

Individual homework. All the answers have to be mathematically justified (unless specified explicitly).

Exercise 1.

Two fair dice are rolled. Let X be the product of the 2 dice. Compute $\mathbb{P}(X = i)$ for each possible value of i .

Exercise 2.

Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed n times. What are the possible values of X ?

Exercise 3. A salesman has scheduled two appointments to sell encyclopedias. His first appointment will lead to a sale with probability .3, and his second will lead independently to a sale with probability .6. Any sale made is equally likely to be either for the deluxe model, which costs \$1000, or the standard model, which costs \$500. Determine the probability mass function of X , the total dollar value of all sales.

Exercise 4. A gambling book recommends the following “winning strategy” for the game of roulette: Bet \$1 on red. If red appears, which has probability $\frac{18}{38}$ then take the \$1 profit and quit. If red does not appear and you lose this bet, which has probability, $\frac{20}{38}$ of occurring, make additional \$1 bets on red on each of the next two spins of the roulette wheel and then quit. Let X denote your winnings when you quit.

1. Compute $\mathbb{P}(X > 0)$.
2. Are you convinced that the strategy is indeed a “winning” strategy? Explain your answer!
3. Compute $\mathbb{E}[X]$.

Exercise 5. Four buses carrying 148 students from the same school arrive at a football stadium. The buses carry, respectively, 40, 33, 25, and 50 students. One of the students is randomly selected. Let X denote the number of students that were on the bus carrying the randomly selected student. One of the 4 bus drivers is also randomly selected. Let Y denote the number of students on her bus. Compute $\mathbb{E}[X]$ and $\mathbb{E}[Y]$, compare the two quantities and explain why we expect this result.

Exercise 6.

Two coins are to be flipped. The first coin will land on heads with probability .6, the second with probability .7. Assume that the results of the flips are independent, and let X equal the total number of heads that result.

1. Compute $\mathbb{P}(X = i)$ for all the possible values taken by X .
2. Compute $\mathbb{E}[X]$.