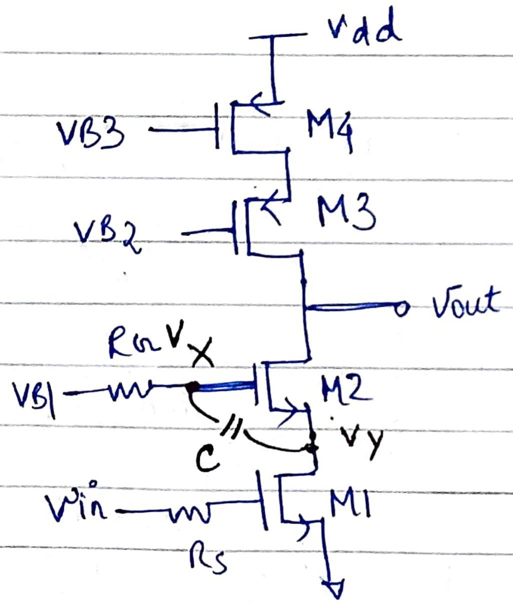
Tutorial-6 Solutions

Q1) After laying out the given typical single-stage cascode amplifier, a parasitic resistance $R_G = 1K\Omega$ was observed at the gate of M2 as shown. Assume that C_{gs} of M2 is the only other parasitic significant enough to contribute, find the magnitude $|A_v|$ of gain at 5GHz. (with at most 1% deviation from exact answer). Assume $R_S = 8K\Omega$, $g_{m3} = g_{m4} = 1mS$, $g_{m1} = g_{m2} = 8mS$, r_{ds} of all transistors as $484\ K\Omega$, $C_{gs2} = 76pF$.

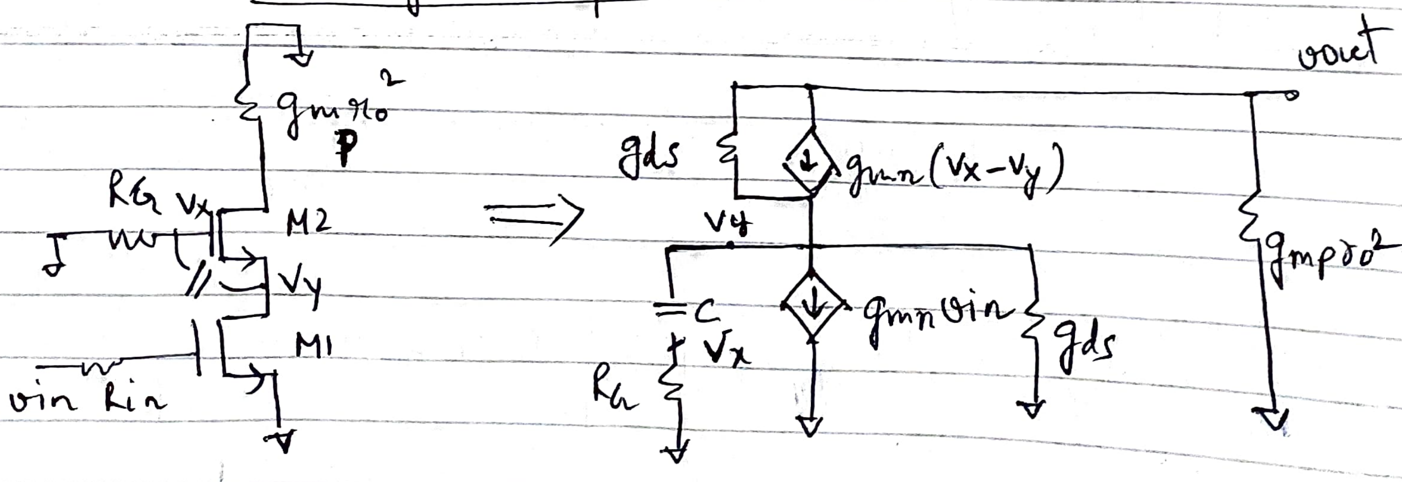


$g_{mp} = g_{m3} = g_{m4} = 1mS$

$g_{mn} = g_{m1} = g_{m2} = 8mS$

Solution

Small signal picture:



KCL at node out,

$$\frac{v_{out}}{g_{mp} r_o^2} + g_{ds}(v_{out} - v_y) + g_{mn}(v_x - v_y) = 0 \quad \text{--- (1)}$$

And, $\dfrac{v_x}{R_G} = (v_y - v_x)\, sC$

$$\Rightarrow v_y = \left(\frac{1}{RsC} + 1\right) v_x \quad \text{--- (2)}$$

using (2) in (1),

$$v_{out}\left[\frac{1}{g_{mp} r_o^2} + g_{ds}\right] - g_{ds}\left(\frac{1}{R_G sC} + 1\right) \cancel{v_{out}}\, v_x - \frac{g_{mn} v_x}{R_G sC} = 0$$

$$\Rightarrow \frac{v_{out}}{v_x} = \frac{\left[\dfrac{g_{mn} + g_{ds}}{R_G sC} + g_{ds}\right]}{\left[\dfrac{1}{g_{mp} r_o^2} + g_{ds}\right]} \approx \frac{\dfrac{g_{mn}}{R_G sC} + g_{ds}}{\dfrac{1}{g_{mp} r_o^2} + g_{ds}} \quad \text{--- (3)}$$

KCL at node Y,

$$\frac{v_y}{R_G + \frac{1}{sC}} + g_{mn} v_{in} + g_{ds} v_y = \frac{0 - v_{out}}{g_{mp} r_o^2}$$

$$\Rightarrow v_y\left[g_{ds} + \frac{1}{R_G + \frac{1}{sC}}\right] + g_{mn} v_{in} = \frac{-v_{out}}{g_{mp} r_o^2} \quad \text{--- (4)}$$

using (2) in (4),

$$v_x\left(1+\frac{1}{R_a sC}\right)\left(g_{ds}+\frac{sC}{1+R_a sC}\right)+g_{mn}v_{in}=\frac{-v_{out}}{g_{mp}r_o^2}$$

$$\Rightarrow v_x\left[\frac{\cancel{R_a sC+1}}{R_a sC}\right]\left[\frac{g_{ds}(1+R_a sC)+sC}{\cancel{1+R_a sC}}\right]+g_{mn}v_{in}=\frac{-v_{out}}{g_{mp}r_o^2}$$

$$\Rightarrow v_x\left[\frac{g_{ds}[1+R_a sC]+sC}{R_a sC}\right]+g_{mn}v_{in}=\frac{-v_{out}}{g_{mp}r_o^2} \quad -(5)$$

using (3) in (5),

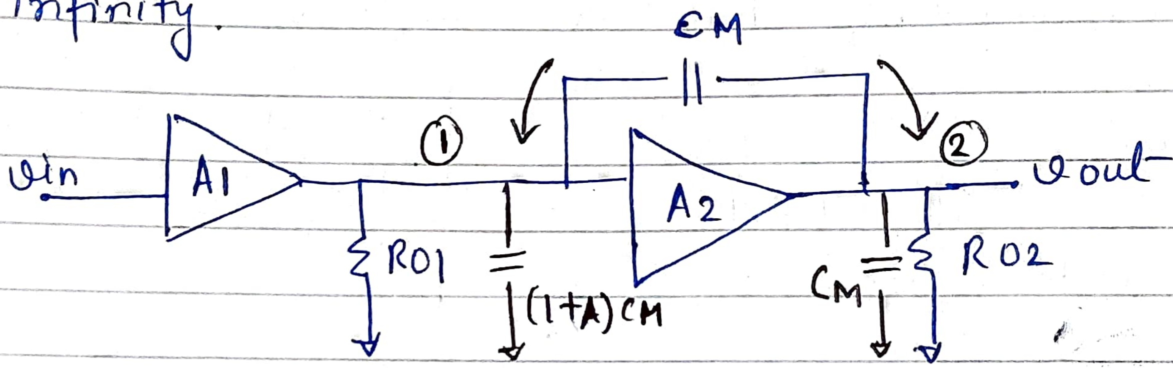
$$\Rightarrow g_{mn}v_{in}+\frac{v_{out}}{R_a sC}\left[\frac{g_{ds}(1+R_a sC)+sC}{\frac{g_{mn}}{R_a sC}+g_{ds}}\right]\left[\frac{1}{g_{mp}r_o^2}+g_{ds}\right]=\frac{-v_{out}}{g_{mp}r_o^2}$$

$$g_{mn} v_{in} = -v_{out}\left[\frac{g_{ds}(1+R_4 sC)+sC}{g_{mn}+g_{ds}(R_4 sC)}\left(\frac{1}{g_{mp} r_o^2}+g_{ds}\right)+\frac{1}{g_{mp} r_o^2}\right]$$

$$\Rightarrow \frac{v_{out}}{v_{in}} = \frac{-g_{mn}}{\left[\frac{g_{ds}(1+R_4 sC)+sC}{g_{mn}+g_{ds} R_4 sC}\left(\frac{1}{g_{mp} r_o^2}+g_{ds}\right)+\frac{1}{g_{mp} r_o^2}\right]}$$

(Ans)

Q2) Find the 3dB bandwidth of the circuit shown below in Hz. Given A_1 is a non inverting amplifier with a gain of 464 and output impedance $R_{O1} = 13\,M\Omega$. A_2 is an inverting amplifier with a gain of 16, output impedance $R_{O2} = 20\,M\Omega$, $C_M = 5\,pF$ and assume input impedance of both amplifiers as infinity.



Solution:

$$\text{Pole frequency at } ① \Rightarrow \frac{1}{2\pi R_{O1}(1+A)C_M}$$

$$= \frac{1}{2\pi(13\,M\Omega)(1+16)(5\,pF)}$$

$$= 144.104\ Hz.$$

$$\text{Pole frequency at } ② \Rightarrow \frac{1}{2\pi R_{O2} C_M}$$

$$= \frac{1}{2\pi(20\,M\Omega)(5\,pF)} = 1.592\ KHz.$$

$\therefore$ 3dB bandwidth / freq = dominant pole frequency

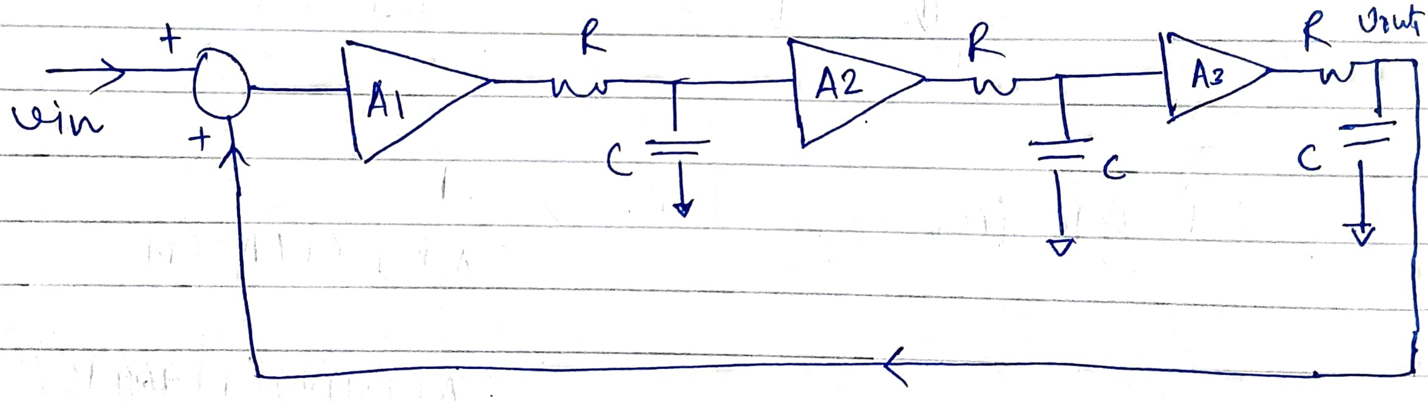
$= 144.104\ Hz$

(Ans)

Q3) In the circuit shown below, A1, A2 and A3 are 3 ideal voltage amplifiers having DC gain of K, −K and K respectively. Then the total phase shift obtained (in degrees) in the loop at a frequency of 2.2 MHz when they are connected in unity feedback is given by $\pm 180° - X°$. Find the value of X.

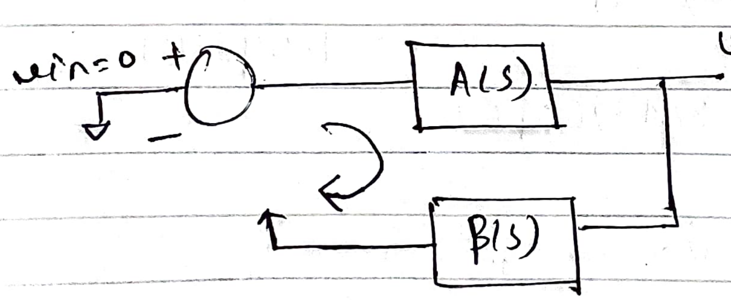
Given : $R = 2.2\,K\Omega$
$C = 19\,pF$
(Ans. upto 3 decimal places)

Ideal volt ampl.
$R_{in} \to \infty$
$R_{out} \to 0$



Solution : Find total phase shift in loop :-

Find Loop transfer function,



Here, loop transfer function $= A(s)\,\beta(s)$.

For given circuit,
$\beta(s) = 1$

$$A(s) = \frac{A_1 A_2 A_3}{(1+Rsc)^3}$$

$$\boxed{A(s)\beta(s) = \frac{-k^3}{(1+Rsc)^3}}$$

~~Total phase shift = ±180° − tan⁻¹~~

Putting $s = j\omega$,

$$A(j\omega)\beta(j\omega) = \frac{-k^3}{(1+j\omega RC)(1+j\omega RC)(1+j\omega RC)}$$

$\therefore$ Phase shift in loop $= \pm 180° - \tan^{-1}(\omega RC) - \tan^{-1}(\omega RC) - \tan^{-1}(\omega RC)$

$$= \pm 180° - \underbrace{3\tan^{-1}(\omega RC)}_{x°}$$

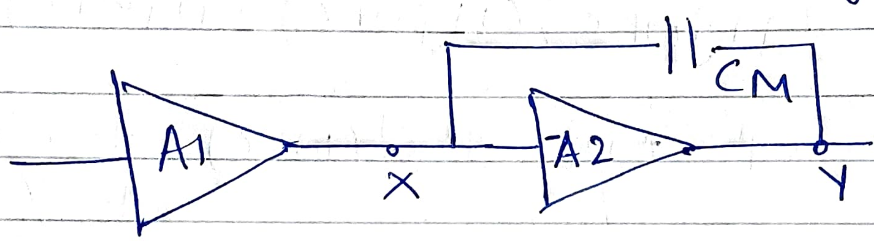
$$x° = 3\tan^{-1}\left(2\pi\,(2.2\,MHz)(2.2\,k\Omega)(19\,pF)\right)$$

$\Rightarrow x° = 3\tan^{-1}(0.5775)$

$\Rightarrow x° = 90.02°$ (Ans)

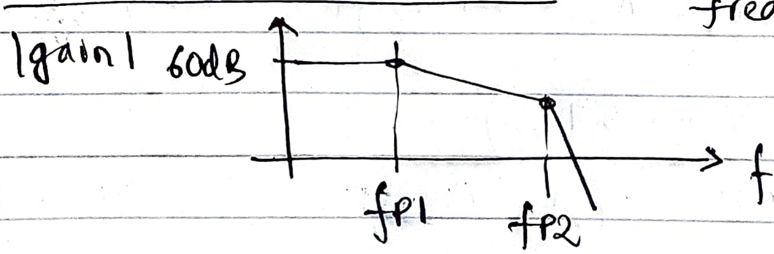
Q4) Given below is an amplifier A, containing 2 stages, A_1 and A_2, with an open loop DC gain of 60 dB. The first dominant pole f_{P1} is at node X and is equal to 6 MHz. The second dominant pole frequency f_{P2} is at node Y and is equal to 25.4 MHz. If this amplifier is needed to be operated in closed loop, some frequency compensation must be done in A to ensure its stability. To do so, the first dominant pole is lowered to a frequency f_{P1}' by adding a capacitor C_M between nodes X and Y.

Determine the factor by which capacitance at node X increases after performing frequency compensation. Assume second dominant pole's location remains unchanged.



Solution: Amplifier A's open loop dc gain is 60dB (overall).

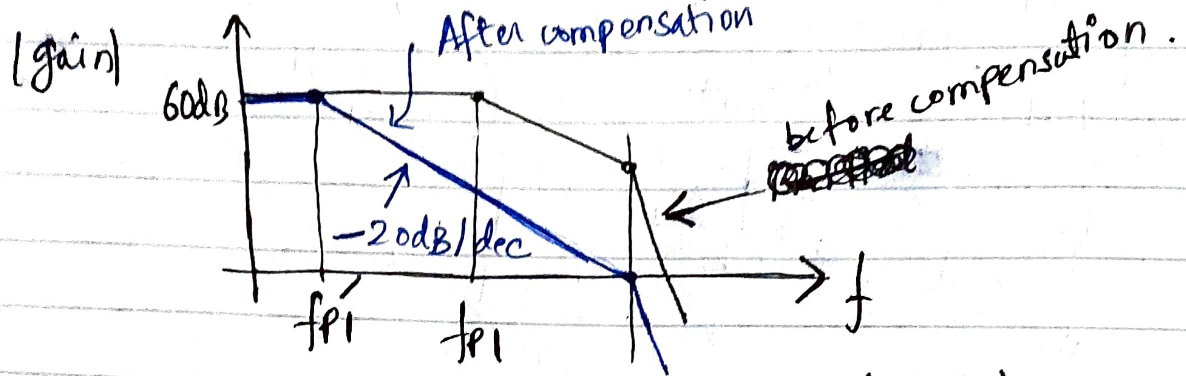
Before compensation suppose: (open loop amp's freq. response)



To ensure stability of A in closed loop means effectively it is desired to make A a first order system (single pole only before unity gain frequency). ~~and then use~~ using compensation and then use it in closed loop.

Limiting case: Make gain at $f_{P2} \to 0dB$ to make A a first order system.

After compensation : (still open loop amp's frequency response).



∴ From f_{P1}' to f_{P2}, there is a 60 dB fall in magnitude.

$$\therefore f_{P1}' = \frac{f_{P2}}{10^3} \qquad \left[\text{per decade, } -20\,dB\right]$$

Since $f_{P1}' < f_{P1}$, $C_x' > C_x$ and $f = \frac{1}{2\pi RC}$,

$$\frac{C_x'}{C_x} = \frac{f_{P1}}{f_{P1}'}$$

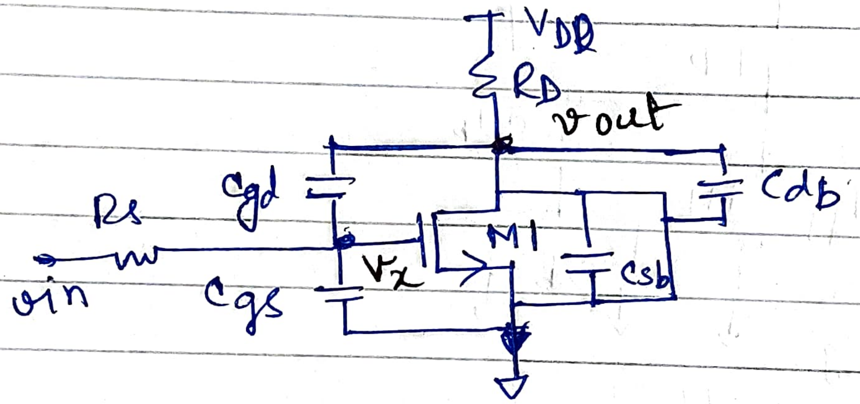
$$\Rightarrow \frac{C_x'}{C_x} = \frac{f_{P1}}{f_{P2}} \times 1000$$

$$\Rightarrow \frac{C_x'}{C_x} = \frac{6\,MHz}{25.4\,MHz} \times 1000 = 236.22$$

(Ans)

Q5) A simple common source amplifier, like the one shown below, starts behaving like a multi-pole system at high frequencies. Try to locate the dominant pole (closest to origin) in M rad/sec.

$C_{gs} = 6fF$, $C_{db} = 17fF$, $C_{sb} = 6fF$, $C_{gd} = 17fF$, $g_m = 19\,mS$, $R_S = 18\,K\Omega$, $R_D = 18\,K\Omega$.



Solution: 2 methods for solving this,

a) Direct approach

$$\frac{(v_x - v_{in})}{R_s} + v_x s C_{gs} + (v_x - v_{out}) s C_{gd} = 0 \quad —(1)$$

and, $(v_{out} - v_x) s C_{gd} + g_m v_x + v_{out}\left(\frac{1}{R_D} + s C_{db}\right) = 0$

$$\Rightarrow v_x = \frac{v_{out}\left(s C_{gd} + \frac{1}{R_D} + s C_{db}\right)}{g_m - s C_{gd}} \quad —(2)$$

substituting (2) in (1),

$$\frac{v_{out}}{v_{in}} = \frac{-(g_m - sC_{gd})R_D}{s^2(R_S R_D C') + s(g_m R_D R_S C_{gd} + R_S C_{gs} + R_D(C_{gd}+C_{db})) + 1}$$

where, $C' = C_{gs}C_{gd} + C_{db}C_{gs} + C_{gd}C_{db}$

Roots of denominator → gives you pole frequencies.

b) <u>Second method (Approximate but quick approach)</u>.

Equivalent capacitance at node X $= C_{gs} + (1+|A_v|)C_{gd}$

$$\Rightarrow C_X = C_{gs} + (1 + g_m R_D)C_{gd}$$

Equivalent resistance across C_X (R_X) $= R_S \parallel \infty$

$= R_S$

$$\boxed{\therefore \text{Pole freq at } X = \frac{1}{R_X C_X} = \frac{1}{R_S\left[C_{gs} + g_m R_D C_{gd}\right]}}$$

Similarly,

Eq. cap at Vout,

$$C_{out} = C_{db} + C_{gd}\left(1 + \frac{1}{|A_v|}\right)$$

$$= C_{db} + C_{gd}\left(1 + \frac{1}{g_m R_D}\right)$$

$$\approx C_{db} + C_{gd}$$

$\therefore$ Pole at vout freq. $= \dfrac{1}{R_D(C_{db}+C_{gd})}$

$\therefore$ pole(X) = 9.545 Mrad/sec

pole(out) = 1.633×10^6 Mrad/sec

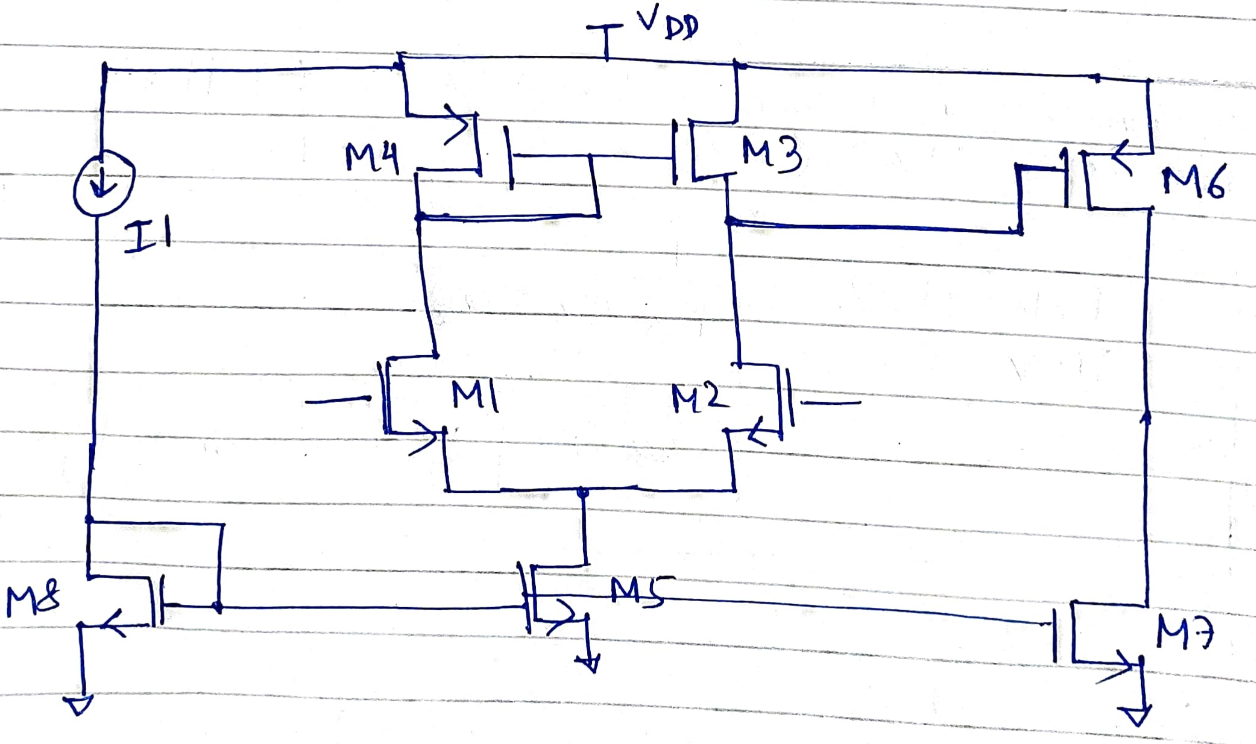
Pole(X) → closest to origin → dominant pole (ans)

Q6) Calculate the aspect ratio $\left(\frac{W}{L}\right)$ of M5.

Given $V_{SG4} = V_{SG6}$, $L6 = L4$, $W6/W4 = 23.2$,

$W7/L7 = 2$, $|V_{TP3}| = |V_{TP4}| = |V_{TP6}|$,

$V_{TN8} = V_{TN5} = V_{TN7}$.



Solution:

$$V_{SG4} = V_{SG6}$$

$$\Rightarrow |V_{TP4}| + V_{SD,sat\,4} = |V_{TP6}| + V_{SD,sat6}$$

$$\Rightarrow V_{SD,sat\,4} = V_{SD,sat\,6}$$

$$\Rightarrow \sqrt{\frac{2I_4}{\mu_p C_{ox}\left(\frac{W}{L}\right)_4}} = \sqrt{\frac{2I_6}{\mu_p C_{ox}\left(\frac{W}{L}\right)_6}}$$

$$\Rightarrow \frac{(W/L)_6}{(W/L)_4} = \frac{I_6}{I_4} = \frac{I_7}{I_5/2} = \frac{2I_7}{I_5} \quad — (1)$$

And, $V_{GS5} = V_{GS7}$

$$\frac{(W/L)_7}{(W/L)_5} = \frac{I_7}{I_5} = \frac{1}{2}\underbrace{\left[\frac{(W/L)_6}{(W/L)_4}\right]}_{\text{from (1)}}$$

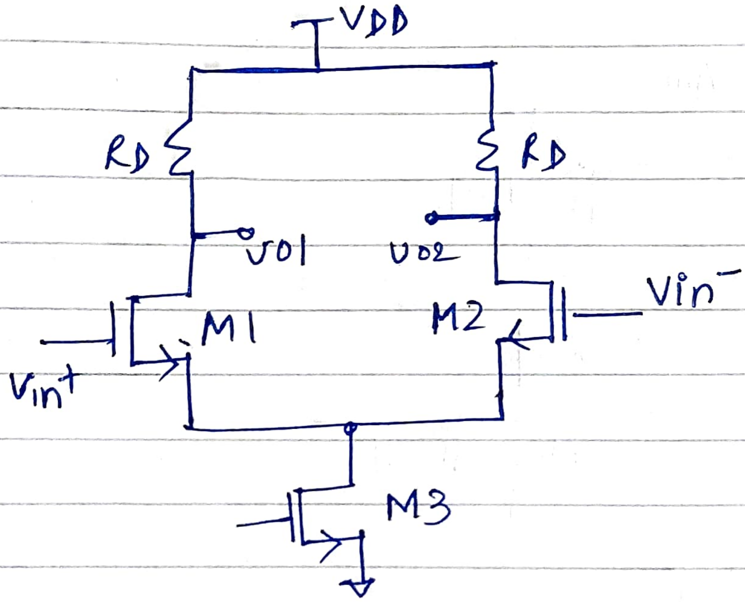
$$\therefore (W/L)_5 = 2\,(W/L)_7\,\frac{(W/L)_4}{(W/L)_6}$$

$$\Rightarrow \left(\frac{W}{L}\right)_5 = \frac{2\times 2}{23.2} = 0.172 \;\; (\text{Ans})$$

Q7) Calculate the magnitude $|A_{CM}| = \left|\frac{V_{O1} - V_{O2}}{V_{CM,in}}\right|$ if there is a mismatch in the transconductance of M1 and M2. Assume $g_{m1} = g_m + \frac{\delta g_m}{2}$ and

$g_{m2} = g_m - \frac{\delta g_m}{2}$, where $\frac{\delta g_m}{g_m} = 0.01$. Take

$g_m = 200\ \mu A/V$, $r_{o3} = 85\,k\Omega$, $R_D = 58\,k\Omega$.



Solution:

$$|A_{CM}| = \frac{v_{o1} - v_{o2}}{v_{cm}}$$

$$v_{o1} - v_{o2} = \frac{-\left(g_m + \frac{\Delta g_m}{2}\right) R_D v_{cm}}{1 + g_m(2R_S)} + \frac{\left(g_m - \frac{\Delta g_m}{2}\right) R_D v_{cm}}{1 + g_m(2R_S)}$$

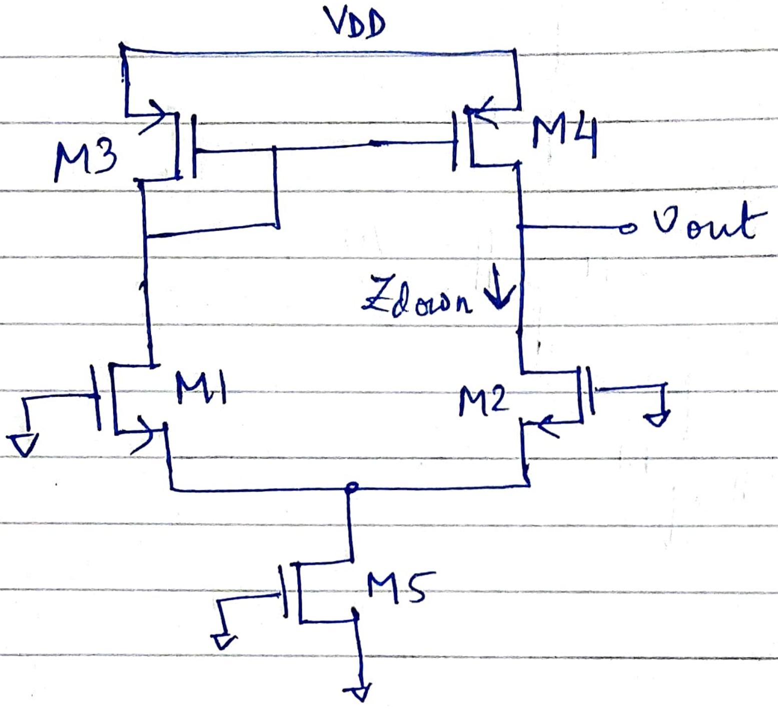
$$\Rightarrow |A_{cm}| = \left|\frac{v_{o1} - v_{o2}}{v_{cm}}\right| = \left|\frac{-\Delta g_m R_D}{1 + g_m(2R_S)}\right|$$

$$\Delta g_m = (0.01)(g_m) = (0.01)(200\ \mu A/V)$$

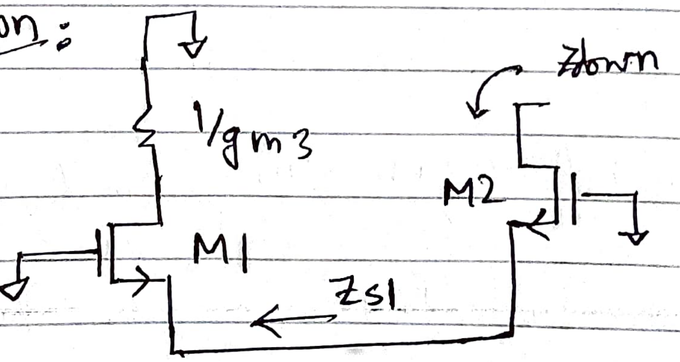
$$\Rightarrow |A_{CM}| = \frac{(2 \times 10^{-6})(58 \times 10^3)}{1 + (200 \times 10^{-6})(2 \times 85 \times 10^3)} = 0.003314$$

(Ans)

Q8) In the below figure, calculate the downward impedance from Vout node in MΩ. Given $g_{m1} = g_{m2} = 735\,\mu S$, $g_{m3} = 589.5\,\mu S$, $I_5 = 32.6\,\mu A$, $\lambda_1 = \lambda_2 = \lambda_3 = \lambda_4 = 10\,mV^{-1}$.

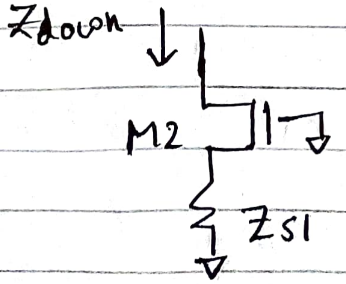


Solution:



$$Z_{S1} = \frac{\frac{1}{g_{m3}} + r_{o1}}{1 + g_{m1} r_{o1}}$$

$$\boxed{Z_{S1} \approx \frac{1}{g_{m1}}}$$



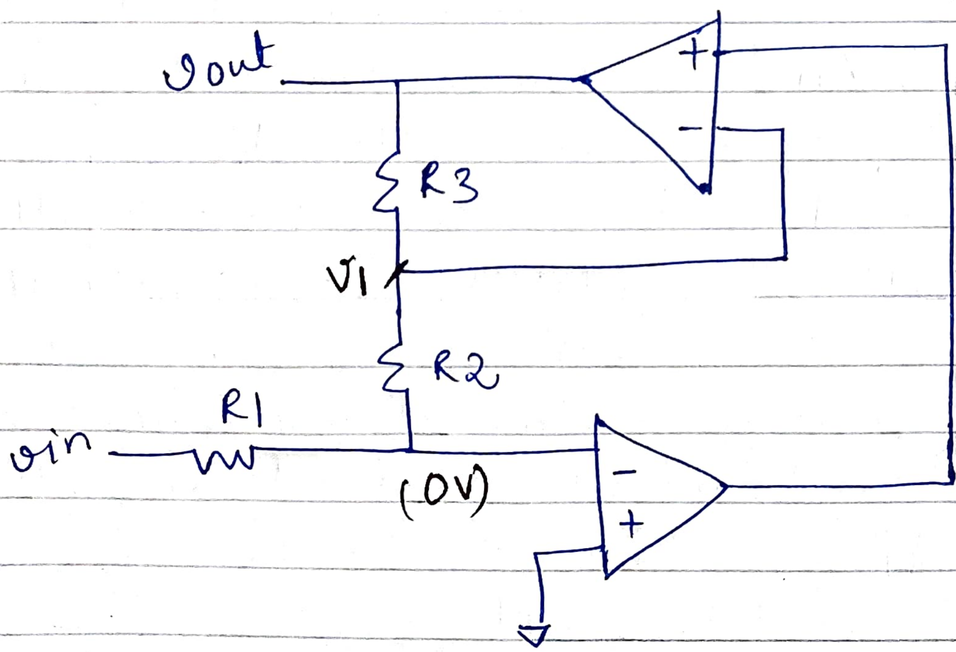
$$Z_{down} = (1 + g_{m2} r_{o2}) Z_{S1} + r_{o2}$$

$$\Rightarrow Z_{down} = \frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}} r_{o2} + r_{o2}$$

$$Z_{down} = r_{o1} + r_{o2} \quad (\text{as } g_{m1} = g_{m2})$$
$$= 2 r_{o2} \quad (r_{o1} = r_{o2})$$
$$= \frac{2}{\lambda_2 \times \left(\frac{I_5}{2}\right)} = \frac{2}{(10mV^{-1})\left(\frac{32.6\mu A}{2}\right)}$$

$$Z_{down} = 12.26\ M\Omega \quad (\text{Ans})$$

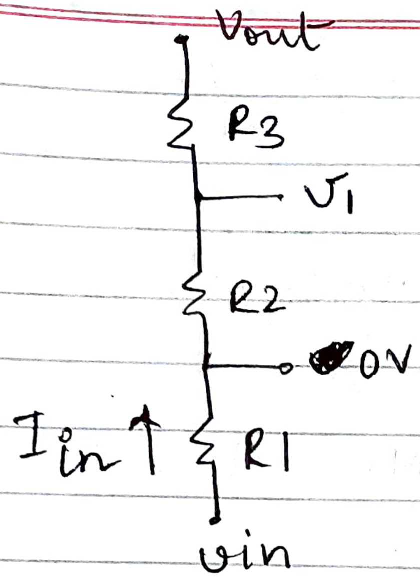
Q9) In the circuit shown below, opamp is ideal. Find V_{out}. Given $R1 = 3K\Omega$, $R2 = 1K\Omega$, $R3 = 9K\Omega$, $V_{in} = 3V$. (Accuracy = ± 0.001)



Solution :

Ideal opamp $\Rightarrow$ $A \to \infty$, $R_{in} \to \infty$, $R_{out} \to 0$.

In the given circuit, virtual short is applicable.

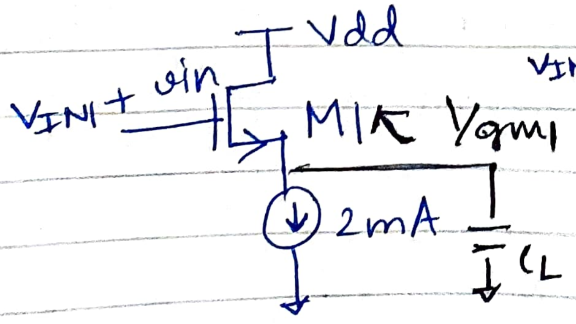


$$I_{in} = \frac{v_{in} - 0}{R1} = \frac{3}{3k\Omega} = 1mA.$$

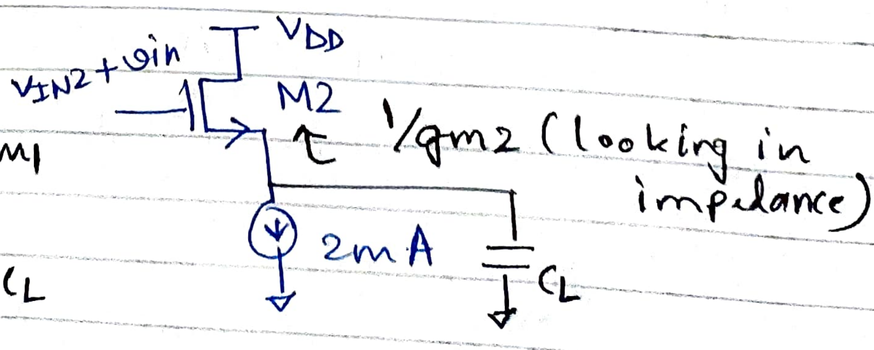
$$v_{out} = -I_{in}\,(R2+R3)$$
$$= -(1mA)(10k\Omega)$$
$$v_{out} = -10V \text{ (Ans)}$$

Q10) In the circuit shown below, current source is ideal and the transistor is in saturation. Circuit 1 and circuit 2 have different DC levels V_{IN1}, V_{IN2} with small signal v_{in} and same output common mode at source node. Circuit 1 and circuit 2 have a 3dB bandwidth (BW) of 100 Mrad/sec, 187 Mrad/sec respectively while driving the same load of 6 pF. New circuit is made by connecting circuit 1 and circuit 2. Find the 3dB BW (in Mrad/sec) of new CIRCUIT which is driving same load 6pF.

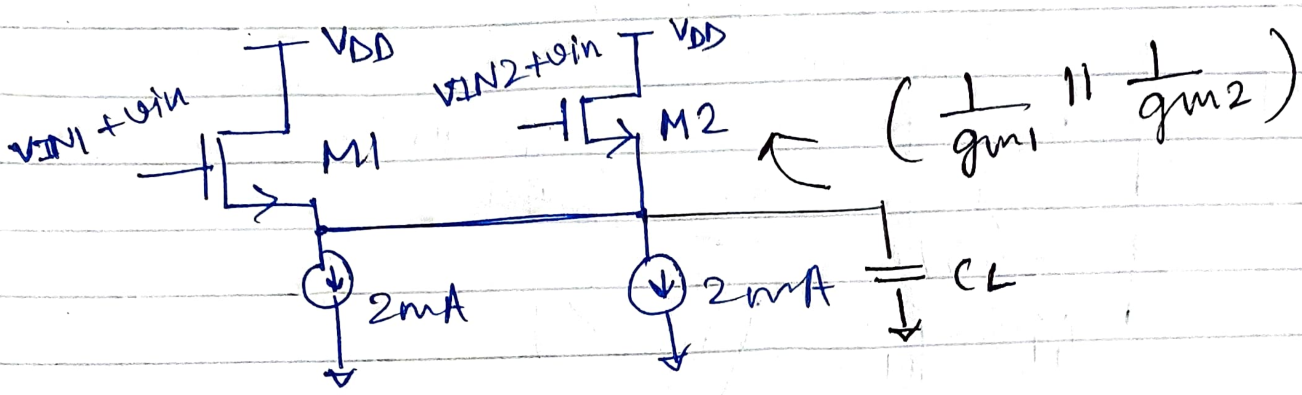
CIRCUIT 1



CIRCUIT 2



NEW CIRCUIT



Solution :

$$3\text{dB BW of circuit } 1 = \omega_1 = \frac{g_{m1}}{C_L}$$

$$\Rightarrow g_{m1} = \omega_1 C_L$$
$$= (100M)(6P)$$
$$\Rightarrow g_{m1} = 600\,\mu S$$

$$3\text{dB BW of circuit } 2 = W_2 = \frac{g_{m2}}{C_L}$$

$$\Rightarrow g_{m2} = W_2 C_L$$
$$= (187M)(6P)$$
$$\Rightarrow g_{m2} = 1122\,\mu S$$

$$\therefore 3\text{dB BW of new circuit } (\omega) = \frac{g_{m1} + g_{m2}}{C_L}$$

$$\Rightarrow \omega = \frac{\omega_1 C_L + W_2 C_L}{C_L} = W_1 + W_2$$

$$= 287 \text{ Mrad/sec}$$

(Ans)