Digital logic design
Unit 1

lecture 1 : conversion (Number system)

lecture 2 : 1's complement
2's complement (Gross half Full concept)
Binary arithmatic - addition, multiplication, subtraction.

15 Sept Lecture 3 :

Subtraction :

NORMAL

$7 - 3 = 4$

0111 $(7)_{10}$
− 0011 $(3)_{10}$
0100 $(4)_{10}$

1st method
1's complement method

3 + (−7)

0011
+ 1000
1011

no carry

0100 (−4)

if carry
add it to L.S.B

2nd method
But for
3 − 7
3 + (−7)
take 2's complement
0011 $(3)_{10}$
+ 1001 — 2's complement of 7
1100

no carry, so problem (sign, number)
2's complement
= 0100 (−4)

if carry then ignore

7 + (−3)

0111
+ 1101
~~1~~ 0100 (4)
ignore

qus ① simplifying functions.
② using truth of fun^c to find fun^n

* Boolean algebra and exp.

literals/variables

A	B	A+B	A or B	AB and
0	0	0	0	0
0	1	1	1	0
1	0	1	1	0
1	1	$(10) = (2)_{10}$	1	1

Laws :-

1. $A + A = A$

$A \cdot A = A$

2. $A + A' = 1$

$A \cdot A' = 0$

3. $1 + A = 1$

$1 \cdot A = A$

4. $A + AB = A$

$A \cdot (1 + B)$

$A \cdot 1$

A

5. $A + A'B = A + B$

De morgan's law

⑥ $(A + B)' = A' \cdot B'$

$(A \cdot B)' = A' + B'$

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Truth table of DeMorgan's law

A	B	(A+B)	(A+B)'	A'	B'	A'·B'
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

onsensus Thm for minimisation

condⁿ — each variable repeated twice

three variable.

only one variable repeated with complement

e.g. $AB + A'C + BC$ ✓

$AB + A'C + B'C$ ✗ redundant term

minimization $= (AB + A'C)$

variable and complement

e.g. $AB + \underbrace{A'C}_{R.T} + B'C$

$= AB + B'C$

or $\underbrace{AB}_{R.T} + AC' + BC$

$= AC' + BC$

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Q. $x'y + yz' + xy'z'$

$= x'y + z'(y + xy')$
$= x'y + z'(y + x)$

$= x'y + z'y + xz'$
$= y(x' + z') + xz'$
$= y(x \cdot z)' + xz'$

By congensus thm
$= x'y + xz'$

Q. $x'y'z + yz + xz$

$\cancel{= z(x'y' + y) + xz}$
$\cancel{= z(x + y) + xz}$
$\cancel{= xz + zy}$
$\cancel{= z(x + y)}$
$= z(x'y' + y + x)$
$= z(x' + y + x)$
$= z(1 + y)$
$= z(1)$
$= z$

$= z(x'y' + y + x)$
$= z((x+y)' + (y+x))$
$= z(1)$
$= z$

Q. $XY + X(Y+Z) + Y(Y+Z)$

$= XY + XZ + Y + YZ$

$= (X+1)Y + Z(X+Y)$

$= Y + Z(X+Y)$

$= Y + \cancel{Z}Y + ZX$

$= Y + ZX$

Q. $(AB'(C + BD) + A'B')C$

$= (AB'C + AB'.B.D + A'B')C$

$= (B'(A'+AC))C$

$= (B'(A'+C))C$

$= (B'A' + B'C)C$

$= \cancel{A'B'C + B'C} \quad B'C(A'+C)$

$= \cancel{B'(A'C + C)} \quad B'C \cdot 1$

$= \cancel{B'} \quad B'C$

Important concepts.

Online class

18 September

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Minterm, Maxterm

for minterm	for maxterm
assume $A \to 1$ $A' \to 0$	assume $A \to 0$ $A' \to 1$
$B \to 1$ $B' \to 0$	$B \to 0$ $B' \to 1$

A	B	minterm (·)	maxterm (+)
0	0	$A'B'$	$A + B$
0	1	$A'B$	$A + B'$
1	0	AB'	$A' + B$
1	1	AB	$A' + B'$
		(mult)	(add).

for eg.

A B C	minterm	maxterm
0 1 0	$A'BC'$	$A + B' + C$

A B C	minterm	maxterm
1 1 1	ABC	$A' + B' + C$

Note to self: max - 1 - complement my trick.
min - 0 - complement

By De morgan's law we know

$$(A \cdot B)' = A' + B'$$

$$(A + B)' = A' \cdot B'$$

$$(A \cdot B \cdot C)' = A' + B' + C'$$

minterm and maxterm are related to this.

SOP and POS

Sum of products - sum of minterm
product of sums - product of maxt

A B - how many input combi
00, 11, 01, 10 - 4 - 2^2

A B C
000 001 010 011 100 101
110 111 - 8 - 2^3

for 4(A, B, C, D) → 2^4 = 16 combinat

Pattern 8 4 2 1

A	B	C	dec
0	0	0	0
0	0	1	1
0	1	0	2
0	1	1	3
1	0	0	4
1	0	1	5
1	1	0	6
1	1	1	7

a boolean expression ~~for~~ in terms of sum of products or product of sums.

A	B	output	minterms ~~SOP~~	SOP
0	0	0	~~A'B'~~	output = 1
0	1	1 ✓	A'B	$f = A'B + AB' + AB$
1	0	1 ✓	AB'	
1	1	1 ✓	~~AB~~	consider output = 1

OR

Now product of sum/maxterms.

output = 0.

$$f = (A + B)(A' + B')$$

note to self: addition to trick:

~~min - 0 - 0 complement → output = 1 consider~~

max - 1 complement → output - 0 consider

~~AND function~~

min	max	A	B	Y	SOP	POS
A'B'	A+B	0	0	0		
A'B	A+B'	0	1	0	$f = A \cdot B$	$f = (A+B) \cdot (A+B') \cdot (A'+B)$
AB'	A'+B	1	0	0		
AB	A'+B'	1	1	1 ✓		

//{ppt example answer by me.
F(S) A'B'C + A'BC + AB'C }
SOP

.POS

//F(S) = (A+B+C)(A+B+C')(A+B'+C)(A'+B+C)

~~conversion~~

Two forms of SOP/POS — Canonical - containing all variables in each term
Non-canonical - simplified/minimized of canonical

Conversion of Non-canonical/Non-standard to canonical

$$F = AB + AC + BC$$

missing C, missing B, A

since ($A + A' = 1$, always)

$$F = AB(C + C') + A(B + B')C + (A + A')BC$$

$$= ABC + ABC' + ABC + AB'C + ABC + A'BC$$

$$= ABC + ABC' + AB'C + A'BC$$

Reverse process

$$= AB(C + C') + C(AB' + A'B)$$

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// { ppt example answer by me.
$F(S) = A'B'C + A'BC + AB'C$ }
SOP.

POS.

// $F(S) = (A+B+C)(A+B+C')(A+B'+C)(A'+B+C)$

Conversion

Two forms of SOP/POS
- canonical – contains all variables in each term — perfect.
- Non-canonical – simplified/minimized of canonical — not big, not perfect

Conversion of Non-canonical/Non-standard to canonical

$$F = AB + AC + BC$$

missing C, missing B, A

since ($A + A' = 1$, always)

$F = AB(C+C') + A(B+B')C + (A+A')BC$

$= ABC + ABC' + ABC + AB'C$
$+ ABC + A'BC$

$= ABC + ABC' + AB'C + A'BC$

Reverse process

$= AB(C+C') + C(AB' + A'B)$

$= AB + C((AB' + A')(AB' + B))$

$= AB + C(A' + B')(A + B)$

$= AB + C(\underbrace{A'A}_{0} + A'B + B'A + \underbrace{B'B}_{0})$

Now for POS

$F = (p' + q + r) * (q' + r + s') * (p + q' + r' + s)$

perfect.

$= (p' + q + r + s.s') * (q' + r + s' + pp') *$ (p + q' + r' + s)

- not big
not perfect.

cannonical form.

$= (p' + q + r + s) * (p' + q + r + s')$

$* (p + q' + r + s') * (p' + q' + r + s')$

$* (p + q' + r' + s)$

law.

$A + BC = (A + B)(AC)$?o. proof - doubt

$= A^2 + AC + AB + BC$

$= A^2 + A(B + C) + BC$

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$F(P,Q,R) = PQ + PR' + QR'$

$\downarrow \quad \downarrow \quad \downarrow$

$R \quad Q \quad P$

$F = PQ(R+R') + PR(Q+Q')R' + (P+P')$

$= \cancel{PQR} + \cancel{PQR'} + \cancel{PQR'} + PQ'R'$

$+ \cancel{PQR'} + P'QR'$

$= PQR + PQR' + PQ'R' + P'QR'$

methods
To minimize Boolean exp

Boolean laws. | K-maps
Karnaugh maps.

K-map → graphical representation

For example

OR

A	B	f
0	0	0
0	1	1
1	0	1
1	1	1

SOP

$= A'.B + A.B' + AB$

$= A'B + A(B+B')$

$= A'B + A$

$= (A+B)(A+A')$

$= (A+B)$ OR

Rules of K-Map :-

A \ B	$\overline{B}$	B
$\overline{A}$	0	1
A	2	3

minterm

or

A \ B	B-0	$\overline{B}$-1
0-A	0	01 1
1-$\overline{A}$	~~0~~0 2	11 3

maxterm

Two variable kmap.
cells $= 2^2 = 4$

Three variable kmap
cells $= 2^3 = 8$

A \ B	$\overline{B}\overline{C}$=00	$\overline{B}C$=01	BC=11	$B\overline{C}$=~~0~~0
0=$\overline{A}$	000 0	001 1	011 3	010 2
1=A	100 4	101 5	111 7	110 6

Four variable K-map

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7	6
AB	12	13	15	14
$A\bar{B}$	8	9	11	10

For K-map of OR

dec	A	B	f
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	1

A \ B	$\bar{B}$	B
$\bar{A}$	0_0	1_1
A	1_2	1_3

~~for A+AB+A~~

Square or rectangle

cells = 2, 4, 8, 16

when they are close form rectangle, cover all one

	$\bar{B}$	B
$\bar{A}$	0	1_1
A	1_2	1_3

- no diagonals
- vertical or horizontal

Horizontal rectangle

$$AB' + AB = A(B+B') = A$$

vertical rectangle

$$A'B + AB = B(A'+A) = B$$

Non-Canonical = $A + B$.

Rule 01: We can either group 0's with 0's or 1's with 1's but we cannot group 0' and 1's together

Step ① For K-map table

0	1
2	3

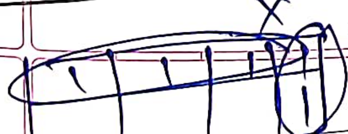
Step ② for SOP ~~cons~~ group 1's

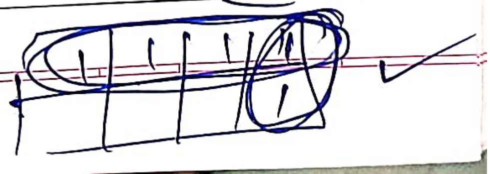
Step ③ for POS group 0's

Rule 02: Groups may overlap

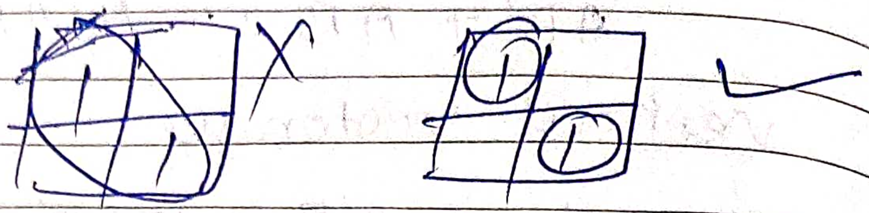
Rule 03: groups of cells having number of $= 2^n$

X 2, 4, 8, 16

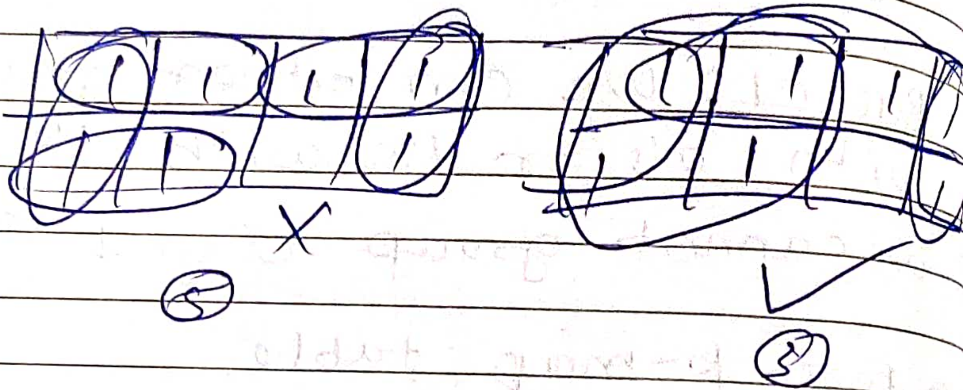




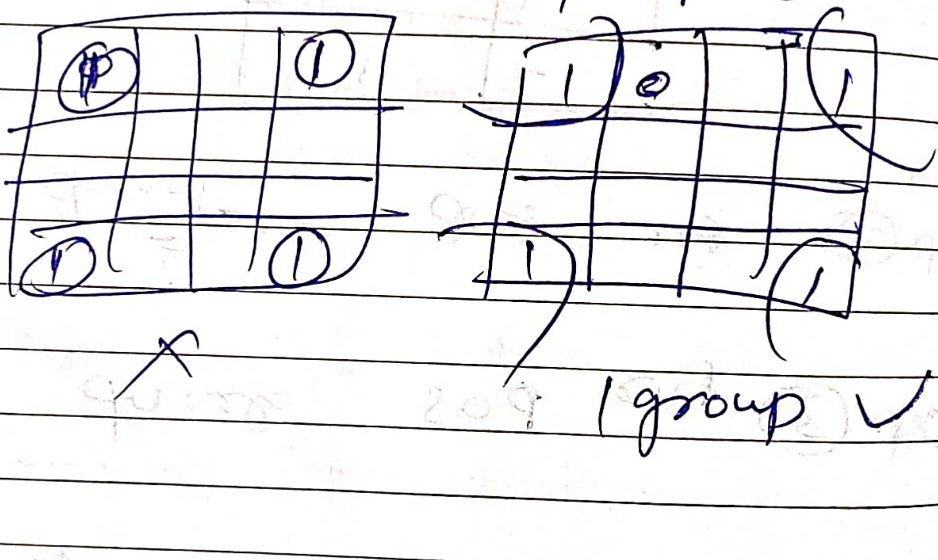
Rule 04 : No diagonal groups.



Rule 05: Each group should be as large as possible



Rule 06: corner grouping



ex. 1 question OR gate

X	Y	F
0	0	0
0	1	1
1	0	1
1	1	1

	$\bar{B}$	B
$\bar{A}$	0 (0)	1 (1)
A	1 (2)	1 (3)

for SOP consider 1

$$= \underline{A'B + AB} + \underline{AB' + AB}.$$

$$= A + B.$$

ex. 3

	$\bar{Y}\bar{Z}$	$\bar{Y}Z$	YZ	$Y\bar{Z}$
$\bar{X}$	1 (0)	1 (1)	1 (3)	1 (2)
X	0 (4)	0 (5)	0 (7)	0 (6)

→ common is only $\bar{X}$

	X	Y	Z	f
0	0	0	0	1
1	0	0	1	1
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	0
7	1	1	1	0

$$= \bar{X}\bar{Y}\bar{Z} + \bar{X}\bar{Y}Z + \bar{X}YZ + \bar{X}Y\bar{Z}$$

$$X' = (\bar{Y}\bar{Z} + \bar{Y}Z + YZ + Y\bar{Z})$$

$$X'(Y' + Y)$$

$$X'$$

Q. minimize (sum of products)

$f(A,B,C,D) = \sum m(0,1,2,5,7,8,9,10,13,15)$

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 (0)	1 (1)	(3)	1 (2)
$\bar{A}B$	(4)	1 (5)	1 (7)	(6)
AB	(12)	1 (13)	1 (15)	(14)
$A\bar{B}$	1 (8)	1 (9)	(11)	1 (10)

$\bar{C}D$

corners = $\bar{B}\bar{D}$

rectangle = $\bar{C}D$

square = BD

ans = $BD + \bar{B}\bar{D}$

Q.

AB \ CD	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1	1	1 (3)	(2)
$\bar{A}B$	(4)	1 (5)	1 (7)	(6)
AB	(12)	1 (13)	1 (15)	(14)
$A\bar{B}$	1 (8)	1 (9)	1 (11)	(10)

$\bar{B}\bar{C}$ $\qquad$ D

$= D + \bar{B}\bar{C}$

Product of sums.

$f(A,B) = \pi m(0 \ldots)$

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Q.s Pattern.

① Exp o/p in boolean term

② write in SOP/POS term

③ expand/write in canon form

④ wat are minterms/max terms of $f = \bar{A} + B$

⑤ what are EPI/PI/RPI ?

	A	B	C	Y	
0	0	0	0	1	$- A'B'C'$
1	0	0	1	1	$- A'B'C$
2	0	1	0	1	$- A'B'C'$
3	0	1	1	1	$- A'BC$
4	1	0	0	0	
5	1	0	1	1	$- AB'C$
6	1	1	0	0	
7	1	1	1	0	

canonical $f = A'B'C' + A'B'C + A'BC' + A'BC + AB'C$

min $= A'B' + A'B + AB'C$

$= A' + AB'C$

$= A' + B'C$

using k-map

0	1	3	2
4	5	7	6
12	13	15	14
8	9	11	10

	$\bar{B}\bar{C}$	$\bar{B}C$	BC	$B\bar{C}$	
$\bar{A}$	1 (0)	1 (1)	1 (3)	1 (2)	$\rightarrow \bar{A}$
A	0 (4)	1 (5)	0 (7)	0 (6)	

$\downarrow$ $\bar{B}C$

SOP minimized

$f = A' + B'C$

Q. 7

$A'(B+B')(C+C') + (A+A')B'C$

$(A'B + A'B')(C+C') + AB'C + A'B'C$

$\cancel{A'BC} + \cancel{A'B'C} + A'BC' + \cancel{A'B'C'}$
$+ \cancel{AB'C} + \cancel{A'B'C}$

Q.2 $= A'B'C' + A'B'C + AB'C + A'BC + A'B$

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Using k-map,

	$\bar B \bar C$	$\bar B C$	BC	$B\bar C$
$\bar A$	1 (0)	1 (1)	1 (3)	1 (2)
A	(4)	1 (5)	(7)	(6)

$A' + B'C$

Pu4 | Pu4 |

$$f = \bar A \bar B \bar C + \bar A \bar B C + \bar A B C + \bar A B \bar C + A \bar B C$$

Q.3

$$f = \Sigma m(0\ 1\ 2\ 3\ 5)$$

$\bar B \bar C$ $\bar B C$ BC $B\bar C$

SOP	POS	$B+C$	$B+\bar C$	$\bar B+\bar C$	$\bar B+C$
$\bar A$	A	1 (0)	1 (1)	1 (3)	1 (2)
A	$\bar A$	0 (4)	1 (5)	0 (7)	0 (6)

$\to \bar A + \bar B$

$A' + C$

$$POS = (\bar A + \bar B)(A' + C)$$

$$\text{SOP for } 0 = AB + AC'$$
$$= (A' + B')(A' + C)$$

	$B+C$	$B+\bar C$	$\bar B+\bar C$	$\bar B+C$
A	(0)	(1)	(3)	(2)
$\bar A$	0 (4)	(5)	0 (7)	0 (6)

$\pi M(4,6,7)$

another e.g.

		$Y+Z$	$Y+\bar{Z}$	$\bar{Y}+\bar{Z}$	$\bar{Y}+Z$
		$\bar{Y}\bar{Z}$	$\bar{Y}Z$	YZ	$Y\bar{Z}$
$W+X$	$\bar{W}\bar{X}$	0	1	0 3	2
$W+\bar{X}$	$\bar{W}X$	0 4	5	0 7	6
$\bar{W}+\bar{X}$	WX	0 12	13	15	14
$\bar{W}+X$	$W\bar{X}$	8	9	11	10

SOP for 0 = $X\bar{Y}\bar{Z} + \bar{W}YZ$

POS = $(X'+Y+Z)(W+Y'+Z$

$\Sigma(\cancel{0}, \cancel{2}, \cancel{5}, \cancel{7}, \cancel{8}, \cancel{10}, 1$

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	1 0	1	3	1 2
$\bar{A}B$	4	1 5	1 7	6
AB	12	1 13	1 14	15
$A\bar{B}$	1 8	9	11	1 10

SOP = $BD + B'D'$

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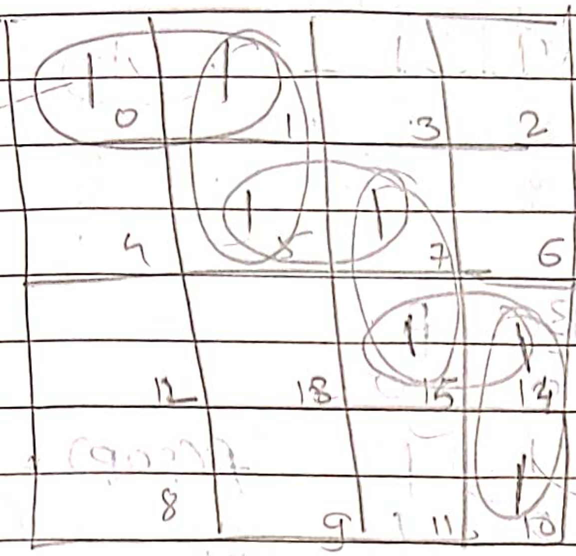
Prime Implicants

- Essential
- Redundant

this is one (EPI) is covered by only one grp



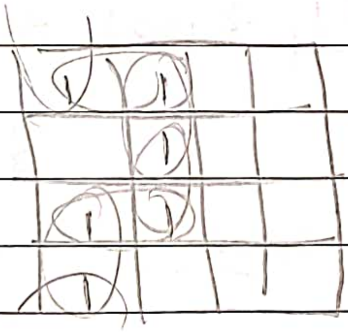
also this 1 is covered by one grp. (EPI)

No. of implicants / minterms in SOP = 7

no. of prime implicants = 6
(max no. of grps)

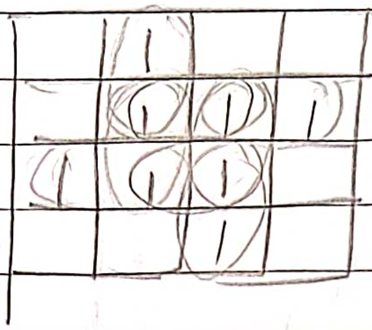
no. of essential prime implicants
= 2

no. of redundant prime implicants
= 2



implicants = 6
~~pr~~ P.I = 6
E.P.I = 0
RPI = 6

~~Prime implica~~



P.I ~~||||~~ |||| - 8

E.P.I - |||| - 4

R.P.I - 5

	A	B	C
0	0	0	0
1	0	1	1
2	1	0	1
3	1	1	X

don't care

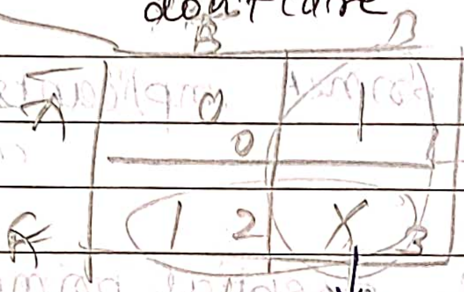
$f(SOP) = A'B + \ldots$

$f(0,0) =$

$f(0,1) =$

$f(1,0) =$

$f(1,1) :$



$= A + B$

$f(0,0)$

$f(0,1)$

$f(1,0)$

suppose to be 1

A

(Don't grp only X.)

$f = \Sigma m(1, 2, 3) + d(0, 4)$

don't care

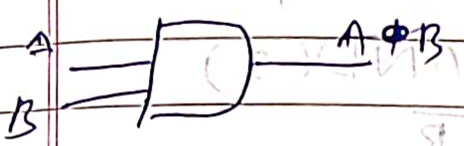
Gates.

- building block ab digital design

· gates → flip flops → registers → storage

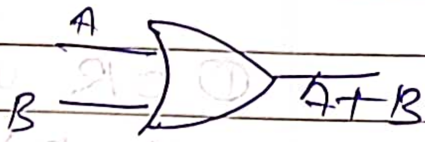
① AND $A \cdot B$

0	0	0
0	1	0
1	0	0
1	1	1



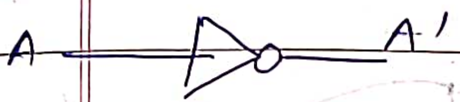
② OR $A + B$

0	0	0
0	1	1
1	0	1
1	1	1



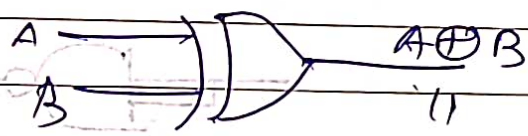
③ NOT

0	1
1	0



④ XOR $A \oplus B$

0	0	0
0	1	1
1	0	1
1	1	0



||

$A'B + AB'$

⑤ XNOR $A \odot B$

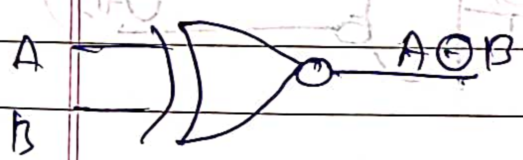
= 0	0	0	1
	0	1	0
	1	0	0
	1	1	1

odd 1's

Note :-

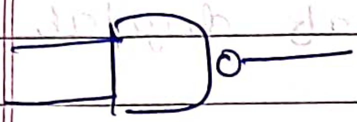
① odd 1's = 1
so 1, 11 = 1

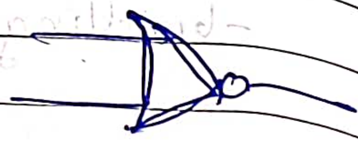
② $X \oplus 1 = \bar{X}$ ☆
$X \oplus 0 = X$



UNIVERSAL

⑥ NAND ⑦ NOR

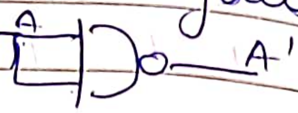




		$(A\cdot B)'$			$(A+B)'$
0	0	1	0	0	1
0	1	1	0	1	0
1	0	1	1	0	0
1	1	0	1	1	0

used to make all other gates.

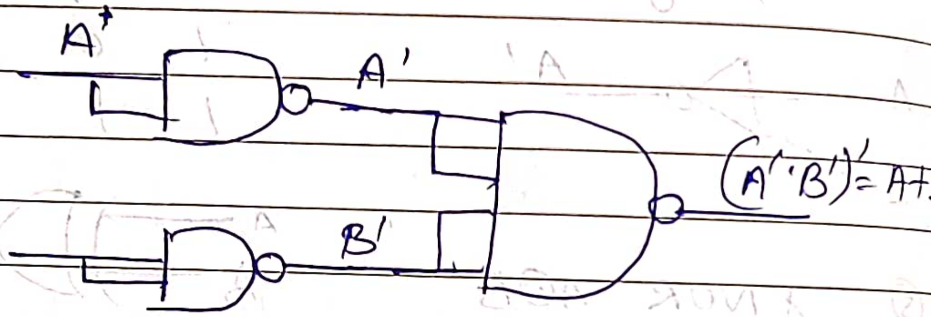
NOR using NAND(1)



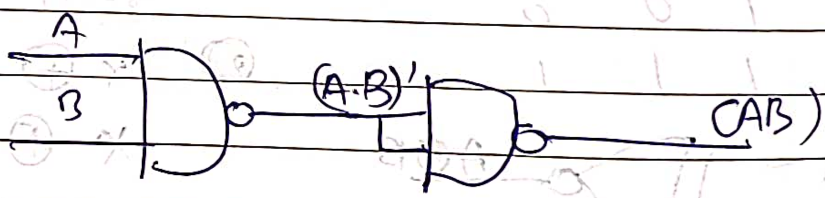
① OR using NAND(3)

$(A+B)$ $\overline{AB}$

$$\overline{(\overline{A}\,\overline{B})} = A + B$$



② AND using NAND(2)



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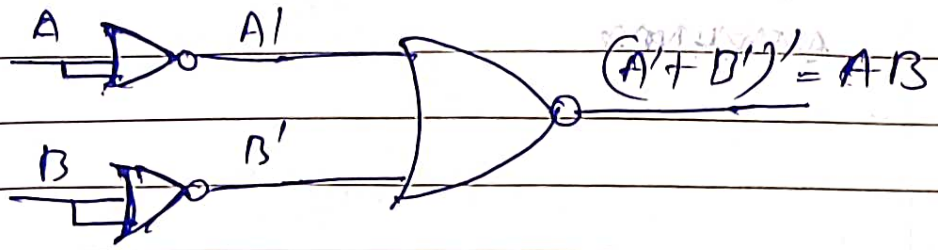
③ ~~NOR using AND~~ AND using NOR (2)

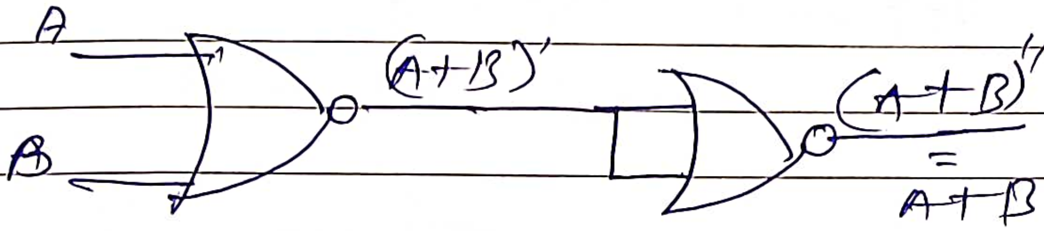
$\overline{(A+B)'} = AB$

$\overline{(\overline{A}+\overline{B})} = A \cdot B$



④ OR using NOR (2)

$A+B$ $(A+B)'$



XOR using NAND
⑤
XNOR

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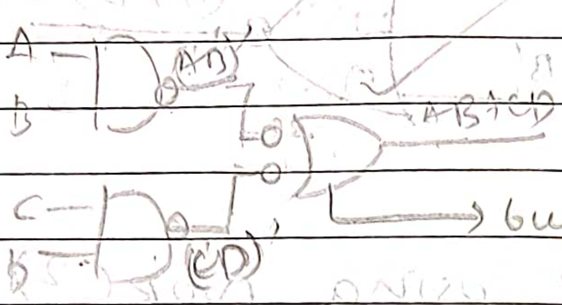
Q. How many NAND gates for this expression $(A \cdot B)'$ be

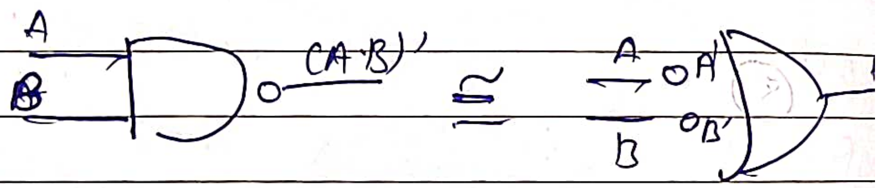
Wee change to SOP Sum of product
Then

AB + ~~B~~CD — 3 Nand gate
~~ABCD~~



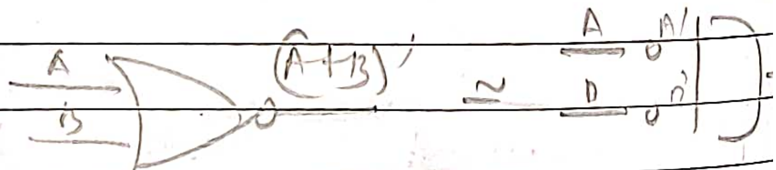
bubbled or ≅ N

Note



nand bubbled or

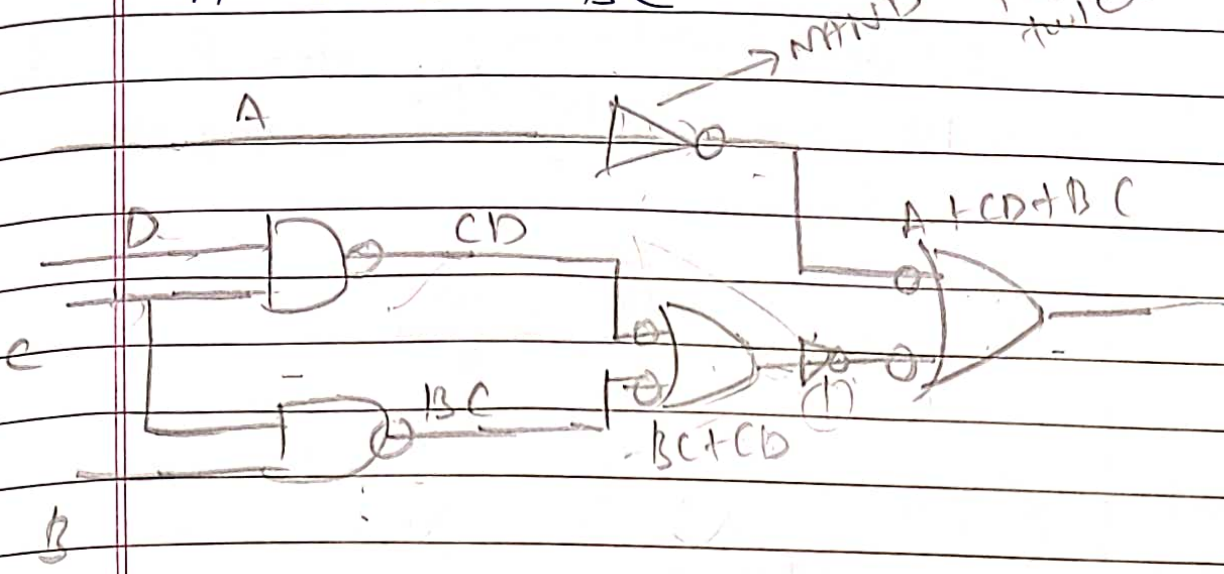
NAND ≅ B OR



NOR ≅ BAND

(6 nand)

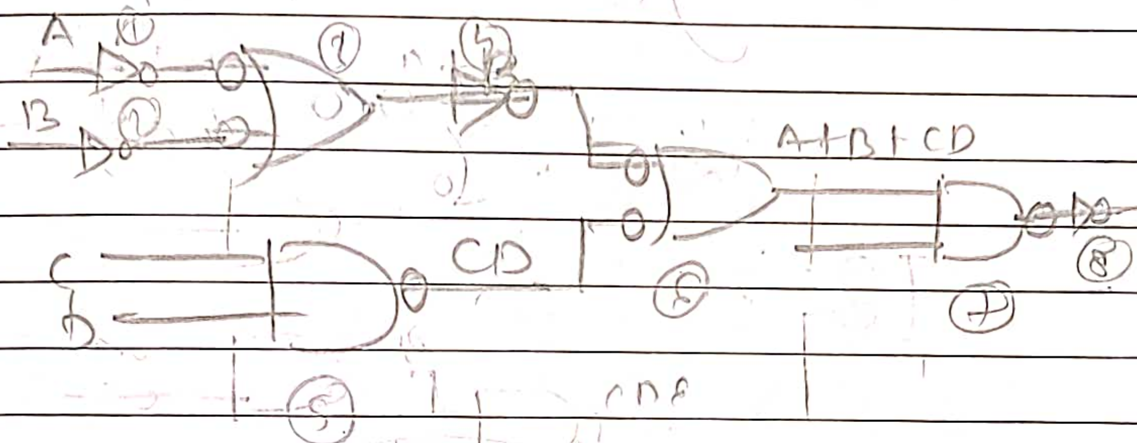
$A + CD + BC$



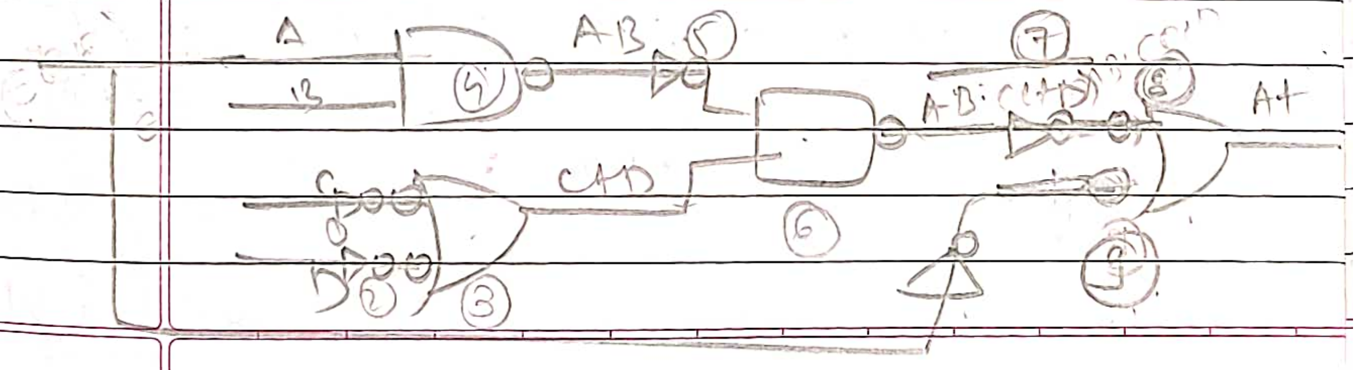
Using nand gate design

$(A + B + CD)E \longrightarrow POS$

$AE + BE + CDE$



$A + \overline{(AB \cdot (C+D))}$ - NAND



BCD - ina

BCD - Binary Coded Decimal

21 hrs → ... + Q + R

Divisible by

register

2 1

0010 0001

A	B	C	D
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
X	X	X	X
X	X	X	X
X	X		

33
+ 21
54

0011 0011
0010 0001
0101 0100
5 4

But 55
45
100

0101 0101
0100 0101
1001 1010
9 BCD
add 6 0110

1010
1 + 0110
0000 0

0101 1
0100
1010 (10)
0110 (6)
1 0000 0

ans.
next carry ⇒ 100

Grey Code

~~Binary code~~ -
Too much change in digits from one decimal to next

why Grey code
from one decimal to next
e.g. 3 → 4 then only one bit changes max only one

Question
- Bin to grey
- Grey to Bin ka circuit.

* Bin to Grey

XOR XOR XOR XOR XOR

MSB 1 0 0 1 1 0 LSB Binary

1 1 0 1 0 1 grey

0010 bin
↓
010 grey

* Grey to Bin

1 1 0 1 0 1

1 0 0 1 1 0

doubt

Bin $\longleftrightarrow$ Grey

	W	X	Y	A	B	C
0	0	0	0	0	0	0
1	0	0	1	0	0	1
2	0	1	0	0	1	1
3	0	1	1	0	1	0
4	1	0	0	1	1	0
5	1	0	1	1	1	1
6	1	1	0	1	0	1
7	1	1	1	1	0	0

For A

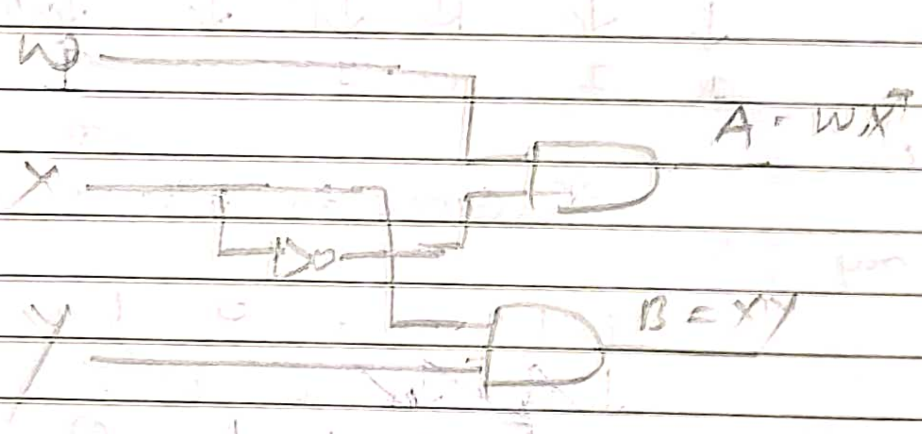
suppose

	$\bar{X}\bar{Y}$	$\bar{X}Y$	XY	$X\bar{Y}$
$\bar{W}$	0	1	3	1 2
W	1 4	1 5	1 7	1 6

$$A = W$$

Say $B = \bar{W}X + W\bar{X} = X \oplus W$

$$C = \bar{X}Y + X\bar{Y} = X \oplus Y$$



why?

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covert BCD to Excess 3 and desiogn circuit

	BCD					Excess 3			
	A	B	C	D		X	Y	Z	W
0	0	0	0	0	+ 3	0	0	1	1
1	0	0	0	1	+ 4	0	1	0	0
2	0	0	1	0	+ 5	0	1	0	1
3	0	0	1	1	+ 6	0	1	1	0
4	0	1	0	0	7	0	1	1	1
5	0	1	0	1	8	1	0	0	0
6	0	1	1	0	9	1	0	0	1
7	0	1	1	1	10	1	0	1	0
8	1	0	0	0	11	1	0	1	1
9	1	0	0	1	12	1	1	0	0

for X

	$\bar{C}\bar{D}$	$\bar{C}D$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	1 5	1 7	1 6
AB	X 12	X 13	X 15	X 14
$A\bar{B}$	1 8	1 9	X 11	X 10

$$X = A\bar{B}\bar{C} + B\bar{C}D + \bar{A}BC$$

$$Y =$$

Unit 2,

sequential Circuits and Combinational

sequential ↓ consider previous state

Combinational ↓ no previous state
- considers only input.

combinational circuit :-

* Half Adder - to add two bits

A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

For sum,

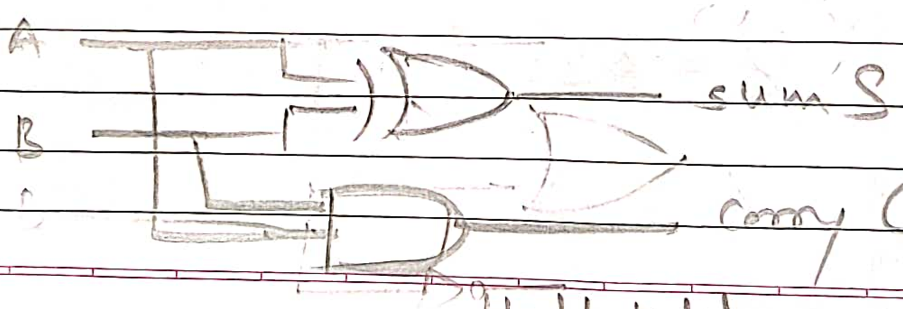
	$\bar{B}$	B
$\bar{A}$	0	1
A	1	0

$$sum = A\bar{B} + \bar{A}B$$
$$= A \oplus B$$

For carry,

	$\bar{B}$	B
$\bar{A}$	0	0
A	0	1

$$carry = AB$$



Half adder.

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* Full Adder – to add two bits and one carry.

	A	B	C	S	C
0	0	0	0	0	0
1	0	0	1	1	0
2	0	1	0	1	0
3	0	1	1	0	1
4	1	0	0	1	0
5	1	0	1	0	1
6	1	1	0	0	1
7	1	1	1	1	1

For sum

	$\bar B \bar C$	$\bar B C$	BC	$B \bar C$
$\bar A$	0	1 (1)	3	1 (2)
A	1 (4)	5	1 (7)	6

$A \oplus B \oplus C$

$$S = A\bar B \bar C + \bar A \bar B C + ABC + \bar A B \bar C$$
$$= \bar B(A\bar C + \bar A C) + BC'$$
$$= B(A \oplus C)$$

	0	1	1 (3)	2
	4	1 (5)	1 (7)	1 (6)

$$C = AC + BC + AB$$
$$= AB + BC + CA$$

Rembes Full Adder circuit diagram

Question no. of XOR and AND or OR gates to add
3 bits.

0 1 0
0 1 1

~~3 full adder~~
1 full adder — how many?
memorize

* Parallel Adder.
— When more than one pair of bits are added simultaneously
always propagate carry

* Ripple carry Adder - Parallel + full adder

Cin = 0

0 1 0 1 ✓ full adder 1
1 0 1 0
1 carry = 0

0 1 0 1 — full adder 2
1 0 1 0
1 1 carry = 0

Q. Carry propagation - time to generate carry
(V) cp time = 5 sec to add 4 bits
sp time = 10 sec. worst delay to add.

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$= \cancel{80\,sec}$

~~total cp time = cp time * n(bits)~~

~~total sp time = 10 sec~~

5×4

~~so worst delay = 80 sec.~~

(n−1) * cp time = (4−1) * 5
= 3 * 5
= 15 sec
for carry to reach to last sum

in last instead of carry time takes worst case time i.e. sp time (if sp time > cp time)

∴ worst delay = 15 sec + 10 sec
= 25 sec.

∴ formula for worst case delay = max (sp, cp) + (n−1) * cp
↓
bits.

sp time.

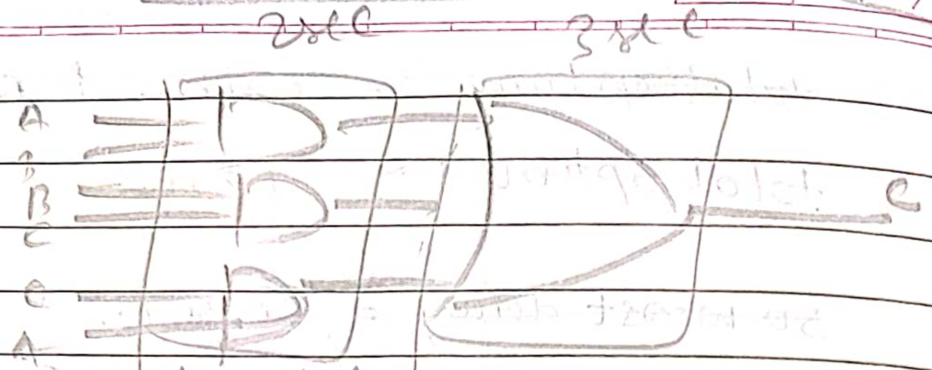
③ eg. XOR − 5 ~~min~~ sec AND − 3 ~~min~~ sec OR − 4 ~~min~~ sec
T delay time = 20

~~sum t = no. of gates * delay time of gate~~

cp time = 7 sec

worst delay = (3 × 7) + 5
= 21 + 5 = 26

∴ cp time = AND + OR time
∴ sp time = XOR time



cp time = 5 sec

Half subtractor   Full subtractor

A	B	D	sim
0	0	0	0
0	1	1	1
1	0	1	0
1	1	0	0

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Q 16 bits :- ripple carry adder
16 identical full adder
cp time = 12 ns of each FA
sp time = 15 ns of each FA

$= (16-1) * 12 + 15$

$= 15 * 12 + 15$

$= 15 * 13$

$= 195$

16 bits
Q. XOR = 20 AND = 15 OR = 10

answer:
cp time = 15 + 10 = 25 nsec

sp time = 20 ns.

Total delay cp time = 16 * 25 = 400 ns.

total sp worst case time delay time
doubt
$= 20 + (15) * 25$

$= 20 + 375$

$= 395$ ns.

ubtractor

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look

☆ Carry ahead adder

$C_3 \quad C_2 \quad C_1 \quad C_0$
$A_3 \quad A_2 \quad A_1 \quad A_0$
$B_3 \quad B_2 \quad B_1 \quad B_0$

$G_i = A_i B_i$

$P_i = A_i \oplus B_i$

$C_1 = C_0 P_0 + G_0$

$C_2 = C_0 P_0 P_1 + G_0 P_1 + G_1$

$C_3 = C_0 P_0 P_1 P_2 + G_0 P_1 P_2 + G_1 P_2 + G_2$

$C_4 = C_0 P_0 P_1 P_2 P_3 + G_0 P_1 P_2 P_3 + G_1 P_2 P_3 + G_2 P_3 + G_3$