The Lorentz force terms implemented in Vlasiator

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Abstract

Following discussions in `https://agile.fmi.fi/browse/QUESPACE-281` based on

- On magnetic reconnection regimes and associated three-dimensional asymmetries: Hybrid, Hall-less hybrid, and Hall-MHD simulations, Karimabadi et al. JGR 109, A09205, 2004;
- The hall effect in magnetic reconnection: Hybrid versus Hall-less hybrid simulations, Malakit et al. GRL 36, L07107, 2009,

the Lorentz force expression used in Vlasiator was changed. This document reviews the change and documents the new implemented terms for reference and verification.

Particularly the sign error spotted by Sebastian in December 2012 was found.

1 Previous expression

Prior to the changes in SVN r1323 the expression used was:

$$\mathbf{a} = \frac{q}{m}\left(\mathbf{E}_{\mathrm{VOL}} + \mathbf{V} \times \mathbf{B}_{\mathrm{VOL}}\right), \tag{1}$$

and the code (`lorentzForceFaceX()`):

```
const REAL VX = blockParams[BlockParams::VXCRD] +
    I*blockParams[BlockParams::DVX];
const REAL VY = blockParams[BlockParams::VYCRD] +
    (J+convert<REAL>(0.5))*blockParams[BlockParams::DVY];
const REAL VZ = blockParams[BlockParams::VZCRD] +
    (K+convert<REAL>(0.5))*blockParams[BlockParams::DVZ];
const REAL EX = cellParams[CellParams::EXVOL];
const REAL EY = cellParams[CellParams::EYVOL];
const REAL EZ = cellParams[CellParams::EZVOL];
const REAL BX = cellParams[CellParams::BXVOL];
const REAL BY = cellParams[CellParams::BYVOL];
const REAL BZ = cellParams[CellParams::BZVOL];
ax = Parameters::q_per_m*(EX + VY*BZ - VZ*BY);
ay = Parameters::q_per_m*(EY + VZ*BX - VX*BZ);
az = Parameters::q_per_m*(EZ + VX*BY - VY*BX);
```

. Arguments in the references above show that using this does not provide a Hall-less hybrid as no bulk forces are present in the description.

2 $\mathbf{J} \times \mathbf{B}$ in the Lorentz force

In this section I derive the term as was implemented in revision r1323 (and has been moved around since, sorry for the delay in writing this up), following Malakit et al. GRL 36, 2009.

Their Eq. (2) is

$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = \mathbf{E}_{ION} + \mathbf{v}_i \times \mathbf{B}, \tag{2}$$

where (Eq. (3) in Malakit)

$$\mathbf{E}_{ION} = -\left(\mathbf{u}_i - \frac{\mathbf{J}}{n}\right) \times \mathbf{B}, \tag{3}$$

so that

$$\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} = -\left(\mathbf{u}_i - \frac{\mathbf{J}}{n}\right) \times \mathbf{B} + \mathbf{v}_i \times \mathbf{B}, \tag{4}$$

where $\mathbf{u}_i$ is the bulk velocity, thus (moving gradually to CellParams notation)

$$\begin{aligned}
\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} &= \left(\frac{\mathbf{J}}{n} - \mathbf{u}_i + \mathbf{v}_i\right) \times \mathbf{B} && (5)\\
&= \left(\frac{\nabla \times \mathbf{B}}{\mu_0 n} - \mathbf{u}_i + \mathbf{v}_i\right) \times \mathbf{B} && (6)\\
&= \frac{\nabla \times \mathbf{B}}{\mu_0 n} \times \mathbf{B} + \left(\mathbf{v}_i - \frac{\mathrm{RHOV}}{\mathrm{RHO}}\right) \times \mathbf{B} && (7)\\
&= \frac{1}{\mu_0 \mathrm{RHO}} \left[\begin{pmatrix}\partial_x\\ \partial_y\\ \partial_z\end{pmatrix} \times \begin{pmatrix}\mathrm{BXVOL}\\ \mathrm{BYVOL}\\ \mathrm{BZVOL}\end{pmatrix}\right] \times \begin{pmatrix}\mathrm{BXVOL}\\ \mathrm{BYVOL}\\ \mathrm{BZVOL}\end{pmatrix} + \left(\mathbf{v}_i - \frac{\mathrm{RHOV}}{\mathrm{RHO}}\right) \times \mathbf{B} && (8)\\
&= \frac{1}{\mu_0 \mathrm{RHO}} \begin{pmatrix}\partial_y \mathrm{BZVOL} - \partial_z \mathrm{BYVOL}\\ \partial_z \mathrm{BXVOL} - \partial_x \mathrm{BZVOL}\\ \partial_x \mathrm{BYVOL} - \partial_y \mathrm{BXVOL}\end{pmatrix} \times \begin{pmatrix}\mathrm{BXVOL}\\ \mathrm{BYVOL}\\ \mathrm{BZVOL}\end{pmatrix} + \left(\mathbf{v}_i - \frac{\mathrm{RHOV}}{\mathrm{RHO}}\right) \times \mathbf{B} && (9)\\
&= \frac{1}{\mu_0 \mathrm{RHO}} \begin{pmatrix}\mathrm{BZVOL}\partial_z\mathrm{BXVOL} - \mathrm{BZVOL}\partial_x\mathrm{BZVOL} - \mathrm{BYVOL}\partial_x\mathrm{BYVOL} + \mathrm{BYVOL}\partial_y\mathrm{BXVOL}\\ \mathrm{BXVOL}\partial_x\mathrm{BYVOL} - \mathrm{BXVOL}\partial_y\mathrm{BXVOL} - \mathrm{BZVOL}\partial_y\mathrm{BZVOL} + \mathrm{BZVOL}\partial_z\mathrm{BYVOL}\\ \mathrm{BYVOL}\partial_y\mathrm{BZVOL} - \mathrm{BYVOL}\partial_z\mathrm{BYVOL} - \mathrm{BXVOL}\partial_z\mathrm{BXVOL} + \mathrm{BXVOL}\partial_x\mathrm{BZVOL}\end{pmatrix} - && (10)\\
&- \begin{pmatrix}\mathrm{VZ}\cdot\mathrm{BY} - \mathrm{BZ}\cdot\mathrm{VY}\\ \mathrm{VX}\cdot\mathrm{BZ} - \mathrm{BX}\cdot\mathrm{VZ}\\ \mathrm{VY}\cdot\mathrm{BY} - \mathrm{BY}\cdot\mathrm{VX}\end{pmatrix} + \frac{1}{\mathrm{RHO}} \begin{pmatrix}\mathrm{BY}\cdot\mathrm{RHOVZ} - \mathrm{BZ}\cdot\mathrm{RHOVY}\\ \mathrm{BZ}\cdot\mathrm{RHOVX} - \mathrm{BX}\cdot\mathrm{RHOVZ}\\ \mathrm{BX}\cdot\mathrm{RHOVY} - \mathrm{BY}\cdot\mathrm{RHOVX}.\end{pmatrix} && (11)
\end{aligned}$$

Note that in r1323 there was a sign error in the last two terms which has been corrected later on in r1497/r1500.

The last term can be replaced by the EVOL terms computed in the field solver as these are corrected, unlike the crude term given here.

A further step discussed is to add a resistivity coefficient to the current term so that one gets:

$$\begin{aligned}
\frac{\mathrm{d}\mathbf{v}_i}{\mathrm{d}t} &= \left(\eta\frac{\mathbf{J}}{n} - \mathbf{u}_i + \mathbf{v}_i\right) \times \mathbf{B} && (12)\\
&= \frac{\eta}{\mu_0 \mathrm{RHO}} \begin{pmatrix}\mathrm{BZVOL}\partial_z\mathrm{BXVOL} - \mathrm{BZVOL}\partial_x\mathrm{BZVOL} - \mathrm{BYVOL}\partial_x\mathrm{BYVOL} + \mathrm{BYVOL}\partial_y\mathrm{BXVOL}\\ \mathrm{BXVOL}\partial_x\mathrm{BYVOL} - \mathrm{BXVOL}\partial_y\mathrm{BXVOL} - \mathrm{BZVOL}\partial_y\mathrm{BZVOL} + \mathrm{BZVOL}\partial_z\mathrm{BYVOL}\\ \mathrm{BYVOL}\partial_y\mathrm{BZVOL} - \mathrm{BYVOL}\partial_z\mathrm{BYVOL} - \mathrm{BXVOL}\partial_z\mathrm{BXVOL} + \mathrm{BXVOL}\partial_x\mathrm{BZVOL}\end{pmatrix} && (13)\\
&- \begin{pmatrix}\mathrm{VZ}\cdot\mathrm{BY} - \mathrm{BZ}\cdot\mathrm{VY}\\ \mathrm{VX}\cdot\mathrm{BZ} - \mathrm{BX}\cdot\mathrm{VZ}\\ \mathrm{VY}\cdot\mathrm{BY} - \mathrm{BY}\cdot\mathrm{VX}\end{pmatrix} && (14)\\
&+ \mathbf{E}_{VOL} && (15)
\end{aligned}$$