\overline{x_1} \wedge x_2 \wedge x_3) \vee (x_1 \wedge x_2 \wedge x_3)$
$\Rightarrow$ **RMF:** $1 \oplus x_2 \oplus x_1 \oplus x_3x_2 \oplus x_2x_1$

b)

$g(x) = \overline{x_3} \oplus \overline{x_1}$

x_1	x_2	x_3	$f(x)$
0	0	0	0
0	0	1	1
0	1	0	0
1	0	0	1
0	1	1	1
1	1	0	1
1	0	1	0
1	1	1	0

$\Rightarrow$ **KNF:** $(x_1 \vee x_2 \vee x_3) \wedge (x_1 \vee \overline{x_2} \vee x_3) \wedge (\overline{x_1} \vee x_2 \vee \overline{x_3}) \wedge (\overline{x_1} \vee \overline{x_2} \vee \overline{x_3})$
$\Rightarrow$ **DNF:** $(\overline{x_1} \wedge \overline{x_2} \wedge x_3) \vee (x_1 \wedge \overline{x_2} \wedge \overline{x_3}) \vee (\overline{x_1} \wedge x_2 \wedge x_3) \vee (x_1 \wedge x_2 \wedge x_3)$
$\Rightarrow$ **RMF:** $x_3 \oplus x_1$

2.

a)

Sei $\overline{\wedge}$ die Schreibweise für NAND:

x	$\overline{x}$	$x \wedge x$	$x \overline{\wedge} x$
0	1	0	1
1	0	1	0

$\Rightarrow \overline{x} = x \overline{\wedge} x$

x	y	$x \wedge y$	$x \overline{\wedge} y$	$(x \overline{\wedge} y) \overline{\wedge} (x \overline{\wedge} y)$
0	0	0	1	0
0	1	0	1	0
1	0	0	1	0
1	1	1	0	1

$\Rightarrow x \wedge y = (x \overline{\wedge} y) \overline{\wedge} (x \overline{\wedge} y) \overset{*}{=} \overline{x \overline{\wedge} y}$

* Da die Negation bereits gezeigt wurde, ist dieser Umformungsschritt legitim.

x	y	$x \vee y$	$x \overline{\wedge} x$	$y \overline{\wedge} y$	$(x \overline{\wedge} x) \overline{\wedge} (y \overline{\wedge} y)$
0	0	0	1	1	0
0	1	1	1	0	1
1	0	1	0	1	1
1	1	1	0	0	1

$\Rightarrow x \vee y = (x \overline{\wedge} x) \overline{\wedge} (y \overline{\wedge} y)$

b)

$$
\begin{aligned}
f(x_3, x_2, x_1) &= (\overline{x_3} \wedge (\overline{x_2} \vee x_1)) \vee (x_1 \wedge (\overline{x_2} \vee x_1)) \\
&= (\overline{x_2} \vee x_1) \wedge (\overline{x_3} \vee x_1) \\
&= x_1 \wedge (\overline{x_2} \vee \overline{x_3}) \\
&= x_1 \wedge (x_2 \barwedge x_3) \\
&= \big(x_1 \barwedge (x_2 \barwedge x_3)\big) \barwedge \big(x_1 \barwedge (x_2 \barwedge x_3)\big)
\end{aligned}
$$

3.

a)

	x_3	x_2	x_1	x_0	A	B
0	0	0	0	0	1	1
1	0	0	0	1	0	1
2	0	0	1	0	1	1
3	0	0	1	1	1	1
4	0	1	0	0	0	1
5	0	1	0	1	1	0
6	0	1	1	0	1	0
7	0	1	1	1	1	1
8	1	0	0	0	1	1
9	1	0	0	1	1	1

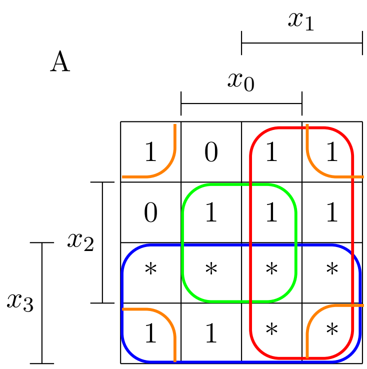
A

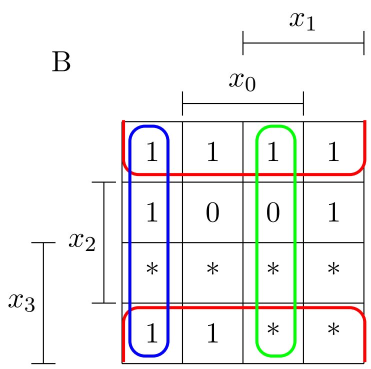
x_3x_2 \ x_1x_0	00	01	11	10
00	1	0	1	1
01	0	1	1	1
11	*	*	*	*
10	1	1	*	*

B

x_3x_2 \ x_1x_0	00	01	11	10
00	1	1	1	1
01	1	0	1	0
11	*	*	*	*
10	1	1	*	*

b)





$A(x) = x_3 \vee x_1 \vee (x_2 \wedge x_0) \vee (\overline{x_2} \wedge \overline{x_0})$
$B(x) = \overline{x_2} \vee (\overline{x_1} \wedge \overline{x_0}) \vee (x_1 \wedge x_0)$

4.

a)

	x_3	x_2	x_1	x_0	y
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	1
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	0
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

b)& c)

A

		x_0	x_0, x_1	x_1
	0	0	0	0
x_2	0	1	1	0
x_2, x_3	1	1	1	1
x_3	0	0	0	0

$$y = (x_3 \wedge x_2) \vee (x_0 \wedge x_2)$$

d)