IEOR240 : Homework 2

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Problem 1 : Giapetto's Woodcarving

We define two numbers : n_S as the number of soldier produced and n_T as the number of train produced. The optimization problem can be written as :

$$\begin{aligned} &\max_{n_S,n_T} 3n_s + 2n_T \\ &\text{s.t. } 2n_S + n_T \leq 100 \\ &n_S + n_T \leq 80 \\ &n_S \leq 40 \end{aligned} \tag{1}$$

My model file : pb1.mod

```
param n;
param f{i in 1..n};
param A{i in 1..n,j in 1..n};
param time{i in 1..n};
param max_Soldier;

var production{i in 1..n};

maximize profit:sum{i in 1..n}f[i]*production[i];


s.t. time_constraints{i in 1..n} :sum{j in 1..n}A[i,j]*production[j]<= time[i];
s.t. max_demand : production[1]<=max_Soldier;
```

My data file : pb1.dat

```
param n:= 2;
param f :=
1 3
2 2;

param A :
1 2 =
1 2 1
2 1 1;

param max_Soldier = 40;

param time:=
1 100
2 80;
```

My run file : pb1.run

```
reset;

model pb1.mod;

data pb1.dat;

option solver cplex;
solve;

display profit, production;
```

And the output is profit = 180 and production = [20,60] where production[1]= n_S and production[2]= n_T.

Problem 2 : Work Scheduling Problem

I introduced a variable nurse : N = $[n_1, n_2, n_3, n_4, n_5, n_6, n_7]^T$ where n_i is the number of nurse starting to work on the i-th day of the week. This problem can be written this way : with $r = [17, 13, 15, 19, 14, 16, 11]^T$ and $A = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$

$$\begin{gathered} \min_N (\textstyle\sum_{i=1}^{7} n_i) \\ \text{s.t. } AN \geq r \end{gathered} \tag{2}$$

My model file : pb2.mod

```
param days;

param week_schedule{i in 1..days,j in 1..days};
param requirement{i in 1..days};

var nurse{i in 1..days};


minimize nurse_needed : sum{i in 1..days}nurse[i];

s.t. day_nurse_need{i in 1..days}:
 sum{j in 1..days}week_schedule[i,j]*nurse[j]>=requirement[i];
s.t. realLife{i in 1..days} :nurse[i]>=0;
```

My data file : pb2.dat

```
param days := 7;

param week_schedule :
1 2 3 4 5 6 7 =
1 1 1 1 1 1 0 0
2 0 1 1 1 1 1 0
3 0 0 1 1 1 1 1
4 1 0 0 1 1 1 1
5 1 1 0 0 1 1 1
6 1 1 1 0 0 1 1
7 1 1 1 1 0 0 1;

param requirement :=
1 17
2 13
3 15
4 19
5 14
6 16
7 11;
```

My run file : pb2.run

```
reset;

model pb2.mod;

data pb2.dat;

option solver cplex;
solve;

display nurse_needed, nurse;
display nurse_needed, nurse > answer_pb2.txt;
```

The solution for this problem is : $N = [7.333, 0, 3.333, 5, 1.333, 5.3333, 0]$ and 22.333 nurse hired in total.
If we require nurses to be integer values : $N = [7, 0, 3, 5, 2, 6, 0]$ and 23 nurses are to be hired

Problem 3 : Blending Problem

This problem can be writen as follows :

$$\begin{gathered}
\max(0.2n_s + 0.25n_E) \\
\text{s.t. } p_{S1}n_S + p_{E1}n_E \leq 100 \\
p_{S2}n_S + p_{E2}n_E \leq 20 \\
p_{S3}n_S + p_{E3}n_E \leq 30 \\
p_{E1} + p_{E2} + p_{E3} = 1 \\
p_{S1} + p_{S2} + p_{S3} = 1, p_{S2} \geq 0.1, p_{S3} \geq 0.1, p_{E2} \geq 0.2 \\
n_S, n_E, p_{E1}, p_{E3}, p_{S1} \geq 0
\end{gathered} \tag{3}$$

Problem 4 : School Districts

In this problem I introduced a tensor $S = (s_{igj})_{i\in[1;I],g\in[1,G],j\in[1,J]}$ such that $\sum_{j=1}^{J} s_{ijg} = S_{ig}$. s_{ijg} reprensents the number of student from neighbourhood i of grade g going to school j so that finding all s_{ijg} identifies the maping between each student and its school.

The optimization problem becomes :

$$\min_{S}\left(\sum_{i,j} d_{ij}\left[\sum_{g} s_{igj}\right]\right) \text{ s.t. } \forall(i,j,g)\in[1;I]\times[1,G]\times[1,J], \sum_{j=1}^{J} s_{igj}=S_{ig}, \sum_{i=1}^{I} s_{igj}\leq C_{jg} \tag{4}$$