IEOR 240 - Fall 2022
Homework 4
Due Date: 10/10/2022 (Monday) at 11:59pm

Homework requirement

- Homework 4 is due on Monday October 10th at 11:59pm through bCourses.
- Submit your homework (typed, scanned or photographed) and AMPL code in a single pdf file. You can choose to merge the pdf files together or include screenshots of your AMPL code and solutions in your homework pdf.
- Make sure your homework is legible, and the AMPL files are well organized
- For problems that require a mathematical formulation, please clearly state the definition of decision variables and the indexes of them. Write a sentence to explain each constraint and objective function. Also include an explanation of the optimal solution when you are required to solve the problem in AMPL.

1. Convert the following problem to a linear program

$$\begin{array}{rr} \min & \max(2x_1, 3x_2, 4|x_3|) \\ s.t. & x_1 + x_2 + x_3 \geq 1 \\ & x_1 \geq 0, x_2 \geq 0 \end{array}$$

   Briefly justify that solving the linear program does indeed solve the original problem

2. Consider the following linear program

$$\begin{array}{rrrrrcll} \max & 7x_1 & +5x_2 & +3x_3 & +2x_4 & & & \\ s.t. & 2x_1 & +5x_2 & +2x_3 & +4x_4 & \leq & 12 & (C1) \\ & 3x_1 & +4x_2 & +x_3 & +x_4 & \leq & 10 & (C2) \\ & x_1 & +x_2 & +x_3 & +x_4 & \leq & 8 & (C3) \\ & \multicolumn{4}{l}{x_i \geq 0, i = 1, 2, 3, 4} & & & \end{array}$$

   Solving the above LP we obtained the following sensitivity report.

Variables

Name	Final Value	Reduced Cost	Objective Coefficients	Upper Bound	Lower Bound
x_1	2	0	7	9	3
x_2	0	-5.5	5	10.5	-1E+20
x_3	4	(a)	3	7	2.3333
x_4	0	-2	2	4	-1E+20

			Constraints		
Name	**Final Value**	**Shadow Price**	**Constraint R.H.Side**	**Upper Bound**	**Lower Bound**
C1	12	0.5	12	16	6.6666
C2	10	2	10	18	6
C3	6	(b)	8	1E+20	(c)

Based on the sensitivity report, answer the following questions:

(a) What is the reduced cost of x_3? (**Explain**)

(b) What is the shadow price of constraint C3? (**Explain**)

(c) What is the allowable decrease for constraint C3 ? (**Explain**)

(d) Suppose the objective function coefficient of x_1 increases from 7 to 8. What will be the new optimal solution and the new optimal objective function value?

(e) Suppose the right hand side of constraint C1 decreases from 12 to 10. What will be the new optimal objective function value? (**Explain**)

(f) Imagine we add a new variable x_5 that has 3 as its cost coefficient and 2, 2, 4 as its coefficients for constraints $C1$, $C2$ and $C3$ respectively. Is the current solution with $x_5 = 0$ still optimal? (**Explain**)

3. For each of the following statements, determine whether it is true or false. Justify your answer by providing either a short proof or an example as appropriate. (**No explanation - no credit**)

(a) If a linear program is infeasible, then its dual must be unbounded.

(b) Suppose the shadow prices for the following linear program

$$\begin{aligned} \min \quad & c_1x_1 + c_2x_2 + \cdots + c_nx_n \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_jx_j \geq b \\ & \sum_{j=1}^{n} -a_jx_j \geq -b \\ & x_j \geq 0, \; j = 1, 2, ..., n. \end{aligned}$$

are $\bar{y}_1, \bar{y}_2$, then $\bar{y}_1 = -\bar{y}_2$.

(c) Suppose the shadow prices for the following linear program

$$\begin{aligned} \max \quad & c_1x_1 + c_2x_2 + \cdots + c_nx_n \\ \text{s.t.} \quad & \sum_{j=1}^{n} a_{ij}x_j \leq b_i, \; i = 1, 2, ..., m \\ & x_j \geq 0, \; j = 1, 2, ..., n. \end{aligned}$$

are $\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_m$. Suppose all objective function coefficients $c_j (j = 1, \ldots, n)$ are multiplied by 2 as

$$\max\ 2c_1x_1 + 2c_2x_2 + \cdots + 2c_nx_n.$$

then in the new problem the shadow prices $\bar{y}_i$ $(i = 1, \ldots, m)$ are unchanged.

(d) Consider the following linear program

$$\begin{aligned} \max \quad & x_1 + x_2 \\ s.t. \quad & a_1x_1 + a_2x_2 \leq 1 \\ & x_1, x_2 \geq 0 \end{aligned}$$

where a_1, a_2 are unknown real numbers. Then the linear program above is feasible for all values of a_1 and a_2.

(e) Consider the following linear program (P)

$$\begin{aligned} \min \quad & \sum_{j=1}^{n} c_jx_j \\ s.t. \quad & \sum_{j=1}^{n} a_{ij}x_j \geq b_i, \quad i = 1, \ldots, m \\ & x_j \geq 0,\ j = 1, \ldots, n \end{aligned}$$

Suppose the only solution to the following system of inequalities

$$\begin{aligned} \textstyle\sum_{i=1}^{m} a_{ij}z_i \leq 0, \quad j = 1 \ldots, n \\ z_i \geq 0, \quad i = 1 \ldots, m \end{aligned}$$

is $z_1 = z_2 = \ldots = z_m = 0$, then (P) must be feasible for all values of $b_i,\ i = 1, \ldots, m$.