

# Homework 7

Due October 19, 2022

## 1 Problem 1

The density function of  $X$  is given by

$$f(x) = \begin{cases} a + bx^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

If  $\mathbb{E}[X] = \frac{3}{5}$ , find  $a$  and  $b$ .

## 2 Problem 2

The lifetime in hours of an electronic tube is a random variable having a probability density function given by

$$f(x) = xe^{-x} \quad x \geq 0.$$

Compute the expected lifetime of such a tube.

## 3 Problem 3

If  $X$  is uniformly distributed over  $(0, 1)$ , find the density function of  $Y = e^X$ .

## 4 Problem 4

The random variable  $X$  has the probability density function

$$f(x) = \begin{cases} a + bx^2 & 0 \leq x \leq 1 \\ 0 & \text{otherwise.} \end{cases}$$

If  $\mathbb{E}[X] = \frac{3}{5}$ , find (a)  $\mathbb{P}\{X < \frac{1}{2}\}$  and (b)  $\text{Var}(X)$ .

## 5 Problem 5

Suppose that 3 balls are chosen without replacement from an urn consisting of 5 white and 8 red balls. Let  $X_i$  equal 1 if the  $i$ th ball selected is white, and let it equal 0 otherwise. Give the joint probability mass function of

- (a)  $(X_1, X_2)$ ;
- (b)  $(X_1, X_2, X_3)$ .

## 6 Problem 6

Calculate  $\text{Var}(X)$  when  $X$  is a general normal random variable (direct computation by integrating the probability density function, without using the centered and reduced normal random variable).

## 7 Problem 7 (Problem 5 of HW6)

Twelve percent of the population is left handed. Approximate the probability that there are at least 20 left-handers in a school of 200 students. State your assumptions.