Technologies for Autonomous Vehicles
Assignment 3

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1 Linear state-space formulation of a single-track model

The first point regards the state-space formulation of a single-track model, that we obtain using a Simulink model that implements the system with the following output

$$y = \begin{bmatrix} \beta \\ \dot{\psi} \\ \rho_G \\ \alpha_F \\ \alpha_R \\ a_y \end{bmatrix} \tag{1}$$

We analyzed how the properties of the single-track model vary with different vehicle velocities. We noticed that the eigenvalues of the matrix A of the state-space model increase when we increase the vehicle speed, while the determinant decreases in the same conditions (**Figure 1**). Then we plotted the step response of the transfer functions of the sidesplip angle β and the yaw rate r, observing increasing underdamped oscillations at large values of vehicle speed.

2 State Feedback Control

In order to control the vehicle trajectory and make it follow the curvature profile a state feedback controller is applied. This type of control is a very basic one which allows to place the eigenvalues and thus imposing a desired behaviour to system through a particular input in the form of $-Kx(t)$. The dynamical system representing the vehicle with e_1 (lateral deviation error), e_2 (orientation error), $\dot{e}_1$ and $\dot{e}_2$ as states collected in the x vector is as follows:

$$\dot{x}(t) = Ax(t) + B_1\delta(t) + B_2\dot{\psi}_{des}(t)$$

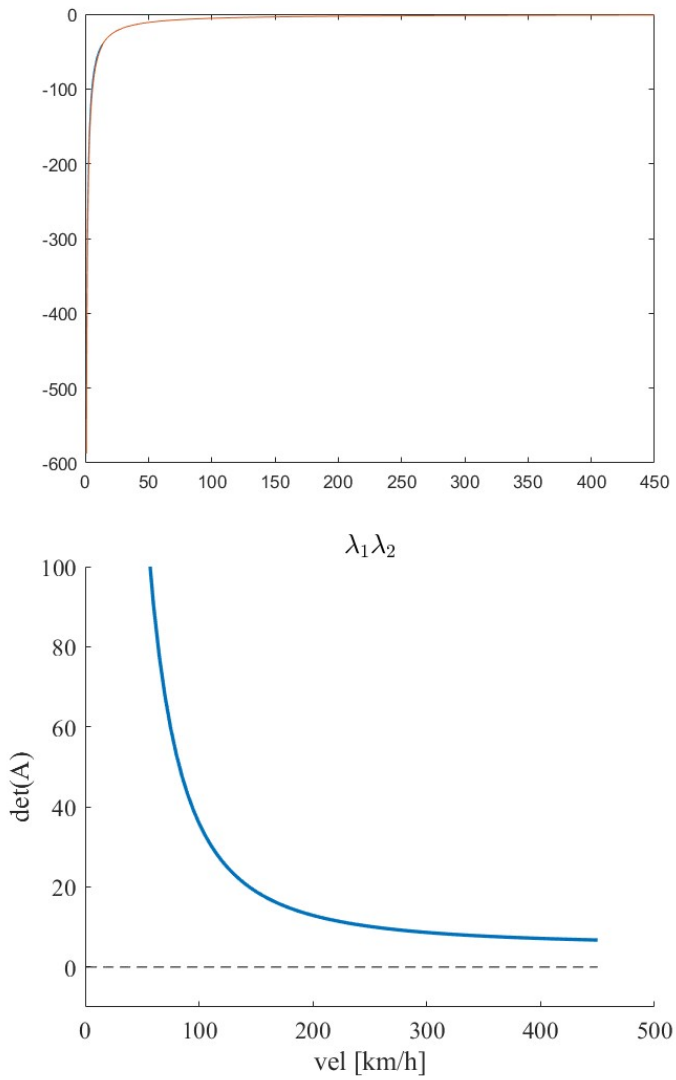


Figure 1: Properties of matrix A with increasing vehicle velocity

$$y(t) = Cx(t)$$

The applied input to the control system is:

$$\delta(t) = -Kx(t) + \delta_{ff}(t)$$

After the application of the controller the new states equation is:

$$[A - B_1 K]x(t) + B_1 \delta_{ff}(t) + B_2 \dot{\psi}_{des}(t)$$

The applied curvature profile is reported in **Figure 2**. This is the reference to which the control system compares the current states to perform the control action.

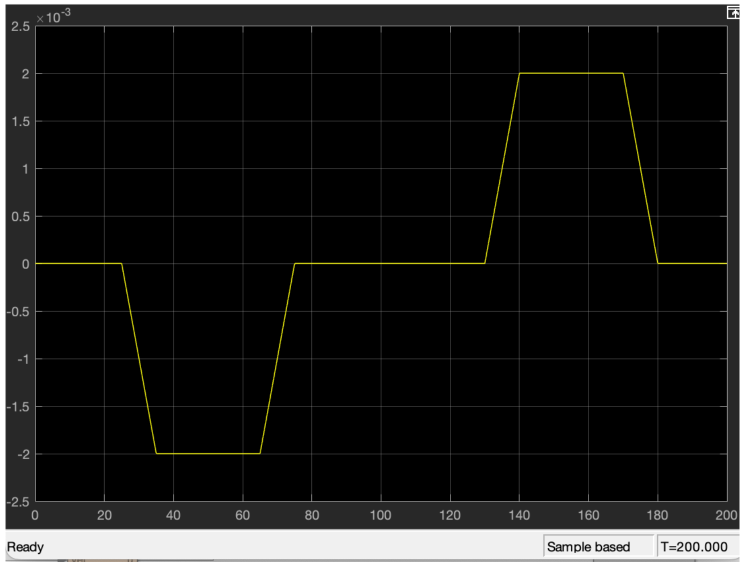
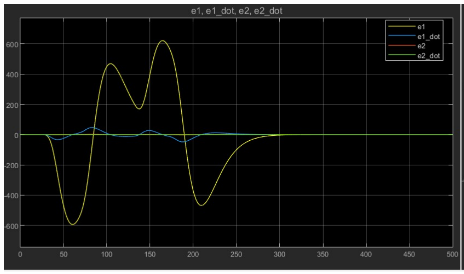


Figure 2: Curvature profile

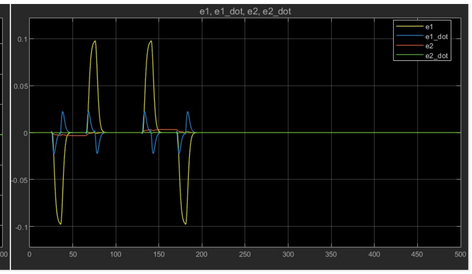
With the purpose of following the trajectory, the eigenvalues are chosen with a pole placement methodology, using a tuning strategy to obtain suitable results. At the first instance, with a vehicle velocity of 80km/h, we try with small values, all equal to each other [-0.1,-0.1,-0.1,-0.1], obtaining a relevant error as output (**Figure 3**).

We notice that, by increasing the eigenvalues while maintaining a constant speed equal to 80km/h, the errors decrease. The difference in magnitude of eigenvalues represents a difference in the system time constants and the smaller the eigenvalues, the bigger the time constants and the slower the system in the reference tracking. Then, we tried to modify the constant speed, using some extreme values (6km/h, 20km/h and 130km/h) in order to see what happens to the control system. With the same poles for the gain matrix, we obtain a larger error in tracking trajectory and we have to use bigger eigenvalues to decrease the errors.
To check the correctness of the controller, we calculate the gain matrix K with the linear quadratic set-up, that we use to plot the variation of control gains as function of vehicle speed (**Figure 4**).
We obtained the same results with both the controllers, using the set of eigenvalues all equal to -10 for the pole placement methodology.

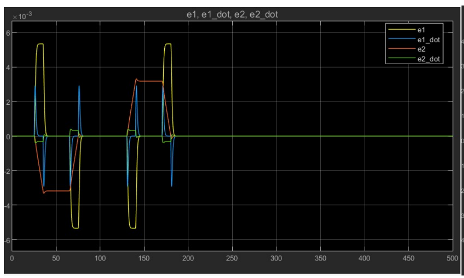
In the analysis of the performance of the controller, we also include other kinds of curvature profiles: skidpad and obstacle avoidance manoeuvres. To



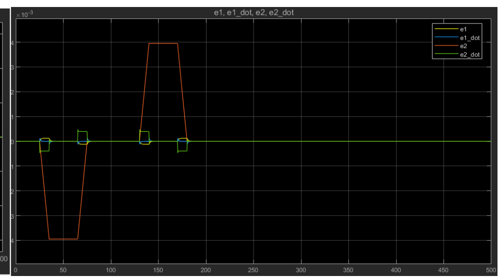
(a) 0.1



(b) -1



(c) -2



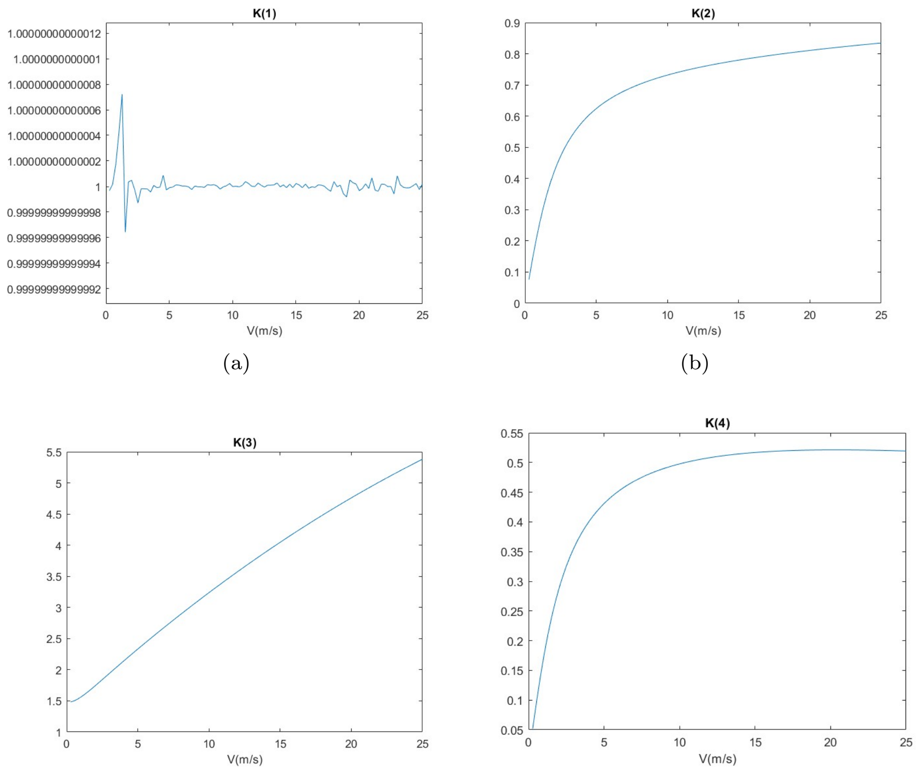
(d) QR controller and -10

Figure 3: Errors

reproduce an obstacle avoidance manoeuvres, we use a very steep and fast curvature profile, as depicted in **Figure 5** and we simulated the vehicle's behaviour at 80km/h velocity, with the same set of eigenvalues. What we observe is that it's necessary to increase in magnitude the value of poles in order to reduce the errors and make the vehicle follows the trajectory. This is due to the fact that the trajectory is more complex to be followed and it requires a faster response (with higher eigenvalues in magnitude).

3 Contribution of feedforward and feedback gain

In nearly-steady state condition, most of the control effort should be carried out by the feedforward contribution. The feedback contribution should just give a correction that should become evident in transient conditions. With the intention of verifying this theory statement, we proceeded by shutting down first the feedforward contribution and we obtain larger error for e_1 than in the initial situation, considering the same set of eigenvalues, **Figure 6**, while the other errors remain the same, since the relative yaw angle e_2 does not depend on the feedforward term. By using the feedforward contribution only, the error grows to large values and there is no control action to diminish it: indeed without the $-Kx$ term there is no way to change the system dynamics represented by matrix A, so there is no guarantee that the system is asymptotically stable



(a) (b) (c) (d)

Figure 4: Control gains wrt vehicle speed

(**Figure 7**). With the only action of state feedback, asymptotic stability is guaranteed but in the period in which K_L is constant, error e_1 is constant as well and is not cancelled. So both contribution are necessary: the feedback to track the reference and the feedforward to strengthen the command input and have better performances such as the suppression of the error in constant K_L periods. To summarize, the feedforward contribution enhances the system response anticipating known disturbances and strengthens the command, while the feedback contribution corrects errors base on system output and betters the system robustness and stability.

4 Relevant vehicle response variables

According to the data provided, the vehicle is in understeering condition, since $C_R * b - C_F * a > 0$ and this is necessary to understand the relevant response variables, obtained after the simulation at 80km/h velocity. The trajectory to

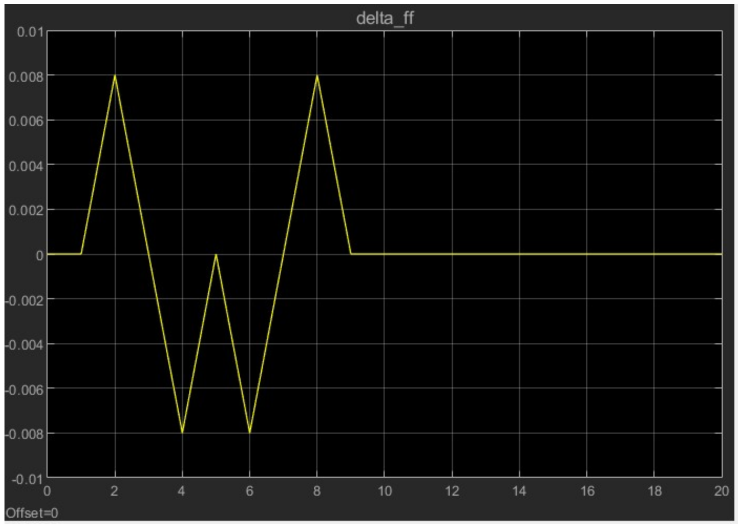


Figure 5: Obstacle avoidance manoeuvres

follow is the same reported in the text homework; the steering angle δ is shown in **Figure 8**. In **Figure 9** we reported the comparison between the sideslip angle β and the yaw rate r. In **Figure 10**, the lateral acceleration and in **Figure 11** the front and rear slip angles are represented, coherent with an understeering vehicle.

We tried to modify the velocity to observe what happens in different operating conditions, with 50 and 130 km/h. We notice that, increasing the vehicle velocity, all the response variables have to increase in magnitude their value to follow the trajectory in a right way. For example, we observed an higher lateral acceleration in situations of higher velocity.

5 Vary the rear cornering stiffness

With the aim of assessing how the control input profile is modified by a variation of the rear axle cornering stiffness, we put the vehicle in neutral condition and then in oversteer (considering that the starting point is understeer). According to the definition of neutral condition, we set

$$C_R = C_F * \frac{a}{b}$$

The two response variables that vary the most are the steering angle, that is smaller than the understeering vehicle, and the front and rear sideslip angles, that are equal to each other (**Figure 18**).

In order to obtain a vehicle in oversteer condition, we impose the condition

$$C_F * a - C_R * b > 0$$

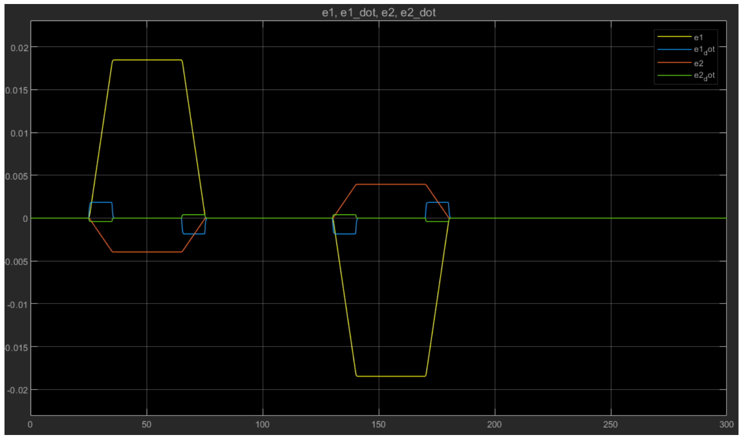
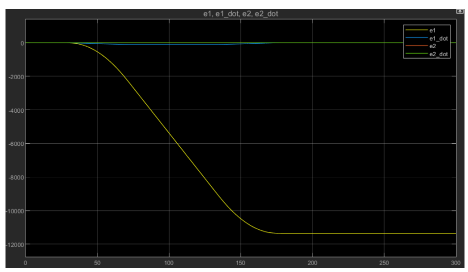
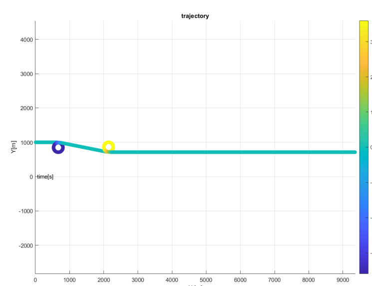


Figure 6: Errors without feedforward contribution



(a)



(b)

Figure 7: Errors and trajectory without feedback control contribution

and we obtain a value of 79.1km/h for the critical speed. This means that, by keeping the velocity of 80km/h, all the response variables diverge; to avoid this behaviour, we reduce the velocity to 60km/h. Even in this case, the response variables that change the most are the steering angle and the sideslip angle, that achieve smaller values than the previous cases, as also the lateral acceleration

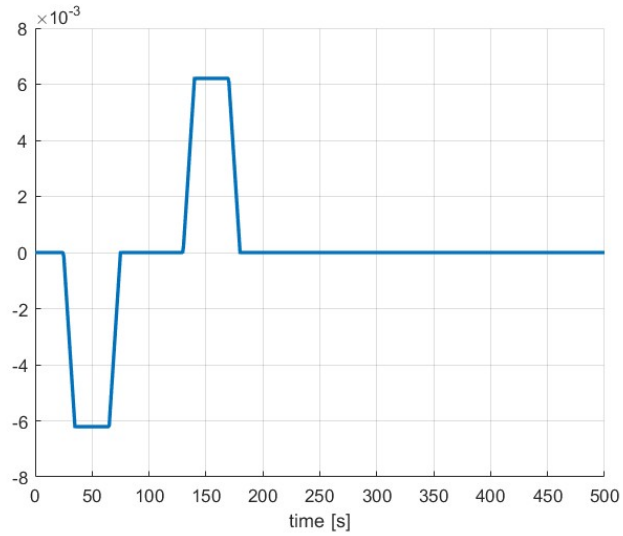


Figure 8: Steering angle

(**Figure 19**). Moreover we observed small increasing errors in the control part, considering always the same set of eigenvalues and the same QR matrices.

6 Presence of pure time delay

The presence of pure time delay in the feedforward part of the input is compared against the case in which no delay is applied for different magnitude of eigenvalues because of the correlation between the eigenvalues and the time constants of the system. No changes are visible for big eigenvalues (magnitude of 10^2). However we noticed changes for small eigenvalues, with increasing errors in tracking the trajectory when the pure time delay is applied (**Figure 20**).

7 Optional: Skidpad Test

In order to implement a skidpad test, a test in which a circular trajectory is performed while accelerating in a smooth way, some edits to the previous model have to be performed: the K_L is simulated as a step of magnitude 0.02 m^{-1} and the speed is now a ramp that goes from $10m/s$ up to $40m/s$, with an acceleration of $1m/s^2$ and then saturates for the rest of simulation time.

Since now $\overline{V}_x$ is no more constant, at each time step a reformulation of the matrices A, B_1 and B_2 is needed and also a new K matrix in order to fit the controller into the evolving scenario. Doing so, e_1 is cancelled while e_2 is reduced to a small value but not suppressed: this means that a small error in the yaw angle with respect to the trajectory still remains implying an erroneous

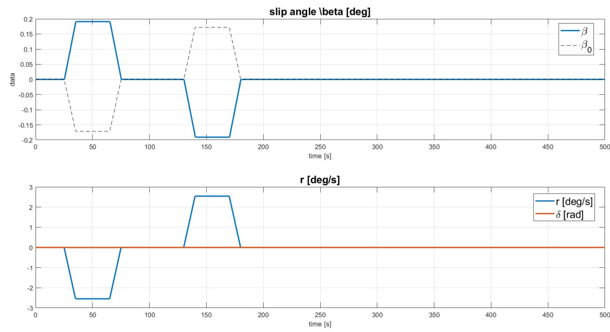


Figure 9: Sideslip angle and Yaw rate

behaviour in following the circumference: indeed, an imperfect trajectory is performed.

However the goal of this simulation is to show how the steering angle changes along time having an increasing speed in the three different conditions of over steer, neutral steer and under steer (**Figure 21**). The most interesting is of course the over steer one in which, since speed goes from $10m/s$ to $40m/s$, critical speed is crossed: in this case it is clearly visible that the steering angle is reduced as speed increases and then is applied in the opposite direction to countersteer. In understeering conditions, δ is increased as speed is increased and in neutral steering δ is kept constant.

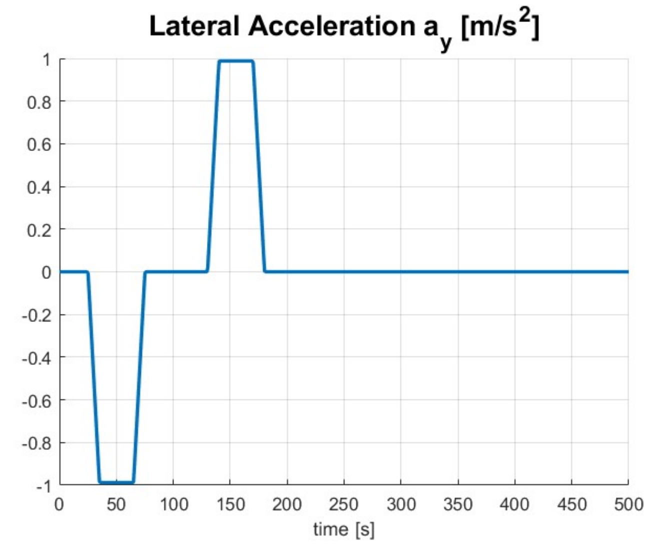


Figure 10: Lateral acceleration

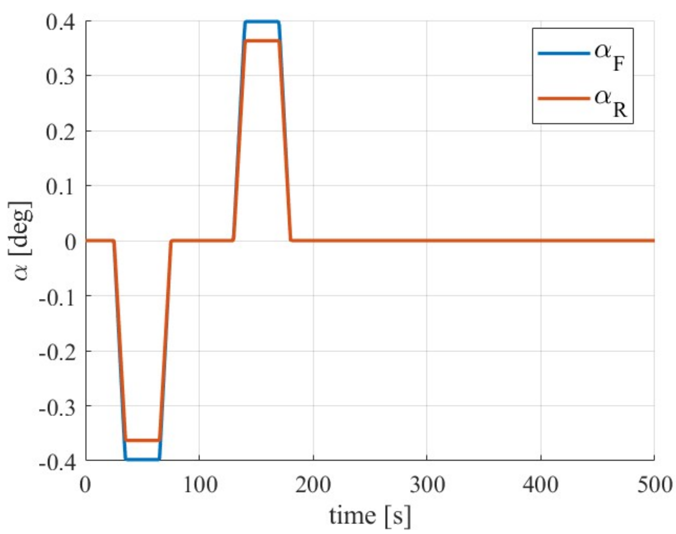


Figure 11: Front and rear slip angles

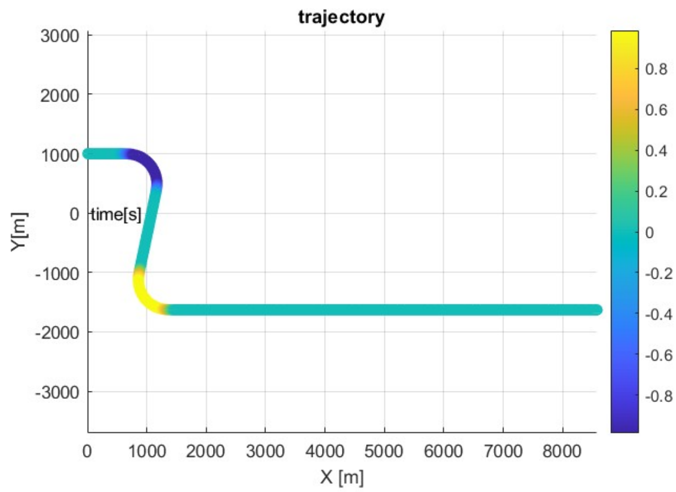
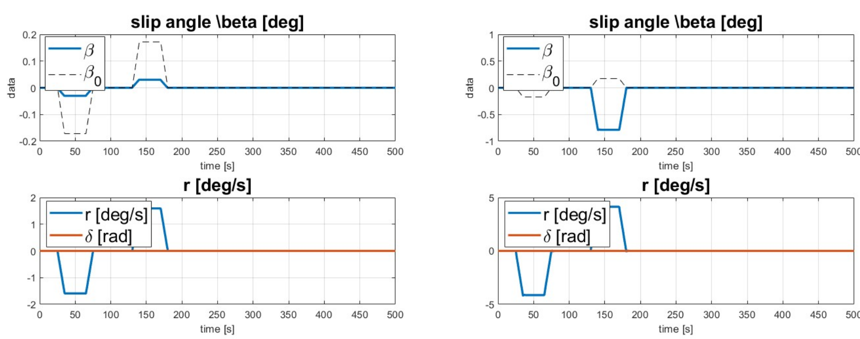


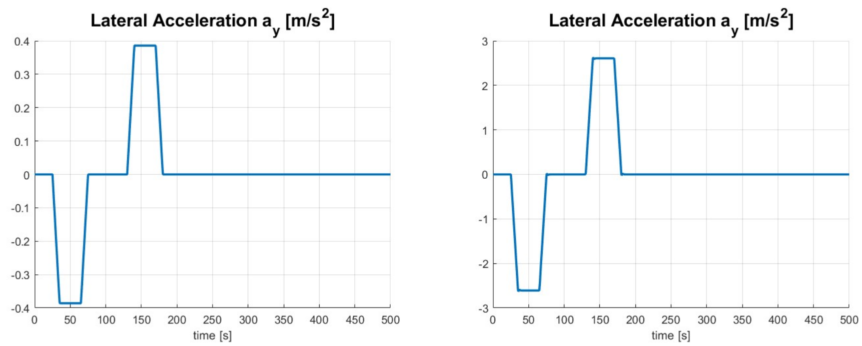
Figure 12: Trajectory



(a) 50 km/h

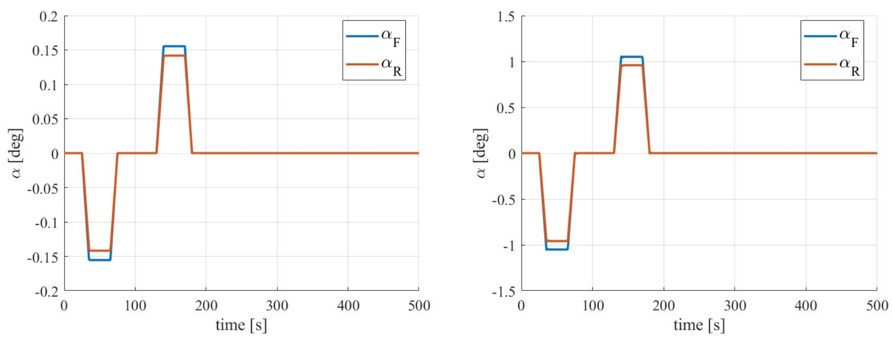
(b) 130 km/h

Figure 13: States



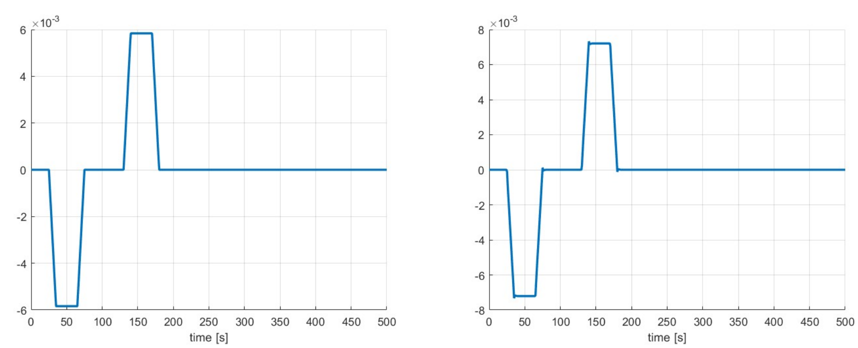
(a) 50 km/h (b) 130 km/h

Figure 14: Lateral acceleration



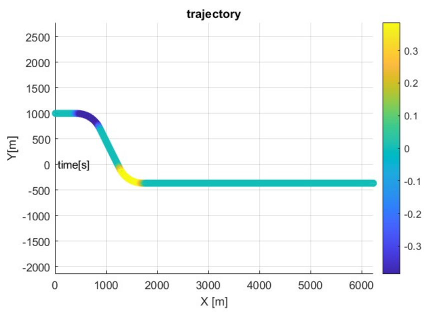
(a) 50 km/h (b) 130 km/h

Figure 15: α front and rear

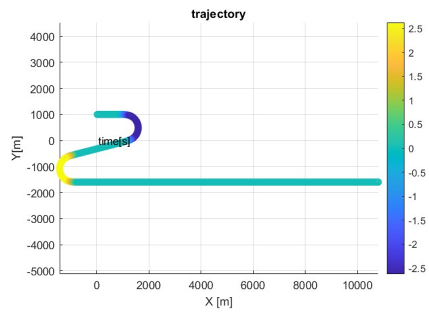


(a) 50 km/h (b) 130 km/h

Figure 16: Steering angle

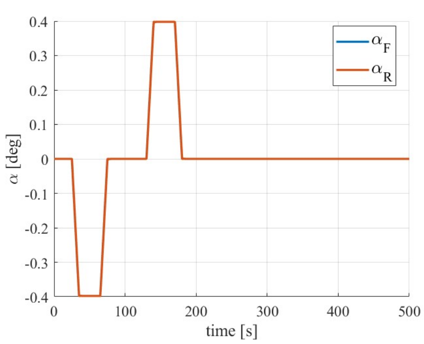


(a) 50 km/h

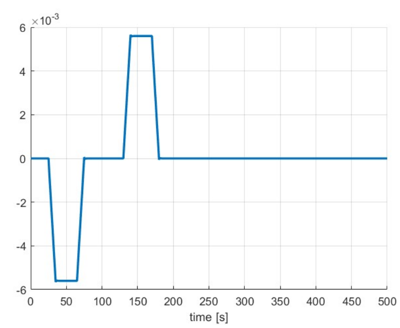


(b) 130 km/h

Figure 17: Trajectory

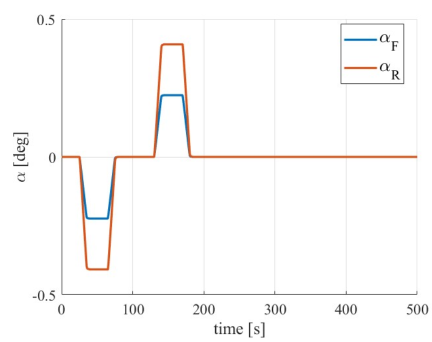


(a) Sideslip angles

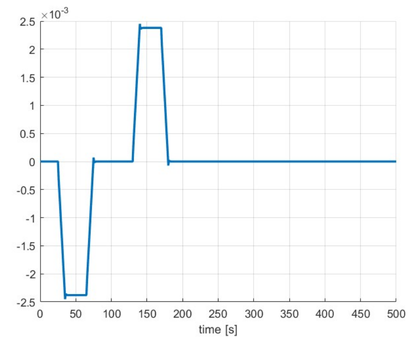


(b) Steering angle

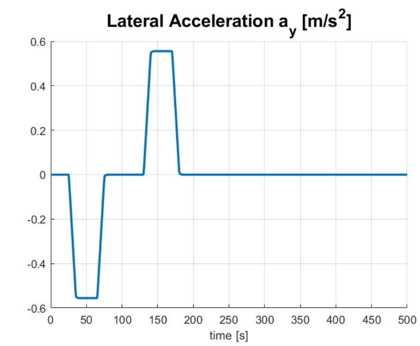
Figure 18: Neutral vehicle



(a) Sideslip angles

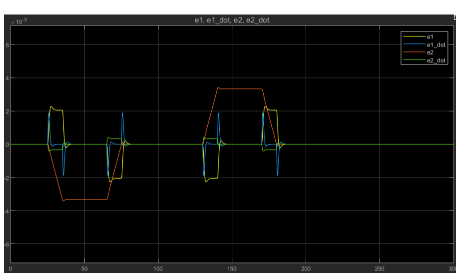


(b) Steering angle

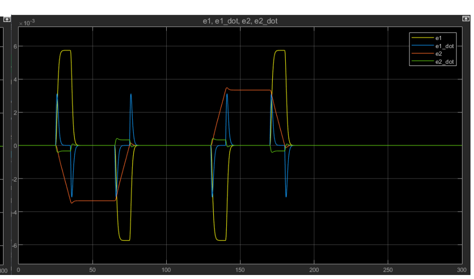


(c) Steering angle

Figure 19: Oversteering vehicle

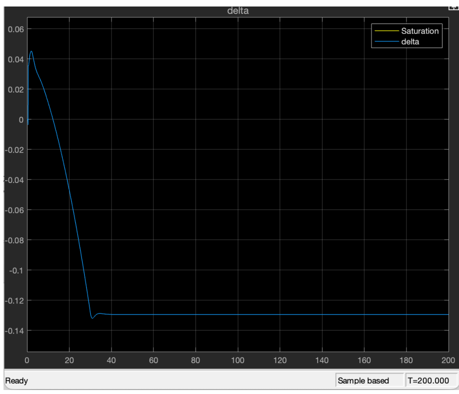


(a) No pure time delay

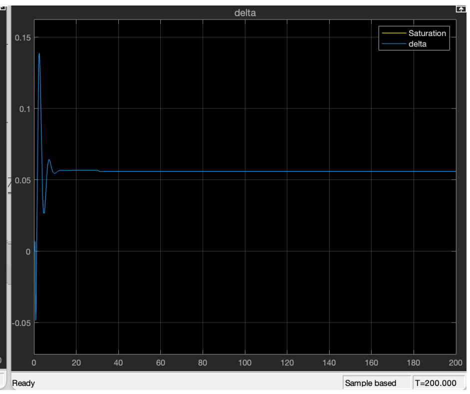


(b) With pure time delay

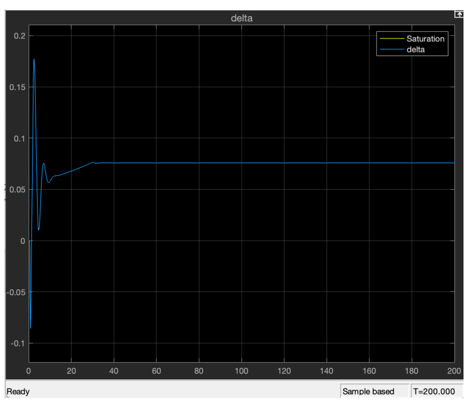
Figure 20: Errors with -2



(a) Over steer



(b) Neutral steer



(c) Under steer

Figure 21: Steering angles