society when the decision is made to invest and hence that society plans on a constant yield LY from the land over a long period of time T. We further assumed that the net present value to society of this sequence of crops can be found by ordinary discounting, using a social discount rate SD. We assumed that the newly developed land enters production at the current average land yield LY. Thus the societal value assigned to this new land is

$$\int_0^T LY \times e^{-SD \times t} \, dt$$

which can be approximated as LY/SD, since typically $SD \times T \gg 1$ ($SD = 0.07$ (year)$^{-1}$ and $T \geq 50$ years, implying $SD \times T \geq 3.5$).

The current marginal productivity of land development MPLD can then be computed from the perceived social benefit, LY/SD, and from the present development cost per hectare DCPH:

```
MPLD.K=LY.K/(DCPH.K*SD)                                    109, A
SD=.07                                                     109.1, C
    MPLD   - MARGINAL PRODUCTIVITY OF LAND DEVELOPMENT
             (VEGETABLE-EQUIVALENT KILOGRAMS/DOLLAR)
    LY     - LAND YIELD (VEGETABLE-EQUIVALENT KILOGRAMS/
             HECTARE-YEAR)
    DCPH   - DEVELOPMENT COST PER HECTARE (DOLLARS/
             HECTARE)
    SD     - SOCIAL DISCOUNT (1/YEAR)
```

Marginal Productivity of Agricultural Inputs MPAI Similarly, the marginal (physical) productivity of agricultural inputs MPAI is the additional amount of crops that will arise from the next dollar invested in agricultural inputs. The immediate increase in food output arising from investment in agricultural inputs can be computed as the partial derivative of food output F with respect to agricultural inputs AI, holding arable land AL constant:

$$\left(\frac{\partial F}{\partial AI}\right)_{AL} = \frac{\partial (LY * AL)}{\partial AI}$$

$$= \frac{\partial LY}{\partial AIPH}$$

$$= LFERT \times LYMAP \times \frac{\partial LYMC}{\partial AIPH}$$

$$= \frac{LY}{LYMC} \times MLYMC,$$

where

AI = agricultural inputs (dollars per year)
AIPH = agricultural inputs per hectare (dollars per hectare-year)
AL = arable land (hectares)
F = food (vegetable-equivalent kilograms per year)
LFERT = land fertility (vegetable-equivalent kilograms per hectare-year)
LY = land yield (vegetable-equivalent kilograms per hectare-year)
LYMAP = land yield multiplier from air pollution (dimensionless)
LYMC = land yield multiplier from capital (dimensionless)
MLYMC = marginal land yield multiplier from capital (hectares per dollar)

The marginal land yield multiplier from capital MLYMC (Figure 4-54) is defined as ∂LYMC/∂AIPH (that is, the derivative of the LYMC graph in Figure 4-48). Again, since some agricultural inputs increase food output beyond one growing season, discounting is necessary to find the present value of the crop increase. In this case the longevity of the agricultural inputs—the average life of agricultural inputs ALAI—is much shorter than the societal planning horizon ($T \approx 50$ years). Thus the present value of a crop increase ΔF is

$$\int_0^{ALAI} \Delta F \times e^{-SD \times t} \, dt,$$

which can be approximated by $\Delta F \times ALAI$ since, typically, $SD \times ALAI \ll 1$ ($SD = 0.07$ (years) $^{-1}$, ALAI = 2 years, implying $SD \times ALAI = 0.14$).

The marginal productivity of agricultural inputs MPAI consequently appears as:

```
MPAI.K=ALAI.K*LY.K*MLYMC.K/LYMC.K                          110, A
    MPAI   - MARGINAL PRODUCTIVITY OF AGRICULTURAL
             INPUTS (VEGETABLE EQUIVALENT KILOGRAMS/
             DOLLAR)
    ALAI   - AVERAGE LIFETIME OF AGRICULTURAL INPUTS
             (YEARS)
    LY     - LAND YIELD (VEGETABLE-EQUIVALENT KILOGRAMS/
             HECTARE-YEAR)
    MLYMC  - MARGINAL LAND YIELD MULTIPLIER FROM CAPITAL
             (HECTARES/DOLLAR)
    LYMC   - LAND YIELD MULTIPLIER FROM CAPITAL
             (DIMENSIONLESS)

MLYMC.K=TABHL(MLYMCT,AIPH.K,0,600,40)                      111, A
MLYMCT=.075/.03/.015/.011/.009/.008/.007/.006/.005/ 111.1, T
    .005/.005/.005/.005/.005/.005/.005
    MLYMC  - MARGINAL LAND YIELD MULTIPLIER FROM CAPITAL
             (HECTARES/DOLLAR)
    TABHL  - A FUNCTION WITH VALUES SPECIFIED BY A TABLE
    MLYMCT - MLYMC TABLE
    AIPH   - AGRICULTURAL INPUTS PER HECTARE (DOLLARS/
             HECTARE-YEAR)
```

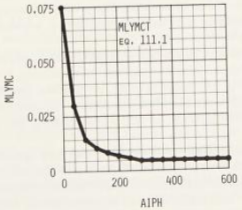

Figure 4-54 Marginal land yield multiplier from capital table