Homework 2

Arnaud Minondo

September 13, 2022

Exercise 1

If $\mathbb{P}(A|B) = 1$ then it means $B \subset A$ so $A^c \subset B^c$ and finally $\mathbb{P}(B^c|A^c) = 1$

Exercise 2

I will denote A :"the two dice fall on different number" and B : "One of the two dice fall on 6"
For A : there are 6 possibilities for the first and 5 for the second.
Thus there are $6 * 5$ possibilities.
B is the contrary of no any of the dice fall on 6 there is 5 possibilities for the first dice and 4 for the second.
Thus there are $6 * 5 - 5 * 4 = 10$ possibilities.
And $\mathbb{P}(B|A) = \frac{10}{30} = \frac{1}{3}$

Exercise 3

The proportion of person having voted is $P = 0.35 * 0.46 + 0.62 * 0.30 + 0.24 * 0.58 = 0.4862$ There is a proportion of : $I = 0.35 * 0.46 = 0.161$ independents, $C = 0.58 * 0.24$ conservatives and $L = 0.62 * 0.30$ liberals.

3.1

$\mathbb{P}(\text{"The person is an Independent"}|\text{"Has voted"}) = \frac{I}{P} = \frac{0.1610}{0.4862} = 0,33.$

3.2

$\mathbb{P}(\text{"The person is a Liberal"}|\text{"Has voted"}) = \frac{L}{P} = \frac{0.186}{0.4862} = 0,38.$

3.3

$\mathbb{P}(\text{"The person is an Conservative"}|\text{"Has voted"}) = \frac{I}{P} = \frac{0.1392}{0.4862} = 0,29.$

3.4

The fraction of voters that participated in the local election is : $P = 0.49$

Exercise 4

4.1

We can deduce that both parents have the pair of genes : (Blue, Brown).
There are 3 possibilities for the genes of Smith : (Blue, Brown), (Brown, Blue), (Brown, Brown)
Only two of them include a blue-eyed gene so the probability is :$\frac{2}{3}$

4.2

I will denote C_1 : "The first Child has brown eyes" and S_{ij} : "Smith has the pair of genes (i, j)" ; B will denote the Brown gene and b the blue one.
The fact that the mother has blue eyes means she has the pair of genes : bb
$\mathbb{P}(\bar{C_1}) = \mathbb{P}(\bar{C_1}|S_{BB})\mathbb{P}(S_{BB}) + \mathbb{P}(\bar{C_1}|S_{Bb})\mathbb{P}(S_{Bb})$
In the case where Smith is BB : his child can't have blue eyes as he will at least have one gene B from Smith. So $\mathbb{P}(C_1|S_{BB}) = 0$.
In the case where Smith is Bb : then there are only two cases : Smith gives b to his child which will result in blue eyes ; or Smith give his B and the child will have brown eyes. Only one on the two cases will result in C_1 so $\mathbb{P}(\bar{C_1}|S_{Bb}) = \frac{1}{2}$.
So $\mathbb{P}(\bar{C_1}) = 0 + \frac{1}{2}\frac{2}{3} = \frac{1}{3}$

4.3

I will denote C_2 : "Their second child has brown eyes".
$\mathbb{P}(C_2|C_1) = \frac{\mathbb{P}(C_2 \cap C_1)}{\mathbb{P}(C_1)}$
$\mathbb{P}(C_2 \cap C_1) = \mathbb{P}(C_2 \cap C_1 \cap S_{Bb}) + \mathbb{P}(C_2 \cap C_1 \cap S_{BB}) = \mathbb{P}(C_2 \cap C_1|S_{Bb})\mathbb{P}(S_{Bb}) + \mathbb{P}(C_2 \cap C_1|S_{BB})\mathbb{P}(S_{BB})$
$= \frac{1}{2^2}\frac{2}{3} + 1 * \frac{1}{3} = \frac{1}{2}$
So finally : $\mathbb{P}(C_2|C_1) = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{3}{4}$

Exercise 5

5.1

I will denote G_{ij} the event : the child is (i,j). For the child there is 2 possibilities : either G_{AA}, G_{Aa}, if we don't care about the order.
Define B : "The off-spring is an albino".
$\mathbb{P}(B) = \mathbb{P}(B|G_{AA})\mathbb{P}(G_{AA}) + \mathbb{P}(B|G_{Aa})\mathbb{P}(G_{Aa})$
$= 0 * \frac{1}{3} + \frac{1}{4}\frac{2}{3} = \frac{1}{6}$

5.2

Define C : "The second offspring is an albino"
$\mathbb{P}(C|\bar{B}) = \frac{\mathbb{P}(C \cap \bar{B})}{\mathbb{P}(\bar{B})}$
We already have $\mathbb{P}(B) = \frac{5}{6}$ then we just have to derive $\mathbb{P}(C \cap \bar{B})$ $\mathbb{P}(C \cap \bar{B}) = \mathbb{P}(C \cap \bar{B} \cap G_{AA}) + \mathbb{P}(C \cap \bar{B} \cap G_{Aa}) = 0 + \frac{1}{4}\frac{3}{4}\frac{2}{3} = \frac{1}{8}$ Then the probability that the second child is an albino is : $\frac{\frac{1}{8}}{\frac{5}{6}} = \frac{3}{20}$

Exercise 6

6.1

I will denote :"The i-th child is a boy" as B_i and "All children are of the same sex" as S.
So let X be the number of boy, then X follows a binomial distribution : $\mathcal{B}(5, \frac{1}{2})$.
$\mathbb{P}(S) = \mathbb{P}((\cap_{i=1}^{5} B_i) \cup (\cap_{i=1}^{5} B_i)) = (\frac{1}{2})^5 + (\frac{1}{2})^5 = \frac{1}{16}$

6.2

I will denote D :" The 3 eldest are boy and the other girls".
$\mathbb{P}(D) = \mathbb{P}(\bar{B}_1 \bar{B}_2 B_3 B_4 B_5) = (\frac{1}{2})^5 = \frac{1}{32}$

6.3

B :"Exactly 3 are boys", the result is given by : $\mathbb{P}(X = 3)$ where X follows a binomial distribution.
$\mathbb{P}(B) = \binom{5}{3}(\frac{1}{2})^5$

6.4

O : “The 2 oldest are girls”
$\mathbb{P}(O) = \mathbb{P}((B_1 \cup \bar{B_1})(B_2 \cup \bar{B_2})(B_3 \cup \bar{B_3})\bar{B_4}\bar{B_5}) = \mathbb{P}(\bar{B_4})\mathbb{P}(\bar{B_5}) = \frac{1}{4}$

6.5

G : “There is at least 1 girl”
$\mathbb{P}(G) = 1 - \mathbb{P}(\bar{G}) = 1 - \mathbb{P}(B_1B_2B_3B_4B_5) = 1 - (\frac{1}{2})^5 = \frac{31}{32}$