Field Solver Test

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1 Introduction

The purpose of the test was to demonstrate that the field solver works, and to test the effects of grid refinement and increase of the order of accuracy of the solver on the results. The field solver is developed by Londrillo and del Zanna [1].

2 Test Case

Field solver test was made by using a simple advection setup in a $(x, y, z) \in [-1, 1] \times [-1, 1] \times [-1, 1]$ grid and by using periodic boundary conditions. The initial condition was $B_x = 1$ in $(y, z) \in [-0.25, 0.25] \times [-0.25, 0.25]$ and zero elsewhere. The field was then propagated with a constant velocity $\mathbf{V} = (0, -1, -1)$.

Test case was run with two different cell sizes, 10×10 (sparse) and 20×20 (dense). Time step in sparse grid was $\Delta t = 0.05$ and in total 40 time steps were calculated. In dense grid the time step was halved, $\Delta t = 0.025$ and in total 80 time steps were calculated. The time step was chosen in such a way that at the end of simulation the advected field was back at its initial position.

3 Results

Figures 1-6 show selected results from field solver test in sparse and dense grid, and with 1^{st}- and 2^{nd}-order accurate solvers. In 2^{nd}-order accurate cases three different slope limiters were used (minmod, MC limiter, Van Leer).

Formal demonstration of solver accuracy would be made by comparing the solutions against the initial state after a full period and by calculating the L^1 error norms. Here the peak value of B_x at the end of simulation is used as a proxy for solver accuracy instead. The L^1 error norms are not diffucult to calculate in this case, however.

The results indicate that moving into 2^{nd}-order accuracy has a larger impact on the quality of the solution than doubling of the grid size (which is what adaptive mesh refinement effectively does). Obviously the best results were obtained with a 2^{nd}-order accurate solver in denser grid. The results are summarized in Table 1.

Solver	sparse grid	dense grid
1^{st}	8.33%	16.1%
2^{nd} (minmod)	21.2%	50.6%
2^{nd} (MC limiter)	38.7%	93.1%
2^{nd} (Van Leer)	15.7%	34.4%

Table 1: Peak value of B_x, as percentage of initial value, with different solver versions in sparse and dense grid.

References

[1] Londrillo and del Zanna, J. Comp. Phys., 195, 2004.

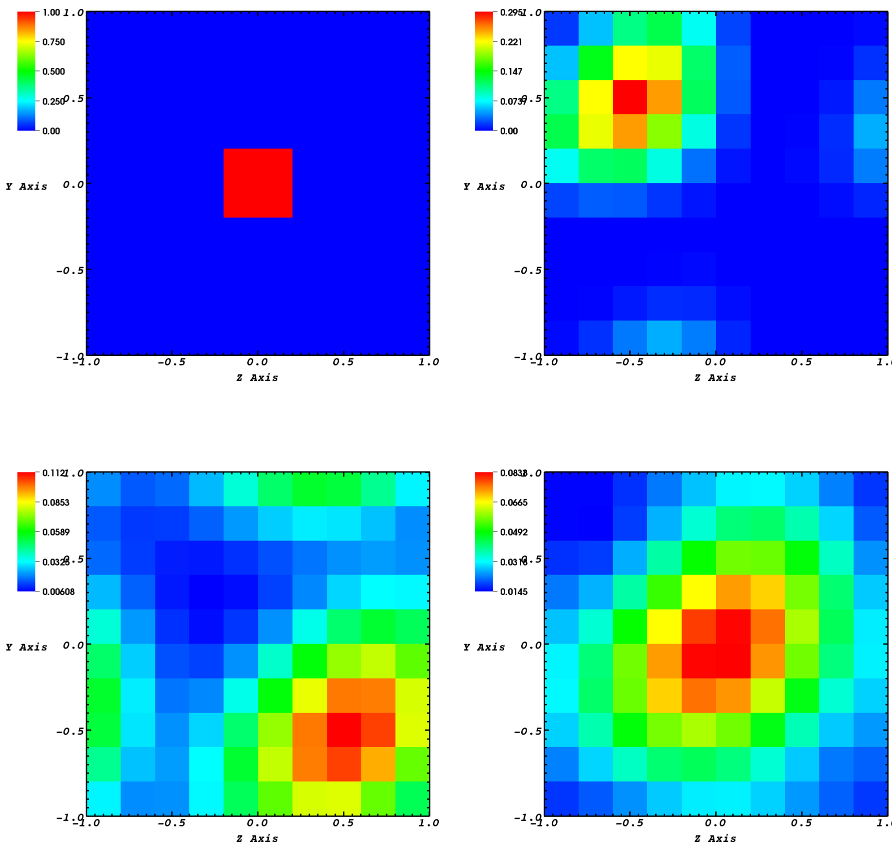

Figure 1: Propagation of B_x with a 1^{st}-order accurate field solver in a sparse grid. The pictures show the initial state, and solutions after 10, 30, and 40 time steps with periodic boundary conditions. Maximum value at 40 time steps is 8.33% of initial value.

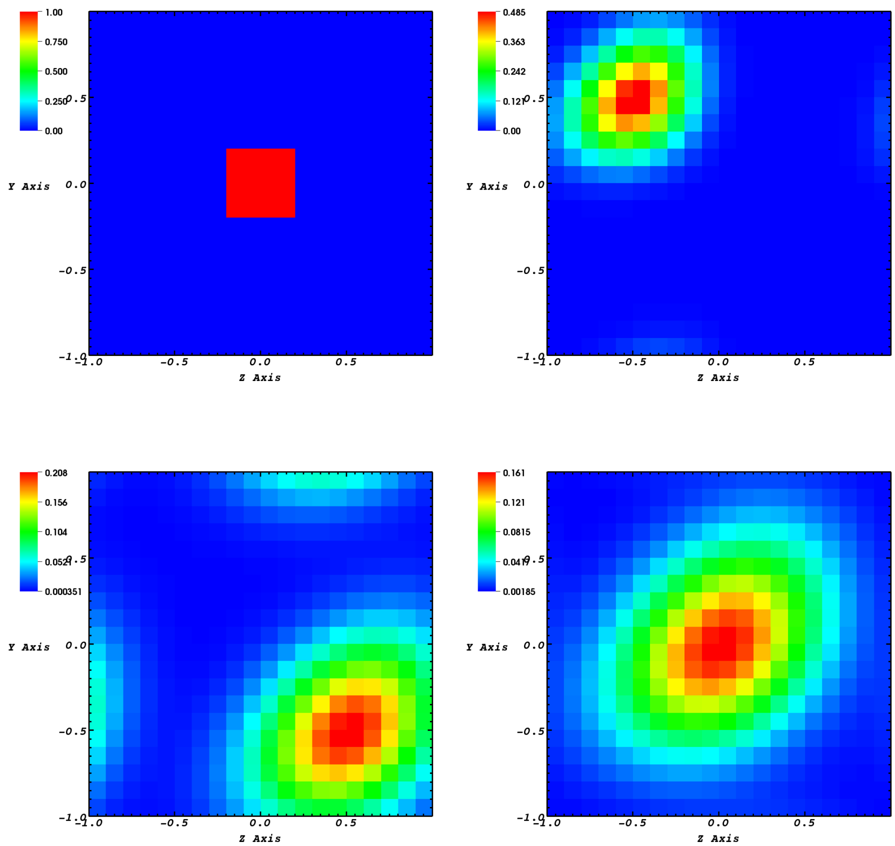

Figure 2: Propagation of B_x with a 1$^{\mathrm{st}}$-order accurate field solver in a dense grid. The pictures show the initial state, and solutions after 10, 30, and 40 time steps with periodic boundary conditions. Maximum value at 80 time steps is 16.1% of initial value.

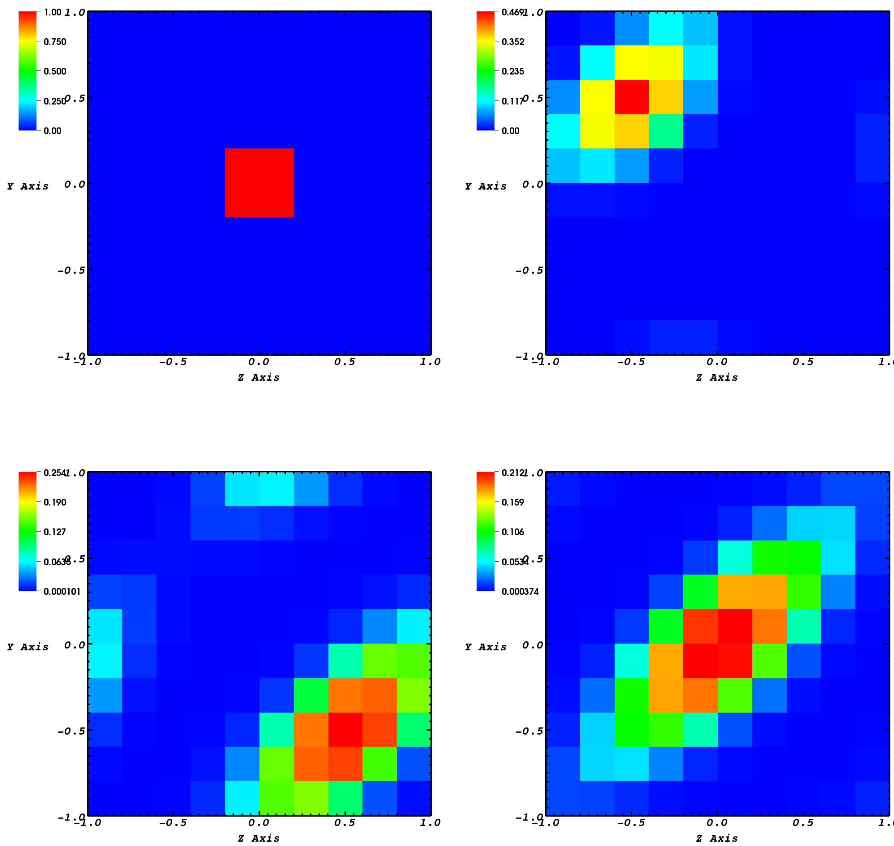

Figure 3: Propagation of B_x with a 2^{nd}-order accurate field solver in a sparse grid. The pictures show the initial state, and solutions after 10, 30, and 40 time steps with periodic boundary conditions. Maximum value at 40 time steps is 21.2% of initial value. Minmod limiter was used.

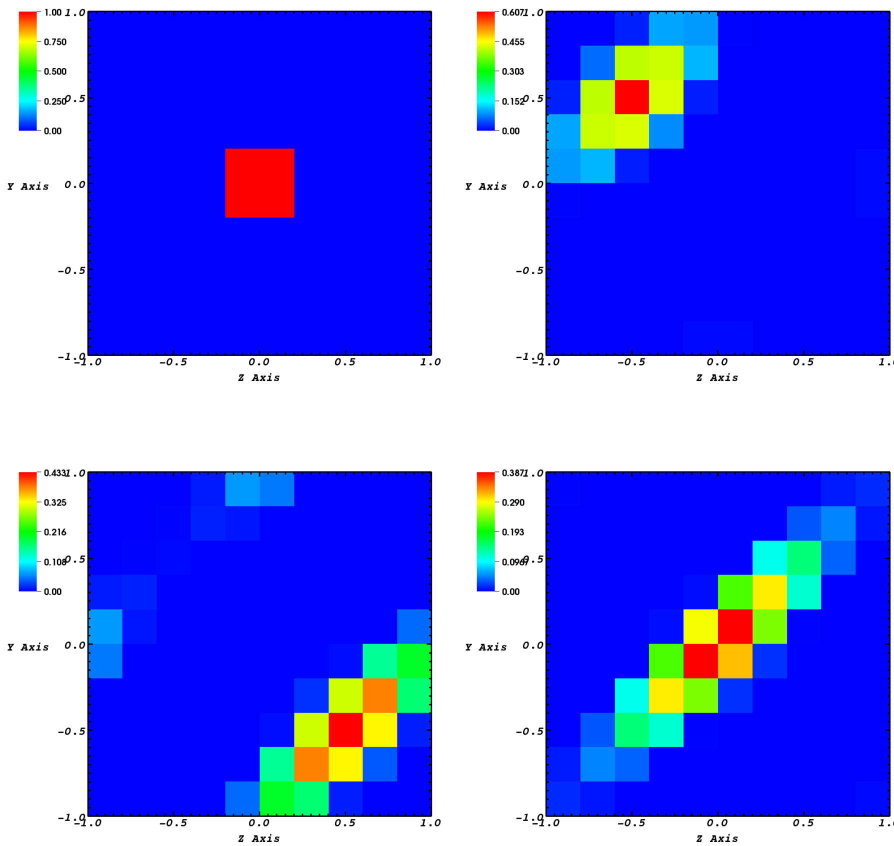

Figure 4: Propagation of B_x with a 2^{nd}-order accurate field solver in a sparse grid. The pictures show the initial state, and solutions after 10, 30, and 40 time steps with periodic boundary conditions. Maximum value at 40 time steps is 38.7% of initial value. MC limiter was used.

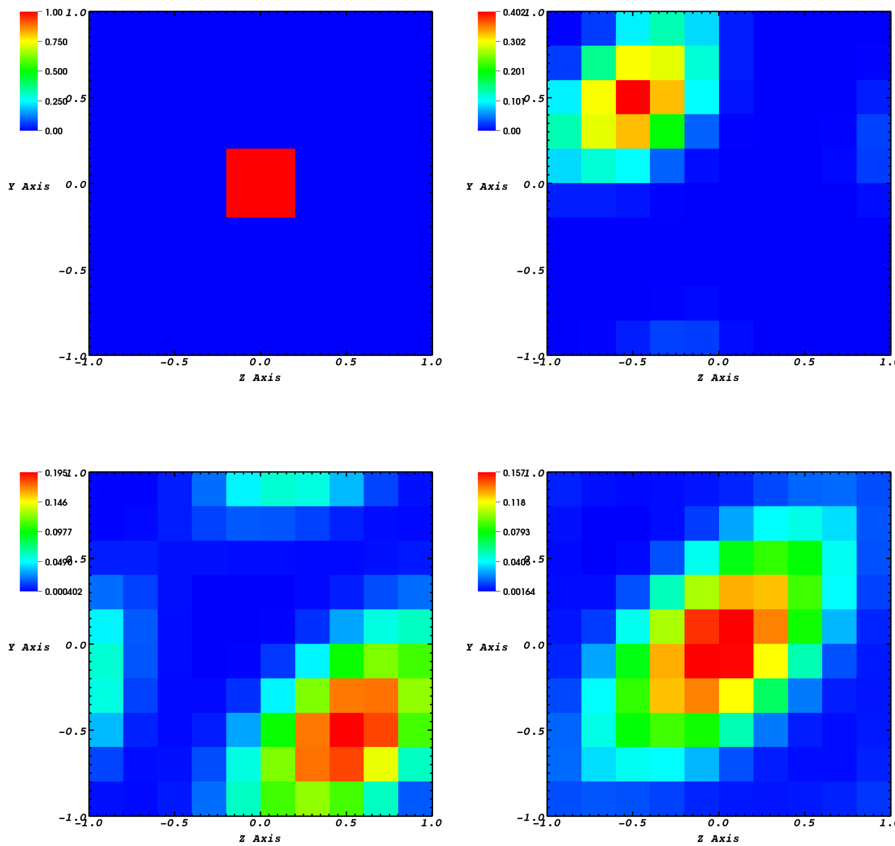

Figure 5: Propagation of B_x with a 2^{nd}-order accurate field solver in a sparse grid. The pictures show the initial state, and solutions after 10, 30, and 40 time steps with periodic boundary conditions. Maximum value at 40 time steps is 15.7% of initial value. Van Leer limiter was used.

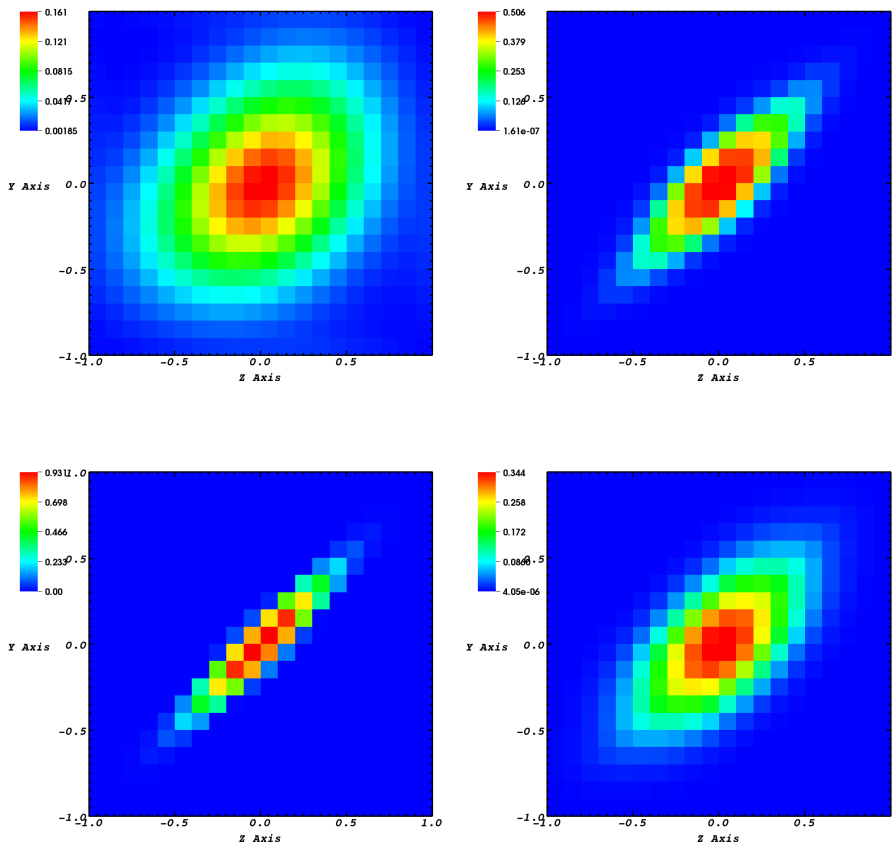

Figure 6: Numerical results of B_x propagation after 80 time steps in dense grid with (top left) 1$^{\text{st}}$-order accurate solver ($B_{x,\max} = 16.1\%$ of initial), and with 2$^{\text{nd}}$-order accurate solver with (top right) minmod limiter ($B_{x,\max} = 50.6\%$ of initial), (bottom left) MC limiter ($B_{x,\max} = 93.1\%$ of initial), (bottom right) Van Leer limiter ($B_{x,\max} = 34.4\%$ of initial).