

Figure 2-2 Global population growth, 1650-2000  
Sources: Carr-Saunders 1936, U.N. 1966, U.N. 1972a.

period. Mathematically, the population increase can be expressed by the following differential equation:

$$\frac{dN}{dt} = rN$$

which states that the rate of change of the population,  $dN/dt$ , is proportional to the size of the population  $N$ . If the factor of proportionality  $r$  is constant, the solution to this equation is

$$N = N_0 e^{rt}$$

where  $N_0$  is the initial population,  $N$  is the population at time  $t$ ,  $r$  is the net population growth rate, and  $e$  is the base of the natural logarithm. The net growth rate  $r$  is usually not constant; it varies as a function of time, and the actual population growth is therefore not perfectly exponential in the strict mathematical sense of the word. However, human population growth is in theory, and in observed fact, a "positive feedback" exponential process. Past increases enhance the probability of further increases, unless strong forces from the rest of the system oppose the tendency toward growth.

For the global population, migration is not a factor. Thus the net growth rate  $r$  is

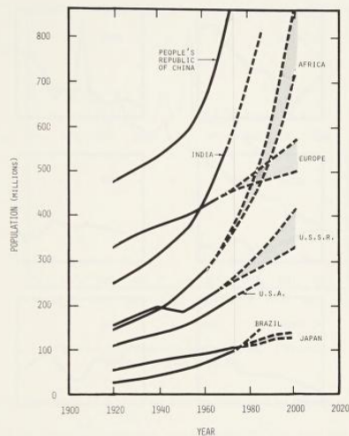


Figure 2-3 Regional population growth, 1920-2000  
Source: U.N. 1966.

simply the difference between the birth rate  $B$  and the death rate  $D$ , both expressed as fractions of the total population:

$$r = B - D.$$

Demographers often refer to  $r$ ,  $B$ , and  $D$  not as fractions of the population (for example, the birth rate was 0.02 of the population last year) but as percentages (2.0 percent of the population) or as ten times the percentages (20 per 1,000 persons in the population). The latter designation is called the "crude birth rate" (births per 1,000 persons per year) or the "crude death rate" (deaths per 1,000 persons per year).

The net growth rate could conceivably be positive ( $B > D$ , positive exponential growth), negative ( $B < D$ , exponential decay), or zero ( $B = D$ , constant population). Actual population growth rates have only rarely and temporarily been zero or negative, as Figure 2-4 indicates.