IEOR 241 : Homework 6

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Exercise 1 :

(a)

$\int_{-1}^{1} c(1-x^2)dx = c(2-\frac{2}{3}) = c\frac{4}{3}$ so $c = \frac{3}{4}$.

(b)

Let $t \in [-1;1]$: $\mathbb{P}(X \leq t) = \int_{-1}^{t} \frac{3}{4}(1-x^2)dx = \frac{3}{4}(t+1-\frac{t^3}{3}-\frac{1}{3}) = \frac{1}{2}+\frac{3}{4}t-\frac{t^3}{4}$.

Exercise 2 :

Let S denote the sales of the filling station.
Let $t \in [0;1]$, $\mathbb{P}(S \leq t) = \int_0^t f(x)dx = \int_0^t 5(1-x)^4 dx = -(1-t)^5+1$.
Let c be the capacity of the filling station. $\mathbb{P}(S \geq c) = 1-\mathbb{P}(S \leq c) = 0.01$
Thus : $1-1+(1-c)^5 = 0.01$ which yields :

$$\boxed{c = 1-(0.01)^{\frac{1}{5}} = 0.60}$$

Exercise 3 :

Let P be the position of the point in the line of length L : $P \sim \mathcal{U}([0;L])$ a uniform law.
The ratio of the smaller over the larger is : $Z = \frac{\min(U,L-U)}{\max(U,L-U)}$.

$\mathbb{P}(Z \leq \frac{1}{4}) = \mathbb{P}(Z \leq \frac{1}{4}|U \leq \frac{1}{2})\mathbb{P}(U \leq \frac{1}{2}) + \mathbb{P}(Z \leq \frac{1}{4}|U > \frac{1}{2})\mathbb{P}(U > \frac{1}{2})$

As U and $L-U$ follow the same law they play simetric role and $\mathbb{P}(\frac{U}{L-U} \leq \frac{1}{4}) = \mathbb{P}(\frac{L-U}{U} \leq \frac{1}{4})$
$\mathbb{P}(Z \leq \frac{1}{4}) = \mathbb{P}(\frac{U}{L-U} \leq \frac{1}{4})\frac{1}{2} + \mathbb{P}(\frac{L-U}{U} \leq \frac{1}{4})\frac{1}{2} = 2\mathbb{P}(\frac{5}{4}U \leq \frac{L}{4}) = 2\mathbb{P}(U \leq \frac{L}{5}) = 2\frac{L}{5}\frac{1}{L} = \frac{2}{5}$

Exercise 4 :

$\mathbb{P}(X > 5) = \mathbb{P}(Z > -\frac{5}{6}) \simeq 0.79$
$\mathbb{P}(X < 8) = \mathbb{P}(Z < -\frac{1}{3}) \simeq 0.30$
$\mathbb{P}(X > 16) = \mathbb{P}(Z > 1) \simeq 0.16$

Exercise 5 :

Let X be the number of left handed in the school. $X \sim \mathcal{B}(200, \frac{1}{5})$
The probability that there is at least 20 left handed is : $\mathbb{P}(X \geq 20) = \sum_{i=20}^{200} \binom{200}{i}(\frac{1}{5})^i(\frac{4}{5})^{200-i} = 0.99$
You can approximate the law of X by a $\mathcal{N}(40, 32)$ normal distribution. The approximation gives $\mathbb{P}(X > 20) \simeq \mathbb{P}(Z > -3.53) = 0.99$

Exercise 6 :

(a)

Let T be the repair time, $T \sim \mathcal{E}(\frac{1}{2})$.
$\mathbb{P}(T > 2) = \exp(-\frac{1}{2}2) = \exp(-1) \simeq 0.37$

(b)

$\mathbb{P}(T > 10 | T > 9) = \mathbb{P}(T > 1) = \exp(-\frac{1}{2}) \simeq 0.61$ because the exponantial law is memoryless.

Exercise 7 :

Let T the function time of the bought radio. $\mathbb{P}(T > 8) = \exp(-\frac{1}{8}8) = \exp(-1) \simeq 0.37$