All the answer have to be mathematically justified (unless specified explicitly).

Exercise 1
A small community organization consists of 20 families, of which 4 have one child, 8 have two children, 5 have three children, 2 have four children, and 1 has five children.

1. If one of these families is chosen at random, what is the probability it has $i$children,$i = 1,2,3,4,5$?
2. If one of the children is randomly chosen, what is the probability that child comes from a family having i children,$i = 1,2,3,4,5$?

Exercise 2 Two dice are thrown n times in succession. Compute the probability that double 6 appears at least once. How large need n be to make this probability at least $1/2$?

Exercise 3 Five balls are randomly chosen, without replacement, from an urn that contains 5 red, 6 white, and 7 blue balls. Find the probability that at least one ball of each color is chosen.

Exercise 4 A family is composed by three children, each of which is equally likely to be a boy or a girl independently of the others. We define the events: $A = \{$ all the children are of the same sex$\}$, $B = \{$ there is at most one boy$\}$, $C = \{$ the family includes a boy and a girl$\}$.

1. Show that A is independent of B, and that B is independent of C.
2. Is A independent of C?
3. Do these results hold if boys and girls are not equally likely?
4. Do these results hold if the family is composed by four children?

Exercise 5 Let G be the event that an accused is guilty, and T the event that some testimony is true. Show that $\mathbb{P}(G|T) = \mathbb{P}(T|G)$ if and only if $\mathbb{P}(G) = \mathbb{P}(T)$.

Exercise 6 An ordinary deck of 52 cards is composed by four French suits: clubs, diamonds, hearts and spades. Each suit includes three court cards (face cards), King, Queen and Jack, with reversible (double-headed) images. Each suit also includes ten numeral cards or pip cards, from one to ten. The one is named the Ace.

Two cards are randomly chosen without replacement from an ordinary deck of 52 cards. Let B be the event that both cards are aces, let As be the event that the ace of spades is chosen, and let A be the event that at least one ace is chosen. Find

1. $\mathbb{P}(B|As)$
2. $\mathbb{P}(B|A)$