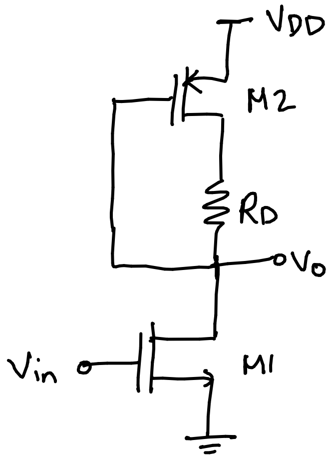
Tutorial 3 solutions : ELL304

Q Find the small signal gain A_v, assuming that both M1 & M2 are in saturation.



$r_{o1} = r_{o2} = 1M\Omega$, $R_D = 10K\Omega$

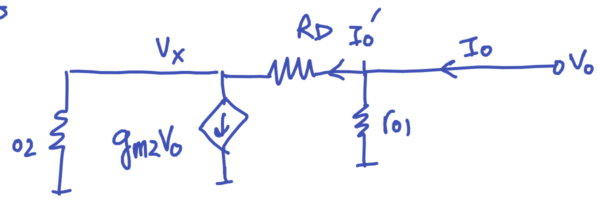
$\left(\frac{W}{L}\right)_1 = 10$, $\left(\frac{W}{L}\right)_2 = 1$

Assume $\mu_n C_{ox} = \mu_p C_{ox} = 100\,\mu A/V^2$

$V_{DSAT,1} = V_{DSAT,2} = 0.5\,V$

The overall $G_m \approx -g_{m1}$ [r_{o1}, R_D, g_{m2}, r_{o2} all vanish for I_{sc}]

For Rout $\longrightarrow$



$$\frac{V_x}{r_{o2}} + g_{m2}V_o = I_o'$$

$$V_o - V_x = I_o' R_D$$

$$V_x = V_o - I_o' R_D$$

$$\frac{V_o - I_o' R_D}{r_{o2}} + g_{m2}V_o = I_o'$$

$$V_o\left(\frac{1}{r_{o2}} + g_{m2}\right) = I_o'\left(\frac{R_D}{r_{o2}} + 1\right)$$

$$R_{out} = r_{o1} \,\Big|\Big|\, \frac{R_D/r_{o2} + 1}{1/r_{o2} + g_{m2}}$$

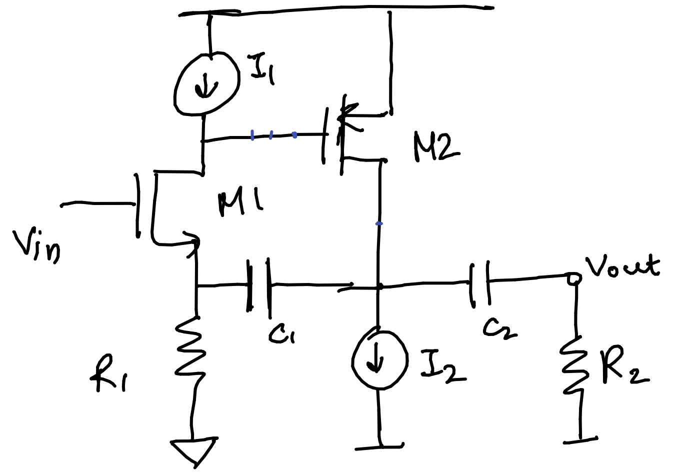
$$= r_{o1} \,\Big|\Big|\, \frac{R_D + r_{o2}}{1 + g_m r_{o2}}$$

small-signal gain $= \frac{V_{out}}{V_{in}} = G_m R_{out} = -g_{m1}\left(r_{o1} \,\Big|\Big|\, \frac{R_D + r_{o2}}{1 + g_m r_{o2}}\right)$

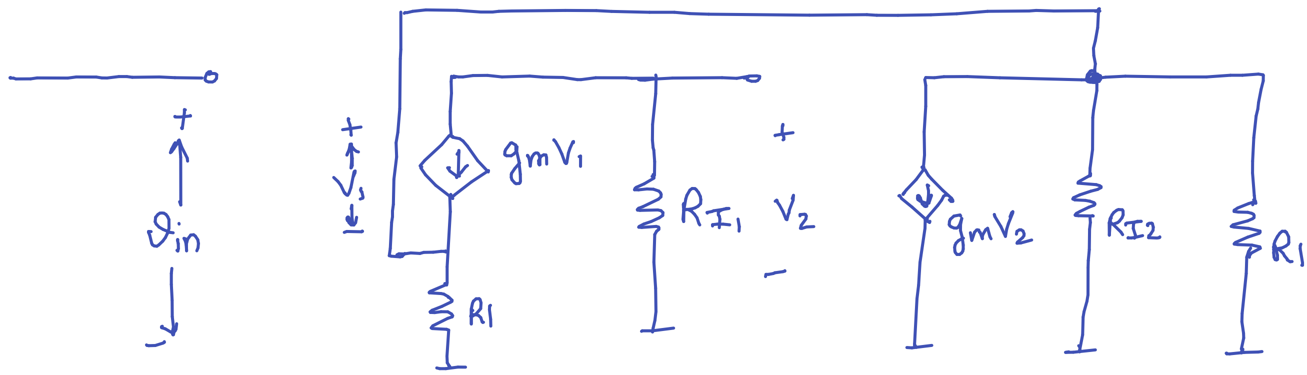
$$\approx -g_{m1}\left(r_{o1} \,||\, 1/g_{m2}\right)$$

$$\approx -g_{m1}/g_{m2}$$

Q2



Assume M1 & M2 are in sat^n. I_1 and I_2 are non ideal current sources with shunt resistance of 10 kΩ. C_1 & C_2 are large. Find the small signal DC gain. $\left(\frac{v_{out}}{v_{in}}\right)$.



$$V_1 = v_{in} - v_{out}$$
$$V_2 = -(g_{m_1} v_1) R_{I1}$$

$$\Rightarrow g_{m_1} v_1 = \frac{v_{out}}{R_1 \| R_2 \| R_{I2}} + g_{m_2} v_2$$

$$g_{m1}(v_{in} - v_{out}) = \frac{v_{out}}{R'} - g_{m2}(g_{m1}(v_{in} - v_{out}) R_{I1})$$

$[R' = R_1 \| R_2 \| R_{I2}]$

$$v_{in}\left(g_m + g_m^2 R_I\right) = v_{out}\left(\frac{1}{R'} + g_m + g_m^2 R_{I_1}\right)$$

$$\frac{v_{out}}{v_{in}} \simeq \frac{g_m(1 + g_m R_{I1})}{\frac{1}{R'} + g_m(1 + g_m R_I)}$$

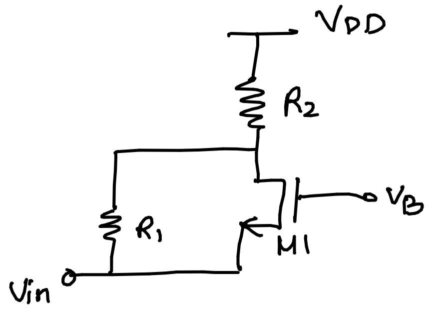
Q3 In the circuit shown below, assume M1 is biased in the saturation region using V_B and some external biasing. The overdrive voltage for M1 is 0.2V. Calculate the incremental input impedance (in Ω) for R_1 = [ ] and $R_2 = 10K\Omega$.

Other parameters:

$V_{DD} = 3.3V$

$\mu_n C_{ox} = 300\ \mu A/V^2$

$\left(\frac{W}{L}\right)_1 = 2.$



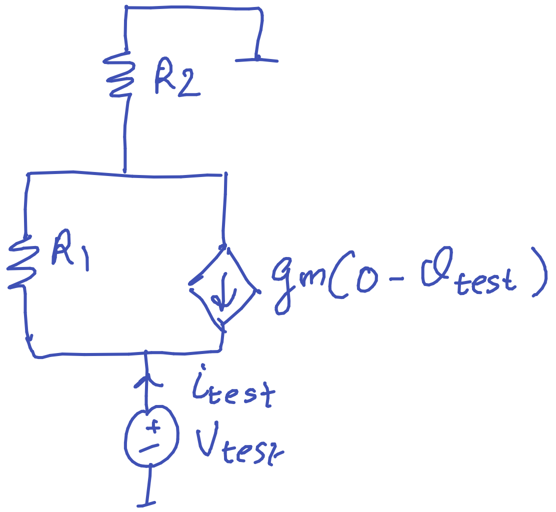
soln

Assume $R_1 = a$

$a = 1000 * x$ where $0.12 < x < 0.3\ M\Omega$

$$g_{m1} = \mu_n C_{ox}\left(\frac{W}{L}\right)_1 V_{ov1} = 120\ \mu S$$

For small signal Rin



$$i_{test} = g_m v_{test} + \frac{v_{test} - R_2 i_{test}}{R_1}$$

$$\Rightarrow i_{test}\left(1 + \frac{R_2}{R_1}\right) = \left(g_m + \frac{1}{R_1}\right) V_{test}$$

$$\Rightarrow R_{in} = \frac{R_1 + R_2}{1 + g_m R_1}$$

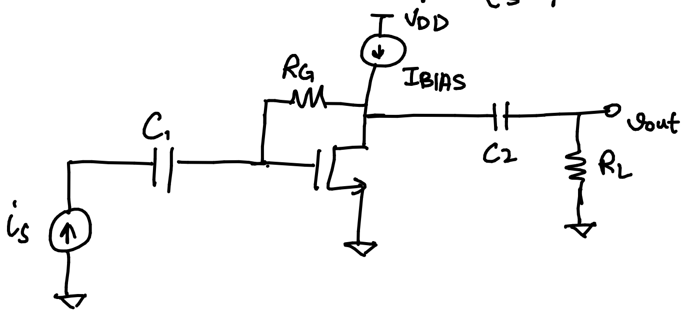
$$= \frac{1000 * (x + 0.01)}{1 + (120\mu S)(x)}$$

Q4 In the circuit shown below, $I_{BIAS} = 200\ \mu A$. On injecting an incremental current i_s, an incremental voltage v_{out} is obtained accross R_L. Consider R_L and R_G to be [[rl]] $K\Omega$, and [[rg]] $K\Omega$, find the ratio $\left|\frac{v_{out}}{i_s}\right|$ in $K\Omega$.

$V_{DD} = 6V$

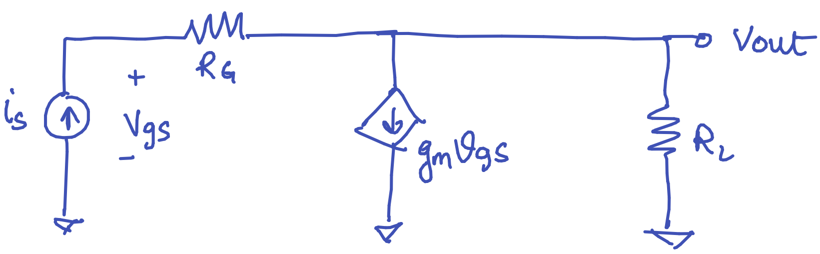
$V_{TN} = 1V$

$(W/L)_1 = 1$



Assume C_1 & C_2 are very large and $\lambda = 0$.

Soln



$$v_g = v_{out} + i_s R_G$$

$$g_m v_{gs} = g_{m1}(v_{out} + i_s R_G)$$

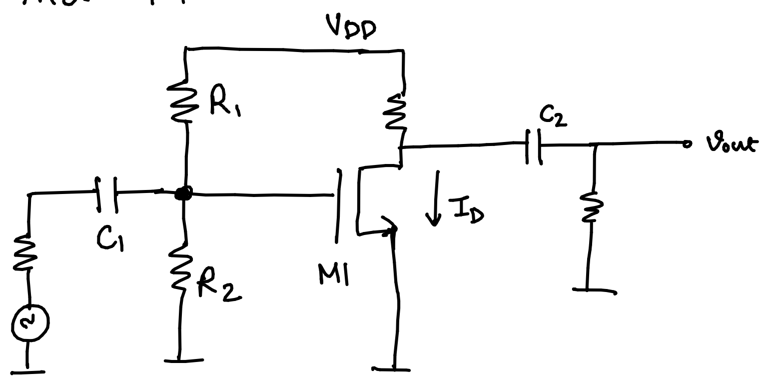
$$i_s = g_{m1}(v_{out} + i_s R_G) + \frac{v_{out}}{R_L}$$

$$v_{out}\left(g_{m1} + \frac{1}{R_L}\right) = i_s(1 - g_m R_G) / 1 + g_m R_L$$

$$\boxed{\left|\frac{v_{out}}{i_s}\right| = \left|\frac{R_L(1 - g_m R_G)}{1 + g_m R_L}\right|}$$

$$g_m = \sqrt{2 I_{BIAS} \mu_n C_{ox} \left(\frac{W}{L}\right)}$$

0 In the circuit shown below, $V_{DD} = 5V$, $R_1 = 6.3M\Omega$, $R_2 = 2.7M\Omega$ $R_L = R_D$. Parameters of M1 are $\mu_n C_{ox} = 200\mu A/V^2$, $\left(\frac{W}{L}\right) = 6$ and $V_{TH} = 0.5V$. To prevent M1 from going into triode, $V_{in} < V_1$, find V_1.



$$V_G = V_{DD}\left[\frac{R_2}{R_1 + R_2}\right] = 1.5V$$

$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T)^2 = 0.6\,mA$$

$$V_{DS} = V_{DD} - I_D R_D = 3.44V$$

$$g_m = \mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS} - V_T) = 1.2mS$$

$$\text{Gain} = -g_m(R_D \| R_L) = -1.56$$

To prevent M_1 from going into triode,

$$V_{DS,tot} > V_{GS,tot} - V_T$$

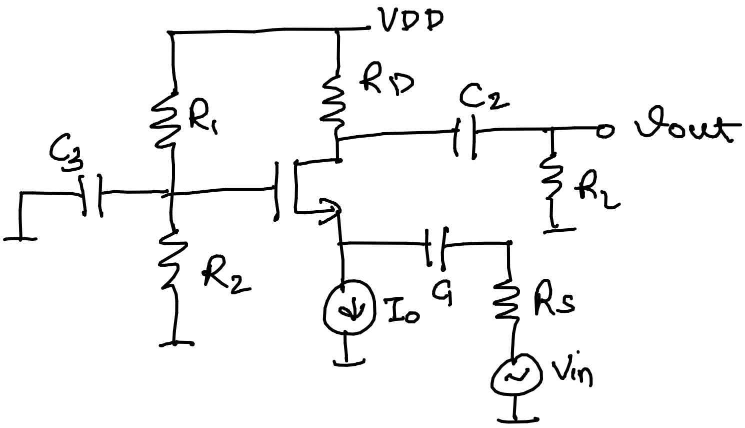
$$V_{DSO} - g_m(R_D \| R_L)v_g > V_{GSO} + v_g - V_{TH}$$

$$\Rightarrow v_g < \frac{V_{DSO} - (V_{GSO} - V_{TH})}{1 + g_m(R_D \| R_L)} = 0.953$$

$$\because v_g = \frac{v_{in}\,(R_1 \| R_2)}{R_S + (R_1 \| R_2)} \approx v_{in}$$

$$\Rightarrow \boxed{v_{in} < 0.953}$$

Q $V_{DD} = 10V$, $R_1 = 4.2M\Omega$, $R_2 = 2.8M\Omega$, $R_S = 8.5K\Omega$, $R_L = R_D$. Source of M1 is at 1.4V. Drain and gate voltages are equal and $I_D = I_0$. $\mu_n C_{ox} = 100\,\mu A/V^2$, $\left(\frac{W}{L}\right)_1 = 1$ and $V_T = 1V$. Find small signal gain $\frac{v_{out}}{v_{in}}$.



$$V_G = \frac{R_2}{R_1 + R_2} V_{DD} = 4V$$

$$V_S = 1.4V \longrightarrow V_{GS} = 2.6V$$

$$I_D = \frac{1}{2} \times (100 \times 10^{-6})(2.6 - 1)^2 = 128\,\mu A$$

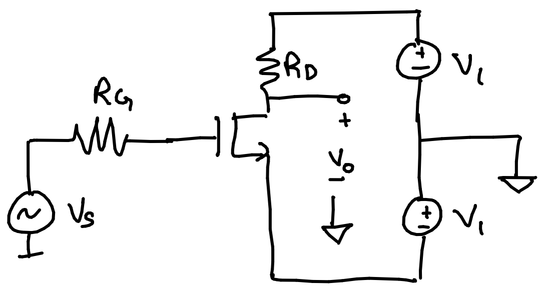
$$g_m = \sqrt{2 I_D \mu_n C_{ox} \left(\frac{W}{L}\right)} = 160\,\mu S$$

$$\frac{1}{g_m} = 6.25\,K\Omega \quad \text{(impedance looking into Source)}$$

$$R_D = \left(\frac{V_{DD} - V_D}{I_D}\right) = \left(\frac{V_{DD} - V_G}{I_D}\right) = 46.875\,K\Omega$$

$$\boxed{\frac{v_{out}}{v_{in}} = \left(\frac{1/g_m}{R_S + 1/g_m}\right) \cdot g_m (R_D \| R_L) = 1.588}$$

Q The circuit shown below is a configuration where coupling capacitors can be eliminated. Find R_D (in $K\Omega$) such that voltage between drain and ground consists of only signal component. $V_1 = 2V$, $R_G = 55K\Omega$, $\mu_n C_{ox} = 50\mu A/V^2$, $\left(\frac{W}{L}\right) = 9$, $V_T = 1V$.



Input DC voltage = 0V

$V_{GS} = 0-(-2) = 2V.$

In order to eliminate requirement of coupling capacitors, DC potential at drain node should be 0V.

$$V_{DS} = (0-(-2)) = 2V$$

$$V_{DS} = V_{GS} \quad (\text{saturation})$$

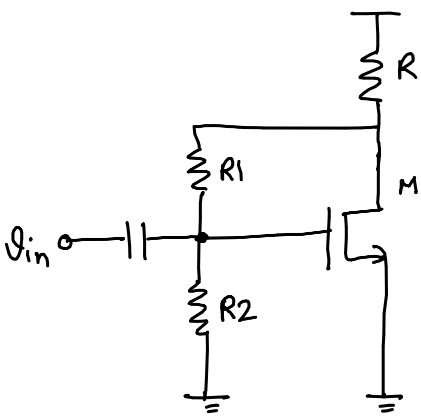
$$I_D = \frac{1}{2}\mu_n C_{ox}\left(\frac{W}{L}\right)(V_{GS}-V_T)^2 = 225\mu A$$

Voltage accross $R_D = I_D R_D$

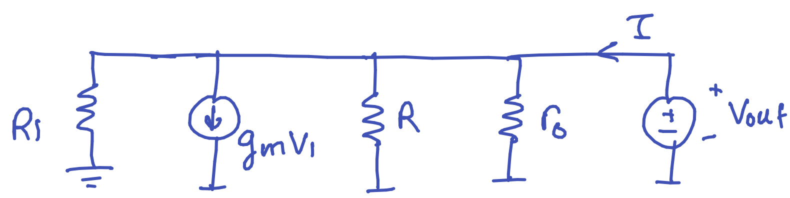
$$(2-0) = (225\mu A) R_D$$

$$\therefore R_D = 8.88 K\Omega.$$

Q For the given common-source amplifier, calculate the output impedance (in Ω). $R1 = 168\Omega$, $R2 = 200K\Omega$, $g_m = 5.01 mS$, $R = 115K\Omega$, $I_D = 50\mu A$, $\lambda = 1mV^{-1}$.



sol^n



$V_1 = 0 \Rightarrow g_m V_1 = 0$

$$\frac{V_{out}}{I} = r_o \| R \| R_1$$

$$= R_1 \quad (r_o >> R > R_1)$$

$$\approx 168\Omega$$