

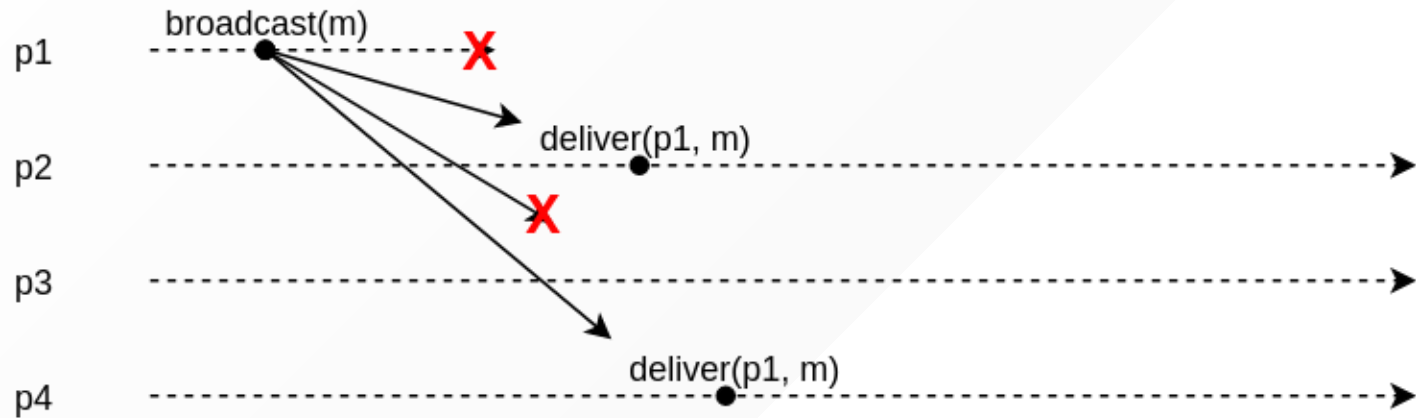
# Large-scale Distributed Systems

Lecture 3: Reliable broadcast

# Today

- How do you talk to **multiple machines** at once?
- What if some of them **fail**?
- Can we guarantee that correct nodes all receive the same messages?
- What about **ordering**?
- What about **performance**?

# Unreliable broadcast



# Reliable broadcast abstractions

# Reliable broadcast abstractions

- **Best-effort broadcast**
  - Guarantees reliability only if sender is correct.
- **Reliable broadcast**
  - Guarantees reliability independent of whether sender is correct.
- **Uniform reliable broadcast**
  - Also considers the behavior of failed nodes.
- **Causal reliable broadcast**
  - Reliable broadcast with causal delivery order.

# Best-effort broadcast

## Module:

**Name:** BestEffortBroadcast, **instance** *beb*.

## Events:

**Request:**  $\langle \text{beb}, \text{Broadcast} \mid m \rangle$ : Broadcasts a message  $m$  to all processes.

**Indication:**  $\langle \text{beb}, \text{Deliver} \mid p, m \rangle$ : Delivers a message  $m$  broadcast by process  $p$ .

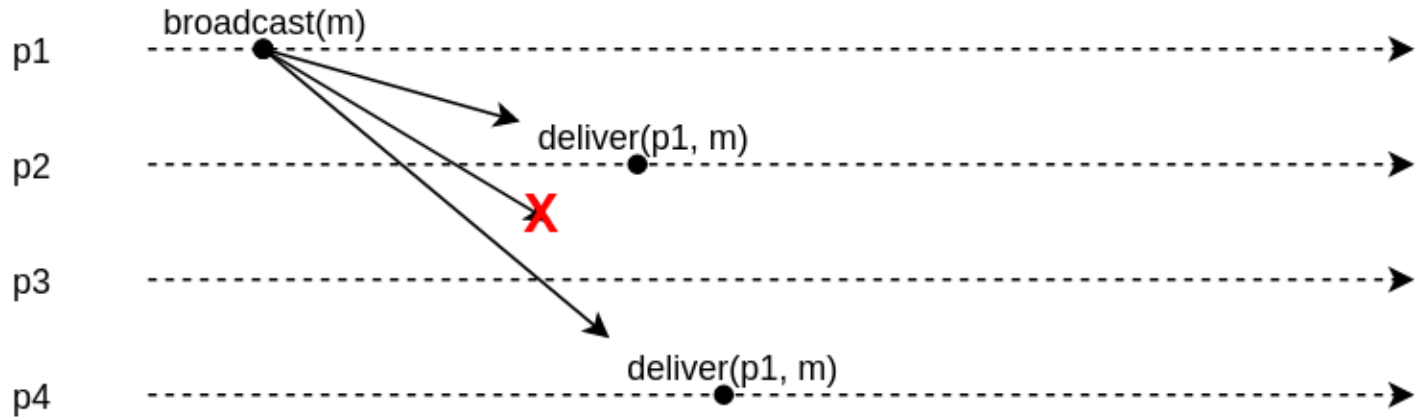
## Properties:

**BEB1: Validity:** If a correct process broadcasts a message  $m$ , then every correct process eventually delivers  $m$ .

**BEB2: No duplication:** No message is delivered more than once.

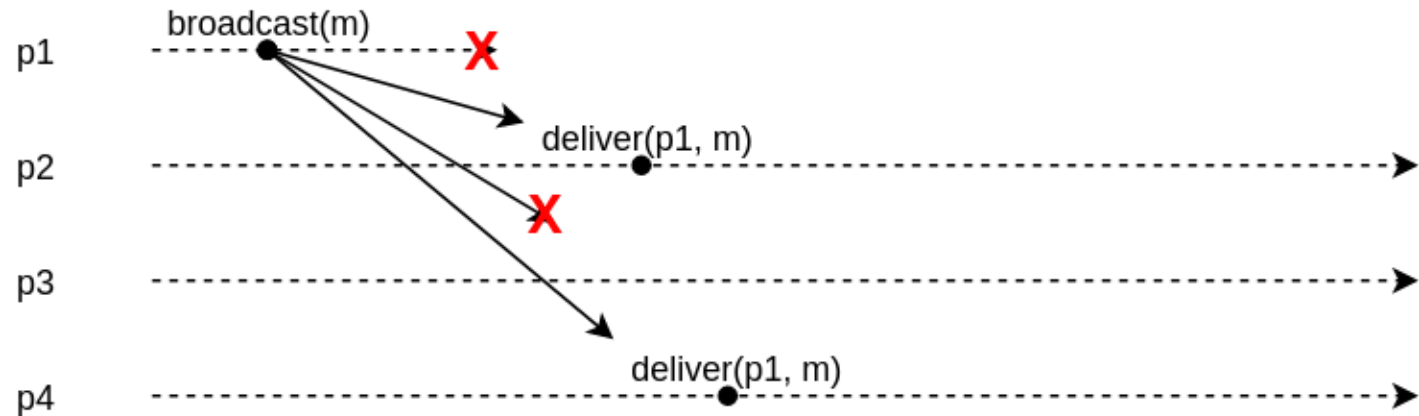
**BEB3: No creation:** If a process delivers a message  $m$  with sender  $s$ , then  $m$  was previously broadcast by process  $s$ .

# BEB example (1)



[Q] Is this allowed?

# BEB example (2)



[Q] Is this allowed?



# Reliable broadcast

- Best-effort broadcast gives no guarantees if **sender crashes**.
- **Reliable broadcast:**
  - Same as best-effort broadcast +
  - If sender crashes, ensure **all or none** of the correct node deliver the message.

# Reliable broadcast

## Module:

**Name:** ReliableBroadcast, **instance** *rb*.

## Events:

**Request:**  $\langle rb, \text{Broadcast} \mid m \rangle$ : Broadcasts a message *m* to all processes.

**Indication:**  $\langle rb, \text{Deliver} \mid p, m \rangle$ : Delivers a message *m* broadcast by process *p*.

## Properties:

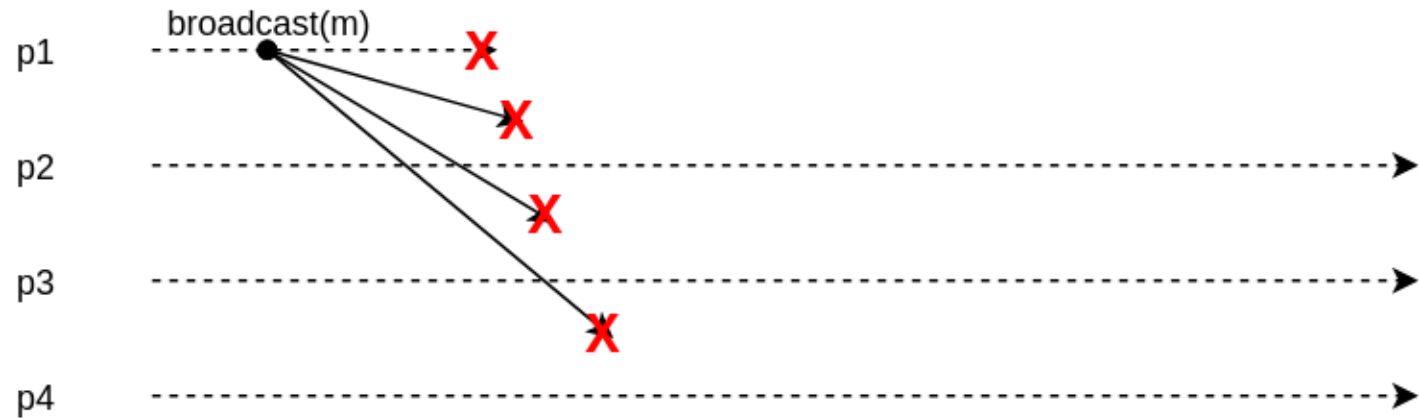
**RB1: Validity:** If a correct process *p* broadcasts a message *m*, then *p* eventually delivers *m*.

**RB2: No duplication:** No message is delivered more than once.

**RB3: No creation:** If a process delivers a message *m* with sender *s*, then *m* was previously broadcast by process *s*.

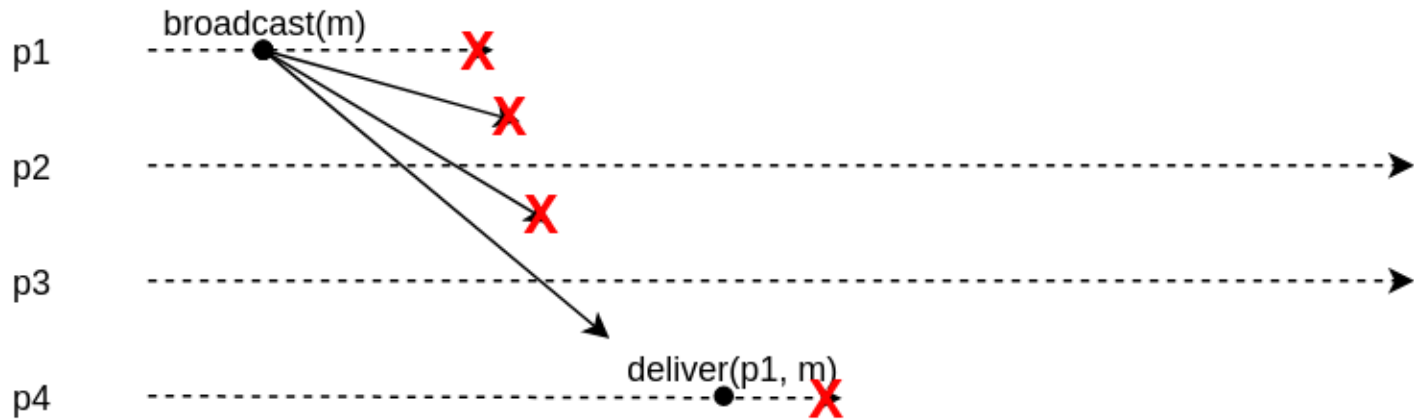
**RB4: Agreement:** If a message *m* is delivered by some correct process, then *m* is eventually delivered by every correct process.

# RB example (1)



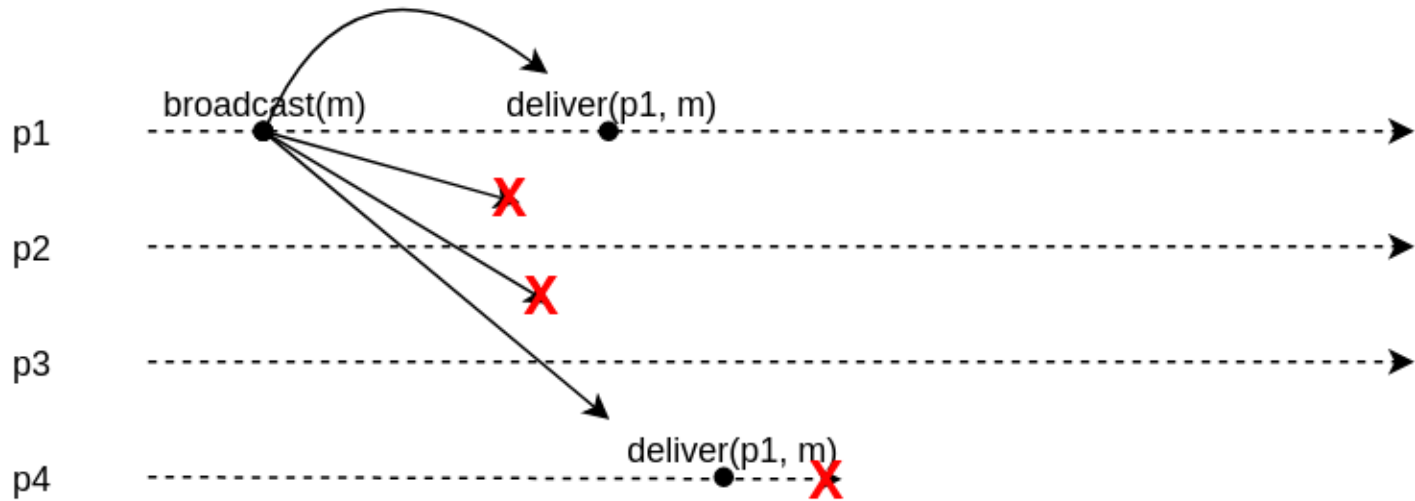
[Q] Is this allowed?

# RB example (2)



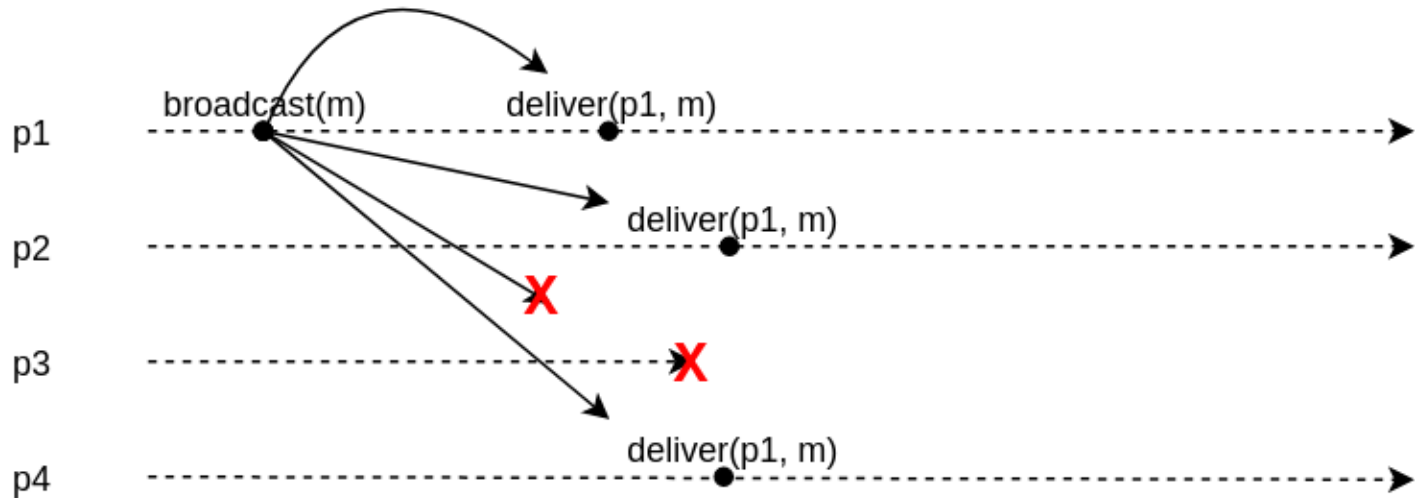
[Q] Is this allowed?

# RB example (3)



[Q] Is this allowed?

# RB example (4)



[Q] Is this allowed?

# Uniform reliable broadcast

- Assume the broadcast enforces
  - Printing a message on paper
  - Withdrawing money from account in variable
- Assume sender broadcasts a message
  - Sender fails
  - No correct node delivers the message
  - Failed nodes deliver the message, is this OK?
- **Uniform** reliable broadcast ensures that if a message is delivered (by a correct **or faulty** process), then all correct processes deliver.

# Uniform reliable broadcast

## Module:

**Name:** UniformReliableBroadcast, **instance** *urb*.

## Events:

**Request:**  $\langle \text{urb}, \text{Broadcast} \mid m \rangle$ : Broadcasts a message  $m$  to all processes.

**Indication:**  $\langle \text{urb}, \text{Deliver} \mid p, m \rangle$ : Delivers a message  $m$  broadcast by process  $p$ .

## Properties:

**URB1–URB3:** Same as properties RB1–RB3 in (regular) reliable broadcast (Module 3.2).

**URB4:** *Uniform agreement:* If a message  $m$  is delivered by some process (whether correct or faulty), then  $m$  is eventually delivered by every correct process.



# Implementations

# Basic broadcast

## Implements:

BestEffortBroadcast, **instance** *beb*.

## Uses:

PerfectPointToPointLinks, **instance** *pl*.

```
upon event  $\langle beb, Broadcast \mid m \rangle$  do  
  forall  $q \in \Pi$  do  
    trigger  $\langle pl, Send \mid q, m \rangle$ ;
```

```
upon event  $\langle pl, Deliver \mid p, m \rangle$  do  
  trigger  $\langle beb, Deliver \mid p, m \rangle$ ;
```

## Correctness:

- **BEB1. Validity**
  - If sender does not crash, every other correct node receives message by perfect channels.
- **BEB2+3. No duplication + no creation**
  - Guaranteed by perfect channels.

# Lazy reliable broadcast

- Assume a **fail-stop** distributed system model.
  - i.e., crash-stop processes, perfect links and a perfect failure detector.
- To broadcast  $m$ :
  - best-effort broadcast  $m$
  - Upon bebDeliver:
    - Save message
    - rbDeliver the message
- If sender  $s$  crashes, detect and relay messages from  $s$  to all.
  - case 1: get  $m$  from  $s$ , detect crash of  $s$ , redistribute  $m$
  - case 2: detect crash of  $s$ , get  $m$  from  $s$ , redistribute  $m$ .
- Filter duplicate messages.

# Lazy reliable broadcast

## Implements:

ReliableBroadcast, instance *rb*.

## Uses:

BestEffortBroadcast, instance *beb*;

PerfectFailureDetector, instance  $\mathcal{P}$ .

**upon event**  $\langle rb, Init \rangle$  **do**

$correct := \Pi$ ;

$from[p] := [\emptyset]^N$ ;

**upon event**  $\langle rb, Broadcast \mid m \rangle$  **do**

**trigger**  $\langle beb, Broadcast \mid [DATA, self, m] \rangle$ ;

**upon event**  $\langle beb, Deliver \mid p, [DATA, s, m] \rangle$  **do**

**if**  $m \notin from[s]$  **then**

**trigger**  $\langle rb, Deliver \mid s, m \rangle$ ;

$from[s] := from[s] \cup \{m\}$ ;

**if**  $s \notin correct$  **then**

**trigger**  $\langle beb, Broadcast \mid [DATA, s, m] \rangle$ ;

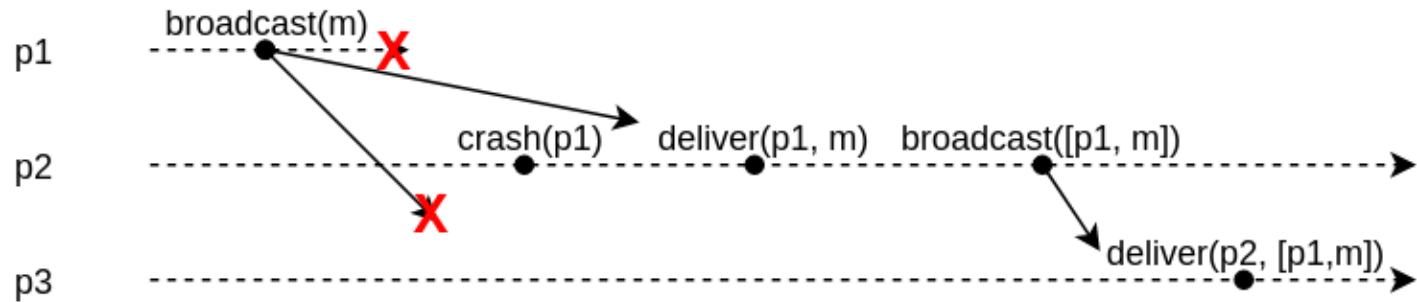
**upon event**  $\langle \mathcal{P}, Crash \mid p \rangle$  **do**

$correct := correct \setminus \{p\}$ ;

**forall**  $m \in from[p]$  **do**

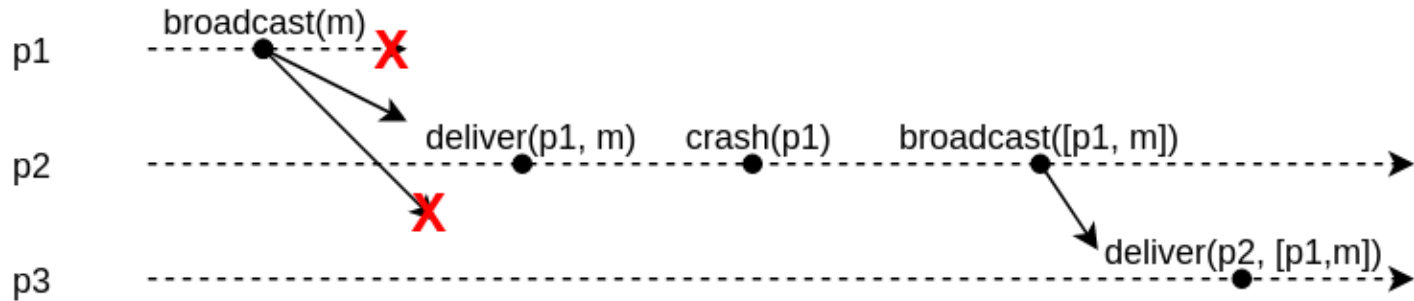
**trigger**  $\langle beb, Broadcast \mid [DATA, p, m] \rangle$ ;

# LRB example (1)



[Q] Which case?

# LRB example (2)



[Q] Which case?

# Correctness of LRB

Correctness:

- RB1-RB3
  - Satisfied with best-effort broadcast.
- RB4. Agreement
  - When correct  $p_j$  delivers  $m$  broadcast by  $p_i$ 
    - if  $p_i$  is correct, BEB ensures correct delivery
    - if  $p_i$  crashes,
      - $p_j$  detects this (because of completeness of the PFD)
      - $p_j$  uses BEB to ensure (BEB1) every correct node gets  $m$ .

# Eager reliable broadcast

- What happens if we use instead an **eventually** perfect failure detector?
  - Only affects performance, not correctness.
- Can we modify Lazy RB to not use a perfect failure detector?
  - Assume all nodes have failed.
  - Best-effort broadcast all received messages.



# Eager reliable broadcast

## Implements:

ReliableBroadcast, **instance** *rb*.

## Uses:

BestEffortBroadcast, **instance** *beb*.

**upon event**  $\langle rb, Init \rangle$  **do**

*delivered* :=  $\emptyset$ ;

**upon event**  $\langle rb, Broadcast \mid m \rangle$  **do**

**trigger**  $\langle beb, Broadcast \mid [DATA, self, m] \rangle$ ;

**upon event**  $\langle beb, Deliver \mid p, [DATA, s, m] \rangle$  **do**

**if**  $m \notin delivered$  **then**

*delivered* := *delivered*  $\cup \{m\}$ ;

**trigger**  $\langle rb, Deliver \mid s, m \rangle$ ;

**trigger**  $\langle beb, Broadcast \mid [DATA, s, m] \rangle$ ;

[Q] Show that eager reliable broadcast is correct.

# Uniformity

Neither Lazy RB nor Eager RB ensure **uniform** agreement.

- E.g., sender  $p$  immediately RB delivers and crashes. Only  $p$  delivered the message.

## Strategy for uniform agreement

- Before delivering a message, we need to ensure all correct nodes have received it.
- Messages are **pending** until all correct nodes get it.
  - Collect acknowledgements from nodes that got the message.
- Deliver once all correct nodes acked.

# All-ack uniform reliable broadcast

## Implements:

UniformReliableBroadcast, **instance** *urb*.

## Uses:

BestEffortBroadcast, **instance** *beb*.

PerfectFailureDetector, **instance**  $\mathcal{P}$ .

**upon event**  $\langle \textit{urb}, \textit{Init} \rangle$  **do**

*delivered* :=  $\emptyset$ ;

*pending* :=  $\emptyset$ ;

*correct* :=  $\Pi$ ;

**forall** *m* **do** *ack*[*m*] :=  $\emptyset$ ;

**upon event**  $\langle \textit{urb}, \textit{Broadcast} \mid m \rangle$  **do**

*pending* := *pending*  $\cup \{(self, m)\}$ ;

**trigger**  $\langle \textit{beb}, \textit{Broadcast} \mid [DATA, self, m] \rangle$ ;

**upon event**  $\langle \textit{beb}, \textit{Deliver} \mid p, [DATA, s, m] \rangle$  **do**

*ack*[*m*] := *ack*[*m*]  $\cup \{p\}$ ;

**if**  $(s, m) \notin \textit{pending}$  **then**

*pending* := *pending*  $\cup \{(s, m)\}$ ;

**trigger**  $\langle \textit{beb}, \textit{Broadcast} \mid [DATA, s, m] \rangle$ ;

**upon event**  $\langle \mathcal{P}, \textit{Crash} \mid p \rangle$  **do**

*correct* := *correct*  $\setminus \{p\}$ ;

**function** *candeliver*(*m*) **returns** Boolean **is**

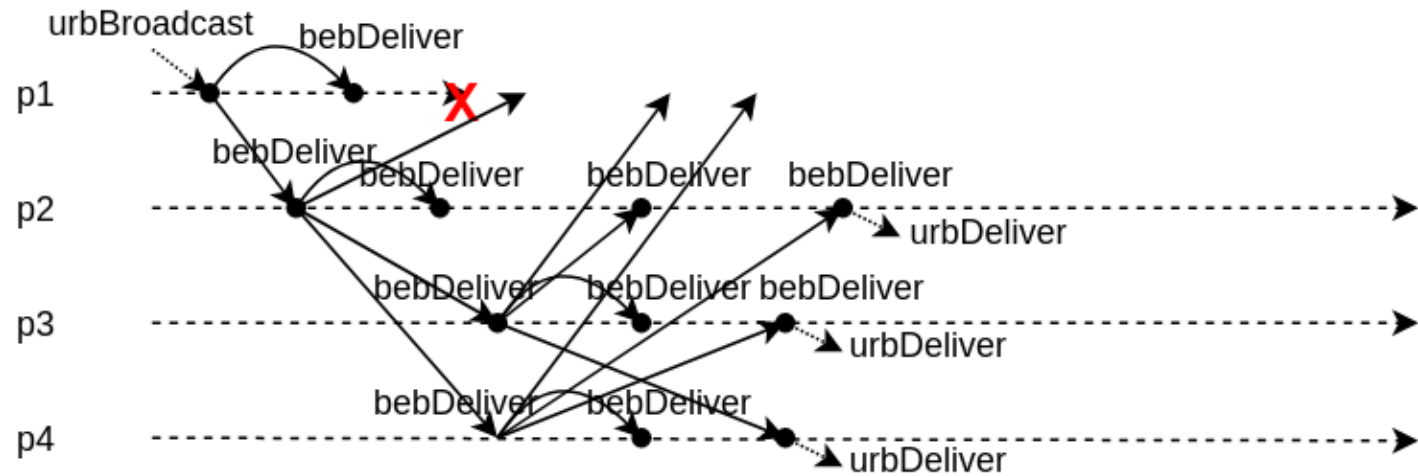
**return** (*correct*  $\subseteq \textit{ack}[m]$ );

**upon exists**  $(s, m) \in \textit{pending}$  such that *candeliver*(*m*)  $\wedge m \notin \textit{delivered}$  **do**

*delivered* := *delivered*  $\cup \{m\}$ ;

**trigger**  $\langle \textit{urb}, \textit{Deliver} \mid s, m \rangle$ ;

# All-ack URB example



# Correctness of All-ack URB

## Lemma

If a correct node  $p$  BEB delivers  $m$ , then  $p$  eventually URB delivers  $m$ .

Proof:

- A correct node  $p$  BEB broadcasts  $m$  as soon as it gets  $m$ .
- By BEB1, every correct node gets  $m$  and BEB broadcasts  $m$ .
- Therefore  $p$  BEB delivers from every correct node by BEB1.
- By completeness of the perfect failure detector,  $p$  will not wait for dead nodes forever.
  - $\text{canDeliver}$  becomes true and  $p$  URB delivers  $m$ .

# Correctness of All-ack URB

- **URB1. Validity**
  - If sender is correct, it will BEB delivers  $m$  by validity (BEB1)
  - By the lemma, it will therefore eventually URB delivers  $m$ .
- **URB2. No duplication**
  - Guaranteed because of the delivered set.
- **URB3. No creation**
  - Ensured from best-effort broadcast.
- **URB4. Uniform agreement**
  - Assume some node (possibly failed) URB delivers  $m$ .
    - Then `canDeliver` was true, and by accuracy of the failure detector, every correct node has BEB delivered  $m$ .
  - By the lemma, each of the nodes that BEB delivered  $m$  will URB deliver  $m$ .

# URB for fail-silent

- All-ack URB requires a perfect failure detector (fail-stop).
- Can we implement URB in [fail-silent](#), without a perfect failure detector?
- **Yes**, provided a majority of nodes are correct.

## **Implements:**

UniformReliableBroadcast, **instance** *urb*.

## **Uses:**

BestEffortBroadcast, **instance** *beb*.

// Except for the function *candeliver*( $\cdot$ ) below and for the absence of  $\langle \text{Crash} \rangle$  events  
// triggered by the perfect failure detector, it is the same as Algorithm 3.4.

**function** *candeliver*( $m$ ) **returns** Boolean **is**  
**return**  $\#(\text{ack}[m]) > N/2$ ;

[Q] Show that this variant is correct.

# Causal reliable broadcast



# Motivation



**Mathias Verraes**

@mathiasverraes

Follow



There are only two hard problems in distributed systems:

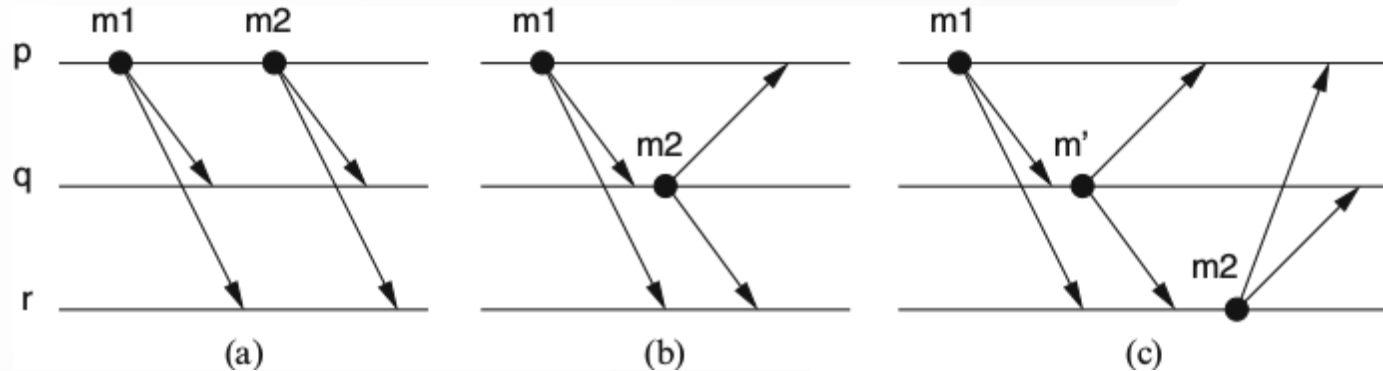
1. Guaranteed order of messages
2. Exactly-once delivery

Reliable broadcast:

- Exactly-once delivery: **guaranteed** by the properties of RB.
- Order of message? **Not guaranteed!**

[Q] Does uniform reliable broadcast remedy this?

# Causal order of messages



A message  $m_1$  may have caused another message  $m_2$ , denoted  $m_1 \rightarrow m_2$  if any of the following relations apply:

- (a) some process  $p$  broadcasts  $m_1$  before it broadcasts  $m_2$ ;
- (b) some process  $p$  delivers  $m_1$  and subsequently broadcasts  $m_2$ ; or
- (c) there exists some message  $m'$  such that  $m_1 \rightarrow m'$  and  $m' \rightarrow m_2$ .

# Causal broadcast

## Module:

**Name:** CausalOrderReliableBroadcast, **instance** *crb*.

## Events:

**Request:**  $\langle crb, Broadcast \mid m \rangle$ : Broadcasts a message  $m$  to all processes.

**Indication:**  $\langle crb, Deliver \mid p, m \rangle$ : Delivers a message  $m$  broadcast by process  $p$ .

## Properties:

**CRB1–CRB4:** Same as properties RB1–RB4 in (regular) reliable broadcast (Module 3.2).

**CRB5:** *Causal delivery*: For any message  $m_1$  that potentially caused a message  $m_2$ , i.e.,  $m_1 \rightarrow m_2$ , no process delivers  $m_2$  unless it has already delivered  $m_1$ .

# No-waiting causal broadcast

## Implements:

CausalOrderReliableBroadcast, **instance** *crb*.

## Uses:

ReliableBroadcast, **instance** *rb*.

**upon event**  $\langle crb, Init \rangle$  **do**

*delivered* :=  $\emptyset$ ;

*past* := [];

**upon event**  $\langle crb, Broadcast \mid m \rangle$  **do**

**trigger**  $\langle rb, Broadcast \mid [DATA, past, m] \rangle$ ;

*append*(*past*, (*self*, *m*));

**upon event**  $\langle rb, Deliver \mid p, [DATA, mpast, m] \rangle$  **do**

**if**  $m \notin delivered$  **then**

**forall**  $(s, n) \in mpast$  **do**

// by the order in the list

**if**  $n \notin delivered$  **then**

**trigger**  $\langle crb, Deliver \mid s, n \rangle$ ;

*delivered* := *delivered*  $\cup \{n\}$ ;

**if**  $(s, n) \notin past$  **then**

*append*(*past*, (*s*, *n*));

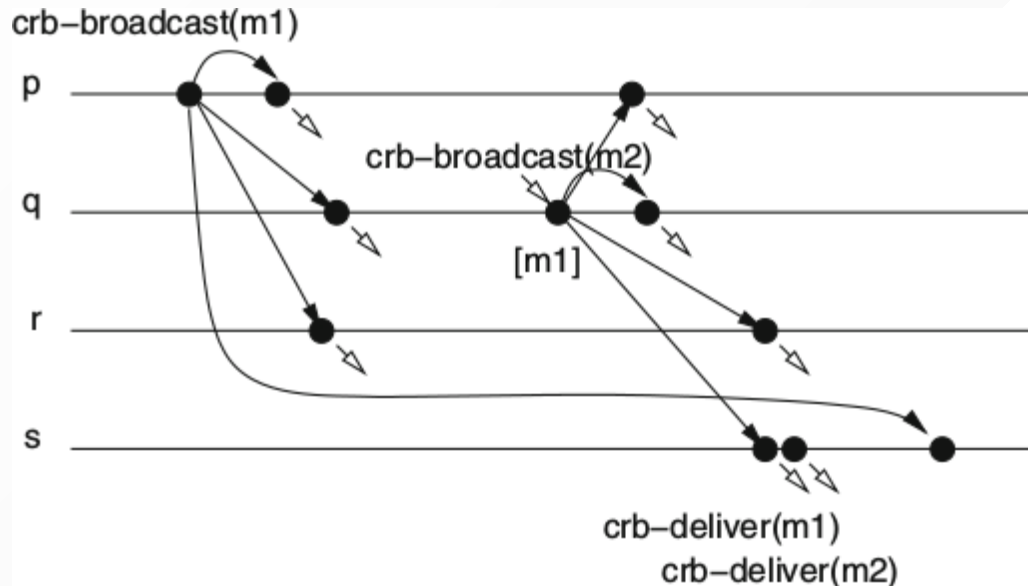
**trigger**  $\langle crb, Deliver \mid p, m \rangle$ ;

*delivered* := *delivered*  $\cup \{m\}$ ;

**if**  $(p, m) \notin past$  **then**

*append*(*past*, (*p*, *m*));

# No-waiting CB example



Issue:

- The size of the message **grows with time**, as messages include their list of causally preceding messages  $mpast$ .
- Solution 1: **Garbage collect** old messages.
- Solution 2: History is a **vector timestamp**!

# Waiting causal broadcast

## Implements:

CausalOrderReliableBroadcast, **instance** *crb*.

## Uses:

ReliableBroadcast, **instance** *rb*.

**upon event**  $\langle crb, Init \rangle$  **do**

$V := [0]^N$ ;

$lsn := 0$ ;

$pending := \emptyset$ ;

**upon event**  $\langle crb, Broadcast \mid m \rangle$  **do**

$W := V$ ;

$W[rank(self)] := lsn$ ;

$lsn := lsn + 1$ ;

**trigger**  $\langle rb, Broadcast \mid [DATA, W, m] \rangle$ ;

**upon event**  $\langle rb, Deliver \mid p, [DATA, W, m] \rangle$  **do**

$pending := pending \cup \{(p, W, m)\}$ ;

**while exists**  $(p', W', m') \in pending$  such that  $W' \leq V$  **do**

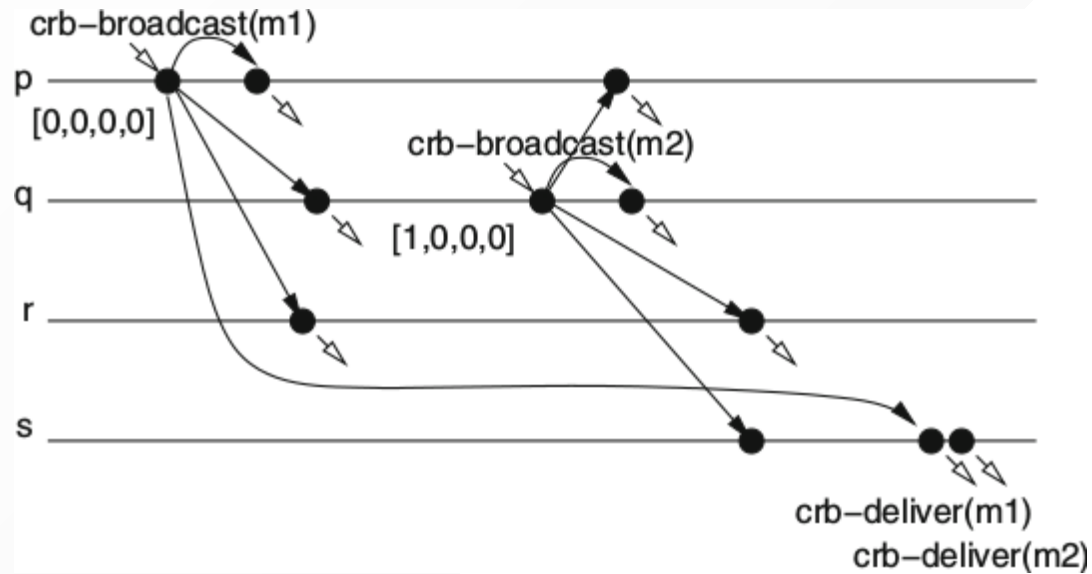
$pending := pending \setminus \{(p', W', m')\}$ ;

$V[rank(p')] := V[rank(p')] + 1$ ;

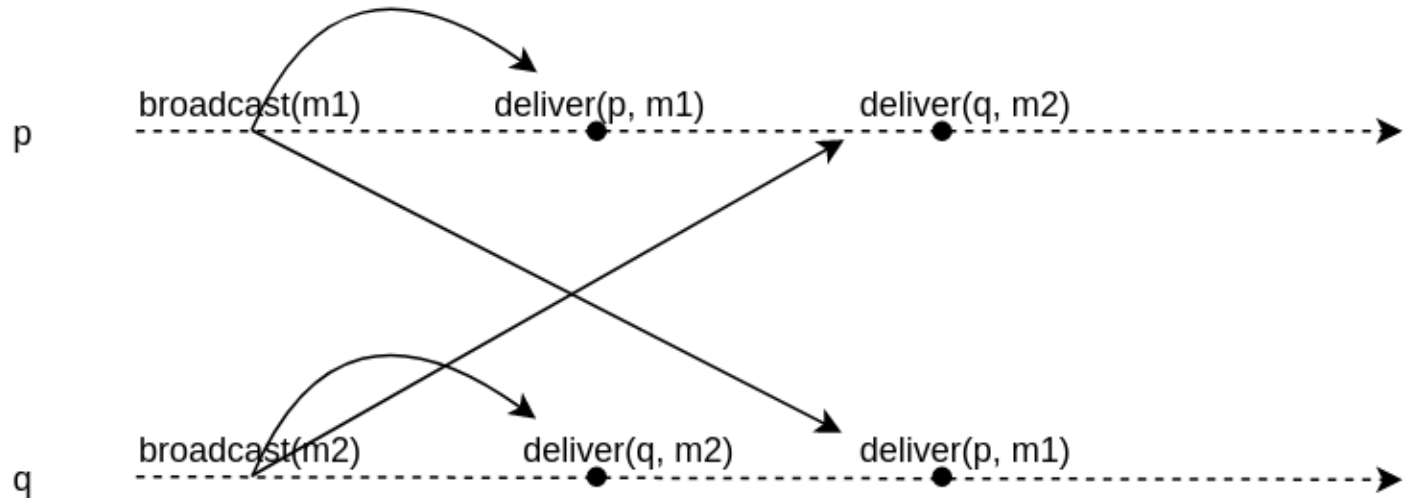
**trigger**  $\langle crb, Deliver \mid p', m' \rangle$ ;

[Q] Show the correctness of the algorithm.

# Waiting CB example



# Possible execution?



[Q] Is this a valid execution? the order of delivery is not the same.

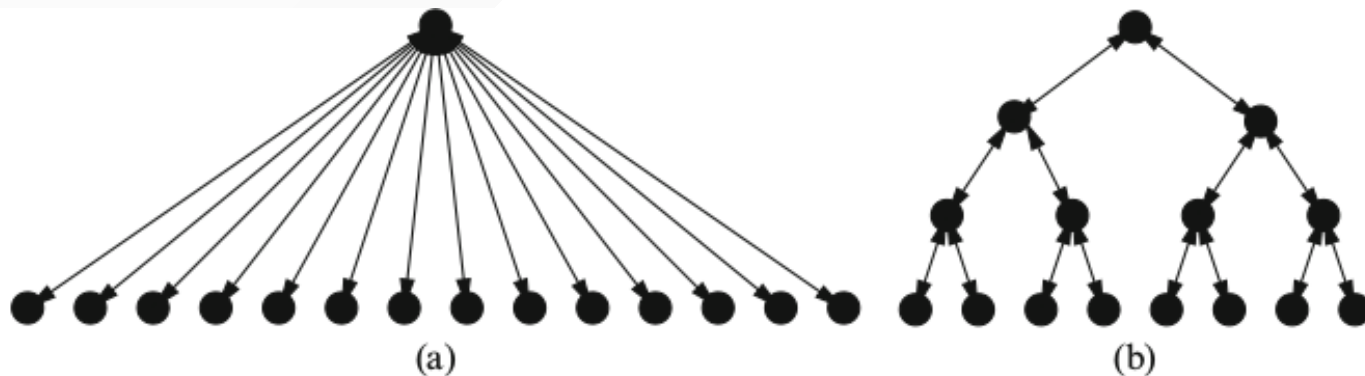


# Probabilistic broadcast

a.k.a. epidemic broadcast or gossip

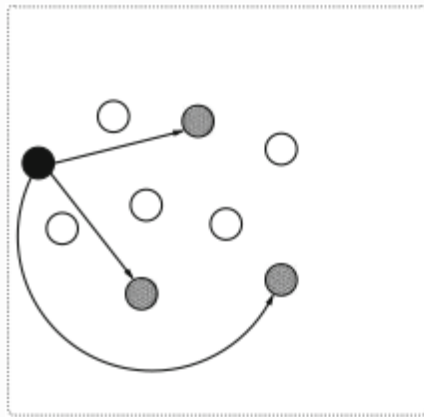
# Scalability of reliable broadcast

- In order to broadcast a message, the sender needs
  - to send messages to all other processes,
  - to collect some form of acknowledgement.
  - $O(N^2)$  are exchanged in total.
    - If  $N$  is large, this can become overwhelming for the system.
- Bandwidth, memory or processing resources may limit the number of messages/acknowledgements that may be sent/collected.
- Hierarchical schemes reduce the total number of messages.
  - This reduces the load of each process.
  - But increases the latency and fragility of the system.

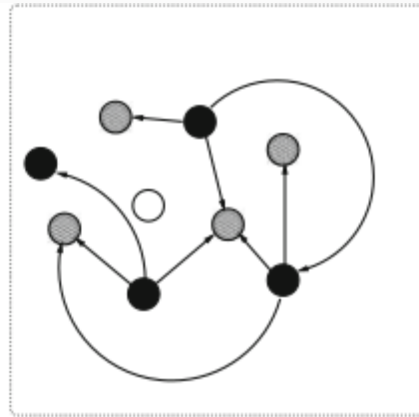


# Epidemic dissemination

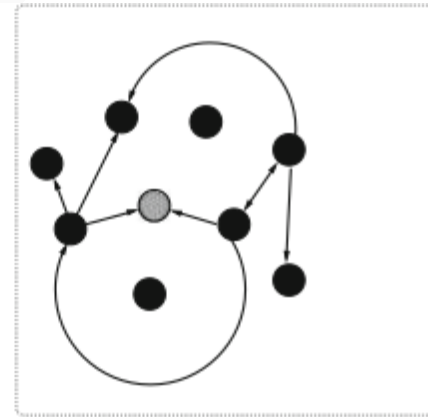
- Nodes infect each other through messages sent in **rounds**.
  - The **fanout**  $k$  determines the number of messages sent by each node.
  - Recipients are drawn **at random** (e.g., uniformly).
  - The **number of rounds** is limited to  $R$ .
- Total number of messages is usually less than  $O(N^2)$ .
- No node is overloaded.



(a) round 1



(b) round 2



(c) round 3

# Probabilistic broadcast

## Module:

**Name:** ProbabilisticBroadcast, **instance**  $pb$ .

## Events:

**Request:**  $\langle pb, \text{Broadcast} \mid m \rangle$ : Broadcasts a message  $m$  to all processes.

**Indication:**  $\langle pb, \text{Deliver} \mid p, m \rangle$ : Delivers a message  $m$  broadcast by process  $p$ .

## Properties:

**PB1: Probabilistic validity:** There is a positive value  $\varepsilon$  such that when a correct process broadcasts a message  $m$ , the probability that every correct process eventually delivers  $m$  is at least  $1 - \varepsilon$ .

**PB2: No duplication:** No message is delivered more than once.

**PB3: No creation:** If a process delivers a message  $m$  with sender  $s$ , then  $m$  was previously broadcast by process  $s$ .

# Eager probabilistic broadcast

## Implements:

ProbabilisticBroadcast, instance *pb*.

## Uses:

FairLossPointToPointLinks, instance *fll*.

**upon event**  $\langle pb, \text{Init} \rangle$  **do**

*delivered* :=  $\emptyset$ ;

**procedure** *gossip*(*msg*) **is**

**forall**  $t \in \text{picktargets}(k)$  **do trigger**  $\langle fll, \text{Send} \mid t, \text{msg} \rangle$ ;

**upon event**  $\langle pb, \text{Broadcast} \mid m \rangle$  **do**

*delivered* := *delivered*  $\cup \{m\}$ ;

**trigger**  $\langle pb, \text{Deliver} \mid \text{self}, m \rangle$ ;

*gossip*([GOSSIP, *self*, *m*, *R*]);

**upon event**  $\langle fll, \text{Deliver} \mid p, [\text{GOSSIP}, s, m, r] \rangle$  **do**

**if**  $m \notin \text{delivered}$  **then**

*delivered* := *delivered*  $\cup \{m\}$ ;

**trigger**  $\langle pb, \text{Deliver} \mid s, m \rangle$ ;

**if**  $r > 1$  **then** *gossip*([GOSSIP, *s*, *m*,  $r - 1$ ]);

# The mathematics of epidemics

- Assume an initial population of  $N$  individuals.
- At any time  $t$ ,
  - $S(t)$  = the number of **susceptible** individuals,
  - $I(t)$  = the number of **infected** individuals.
- $I(0) = 1$
- $S(0) = N - 1$
- $S(t) + I(t) = N$  for all  $t$ .

# The mathematics of epidemics

The dynamics of the SIS model is given as follows:

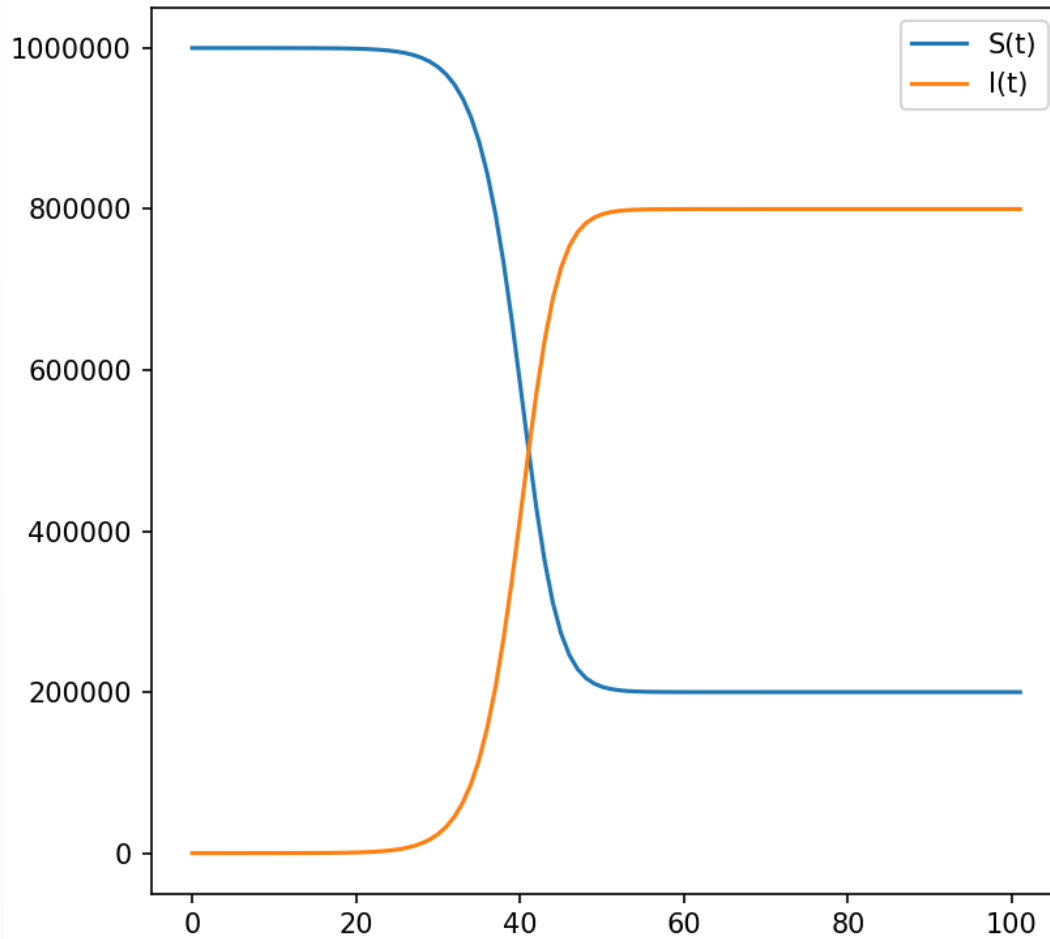
$$S(t + 1) = S(t) - \frac{\alpha \Delta t}{N} S(t) I(t) + \gamma \Delta t I(t)$$

$$I(t + 1) = I(t) + \frac{\alpha \Delta t}{N} S(t) I(t) - \gamma \Delta t I(t)$$

where

- $\alpha$  is the contact rate with whom infected individuals make contact per unit of time.
- $\frac{S(t)}{N}$  is the proportion of contacts with susceptible individuals for each infected individual.
- $\gamma$  is the probability for an infected individual to recover and switch to the pool of susceptibles.





$$N = 1000000, \alpha = 5, \gamma = 0.5, \Delta t = 0.1$$

# The mathematics of epidemics

In eager reliable broadcast,

- $\alpha = k$ 
  - An infected node selects  $k$  nodes among  $N$  to send its messages.
- $\gamma = 1$ 
  - An infected node immediately recovers.

# Probabilistic validity

At time  $t$ , the probability of not receiving a message is

$$\left(1 - \frac{k}{N}\right)^{i(t)}$$

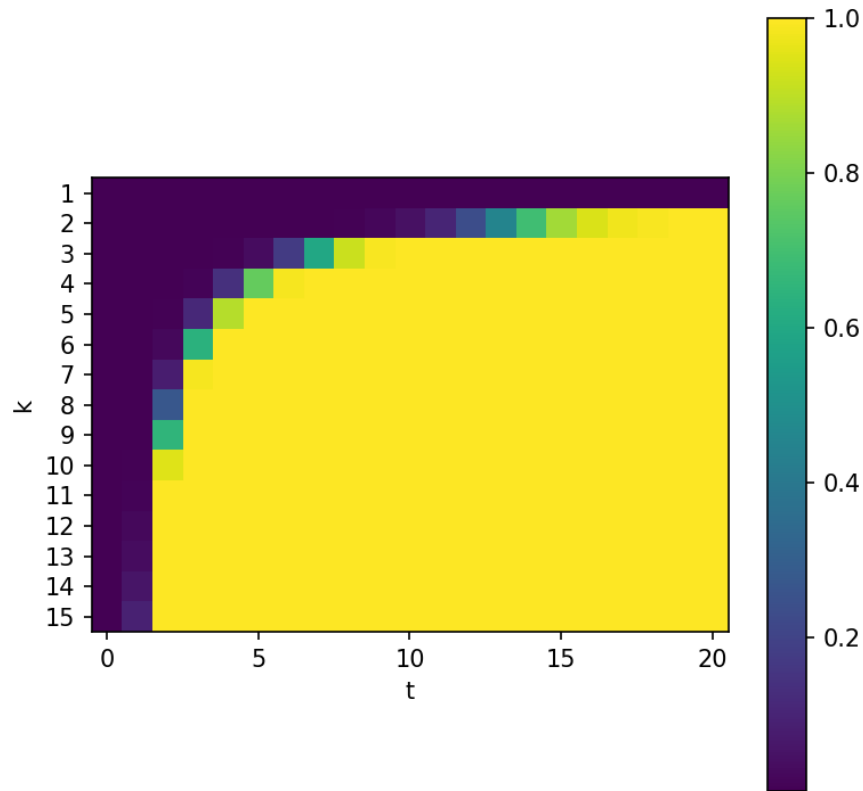
Therefore the probability of having received of one or more gossip messages up to time  $t$ , that is to have PB-delivered, is

$$p(\text{delivery}) = 1 - \left(1 - \frac{k}{N}\right)^{\sum_{t_i=0}^t i(t_i)}$$

[Q] What if nodes fail? if packets are loss?

# Probabilistic validity

$$p(\text{delivery}|k, t)$$



$$N = 1000000, \gamma = 1.0$$

# Probabilistic validity

From this plot, we observe that:

- Within only a few rounds (**low latency**), a large fraction of nodes receive the message (**reliability**)
- Each node has transmitted no more than  $kR$  messages (**lightweight**).

# Lazy Probabilistic broadcast

- Eager probabilistic broadcast consumes **considerable resources** and causes many **redundant transmissions**.
  - in particular as  $r$  gets larger and almost all nodes have received the message once.
- Assume **a stream of messages** to be broadcast.
- Broadcast messages in **two phases**:
  - **Phase 1 (data dissemination)**: run probabilistic broadcast with a large probability  $\epsilon$  that reliable delivery fails. That is, assume a constant fraction of nodes obtain the message (e.g.,  $\frac{1}{2}$ ).
  - **Phase 2 (recovery)**: upon delivery, detect omissions through sequence numbers and initiate retransmissions with gossip.

# Lazy Probabilistic broadcast

## Phase 1: data dissemination

**Implements:**

ProbabilisticBroadcast, **instance** *pb*.

**Uses:**

FairLossPointToPointLinks, **instance** *fl*;

ProbabilisticBroadcast, **instance** *upb*.

// an *unreliable* implementation

**upon event**  $\langle pb, Init \rangle$  **do**

$next := [1]^N$ ;

$lsn := 0$ ;

$pending := \emptyset$ ;  $stored := \emptyset$ ;

**procedure** *gossip*(*msg*) **is**

**forall**  $t \in picktargets(k)$  **do trigger**  $\langle fl, Send \mid t, msg \rangle$ ;

**upon event**  $\langle pb, Broadcast \mid m \rangle$  **do**

$lsn := lsn + 1$ ;

**trigger**  $\langle upb, Broadcast \mid [DATA, self, m, lsn] \rangle$ ;

**upon event**  $\langle upb, Deliver \mid p, [DATA, s, m, sn] \rangle$  **do**

**if**  $random([0, 1]) > \alpha$  **then**

$stored := stored \cup \{[DATA, s, m, sn]\}$ ;

**if**  $sn = next[s]$  **then**

$next[s] := next[s] + 1$ ;

**trigger**  $\langle pb, Deliver \mid s, m \rangle$ ;

**else if**  $sn > next[s]$  **then**

$pending := pending \cup \{[DATA, s, m, sn]\}$ ;

**forall**  $missing \in [next[s], \dots, sn - 1]$  **do**

**if** no  $m'$  exists such that  $[DATA, s, m', missing] \in pending$  **then**

$gossip([REQUEST, self, s, missing, R - 1])$ ;

$starttimer(\Delta, s, sn)$ ;

# Lazy Probabilistic broadcast

## Phase 2: recovery

```
upon event  $\langle fl, Deliver \mid p, [REQUEST, q, s, sn, r] \rangle$  do
  if exists  $m$  such that  $[DATA, s, m, sn] \in stored$  then
    trigger  $\langle fl, Send \mid q, [DATA, s, m, sn] \rangle$ ;
  else if  $r > 0$  then
    gossip( $[REQUEST, q, s, sn, r - 1]$ );

upon event  $\langle fl, Deliver \mid p, [DATA, s, m, sn] \rangle$  do
   $pending := pending \cup \{[DATA, s, m, sn]\}$ ;

upon exists  $[DATA, s, x, sn] \in pending$  such that  $sn = next[s]$  do
   $next[s] := next[s] + 1$ ;
   $pending := pending \setminus \{[DATA, s, x, sn]\}$ ;
  trigger  $\langle pb, Deliver \mid s, x \rangle$ ;

upon event  $\langle Timeout \mid s, sn \rangle$  do
  if  $sn > next[s]$  then
     $next[s] := sn + 1$ ;
```



# Summary

- Reliable multicast enable group communication, while ensuring **validity** and (uniform) **agreement**.
- Causal broadcast extends reliable broadcast with **causal ordering** guarantees.
- **Probabilistic broadcast** enable low-latency, reliable and lightweight group communication.

# References

- Allen, Linda JS. "Some discrete-time SI, SIR, and SIS epidemic models." Mathematical biosciences 124.1 (1994): 83-105.