Chirp Spread Spectrum modulation

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A constant frequency sinewave can be described by Equation (1) where $s(t)$ is the amplitude of the sinewave along time t, f is its frequency and ϕ_0 the initial phase.

$$s(t) = \sin 2\pi f t + \phi_0 \tag{1}$$

Another way to see this equation is by looking at the instantaneous phase $\phi(t)$ of the sine. It varies linearly with time and can be described as $\phi(t) = 2\pi f t + \phi_0$. Frequency f can therefore be seen as the time derivative of the phase $\phi(t)$, that is $\phi'(t) = 2\pi f$. The benefit of this approach is that $\phi(t)$ can now be obtained by integrating the frequency : $\phi(t) = 2\pi \int_0^t f\tau \, \mathrm{d}\tau$. We use this observation as a starting point to define the synthesis of (linear) chirps.

1 Linear Chirp synthesis

In a linear *chirp* [BG73, SGH+00], the frequency is no longer constant but it either increases or decreases linearly with time. We use the term *upchirp* to refer to a chirp with linearly increasing frequency and to a *downchirp* when the frequency decreases with time.

In chirps, the derivative of the phase is no longer a constant but a function of time. In a linearly increasing frequency, this derivative is a linear function $\phi'(t) = 2\pi(f_0 + rt)$. The actual phase is obtained by integrating this quantity, as shown in Equation (2). In this presentation, f_0 is the initial frequency and $R_f = f'(t)$ is the rate of frequency change, also called the *chirp rate*. It can also be observed that $\phi''(t) = 2\pi R_f$. Upchirps are obtained with $R_f > 0$, downchirps with $R_f < 0$.

$$\begin{aligned}
\phi(t) &= 2\pi \int_0^t f_0 + R_f \tau \, \mathrm{d}\tau \\
&= 2\pi \left(f_0 t + \frac{R_f t^2}{2} \right)
\end{aligned} \tag{2}$$

Usually, chirps are limited in time. Let T be the duration of a chirp, f_0 the initial frequency and f_1 the final one. The bandwidth $B = f_1 - f_0$ is the portion of frequency spectrum that is occupied by the chirp. The rate of frequency change can therefore be expressed as $R_f = \frac{B}{T}$. Let's also assume for the sake

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of simplicity that the initial frequency, f_0, is zero. Using these conventions, a linear chirp of duration T and bandwidth B can be described by Equation (3).

$$s(t) = \sin 2\pi \frac{Bt^2}{2T}, \quad t \in [0, T) \tag{3}$$

Figure 1 shows an example upchirp that spans a bandwidth of $B = 32\,\mathrm{Hz}$ over a period of $T = 0.5\,\mathrm{s}$. The left part of the figure represents the chirp waveform which is a sinusoid that starts with a frequency of $0\,\mathrm{Hz}$ and ends with a frequency of $32\,\mathrm{Hz}$. The right part of the figure shows how the frequency increases linearly with time. The slope of the curve is given by $R_f = \frac{B}{T} = 64\,\mathrm{Hz/s}$.

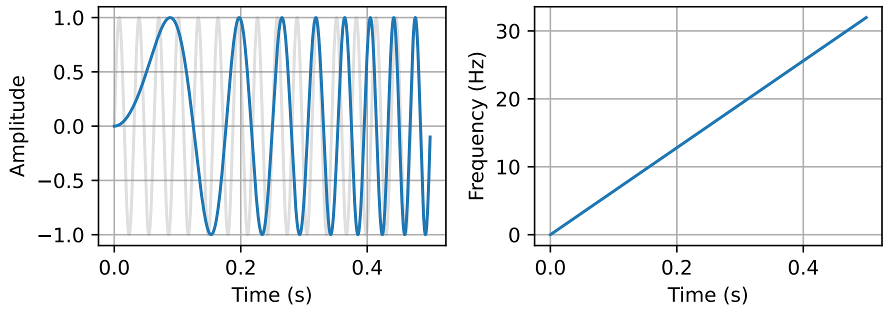

Figure 1: Linear chirp of duration $T = 0.5\,\mathrm{s}$ and bandwidth $B = 32\,\mathrm{Hz}$.

2 Chirp Spread Spectrum modulation

Frequency Shift Chirp Spread Spectrum (FSCSS) is a modulation technique [RP16, Van17] where data is carried over upchirps by shifting them cyclically by a discrete amount of time. Each upchirp typically carries a sequence of N bits. To this end, an upchirp is divided in 2^N *chips* which are the positions in time when frequency is adjusted.

Other chirp based digital modulations have been proposed in the past such as *slope-shifting* where upchirps represent binary 1s and downchirps binary 0s.

To perform modulation, the starting frequency of an upchirp is now a function of the N-bits value to be transmitted. Let $k \in \{0, \ldots, 2^{\mathrm{N}} - 1\}$ be the value to be transmitted. The upchirp starting frequency is equal to $\frac{kB}{2^{\mathrm{N}}}$. It then increases linearly at a rate of $\frac{B}{T}$. However, once the frequency has reached its maximum value, it wraps around and continues from zero upwards. The time at which the wrap-around occurs is equal to $t_{\mathrm{wrap}} = T\left(1 - \frac{k}{2^{\mathrm{N}}}\right)$.

Figure 2 shows a modulated upchirp of duration $T = 0.5\,\mathrm{s}$ and bandwidth $B = 32\,\mathrm{Hz}$. The carried value is $k = 2$ (seen as a 3-bits value). The left part shows the upchirp waveform. It starts with a frequency of $8\,\mathrm{Hz}$, then increases linearly to a maximum frequency of $32\,\mathrm{Hz}$ which is reached after $t_{\mathrm{wrap}} = 0.375\,\mathrm{s}$. At this point, the frequency immediately drops down to the lowest frequency of

0 Hz to finally increase again up to 8 Hz, spanning a full 32 Hz bandwidth over 0.5 s. Figure 3 shows the waveform obtained with the same parameters, but carrying value $k = 5$.

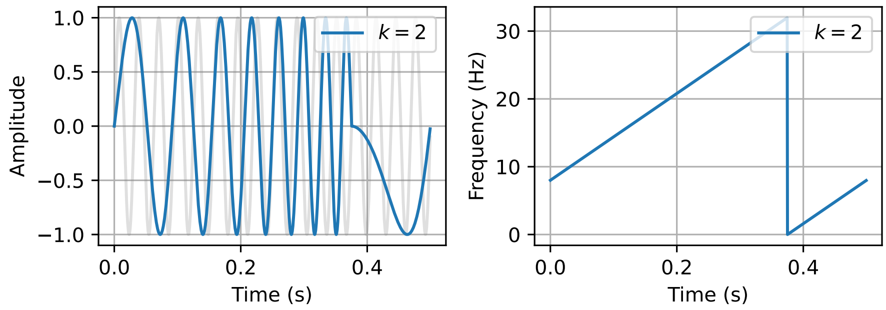

Figure 2: Modulated upchirp of duration $T = 1\,\mathrm{s}$ and bandwidth $B = 32\,\mathrm{Hz}$ carrying value $k = 2$ (among 2^3 possible values).

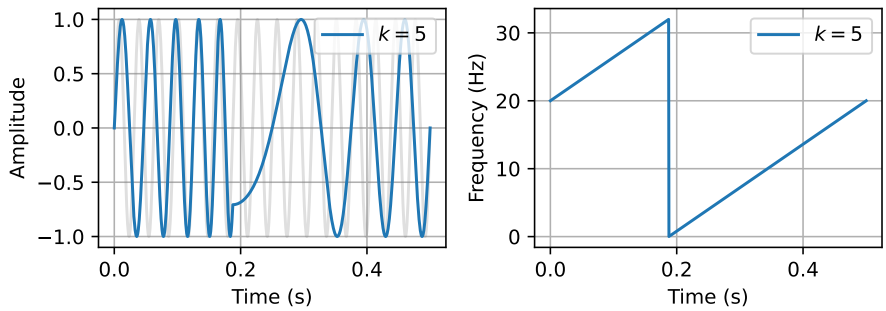

Figure 3: Modulated upchirp of duration $T = 1\,\mathrm{s}$ and bandwidth $B = 32\,\mathrm{Hz}$ carrying value $k = 5$ (among 2^3 possible values).

We split the derivation of the modulated upchirp equation into two cases : before and after the wrap-around.

Before wrap-around ($0 \le t < t_{\text{wrap}}$). Before the wrap-around time, the derivation is similar to Equation (3) with the exception that a shift in frequency is added that is proportional to the carried value k.

$$\begin{aligned}\phi(t) &= 2\pi \int_0^t \frac{kB}{2^{\mathrm{N}}} + \frac{B\tau}{T}\,\mathrm{d}\tau \\ &= 2\pi B t \left(\frac{k}{2^{\mathrm{N}}} + \frac{t}{2T} \right)\end{aligned}$$

After wrap-around ($t_{\text{wrap}} \leq t < T$). After the wrap-around, the frequency must start over with the initial frequency, hence we shift backward by an amount equal to the full bandwidth B.

$$\begin{aligned}\phi(t) &= 2\pi \int_0^t \frac{kB}{2^{\mathrm{N}}} - B + \frac{B\tau}{T}\, \mathrm{d}\tau \\ &= 2\pi Bt \left(\frac{k}{2^{\mathrm{N}}} - 1 + \frac{t}{2T} \right)\end{aligned}$$

3 LoRa CSS

The LoRa (Long-Range) communication technology relies on CSS as introduced in Section 2, but with a twist. LoRa imposes that the bandwidth of an upchirp be equal to the rate of chips, i.e. $B = R_{\mathrm{c}} = \frac{2^N}{T}$, or equivalently, the duration of a chip is equal to the inverse of the bandwidth, that is $\frac{1}{B}$. As a consequence, the chirp duration cannot be directly controlled, but instead the number N of bits sent by upchirp is a LoRa parameter named the *Spreading Factor* (SF)[1]. The duration of a chirp depends on the bandwidth and SF parameters, as shown in Equation (4).

$$T = 2^{\mathrm{SF}} \times R_{\mathrm{c}} = \frac{2^{\mathrm{SF}}}{B} \tag{4}$$

Show same symbol value carried with different SF values ?

References

[BG73] A. Berni and W. Gregg. On the utility of chirp modulation for digital signaling. *IEEE Transactions on Communications*, 21(6):748–751, 1973.

[Net] The Things Network. `https://www.thethingsnetwork.org/docs/lorawan/spreading-factors/`.

[RP16] Brecht Reynders and Sofie Pollin. Chirp spread spectrum as a modulation technique for long range communication. In *2016 Symposium on Communications and Vehicular Technologies (SCVT)*, pages 1–5, 2016.

[Sem15] Semtech. Application Note AN1200.22 – LoRa Modulation Basics. `https://www.frugalprototype.com/wp-content/uploads/2016/08/an1200.22.pdf`, May 2015.

[1] At first sight, it might not be obvious why this parameter is named *Spreading Factor*. It is actually linked with the processing gain obtained by spreading the data over a larger bandwidth [Net, Sem15]. This gain is calculated as $G_{\mathrm{p}} = \frac{R_{\mathrm{c}}}{R_{\mathrm{b}}}$ where R_{c} and R_{b} are the chip and bit rates respectively. Higher SF values provide higher processing gain, hence better receiver sensitivity, but at the cost of longer symbol duration, hence higher power consumption.

[SGH+00] A. Springer, W. Gugler, M. Huemer, L. Reindl, C.C.W. Ruppel, and R. Weigel. Spread spectrum communications using chirp signals. In *IEEE/AFCEA EUROCOMM 2000. Information Systems for Enhanced Public Safety and Security (Cat. No.00EX405)*, pages 166–170, 2000.

[Van17] Lorenzo Vangelista. Frequency shift chirp modulation: The lora modulation. *IEEE Signal Processing Letters*, 24(12):1818–1821, 2017.