Assuming that the assimilation half-life AHL is only a function of the quantity of pollutant present, one could conceive of several different relationships between the two variables. Four possibilities are illustrated in Figure 6-20. Line A in each figure represents the case in which the half-life is constant and is independent of the total quantity of pollutant present. If the half-life is constant, the assimilation rate (shown by line A of Figure 6-20B) increases linearly with pollution. Line C (in Figures 6-20A and 6-20B) represents a case in which the assimilation half-life is constant over the lower range of pollution but rises quickly after pollution passes some threshold. In this case the assimilation rate rises linearly at first, as in the case represented by line A, but it ultimately begins to decrease as rising pollution forces the half-life toward infinity and the assimilation rate toward zero. Lines B and D in both figures correspond to other conceivable relationships.

We designed the World3 equations relating AHL to PPOL so that other analysts can easily incorporate the relationship they believe is most reasonable. Because the half-life of each radioisotope is independent of the isotope's quantity, the line relating half-life to the level of a radioactive pollutant is linear with slope zero and thus of the form specified by line A of Figures 6-20A and 6-20B. The relationship between the level of pollutant and the half-life of assimilation through deposition is unclear, though line A of Figures 6-20A and 6-20B may be a useful approximation for that mode of assimilation as well. For information about the form of the relationship between PPOL and AHL that is most appropriate for assimilation through chemical or biological degradation, we employed a simple model of the kinetics of degradation of a single pollutant. The casual reader may omit the mathematical derivation and proceed directly to equation (6.8) of this section.

Some forms of pollution degradation may be carried out by "degraders," which may be small molecules such as oxygen or water, large molecules such as enzymes,

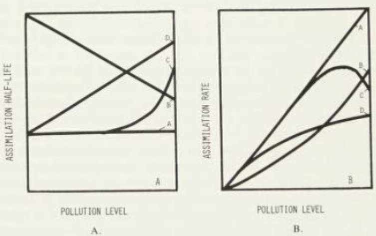

Figure 6-20 Alternative possible relationships between pollution level and assimilation half-life, together with the corresponding rate of pollution assimilation Note: PPASR = PPOL/(AHL × 1.4).

or entire organisms such as bacteria. For all these the process of pollution degradation could proceed in two steps as follows:

1. $$P + D \underset{k_1}{\overset{k_1'}{\rightleftharpoons}} PD$$

This is a reversible step in which a pollutant P and a degrader D form some kind of intermediate complex.

2. $$PD \underset{k_2}{\rightarrow} H + D$$

This is a step in which the complex PD decomposes irreversibly to yield a harmless species H and the degrader D. The degrader is then free for another catalytic cycle.

This formulation is identical to that proposed by Michaelis-Mention to represent enzyme-catalyzed kinetics (Fruton and Simmonds 1958). Following the usual pattern of chemical-kinetic analysis, one presumes a steady-state concentration of the complex PD:

$$\frac{d[PD]}{dt} = 0 = k_1[P][D] - k_1'[PD] - k_2[PD], \qquad 6.5$$

where k_1, k_1', and k_2 are the rate constants for the indicated reactions. The total degrader concentration, uncomplexed and complex, $[D^*]$ is defined by:

$$[D^*] = [D] + [PD]. \qquad 6.6$$

From equation (6.5) at steady state the concentration of the complex PD is

$$[PD] = \frac{k_1[P][D^*]}{k_1' + k_2 + k_1 (P)}. \qquad 6.7$$

The rate at which the pollutant is assimilated PPASR is the rate of the second step in the reaction mechanism. Thus PPASR = $k_2[PD]$ or

$$PPASR = \frac{k_1 k_2 [P][D^*]}{k_1' + k_2 + k_1 [P]}. \qquad 6.8$$

From equation (6.8) it is possible to calculate the pollutant's degradation or assimilation half-life AHL:

$$AHL = \frac{[P]}{PPASR\,(1.4)} = \frac{k_1' + k_2 + k_1 [P]}{k_1 k_2 [D^*]\,(1.4)}. \qquad 6.9$$

This theoretical model yields AHL as a linear function of [P], the concentration of the persistent pollutant, as shown by Figure 6-21, where $\alpha = (k_2' + k_2)/k_1 k_2 [D^*]\,(1.4)$ and $\beta = 1/k_2[D^*]\,(1.4)$.

Equation (6.8) is a general expression for pollution degradation for which there are two limiting cases. First, if pollution is small relative to the number of degraders, so that $(k_1' + k_2) >> k_1 [P]$, then:

$$PPASR = \frac{k_1 k_2 [P][D^*]}{k_1' + k_2} \qquad 6.10$$

and

$$AHL = \frac{k_1' + k_2}{k_1 k_2 [D^*]\,(1.4)}, \qquad 6.11$$