IEOR 241 Homework 6

1. Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1 - x^2) & -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

    (a) What is the value of c ?
    (b) What is the cumulative distribution function of X ?

2. A filling station is supplied with gasoline once a week. If its weekly volume of sales in thousands of gallons is a random variable with probability density function

$$f(x) = \begin{cases} 5(1 - x)^4 & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

what must the capacity of the tank be so that the probability of the supply's being exhausted in a given week is $.01$?

3. A point is chosen at random on a line segment of length L. Interpret this statement, and find the probability that the ratio of the shorter to the longer segment is less than $\frac{1}{4}$.

4. If X is a normal random variable with parameters $\mu = 10$ and $\sigma^2 = 36$, compute
(a) $P\{X > 5\}$;
(b) $P\{X < 8\}$
(c) $P\{X > 16\}$.

5. Twelve percent of the population is left handed. Approximate the probability that there are at least 20 left-handers in a school of 200 students. State your assumptions.

6. The time (in hours) required to repair a machine is an exponentially distributed random variable with parameter $\lambda = \frac{1}{2}$. What is
(a) the probability that a repair time exceeds 2 hours?
(b) the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 9 hours?

7. The number of years a radio functions is exponentially distributed with parameter $\lambda = \frac{1}{8}$. If Jones buys a used radio, what is the probability that it will be working after an additional 8 years?