ARTISHOW PROJECT

LAGRANGE2

SETTING A CARBON TAX USING LAGRANGE'S DUALITY

First year project supervised by : Iyad Walwil

Made by Arnaud Capitan, Louis Maillard, Arthur Hottier, Cyprien Braunwald

OBJECTIVE : REDUCING CARBON EMISSIONS

In a population, we aim to optimize the cost of products for each individual while establishing a global limit on carbon emissions. Each product has its own cost and carbon footprint, and each individual has their own consumption preferences, making it challenging to balance low costs, low carbon emissions, and high satisfaction.

1. Mathematical approach : an optimization problem

- Agent ***a*** in population ***A*** : $a \in A$
- Product ***i*** in products ***I*** : $i \in I$
- Carbon footprint of product ***i*** : c_i
- Price of product ***i*** : p_i
- Bound of carbon emissions : C
- Quantity of product ***i*** used by agent ***a*** : $x_{a,i}$
- Utility function of agent ***a*** : $U_a((x_{a,i})_{i \in I})$
  - Represents the satisfaction of agent ***a*** according to what products he is using

Objective : Minimizing the global cost

$$min_{x \geq 0} \sum_{a \in A} \sum_{i \in I} x_{a,i} p_i$$

Constraint 1 : Satisfaying every agent

$$U_a\left(\{x_{a,i}\}_{i \in I}\right) \geq 1 \quad \forall a \in A$$

Constraint 2 : Carbon emissions

$$\sum_{a \in A} \sum_{i \in I} x_{i,a} c_i \leq C$$

Lagrange's duality gives us this equivalent saddle point problem :

$$\min_{x \geq 0} \max_{\lambda \geq 0} \left(\sum_{a \in A} \sum_{i \in I} x_{a,i} p_i + \lambda \left(\sum_{a \in A} \sum_{i \in I} x_{i,a} c_i - C \right) \right)$$

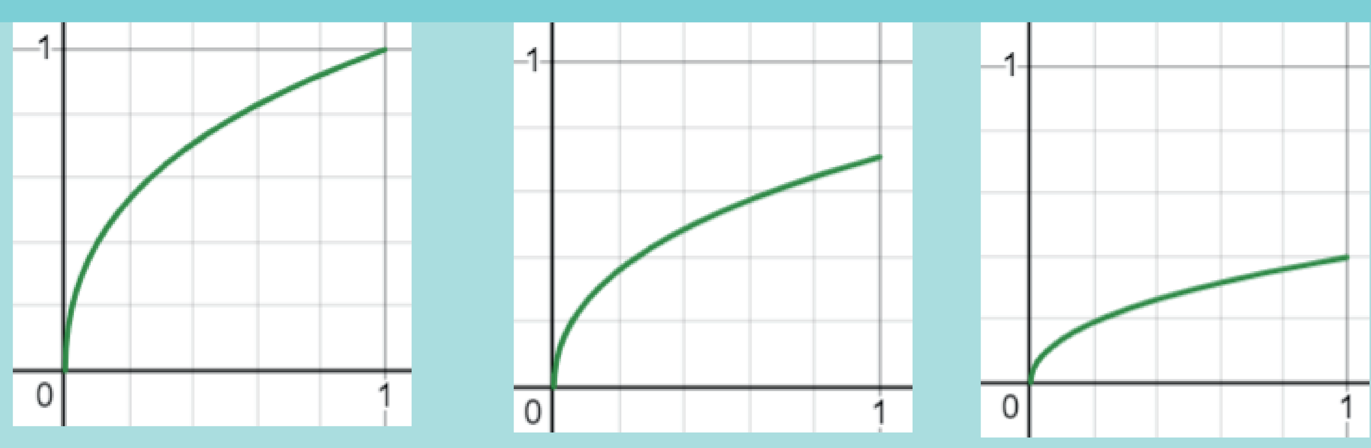
2. Our model : The clothing industry

- Goal : Optimizing the satisfaction of every client (Politician, Office worker, Artist, Influencer, Craftsman, Sportsman) while limiting the carbon impact of clothes with C bound
- Multiple items of clothing, ranging from Basic shirt to three-piece suit and swimsuit
- Data gathered from multiple sources and different location (France, China, United-States)

3. Our own utility function - Solving this problem using Python

Thanks to Python Library CVXPY, we can solve this saddle point problem. However, the solver required convex functions to work properly. We decided to use our own function :

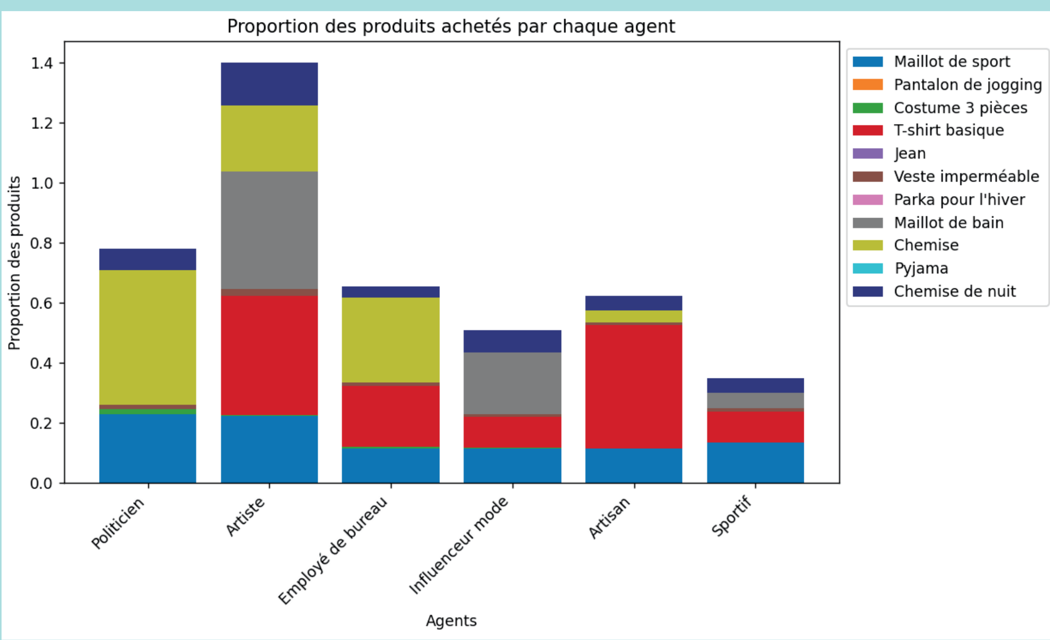
$$U_a(x) = \sum_{i \in I} \log_2(\sqrt{a x_i} + 1)$$



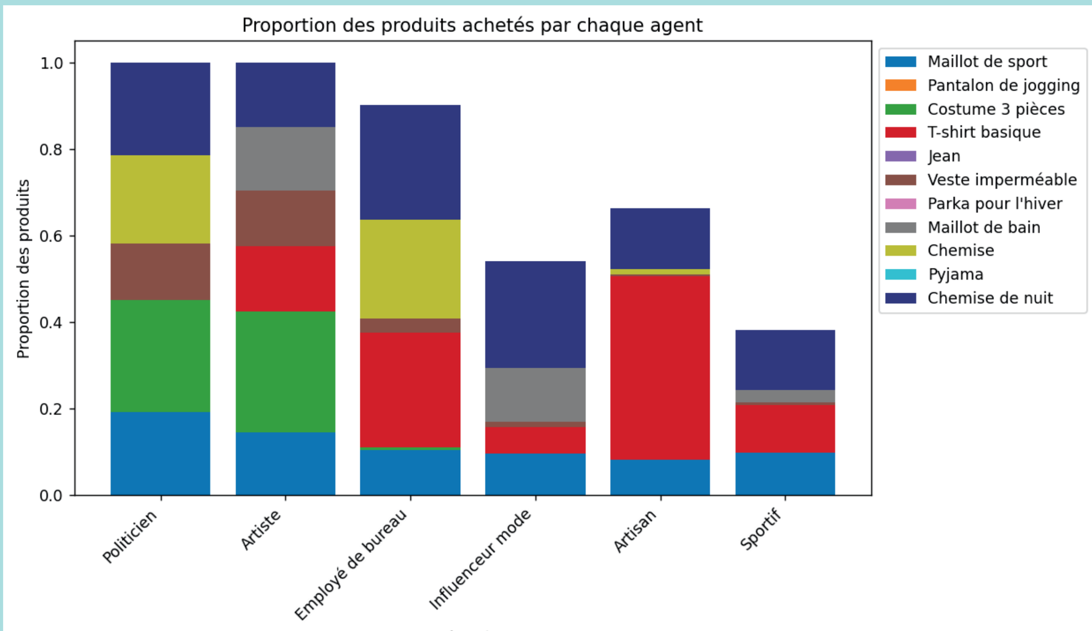
Graph plot for a = {1,0.4,0.2}
Why this function ? Ask us !

4. Results :

- Without constraints, everyone uses whatever they want
- When we implement the carbon tax, agents only use what they really need



Striking example : The politician



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