215 : Homework 3

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Exercise 1 :

$p \vee q \rightarrow r \equiv \neg(p \vee q) \vee r \equiv (\neg p \wedge \neg q) \vee r$ after De Morgan's law.

Thus : $p \vee q \rightarrow r \equiv (\neg p \vee r) \wedge (\neg p \vee r) \equiv (p \rightarrow r) \wedge (q \rightarrow r)$.

Finally we have obtained that :
$$\boxed{p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)}$$

Exercise 2 :

Let $\{\neg p \rightarrow (r \wedge \neg s), t \rightarrow s, u \rightarrow \neg p, \neg w, u \vee w\} \subset F$:

As $\neg w \wedge (u \vee w) \models u$, $u \wedge (u \rightarrow \neg p) \models \neg p$, $\neg p \wedge (\neg p \rightarrow (r \wedge \neg s)) \models r \wedge \neg s$.

Thus as $r \wedge \neg s \models \neg s$ and $\neg s \wedge (t \rightarrow s) \models \neg t$ by contrapositive, and finally $\neg t \vee w$ is always true.

As $\neg t \vee w \equiv t \rightarrow w$ then :
$$\boxed{F \models (t \rightarrow w)}$$