**Morlet Wavelet:** $\psi(t) = k e^{i\omega_0 t} e^{-\frac{t^2}{2\sigma^2}}$  $k \in \mathbb{R}$, $\omega_0 > 0$  $\sigma^2 > 0$

**Admissibility condition:** $\int_{-\infty}^{+\infty} \psi(t)\,dt = 0$

We define $I = \int_{-\infty}^{+\infty} \psi(t)\,dt = a + ib$,  $a = \mathrm{Re}(I) \in \mathbb{R}$
$b = \mathrm{Im}(I) \in \mathbb{R}$

$$I = \int_{-\infty}^{+\infty} k e^{i\omega_0 t} e^{-\frac{t^2}{2\sigma^2}}\,dt = \int_{-\infty}^{+\infty} k(\cos(\omega_0 t) + i\sin(\omega_0 t))\, e^{-\frac{t^2}{2\sigma^2}}\,dt$$

We have $t \to k i \sin(\omega_0 t) e^{-\frac{t^2}{2\sigma^2}} dt$ an odd integrable function, so:

$$\int_{-\infty}^{+\infty} k i \sin(\omega_0 t) e^{-\frac{t^2}{2\sigma^2}}\,dt = 0$$

$\int_{-\infty}^{+\infty} |k i \sin(\omega_0 t) e^{-\frac{t^2}{2\sigma^2}}|\,dt \leq |k| \int_{-\infty}^{+\infty} e^{-\frac{t^2}{2\sigma^2}}\,dt = |k|\sqrt{2\pi\sigma^2} < +\infty$

Thus $I = \int_{-\infty}^{+\infty} k \cos(\omega_0 t) e^{-\frac{t^2}{2\sigma^2}}\,dt$

Euler's identity: $\cos(\omega_0 t) = \dfrac{e^{i\omega_0 t} + e^{-i\omega_0 t}}{2i}$

$$I = \int_{-\infty}^{+\infty} k \frac{e^{i\omega_0 t} + e^{-i\omega_0 t}}{2i} e^{-\frac{t^2}{2\sigma^2}}\,dt = \frac{I}{2i} + \frac{1}{2i}\int_{-\infty}^{+\infty} \overline{\psi(t)}\,dt$$

We have $2iI = I + \int_{-\infty}^{+\infty} \overline{\psi(t)}\,dt$.

If $\int_{-\infty}^{+\infty} |\psi(t)|\,dt < +\infty$, we can write $\int_{-\infty}^{+\infty} \overline{\psi(t)}\,dt = \overline{\int_{-\infty}^{+\infty} \psi(t)\,dt} = \overline{I}$

**Proof:** $\int_{-\infty}^{+\infty} |\psi(t)|\,dt = \int_{-\infty}^{+\infty} |k| e^{-\frac{t^2}{2\sigma^2}}\,dt = |k|\sqrt{2\pi\sigma^2} < +\infty$

Gaussian integral

Thus $2iI = I + \overline{I}$, $\iff 2i(a+ib) = 2a \iff \begin{cases} 2a(i-1) = 0 \\ -2b = 0 \end{cases}$

$\iff a = b = 0$

$\iff I = 0$

$$\boxed{\int_{-\infty}^{+\infty} \psi(t)\,dt = 0}$$

Finite energy condition :

$$\int_{-\infty}^{+\infty} |\psi(t)|^2 dt = \int_{-\infty}^{+\infty} \left| k\, e^{i\omega_0 t} e^{-\frac{t^2}{2\sigma^2}} \right|^2 dt = |k|^2 \int_{-\infty}^{+\infty} e^{-\frac{t^2}{\sigma^2}} dt = |k|^2 \sqrt{\pi\sigma^2} < +\infty$$