IEOR 240 - Fall 2022
Homework 3
Due Date: 10/03/2022 (Monday) at 11:59pm (23:59)

Submission requirement

- Submit your homework (typed, scanned or photographed) and AMPL code in a single pdf file. You can choose to merge the pdf files together or include screenshots of your AMPL code and solutions in your homework pdf.
- Make sure your homework is legible, and the AMPL files are well organized
- For problems that require a mathematical formulation, please clearly state the definition of decision variables and the indexes of them. Write a sentence to explain each constraint and objective function. Also include an explanation of the optimal solution when you are required to solve the problem in AMPL.

1. **(True or False)**
   Determine whether the following statement is true or false, and justify your answer by providing either a short explanation or a counterexample. No credit is given without justification.

   (a) If we have an Unboundedness certificate for a linear program in symmetric form, then we know it must be unbounded.

   (b) All linear programs with optimal solutions are bounded.

   (c) The only way to show that a linear program is feasible is to find the optimal solution.

   (d) If a linear program is feasible and bounded than it must have an optimal solution.

   (e) If the feasible region of a linear program is unbounded, than the linear program itself is unbounded.

   (f) If a linear program has two different optimal solutions, then it has infinitely many optimal solutions.

   (g) If a linear program has infinitely many optimal solutions, then it must be unbounded.

   (h) A linear program can be both infeasible and unbounded.

   (i) There exists a linear program that has neither a boundedness nor an unboundedness certificate.

   (j) All linear programs can be converted to symmetric form.

2. For the following Linear Program:

$$\begin{aligned} \min \ & 2x_1 - 3x_2 + 5x_3 + 6x_4 \\ \text{s.t.} \ & x_1 - x_2 \leq 4 \\ & -2x_2 + 3x_3 \geq 2 \\ & x_1 + x_4 \geq 5 \\ & x_1 \geq 0, x_3 \geq 0, x_4 \geq 0 \end{aligned}$$

Do the following:

(a) Change it to Symmetric form.

(b) Write the mathematical formulation of its dual.

(c) Solve both the dual and the original problem in AMPL and report their results.

3. Consider the general problem

$$\begin{aligned} (\mathcal{P}): \ \max \ & \mathbf{c}^T\mathbf{x} \\ \text{s.t.} \ & \mathbf{A}\mathbf{x} \leq \mathbf{0} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

(a) Can the dual of $(\mathcal{P})$ be unbounded?

(b) Suppose $(\mathcal{P})$ is bounded, then what is the optimal objective function value of $(\mathcal{P})$?

4. Consider the following linear programming problem:

$$\begin{aligned} (\mathcal{P}): \ \min \ & \mathbf{c}^T\mathbf{x} \\ \text{s.t.} \ & \mathbf{A}\mathbf{x} \leq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

where both c and b are nonnegative (that is, all the entries in these vectors are nonnegative). Then this LP must have an optimal solution.