Case Study: Calgary Desk Company

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1 Problem Formulation

Calgary Desk Company (CALDESCO) is a well-established manufacturer and is about to plan the production schedule for its entire line of desks for September. CALDESCO seeks a production schedule recommendation with a detailed report of the required resources for maximising profit.

The purpose of this report is to analyse available resources, costs, and profits to provide CALDESCO an optimal production schedule recommendation. Based on the company's internal policies, all desks previously manufactured have been sold and we assume all future desks will be sold in a given month. Hence, the unit profit generated by each desk is considered reliable.

1.1 Variables and Parameters

The following decision variables are used in the mathematical problem formulation. This key may be referenced throughout the report to determine a particular desk line and size manufactured by CALDESCO.

Decision Variables	Desk(Line, Size)
x_{11}	Economy, Student
x_{12}	Economy, Standard
x_{13}	Economy, Executive
x_{21}	Basic, Student
x_{22}	Basic, Standard
x_{23}	Basic, Executive
x_{31}	Hand-crafted, Student
x_{32}	Hand-crafted, Standard
x_{33}	Hand-crafted, Executive

1.2 Objective Function

The objective function is mathematical notation representing the problem CALDESCO desires to solve. Every decision variable coefficient is the profit in U.S. Dollars each desk is known to yield. Hence, the maximisation problem:

$$\max \quad 20x_{11} + 30x_{12} + 40x_{13} + 50x_{21} + 80x_{22} + 125x_{23} + 100x_{31} + 250x_{32} + 325x_{33}$$

1.3 Constraints

CALDESCO manufactures desks subject to certain limitations on resources. The following subsections outline the available resources to be used in production and lists these constraints in mathematical form. Additional details are discussed later in Subsection 3.2.

1.3.1 Monthly Production

An in-house minimum production quota has been set by CALDESCO according to Table 1 below.

Table 1: Minimum number of desks to be manufactured

Decision Variables	x_{11}	x_{12}	x_{13}	x_{21}	x_{22}	x_{23}	x_{31}	x_{32}	x_{33}
Total Desks ($\geq$)	750	1500	100	400	1500	100	25	150	50

1.3.2 Materials

Three material sources are used to manufacture desks. The amounts of each material are given in square feet.

Aluminium: $14x_{11} + 24x_{12} + 30x_{13} \leq 65,000$

Particle board: $8x_{11} + 15x_{12} + 24x_{13} \leq 60,000$

Pine sheet: $22x_{21} + 40x_{22} + 55x_{23} + 25x_{31} + 45x_{32} + 60x_{33} \leq 175,000$

1.3.3 Labour

CALDESCO currently employs a workforce of 30 craftsmen. The time required for making any model desk, running of the manufacturing lines, assembling the product, or performing the detailed operations necessary to produce the hand-crafted models is constrained by the formula below. Labour time is in man-minutes.

$$15x_{11} + 17x_{12} + 19x_{13} + 23x_{21} + 28x_{22} + 32x_{23} + 76x_{31} + 93x_{32} + 110x_{33} \leq 230,400$$

1.3.4 Line Production Time

The manufacturing times available on the three production lines are constrained by the summary below. Production line times are in minutes.

Line 1: $1.5x_{11} + 2.0x_{12} + 2.5x_{13} \leq 9,600$

Line 2: $x_{21} + x_{22} + x_{23} + x_{31} + x_{32} + x_{33} \leq 9,600$

Line 3: $3x_{21} + 4x_{22} + 5x_{23} + 3x_{31} + 4x_{32} + 5x_{33} \leq 19,200$

1.3.5 Production Quotas

By adhering to internal policies of production quotas, CALDESCO has been able to sell all desks it manufactures in a particular month and maintain its profit margins. To ensure these sales and margins continue, the ratios of total production hereunder are to be sustained.

Economy:

$$0.2 \leq \frac{x_{11} + x_{12} + x_{13}}{\sum_{i=1}^{3} \sum_{j=1}^{3} x_{i,j}} \leq 0.5$$

Basic:

$$0.4 \leq \frac{x_{21} + x_{22} + x_{23}}{\sum_{i=1}^{3} \sum_{j=1}^{3} x_{i,j}} \leq 0.6$$

Hand-crafted:

$$0.1 \leq \frac{x_{31} + x_{32} + x_{33}}{\sum_{i=1}^{3} \sum_{j=1}^{3} x_{i,j}} \leq 0.2$$

Student:

$$0.2 \leq \frac{x_{11} + x_{21} + x_{31}}{\sum_{i=1}^{3} \sum_{j=1}^{3} x_{i,j}} \leq 0.35$$

Standard:

$$0.4 \leq \frac{x_{12} + x_{22} + x_{32}}{\sum_{i=1}^{3} \sum_{j=1}^{3} x_{i,j}} \leq 0.7$$

Executive:

$$0.05 \leq \frac{x_{13} + x_{23} + x_{33}}{\sum_{i=1}^{3} \sum_{j=1}^{3} x_{i,j}} \leq 0.15$$

2 Recommendation

After building our model and executing computations, we calculate that the maximum profit will be **$612,113.38** for the month of September. In order to realise this optimal value, the following units should be produced:

Table 2: Minimum monthly desk production required for optimality

SIZE\LINE	ECONOMY	BASIC	HAND-CRAFTED
STUDENT	750	525	25
STANDARD	1500	1657	1069
EXECUTIVE	100	825	50

Note that the raw results give an optimal solution where some of the desk production requirements returned as fractional units. Since desks cannot be sold as part of a whole unit, the values shown in Table 2 were rounded down to the closest integer.

3 Sensitivity Report

A sensitivity report is used to identify how much variations in the input values for a given variable impact the results of a mathematical model. This research team used AMPL software to compute the linear optimisation problem and generate a sensitivity analysis. The results in Tables 3 and 4 display the output produced by AMPL. Note that the column headers specifically reference AMPL code syntax. The header meanings are discussed in Subsections 3.1 and 3.2.

Table 3: Variables recorded from AMPL output

DESK(Line,Size)	var.val	var.rc	var.current	var.down	var.up
ECONOMY, STUDENT	750	0	20	0	24.06
ECONOMY, STANDARD	1500	0	30	0	42.05
ECONOMY, EXECUTIVE	100	0	40	0	78.34
BASIC, STUDENT	525.54	0	50	45.40	61.67
BASIC, STANDARD	1657.50	0	80	72.03	83.06
BASIC, EXECUTIVE	825.40	0	125	121.22	171.26
HAND-CRAFTED, STUDENT	25	0	100	0	188.36
HAND-CRAFTED, STANDARD	1069.24	0	250	246.89	270.17
HAND-CRAFTED, EXECUTIVE	50	0	325	0	328.77

3.1 AMPL Variables Explained

var.value = OPTIMAL SOLUTION VALUES

The optimal solution indicates that to maximise profit, a "var.val" number of units should be produced. For example, CALDESCO would need 750 units of Line - ECONOMY, Size - STUDENT; and 1500 units of Line - ECONOMY, Size - STANDARD. The remaining variables can be interpreted in a similar manner.

Mathematically, the "var.val" values multiplied by the corresponding profit for each combination of Line and Size give the optimal solution value of $612,113.38, which is calculated as: $750 \times 20 + 1500 \times 30 + 100 \times 40 + \cdots + 50 \times 325 = \$612,113.38$

var.rc = REDUCED COST

The opportunity/reduced cost of a given decision variable can be interpreted as the rate at which the value of the objective function, i.e., profit, will deteriorate for each unit change in the optimised value of the decision variable with all other data being held constant. In other words, it is the amount by which an objective function coefficient (expected profit for each type) would have to improve (increase) before it would be possible for a corresponding variable to assume a positive value in the optimal solution. This means that if the company begins production of a particular type of desk then the new expected profit will affect the optimal solution.

Suppose that a given desk has a reduced cost of $10, then the objective coefficient (profit) of its decision variable would have to increase by 10 units in a maximisation problem for the variable to become an attractive alternative to enter into the solution. Since the recommended optimal solution requires production of all types of desks (var.val),the reduced cost is zero. For this to hold true, notice that either the optimal solution or the corresponding reduced cost variables must be zero.

var.current = PROFIT COEFFICIENT

The var.current column here represents the current unit profit contributed by each desk model and style sold by the company. For example, when the company sells a Size - STUDENT desk from the Line - ECONOMY, it makes a profit of $20. Likewise, when the company sells a Size - STANDARD from the Line - HAND-CRAFTED it makes a profit of $250.

var.down = LOWER BOUND VALUE

The var.down column represents the lower bound of the profit values in the var.current column for each type of desk sold by the company. For example, if the profit value goes below $45.40 when the company sells the Size - STUDENT from the Line - BASIC, the overall maximum profit (objective value) changes and the problem will have to be recalculated to produce a different number of desks.

var.up = UPPER BOUND VALUE

The var.up column denotes the upper bound value of the profit values in the var.current column for type of desk sold by the company. Similar to the lower bound, if the profit value goes beyond \$328.77 when the company sells a Size - EXECUTIVE in the Line - HANDCRAFTED, the overall maximum profit value changes and the problem will have to be recalculated to produce a different number of desks.

Now that we have an understanding of the various columns in the variable table, let us understand the interpretation of it with an example in detail.

Let us consider a desk of Size - STUDENT in the Line - ECONOMY with an estimated profit of \$20 (var.current). Now, suppose the company wants to increase the maximum profit value with a margin, it must lie between the upper bound value (var.up) and the lower bound value (var.down) if the company wishes to avoid some impact on the overall maximum profit value (objective function value).

For example, if the company wants to profit \$21 for every Size - STUDENT desk sold from the Line - ECONOMY, the number of units that should be produced (var.val) to achieve the desired total maximum profit (objective function value = \$612,113.38) will remain the same. The actual profit that the company makes, however, will increase by an additional \$750. Hence, the new maximum profit made by the company will be \$612,113.38 + \$750 = \$612,863.38.

Still if the company wants to increase the profit margin to \$25, which is more than the upper bound value (var.up), the number of desks from Size - STUDENT from the Line - ECONOMY required for production will need to be recalculated.

3.2 AMPL Constraints Explained

Based on the current availability of resources, optimal restrictions can be set to ensure profits are maximised. The constraints computed by AMPL are shown in Table 4. Note that several values in the lower bounds from AMPL's raw output were originally calculated to approach negative infinity. Resources that were shown to have negative infinite bounds have been limited to zero since it would not be reasonable to assume such losses in time, materials, or production of desks.

con = SHADOW PRICE

The shadow price of a resource constraint in linear programming is defined as the maximum price which should be paid to obtain one additional unit of said resource. In other words, the shadow price indicates how much more profit to expect by increasing the amount of that resource by one unit. A good way to think about shadow price is by asking, "How much should the company be willing to pay for one additional unit of resource?"

In linear programming, shadow prices are constant over a range of possible changes in both unit profit and resource constraints of the objective function. For example, Table 4 shows the current available labour is 230,400 man-minutes. If labour were increased by one unit to 230,401 man-minutes, it would mean that the expected profit (objective value) would increase by the corresponding shadow price of \$2.59735.

Table 4: Optimal model results obtained from AMPL output

RESOURCES	MODEL RESULTS				
	con	con.slack	con.current	con.down	con.up
Aluminium (ft^2)	0	15,500	65,000	49,500	∞
Particle Board (ft^2)	0	29,100	60,000	30,900	∞
Pine Sheets (ft^2)	0.234495	0	175,000	169,646	195,940
Production Line 1 (min)	0	5,225	0	0	0
Production Line 2 (min)	0	4,241.56	9,600	5,358.44	∞
Production Line 3 (min)	0	2,264.39	19,200	16,935.60	∞
Labour (man-min)	2.59735	0	230,400	200,443	239,325
Economy Min	0	1,049.46 0	0	-1,049.46	∞
Economy Max	0	901.343	0	0	901.343
Basic Min	0	407.367	0	-407.367	∞
Basic Min	0	893.17	0	0	893.17
Hand-crafted Min	0	493.976	0	-493.976	∞
Hand-crafted Max	0	156.293	0	0	156.293
Student Min	-12.7924	0	0	-114.494	281.915
Student Max	0	975.403	0	-975.403	∞
Standard Min	0	1,625.67	0	0	1,625.67
Standard Max	0	325.134	0	-325.134	∞
Executive Min	0	650.269	0	0	650.269
Executive Max	31.0932	0	0	-331.807	114.905

con.slack=SLACK

Slack refers to the number of resources that were unused in order to arrive at the maximised profit. For example, Aluminium has an extra 15,500 units. Note that when there is slack, the shadow price is \$0 because buying more of a resource that already exists will not change the result.

con.current = CURRENT AVAILABLE RESOURCES

This column shows the resources currently available to CALDESCO. Table 4, for instance, shows 175,000 ft^2 of pine sheets available to be used in manufacturing. Production Line 3 has 19,200 minutes available and so forth. The number of resources to be allocated for manufacturing in the recommended production schedule are derived from these values as: Current Available Resources - Slack.

con.down = LOWER BOUND of CONSTRAINTS

The lower bound refers to the amount by which a resource could be decreased without changing the shadow price value.

The allowable decrease for a non-binding constraint—resources that have slackness—is equal to the slack in the constraint. Hence, from Table 4 the allowable decrease for Aluminium is 49,500 ft^2. This means that as long the square feet of Aluminium does not decrease below this limit then the optimal basis will not change. In fact, the only part of the solution that changes when modifying amounts of a resource used within its bounds is the value of its slack variable. Saying that the basis does not change means that the variables that were zero in the original solution continue to be zero in the new problem—the constrained resource being the only value which changes.

For a binding constraint, i.e, slack is zero, if its value is decreased to anything less than the lower bound then the optimal basis does not change but the solution of the problem does change. As a result, this may require resolving of the linear program.

con.up = UPPER BOUND of CONSTRAINTS

The upper bound refers to the amount by which the resource could be increased without changing the shadow price value.

The allowable increase for a non-binding constraint is equal to infinity. From Table 4, the allowable increase for Aluminium is infinite square feet due to CALDESCO currently having a surplus of this resource. Increasing resources that already have a surplus will not change the optimal solution.

For a binding constraint on resources, if the constraint is increased to a value greater than the upper bound then the optimal basis does not change but the solution of the problem will change. As a result, this may also require resolving the linear program.

Example: If square feet of pine sheets used in production increases or decreases by one unit within a range from 169,646 ft^2 to 195,940 ft^2, then the profit increases or decreases by the shadow price of $0.234495. However, if the number of square feet used in production go beyond this range then the current optimal solution may not hold true, which would require reformulating and resolving the linear program to get a new solution.

4 "What-if" Analyses

The next three subsections consider the consequences that result from modifying certain parameters at management's discretion. Each scenario focuses on a particular area of production and analyses how the changes would affect company profits.

4.1 ORDERS

Consider the Size - STUDENT desk from the Line - HAND-CRAFTED, which has a current demand for 25 units. If the company wanted to forgo manufacturing these desks, how would profits change assuming no buyer penalties? If CALDESCO decided not to produce this type of desk at all for the month of September, there would be an increase in profit of $2,209.07. This is because the shadow price of a hand-crafted student desk is $-\$88.3629$ (See Appendix D), so $\$-88.3629 \times -25 = \$2,209.0725$.

What would be the change if they did not fulfill these orders but this time incurred a buyer imposed penalty of $10 per desk? The updated profit after not fulfilling the orders is $614,322.46. However, by enforcing a nondelivery penalty the company's profits would only decrease by $25 \times \$10 = \$250$ to $\$614,072.46$, which indicates that a benefit still exists.

Now, if instead CALDESCO decided to fulfill the Size - STUDENT desk from the Line - HAND-CRAFTED with a different model desk, how would the results change? By replacing the order with a different type of desk the effect on profit can be calculated in a similar manner. In this case, assume the order is replaced in its entirety with the new model so that no mixing occurs. Simply multiply the shadow price of the new model by the 25 replacement units.

The results show that offering Size - STUDENT, STANDARD, and EXECUTIVE from the Line - BASIC or Size - STANDARD from the Line - HAND-CRAFTED would have no effect on total profit because they each have slackness (shadow price of $0). On the other hand, replacing the order with either Size - STUDENT from the Line - ECONOMY ($25 \times -\$4.06234 = -\$101.5585$) or Size - STANDARD from the Line - HAND-CRAFTED ($25 \times -\$3.7655 = -\$94.1375$) would have the least impact on profit since these amounts are less than the $250 penalty. In both of these cases, the new profits become $614,220.90 and $614,228.32, respectively.

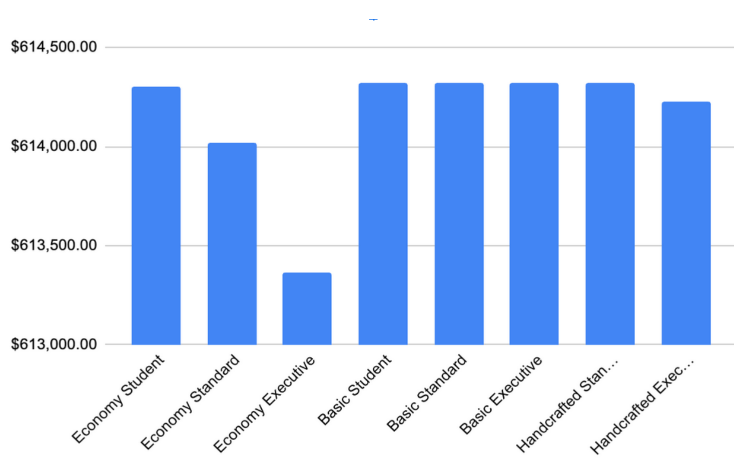

Figure 1: Redistribution of profits for a replacement order given the different desk types

4.2 LABOUR TIME

Table 4 shows labour time can be increased to a maximum of 239,325 man-minutes and still achieve optimality. Using the logic described for shadow price in Subsection 3.2, every man-minute increase in labour time will increase profits by \$2.59735.

The company begins with a total workforce of 30 craftsmen providing 230,400 man-minutes of labour time. One new employee contributes an additional 7,680 man-minutes and generates $7,680 \times \$2.59735 = \$19,947.65$ more in profits.

Suppose CALDESCO hires five new workers so that labour time has increased to 268,800 man-minutes. Then nothing can be said about the new profit because it is greater than the upper bound, which is 239,325 man-minutes. Instead, the company can hire one new craftsman to increase labour time to 238,080 man-minutes whilst collecting a new maximum profit of \$632,061.03.

Ideally, utilising labour time up to the 239,325 man-minute bound would mean CALDESCO can improve its labour resource by 3.87% and the maximum profit generated would be: \$635,294.73. The consequences on profit by increasing labour time until the 239,325 limit is represented graphically in Figure 2.

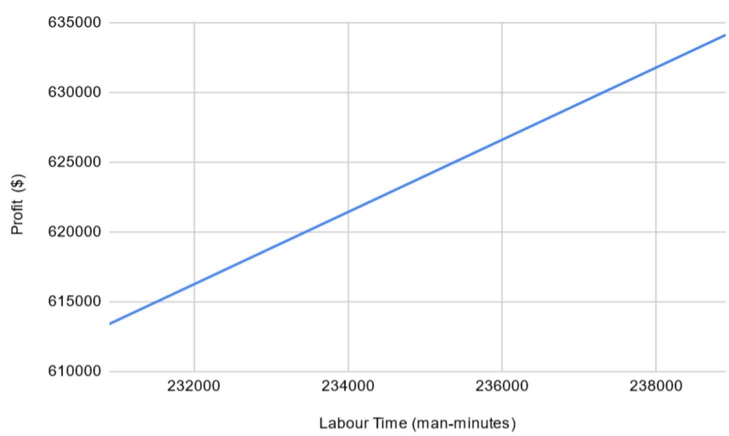

Figure 2: Influence of labour on profit

4.3 PRICING

Suppose CALDESCO wants to explore the effects of changing the sale prices. It does not want any changes to impact the sales mix. If the company wants to increase the price of one desk, which type should be chosen to maximise profit? If they want to decrease the price of one desk, which type should they choose in order to maximise profit?

CALDESCO can decrease prices down to the point where the profit coefficient is at the lower bound (var.down). Any decrease beyond this point would result in a new optimal solution and thus a new sales mix. To find the decrease in total profit take the difference between the current coefficient and lower bound, then multiply by demand for each desk type. Note that the analysis in Appendix D reports some lower bounds as ranging through negative infinity. We assume the company will not sell desks at a loss, hence the profit coefficients are set to be greater than or equal to zero. In Table 5, lower bounds originally output as negative infinity are shown as zero.

Alternatively, CALDESCO can increase prices for each desk up to the point where the profit coefficient reaches the upper bound (var.up). Any increase beyond this point would result in a new optimal solution and thus a new sales mix. See Figure 3 for more details.

Table 5: Minimum/Maximum Pricing

DESK(Line,Size)	Units	Profit	Lower	Profit Decrease	Upper	Profit Increase
ECONOMY, STD.	750	\$20	0	−\$15,000.00	24.0623	+\$3,046.73
ECONOMY, STN.	1500	\$30	0	−\$45,000.00	42.049	+\$18,074.10
ECONOMY, EXC.	100	\$40	0	−\$4,000.00	78.3373	+\$3,833.73
BASIC, STD.	525.537	\$50	45.3989	−\$2,418.05	61.6671	+\$6,131.49
BASIC, STN.	1657.5	\$80	72.0293	−\$13,211.44	83.0628	+\$5,076.59
BASIC, EXC.	825.403	\$125	121.219	−\$3,120.85	171.259	+\$38,182.32
HND-CFD, STD.	25	\$100	0	−\$2,500.00	188.363	+\$2,209.08
HND-CFD, STN.	1069.24	\$250	246.892	−\$3,323.20	270.172	+\$21,568.71
HND-CFD, EXC.	50	\$325	0	−\$16,250.00	328.766	+\$188.30

The results show that a decrease in the price for the Size - STUDENT desk from the Line - BASIC would affect overall production the least. Interestingly, if the Size - EXECUTIVE from the Line - ECONOMY or Size - STUDENT from the Line - HAND-CRAFTED were sold at cost, profits reductions would be less than $5,000. Whereas, an $8 price reduction to the Size - STANDARD from the Line - BASIC would lead to a profit reduction greater than $13,000.

The results above show that the high upper bound of the Size - EXECUTIVE from the Line - BASIC can potentially increase total profits more than $38,000 by increasing sales just over $3. A similar increase in price on the Size - EXECUTIVE from the line HAND-CRAFTED desk would only increase profit by $188.

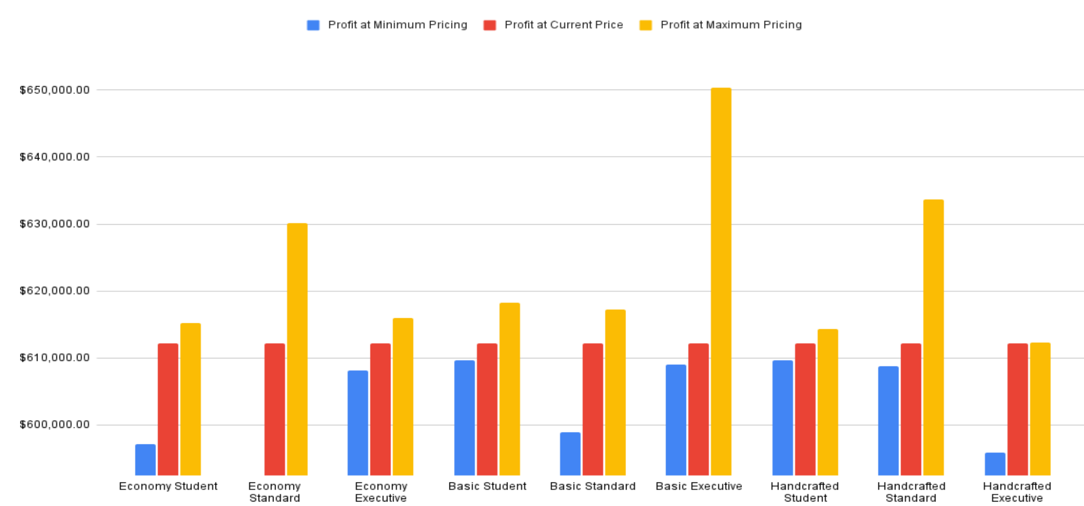

Figure 3: Pricing impact on profit

Appendix A .data file

```
set LINE = L1 L2 L3;
set SIZE = S1 S2 S3;

param profit:
           S1 S2 S3 =
        L1  20 30 40
        L2  50 80 125
        L3  100 250 325;

param s_orders:
           S1 S2 S3 =
        L1  750 1500 100
        L2  400 1500 100
        L3  25 150 50;

param alum:
           S1 S2 S3 =
        L1  14  24  30
        L2  0   0   0
        L3  0   0   0;

param particle:
           S1 S2 S3 =
        L1  8   15  24
        L2  0   0   0
        L3  0   0   0;

param pine:
           S1 S2 S3 =
        L1  0   0   0
        L2  22  40  55
        L3  25  45  60;

param pline_1:
           S1 S2 S3 =
        L1  1.5  2.0  2.5
        L2  0   0   0
        L3  0   0   0;


param pline_2:
           S1 S2 S3 =
        L1  1   1   1
        L2  1   1   1
        L3  0   0   0;
```

```
param pline_3:
           S1 S2 S3 =
        L1  0  0  0
        L2  3  4  5
        L3  3  4  5;

param assembly:
           S1 S2 S3 =
        L1  10 11 12
        L2  15 18 20
        L3  20 25 30;

param hand_craft:
           S1 S2 S3 =
        L1  0  0  0
        L2  0  0  0
        L3  50 60 70;
```

Appendix B .mod file

```
set LINE;
set SIZE;

param profit{LINE,SIZE};

param s_orders{LINE,SIZE};
param alum{LINE,SIZE};
param particle{LINE,SIZE};
param pine{LINE,SIZE};
param pline_1{LINE,SIZE};
param pline_2{LINE,SIZE};
param pline_3{LINE,SIZE};
param assembly{LINE,SIZE};
param hand_craft{LINE,SIZE};
#variables;
var x{LINE,SIZE} >=0;
var total =
#sum{k in LINE,p in SIZE}x[k,p];
#var production_line =
#sum{i in LINE,j in SIZE} x[i,j] * (( pline_1[i,j] + pline_2[i,j] + pline_3[i,j])*2)

#objective
maximize p:
        sum{i in LINE, j in SIZE}x[i,j] * profit[i,j];

subject to sept_orders{i in LINE,j in SIZE}:
        x[i,j] >= s_orders[i,j];

subject to aluminium:
        sum{i in LINE,j in SIZE}x[i,j] * alum[i,j] <= 65000;

subject to particle_board:
        sum{i in LINE,j in SIZE}x[i,j] * particle[i,j] <= 60000;

subject to pine_sheet:
        sum{i in LINE,j in SIZE}x[i,j] * pine[i,j] <= 175000;

subject to productionline_1:
        sum{i in LINE,j in SIZE}x[i,j] * pline_1[i,j] <= 9600;

subject to productionline_2:
        sum{i in LINE,j in SIZE}x[i,j] * pline_2[i,j] <= 9600;

subject to productionline_3:
        sum{i in LINE,j in SIZE}x[i,j] * pline_3[i,j] <= 19200;

subject to labor:
        sum{i in LINE,j in SIZE}(x[i,j]*((pline_1[i,j] + pline_2[i,j] + pline_3[i,j])*2
        + hand_craft[i,j] + assembly[i,j]))  <= 230400;
```

```
s.t. economy_min : 0 <= (sum{j in SIZE}x['L1',j])-total*0.2;
s.t. economy_max : 0 >= (sum{j in SIZE}x['L1',j])-total*0.5;

s.t. basic_min : 0 <= (sum{j in SIZE}x['L2',j])-total*0.4;
s.t. basic_max : 0 >= (sum{j in SIZE}x['L2',j])-total*0.6;

s.t. hf_min : 0 <= (sum{j in SIZE}x['L3',j])-total*0.1;
s.t. hf_max : 0 >= (sum{j in SIZE}x['L3',j])-total*0.2;

s.t. student_min : (sum{j in LINE}x[j,'S1'])-0.2*total>=0;
s.t. student_max : (sum{j in LINE}x[j,'S1'])-0.35*total<=0;

s.t. standard_min :(sum{j in LINE}x[j,'S2'])-0.4*total>=0;
s.t. standard_max : (sum{j in LINE}x[j,'S2'])-0.7*total<=0;

s.t. executive_min :(sum{j in LINE}x[j,'S3'])-0.05*total>=0;
s.t. executive_max : (sum{j in LINE}x[j,'S3'])-0.15*total<=0;
```

Appendix C .run file

```
reset;
reset data;

model casestudy.mod;

data casestudy.dat;
option presolve 10;
option solver cplex;
option cplex_options 'sensitivity';

solve;

display _varname, _var , _var.rc ,_var.current , _var.down, _var.up ;
display _conname, _con, _con.slack,_con.current, _con.down, _con.up;
```

Appendix D Raw AMPL Output

:	_conname	_con	_con.slack	_con.current	_con.down	_con.up
1	'=total'	2.10551	0	0	0	0
2	"sept_orders['L1','S1']"	-4.06234	0	0	0	0
3	"sept_orders['L1','S2']"	-12.0494	0	0	0	0
4	"sept_orders['L1','S3']"	-38.3373	0	0	0	0
5	"sept_orders['L2','S1']"	0	125.537	0	0	0
6	"sept_orders['L2','S2']"	0	157.501	0	0	0
7	"sept_orders['L2','S3']"	0	725.403	0	0	0
8	"sept_orders['L3','S1']"	-88.3629	0	0	0	0
9	"sept_orders['L3','S2']"	0	919.244	0	0	0
10	"sept_orders['L3','S3']"	-3.7655	0	0	0	0
11	aluminium	0	15500	65000	49500	1e+20
12	particle_board	0	29100	60000	30900	1e+20
13	pine_sheet	0.234495	0	175000	169646	195940
14	productionline_1	0	5225	0	0	0
15	productionline_2	0	4241.56	9600	5358.44	1e+20
16	productionline_3	0	2264.39	19200	16935.6	1e+20
17	labor	2.59735	0	230400	200443	239325
18	economy_min	0	1049.46	0	-1049.46	1e+20
19	economy_max	0	901.343	0	-1e+20	901.343
20	basic_min	0	407.367	0	-407.367	1e+20
21	basic_max	0	893.17	0	-1e+20	893.17
22	hf_min	0	493.976	0	-493.976	1e+20
23	hf_max	0	156.293	0	-1e+20	156.293
24	student_min	-12.7924	2.27374e-13	0	-114.494	281.915
25	student_max	0	975.403	0	-975.403	1e+20
26	standard_min	0	1625.67	0	-1e+20	1625.67
27	standard_max	0	325.134	0	-325.134	1e+20
28	executive_min	0	650.269	0	-1e+20	650.269
29	executive_max	31.0932	0	0	-331.807	114.905