IEOR 263A Problem Set 4

Chapter 4 problems: 3, 5, 12 (optional), 14, 15, 17 (optional), 18 (optional). Also:

1. Give an example of a Markov chain that has a stationary distribution, but not a long-run time-average distribution. Give an example of a Markov chain that has a time-average distribution but not a steady-state distribution.

2. (optional) Consider the Gilbert model of channel fading in which the state of a transmission channel is G (good) or B (bad) can be modeled as a Markov chain where P{G to B} = p and P{B to G} = q. Let r = 1-p-q. Show that $P_{GG}{}^{n} = q + rP_{GG}{}^{n-1}$ and $P_{GG}{}^{n} = q(1-r^n)/(1-r) + r^n$.

3. Consider the G/D/1/2 slotted telecommunications model, where the channel can transmit 1 packet in each time slot, the numbers of arrivals during each slot are iid, where the number of arrivals is 0 with probability p_0, 1 with probability p_1 and more than 1 with probability p_2. Think of arrivals as occurring just before the end of a slot, so if a slot starts with 1 packet, and there is 1 arrival, the first packet will be transmitted and the next slot will start with 1 packet. The buffer can only hold 2 packets. Let X_n be the number of packets at the start of slot n. Find the function f such that Xn+1 = f(X_n, U_n) where the U_i's are iid unif(01) random variables.

4. (a) Define an appropriate state and give the transition matrix for the geom(p)/geom(q)/1 queue. (Single server, infinite buffer, inter-arrival times are iid ~ geometric(p), service times are iid ~ geometric(q).)

(b) Give the transition diagram for the geom(p)/D(2)/1 queue. (Single server, infinite buffer, inter-arrival times are iid ~ geom(p), service times are deterministic and identically equal to 2.)