Individual homework. All the answers have to be mathematically justified (unless specified explicitly).

Exercise 1.
Suppose that the distribution function of X is given by

$$F(b) = \begin{cases} 0 & b < 0 \\ \frac{b}{4} & 0 \leq b < 1 \\ \frac{1}{2} + \frac{b-1}{4} & 1 \leq b < 2 \\ \frac{11}{12} & 2 \leq b < 3 \\ 1 & 3 \leq b \end{cases}$$

(a) Find $P\{X = i\}, i = 1, 2, 3$.
(b) Find $P\left\{\frac{1}{2} < X < \frac{3}{2}\right\}$.

Exercise 2. Suppose that two teams play a series of games that ends when one of them has won i games. Suppose that each game played is, independently, won by team A with probability p. Find the expected number of games that are played when (a) $i = 2$ and (b) $i = 3$. Also, show in both cases that this number is maximized when $p = \frac{1}{2}$.

Exercise 3. N people arrive separately to a professional dinner. Upon arrival, each person looks to see if he or she has any friends among those present. That person then sits either at the table of a friend or at an unoccupied table if none of those present is a friend. Assuming that each of the $\binom{N}{2}$ pairs of people is, independently, a pair of friends with probability p, find the expected number of occupied tables. Hint: Let X_i equal 1 or 0 , depending on whether the ith arrival sits at a previously unoccupied table.

Exercise 4. A set of 1000 cards numbered 1 through 1000 is randomly distributed among 1000 people with each receiving one card. Compute the expected number of cards that are given to people whose age matches the number on the card.

Exercise 5. In Example 2h (Section 7.2 of Ross' textbook), say that i and j, $i \neq j$, form a matched pair if i chooses the hat belonging to j and j chooses the hat belonging to i. Find the expected number of matched pairs.

Exercise 6. If $E[X] = 1$ and $\mathrm{Var}(X) = 5$, find
(a) $E\left[(2 + X)^2\right]$
(b) $\mathrm{Var}(4 + 3X)$

Exercise 7. If X and Y are independent and identically distributed with mean μ and variance σ^2, find

$$E\left[(X - Y)^2\right]$$

Exercise 8. Let X be the number of 1's and Y the number of 2 's that occur in n rolls of a fair die. Compute $\mathrm{Cov}(X, Y)$.