INDENG241 : Homework 4

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September 22, 2022

Exercise 1

Let D_1 the result of the roll of the first dice and D_2 the result of the roll of the second dice.
The universe $\Omega(X)$ is described by the following array :

D_1, D_2	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

We can conclude that $\forall i \in \{1, 9, 16, 25, 36\}, \mathbb{P}(X = i) = \frac{1}{36}$, $\forall i \in \{2, 3, 5, 8, 10, 15, 18, 20, 24, 30, \}\mathbb{P}(X = i) = \frac{2}{36} = \frac{1}{23}$, $\mathbb{P}(X = 4) = \frac{3}{36} = \frac{1}{12}$, $\forall i \in \{6, 12\}, \mathbb{P}(X = i) = \frac{4}{36} = \frac{1}{9}$

Exercise 2

If n is odd : $n = 2k + 1$ with $k \in \mathbb{N}$ then $X \in \{-(2k + 1), -(2k - 1), ..., -1, 1, 3, ..., 2k + 1\}$ and if n is even : $n = 2k$ with $k \in \mathbb{N}$ then $X \in \{-2k, -2k - 2, ..., -2, 0, 2, ..., 2k\}$.

Exercise 3

Let S the sales of the salesman. $\mathbb{E}(S) = 0.3 * 0.5 * 500 + 0.3 * 0.5 * 1000 + 0.6 * 0.5 * 500 + 0.6 * 0.5 * 1000 = 675$

Exercise 4

4.1

$\mathbb{P}(X > 0) = \mathbb{P}(X = 1) = \frac{18}{38} + (\frac{18}{38})^2 \frac{20}{38} = 0.59$

4.2

The stratedy don't seem to be a good way of winning because you have less than $\frac{1}{2}$ of winning each turn so is fortune likely to decrease after each round.

4.3

$\mathbb{E}(X) = \mathbb{P}(X = 1) - \mathbb{P}(X = -1) - 3\mathbb{P}(X = -3) = \frac{18}{38} + (\frac{18}{38})^2 \frac{20}{38} - 2(\frac{20}{38})^2 \frac{18}{38} - 3(\frac{20}{38})^3 = -0.108$

Exercise 5

$\mathbb{E}(X) = 50\frac{50}{148} + 33\frac{33}{148} + 25\frac{25}{148} + 40\frac{40}{148} = 39.3$
$\mathbb{E}(Y) = \frac{148}{4} = 37$
Obviously $\mathbb{E}(X) \geq \mathbb{E}(Y)$ because a student randomly chosen has more chance coming from a bus with more student inside.

Exercise 6

6.1

Two coins are flipped, denote X_1 the outcome of the first coin either H or T and X_2 the outcome of the second coin. $\mathbb{P}(X = 0) = \mathbb{P}(X_1 = T, X_2 = T) = \mathbb{P}(X_1 = T)\mathbb{P}(X_2 = T) = 0.12$
$\mathbb{P}(X = 1) = \mathbb{P}(X_1 = T, X_2 = H) + \mathbb{P}(X_1 = H, X_2 = H) = 0.6 * 0.3 + 0.7 * 0.4 = 0.46$
$\mathbb{P}(X = 2) = \mathbb{P}(X_1 = H, X_2 = H) = 0.6 * 0.7 = 0.42$

6.2

$\mathbb{E}(X) = \mathbb{P}(X = 1) + 2\mathbb{P}(X = 2) = 1.3$