Outage probability in OFDMA protocol

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1 Model

In OFDMA systems, resources are allocated in time and frequency.

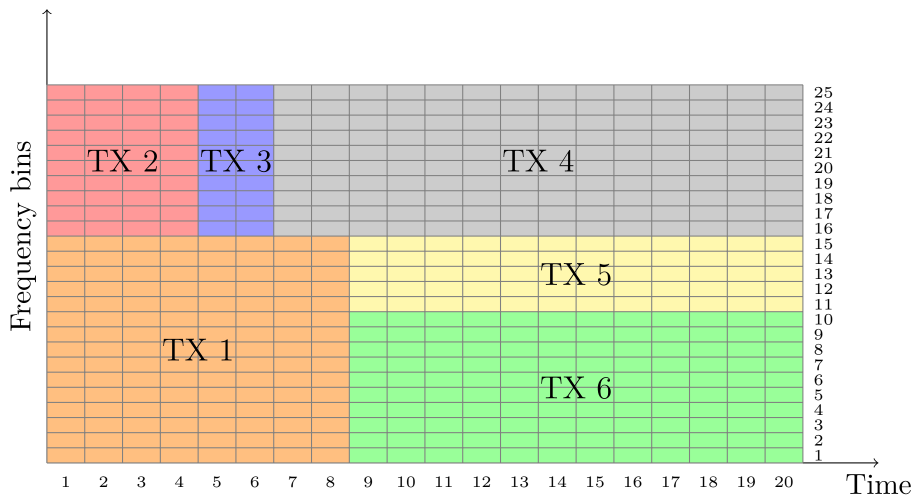

Figure 1: Every square is called a subcarrier. A group of same color subarriers is a subchannel.

For each message, the emitter is given several subcarriers for a few time slots, this builds a subchannel. In principle, the number of allocated frequency bins can vary from one slot to the other for a given user, however, we here consider that this number is fixed during the ON period, hence the rectangular form of the TXs. Let $N(x)$ be the number of subcarriers needed by a user located at x, who wants to communicate at a rate C:

$$N(x) = \left\lceil \frac{C}{W \log_2(1 + \frac{K}{\|x\|^\gamma})} \right\rceil \text{ si } K\|x\|^{-\gamma} > \text{SNR}_{min},$$

where SNR_{min} is the minimum signal over noise ratio necessary to ensure the feasibility of the communication. Moreover, $\lceil z \rceil$ is the least integer greater the real z, W is the bandwidth of each frequency bin and K is a constant which takes into account emitting power, fading and shadowing.

Note that the number of slots per user is bounded by

$$N_{\max} = \left\lceil \frac{C}{W \log_2(1 + \text{SNR}_{\min})} \right\rceil$$

1.1 Numerical values

C	200 kb/s
W	250 kHz
K	10^6
γ	2.8
R	300 m
λ	0,01 m^{-2}
$\text{SINR}_{\min}$	0,1
p	0,01
$S_{\min}$	30
$S_{\max}$	100

2 Questions

We consider that users are represented by a Poisson process of intensity measure $\lambda\, dx$ and that there is a percentage p of active users in a time slot. We want to estimate for one time slot the outage probability, i.e. the probability that the cumulative number of required subchannels by the set of active customer is greater than the number of available resources N_{avail}, which is 25 on figure 1.

The access point is located at (0, 0). The observation window is a circle of radius R.

1. Show that the process of active customers is a Poisson process. Give its intensity measure.

2. What is the mean number of active customers in the cell ?

3. For $k = 1, \cdots, N_{\max}$, characterize geometrically the subset of the cell in which all customers require k subcarriers.

4. What is the distribution of A_3, the number of customers requiring 3 subcarriers ?

5. Using the Kolmogorov-Smirnov test, how can you corroborate this result by simulation ?

Set

$$F(\phi) = \sum_{x\in\phi} N(x).$$

We want to compute the outage probability

$$p_o = \mathbf{P}(F > S)$$

where S is the number of available slots.

6. Show that F can be written as

$$F(\phi) = \sum_{k=1}^{N_{\max}} k\zeta_k$$

   where ζ_k are independent random variables with Poisson distribution of parameter to be computed.

   This kind of distribution is called compound Poisson distribution.

7. Compute $\mathbf{E}\left[e^{\theta X_\mu}\right]$ when X_μ is a Poisson r.v. of parameter μ and find

$$\min_{\theta>0} e^{-K\mu\theta}\mathbf{E}\left[e^{\theta X_\mu}\right].$$

8. Using the fact that

$$\mathbf{P}(X_\mu \geq K\mu) \leq e^{-K\mu\theta}\mathbf{E}\left[e^{\theta X_\mu}\right],$$

   find K_μ such that $\mathbf{P}(X_\mu \geq K\mu) \leq 10^{-4}$.

We deduce from this identity that the distribution of a Poisson r.v. of intensity μ can be represented in a computer by the sequence

$$e^{-\mu}\frac{\mu^k}{k!},\ \text{for } 0 \leq k \leq K$$

9. Using Python,

   - compute $N_{\max}$

- compute the mean value of the r.v. ζ_k for $k = 1, \cdots, N_{\max}$.
- compute the size of the vector which represents the distribution of ζ_k for $k = 1, \cdots, N_{\max}$ and take K to be the supremum of these dimensions.
- construct the vectors representing the distribution of the ζ_ks.
- construct the vectors representing the distribution of the $k\zeta_k$s
- using the *numpy.convolve*, build the vector representing the distribution of F.

10. Compute S such that the outage probability is smaller than 0.01.