

Digital logic design

Unit 1

Lecture 1: A conversion (Number system)

lecture 2: 1's complement

2's complement

(Cross half
Full concept)

Binary arithmetic -
addition, multiplication,
subtraction.

18 Sept. Lecture 3:

$$A = A + A \cdot 1$$

$$A = A \cdot A + 1$$

Subtraction:

Normal

$$7 - 3$$

$$0111 (7)_{10}$$

$$- 0011 (3)_{10}$$

$$0100 (4)_{10}$$

1's complement

$$3 + (-7)$$

$$\begin{array}{r} 0011 \\ + 1000 \\ \hline 1011 \end{array}$$

no carry

$$0100 (-4)$$

1's carry

add it to 1.8.10

$$8 + 10$$

Ignore

1st method

$$3 - 7$$

$$1 - 3 + (-7)$$

$$A = A \cdot 4 \quad \text{Take 2's complement}$$

$$0011 (3)_{10}$$

$$+ 1100 \quad \text{2's complement of 7}$$

no sign number carry, so problem

2's complement

$$= 0100 (-4)$$

if carry then ignore

$$0011 (-3)$$

$$8 + 10 = 0011$$

$$8 + 10 = 0100 (4)$$

- ① simplifying functions
 AND ② using truth table to find function

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* Boolean algebra and exp.

literals/variables

$$\text{Complement} \rightarrow A' \text{ if } A \text{ is true} \quad A + B \text{ or } A \cdot B \quad AB \text{ and}$$

0	0	0	0	0
0	1	1	1	0
1	0	1	0	0

$$\text{arithmetics} \rightarrow (10)_2 = (12)_{10} \quad 1$$

minutability, multiplication

Laws :- minutability

$$1. A + A = A$$

$$2. A \cdot A = A$$

$$3. A + A' = 1$$

$$A \cdot A' = 0$$

$$F = 0$$

$$4. 1 + A = 1$$

$$5. A \cdot 1 = A$$

$$6. A \cdot 0 = 0$$

$$7. A + A + B = A$$

$$= A \cdot (1 + B)$$

$$8. A + A'B = A + B$$

$$(A + B) \cdot (A + B)' = 0$$

$$= A + B$$

$$= A + B$$

$$= A + B$$

$$= A + B$$

$$6. (A + B)' = A' \cdot B'$$

$$(A \cdot B)' = A' + B'$$

truth table ab Demorgan's laws

A	B	$(A+B)$	$(A+B)'$	A'	B'	$A' \cdot B'$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

irredundant Thm for minimization

condⁿ — each variable repeated twice

three variable

only one variable repeated with complement

$$\text{e.g. } AB + A'C + BC \checkmark$$

$$AB + A'C + B'C \times$$

redundant term

$$\text{minimization} = (AB + \underline{A'C})$$

(1) S =

variable and complement

S =

$$\text{e.g. } AB + \overbrace{AC + B'C}^{\text{R.T}} \times =$$

$$= AB + B'C + !S =$$

$$\text{or } AB + AC' + BC =$$

R.T

$$= AC' + BC$$

Q. What is the simplified form of the given expression?

$$Q. x'y + yz' + xy'z'$$

$$= x'y + z'(y + xy')$$

$$= x'y + z'(y + x)$$

$$= x'y + z'y + xz'$$

$$= y(x' + z') + xz'$$

$$= y(x \cdot z)' + xz'$$

Ans: $y(x \cdot z)' + xz'$

By consensus theorem

$$= x'y + xz'$$

$$Q. x'y'z + yz + xz$$

$$= z(xy' + x) + xz = z(x'y' + y + x)$$

$$\Rightarrow z(x + y) + xz = z((x+y)' + (y+x))$$

$$= xz + zy$$

$$= z(1)$$

$$= z$$

$$= z(x'y' + y + x)$$

$$= z(x' + y + x)$$

$$= z(1 + y) + xz =$$

$$= z(1)$$

$$= z(x'y' + y + x)$$

Ans:

$$= z(x'y' + y + x)$$



Q. $XY + X(Y+Z) + Y(Y+Z)$

$$\begin{aligned}
 &= XY + XZ + Y + YZ \\
 &= (X+1)Y + Z(X+Y) \\
 &= Y + Z(X+Y) \\
 &= Y + ZY + ZX \\
 &= Y + ZX
 \end{aligned}$$

Q. $(\bar{A}B' (C'C + BD) + A'B')C$

$$= (\bar{A}B'C + A\bar{B}\cdot B\cdot D + A'B')C$$

$$= (B'(A' + AC))C$$

$$= (B'(A' + C))C$$

$$= (B'A' + B'C)C$$

$$= \cancel{A'B'C} + B'C \quad B'C(A' + C)$$

$$= B'(A'C + C) \quad B'C \cdot I$$

$$= \cancel{B'} \quad B'C$$

Min term, Max term

for min term

assume $A \rightarrow 1 \text{ or } A' \rightarrow 0$

$B \rightarrow 1 \text{ or } B' \rightarrow 0$

for max term

assume $A \rightarrow 0 \text{ or } A' \rightarrow 1$

$B \rightarrow 0 \text{ or } B' \rightarrow 1$

(•)

min term & max term

$$\begin{array}{ccccc} 0 & 0 & 0 & A'B' & A + B \\ 0 & 1 & 1 & A'B & A + B' \\ 1 & 0 & 1 & AB' & A' + B \\ 1 & 1 & 1 & AB & A' + B' \end{array}$$

(mult)

(add)

for ex.

$$\begin{array}{ccc} ABC & \text{min term} & \text{max term} \\ 010 & A'BC' & A + B' + C \end{array}$$

$$\begin{array}{ccc} ABC & \text{min term} & \text{max term} \\ 111 & A + BC & A' + B' + C' \end{array}$$

Note to self: max - 1-complement
min - 0-complement

By de morgan's law we know

$$(A \cdot B)' = A' + B'$$

$$(A' + B)' = A \cdot B'$$

$$(A \cdot B \cdot C)' = A' + B' + C'$$

min term
and max term
are related to
this.

SOP and POS

Sum of products → sum of minterms
Product of sums → product of maxterms

no. of input comb. = 2^n = 2^2 = 4

$$00, 11, 01, 10 \rightarrow 4 = 2^2$$

$$A + A \quad \bar{A} + \bar{A} \quad 1 \quad 0$$

$$A \bar{A} + \bar{A} B + A B + \bar{A} \bar{B} \quad 1 \quad 0 \quad 1 \quad 0$$

$$000 \quad 001 \quad 010 \quad 011 \quad 100 \quad 101$$

$$010 \quad 111 \quad \rightarrow 8 = 2^3$$

from $A + B + C + D \rightarrow 2^4 = 16$ combinations

$$0000 \quad 0001 \quad 0010 \quad 0011 \quad 0100 \quad 0101$$

Pattern 8×2^3

combinations of A, B, C, D in decimal

$$0000 \quad 0001 \quad 0010 \quad 0011 \quad 0100 \quad 0101$$

$$0110 \quad 0111 \quad 1000 \quad 1001 \quad 1010 \quad 1011$$

$$1100 \quad 1101 \quad 1110 \quad 1111$$

number of 1's = 4 = 2²

$$1 \quad 0 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1$$

$$1 \quad 0 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1$$

$$1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

$$8 \cdot A = (8 + \bar{A})$$

$$1 + 8 + A = (1 \cdot B \cdot A)$$

a boolean expression for in terms
of sum of products or product of
minterms.

A OR B

0 0 0

0 1 1 ✓

1 1 1 ✓

output A:

minterms

SOP

$A' + B'$

output - 2

$f = A'B + AB'$

+ AB

$\underline{A} \underline{B}$

consider

output - 1

Now product of sum/maxterms.

output = 0.

$$f = (A' + B)(A' + B')$$

note to
self:

addition to trick: $A = A$

min-0 - 0 complement \rightarrow output = 1 consider

max-2 complement \rightarrow output = 0 consider

AND function

min	max	A	B	y	SOP	- POS
$A'B'$	AB	0	0	$A'B$	$A'B + AB$	
AB'	$A+B'$	0	1	0	$f = A \cdot B$	$f = (A+B)$
AB'	$A+B$	1	0	1	$f = A + B$	$= (A+B')$
AB	$A'+B'$	1	1	1 ✓		$(A'+B)$

$$(A+B)A + (A+B')A =$$

Ques example answer by me.

$$F(S) = A'B'C + A'BC + AB'C + ABC$$

SOP
POS

$$F(S) = (A+B+C)(A+B+C')(A+B+C)(A+B+C')$$

Conversion

Two forms of SOP/POS

canonical

- contains all variables in each term

Non-canonical

- simplified/minimized
of canonical

Conversion of Non-canonical / Non-standard to canonical

$$F = AB + A'C + BC$$

↓
missing terms

$$\rightarrow F = AB + A'C + BC + A'BC + A'BC' + ABC' + ABC$$

$$F = AB(C+C') + A(B+B')C + EA(FA)'$$

$$F = ABC + ABC' + A'BC + A'BC'$$

$$+ A'BC + A'BC' + ABC + ABC'$$

$$F = ABC + ABC' + AB'C + A'BC$$

Reverse process

$$= AB(C+C') + C(A'B'+A'B)$$

1/2 PPT example answer by me!!!

$$P(S) = A'B'C + A'BC + AB'C \quad \text{SOP}$$

. POS.

$$1/P(S) = (A+B+C)(A+B+C')(A'+B+C)$$

conversion

Two forms of SOP/POS

canonical
- contains all variables
in each term
Non-canonical

perfect

- simplified/minimized
of canonical

- not big
not perfect

Conversion of Non-canonical / Non-standard
to canonical

$$F = AB + AC + BC$$

missing $C'A$ since $(A+A')=1$, always

$$F = AB(C+C') + A(B+B')C + (A+A')BC$$

$$= ABC + ABC' + ABC + AB'C$$

$$+ ABC + A'BC$$

$$= ABC + ABC' + AB'C + A'BC$$

$$\begin{aligned} & \text{AT } BC \\ & = C(A+B) \end{aligned}$$

Reverse process

$$= AB(C+C') + A(CAB' + A'B)$$

$$\begin{aligned}
 &= AB + C((AB' + A')(A'B' + B)) \\
 &\Rightarrow AB + C(A' + B')(A + B) \\
 &= AB + C(A'A + A'B + B'A + B'B)
 \end{aligned}$$

Now for POS

$$F = (P' + Q + R)^* (Q' + R + S)^* (P + Q' + R)$$

perfect.

$$\begin{aligned}
 &= (P' + Q + R + S \cdot S')^* (Q' + R + S' + P \cdot P')^* \\
 &\quad * (P + Q' + R' + S)
 \end{aligned}$$

canonical form.

$$\begin{aligned}
 &= (P' + Q + R + S)^* (P + Q + R + S')^* \\
 &\quad * (P + Q' + R + S')^* (P' + Q' + R + S')^* \\
 &\quad * (P + Q' + R' + S)
 \end{aligned}$$

Ans.

AT BC \Rightarrow proof. doubt

$$\begin{aligned}
 &= ((A+B)(A+C)) \\
 &\quad * (A \cdot A + B \cdot C + B \cdot C + C \cdot C)
 \end{aligned}$$

$$= A^2 + B \cdot A + C \cdot A + B \cdot C + C \cdot C$$

$$= A^2 + B \cdot A + C \cdot A + B \cdot C + C^2$$

$$= A^2 + B \cdot A + C \cdot A + B \cdot C + C^2$$

$$= A^2 + B \cdot A + C \cdot A + B \cdot C + C^2$$

$$F(PQ, R) = P \cdot Q + P \cdot R' + Q \cdot R'$$

P Q R R' P

$$\begin{aligned}
 F &= PQ(R+R') + PQR + Q'R \\
 &= \cancel{PQR} + \cancel{PQ'R} + \cancel{PQ'R'} \\
 &\quad + \cancel{PQR'} + P'QR \\
 &= PQR + PQR' + P'Q'R' + P'Q'R
 \end{aligned}$$

Methods

To minimize Boolean exp

Boolean alg + K-maps
Karnaugh maps.

K-map \rightarrow graphical representation

for example

① R

$$\begin{array}{ccc}
 A & B & f \\
 0 & 0 & 0
 \end{array}
 \quad \text{sop}$$

$$\begin{array}{ccc}
 0 & 1 & 1
 \end{array}
 = A' \cdot B + A \cdot B' + AB$$

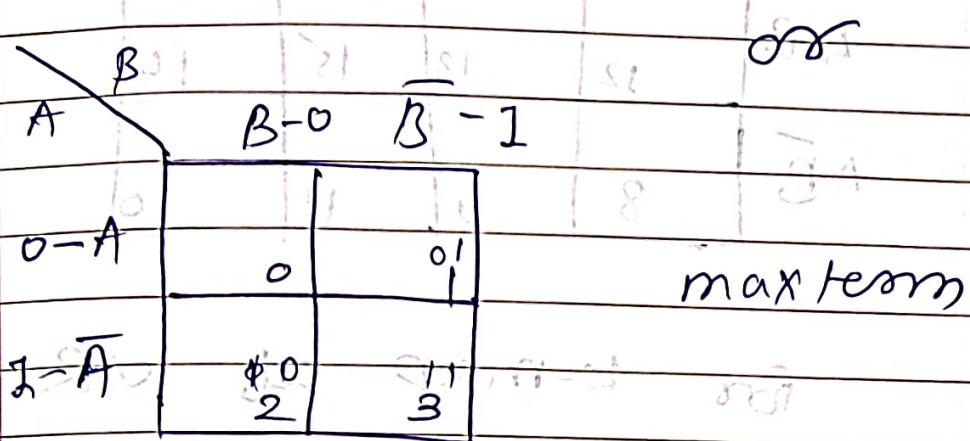
$$\begin{array}{ccc}
 1 & 0 & 1
 \end{array}
 = A' B + A(B+B')$$

$$= A' B + A$$

$$= (A+B)(A+A')$$

$$= (A+B) \quad \textcircled{OR}$$

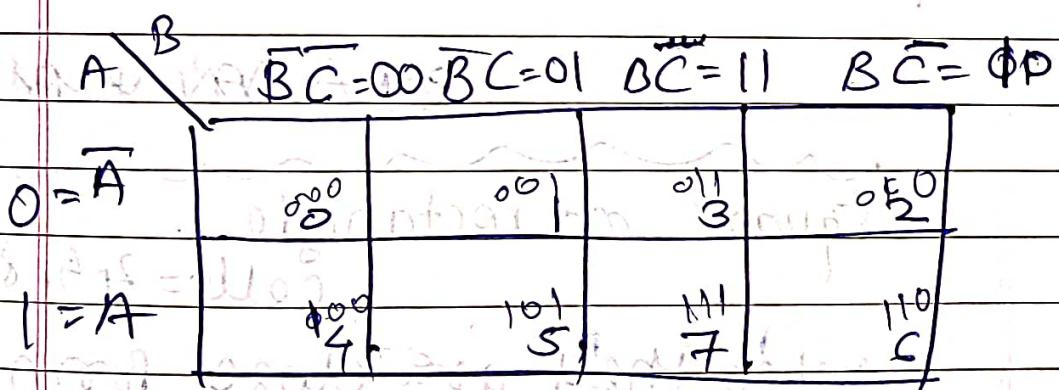
Rules of K-Map



Two variable K-map
cells = $2^2 = 4$

Three variable K-map

cells = $2^3 = 8$



Don't care cases & Aggregation

Given $f(A, B, C) = \bar{A}B + \bar{A}C + BA$

Don't care cases = $\bar{A}\bar{B}\bar{C}$

four variable K-map

27 =

AB	cD	$\bar{c}D$	$\bar{c}\bar{D}$	CD	$C\bar{D}$
$\bar{A}\bar{B}$	0	1	3	2	B
$\bar{A}B$	4	5	7	6	A
$A\bar{B}$	12	13	15	14	
$A\bar{B}$	8	9	11	10	

DRAG X

for K-map of OR

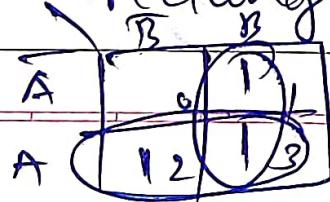
dec	A	B	f
0	0	0	0
1	0	1	1
3	1	0	1

0=0 1=1 3=1

Square or rectangle

cells = 2, 4, 8, 16

when they are close form rectangle, cover all of it



- no diagonals
- vertical or horizontal

Horizontal rectangle

$$A B' + A B = A (B' + B) = A$$

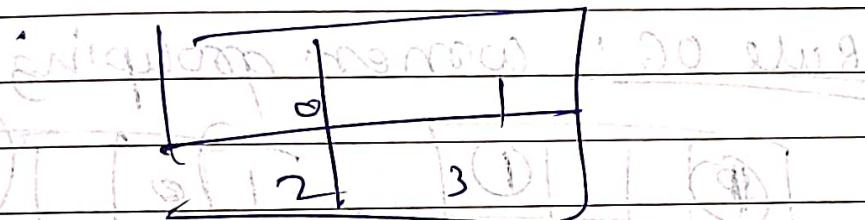
Vertical rectangle

$$A' B + A B = B (A' + A) = B$$

$$\text{Non-Canonical} = A + B.$$

Rule 01: We can either group 0's with 0's or 1's with 1's but we cannot group 0 and 1's together

Step ① for k-map table

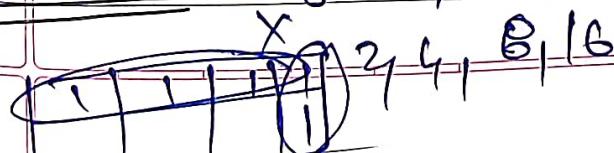


Step ② for sup. cont. group 1's

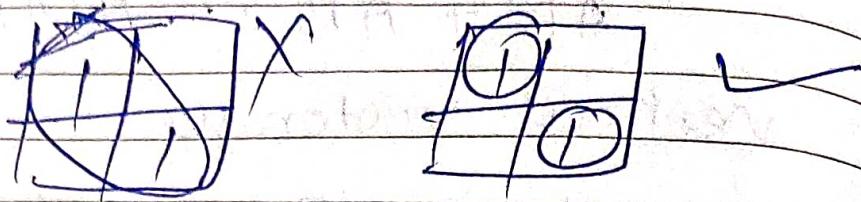
Step ③ for pos. group 0's

Rule 02: Groups may overlap

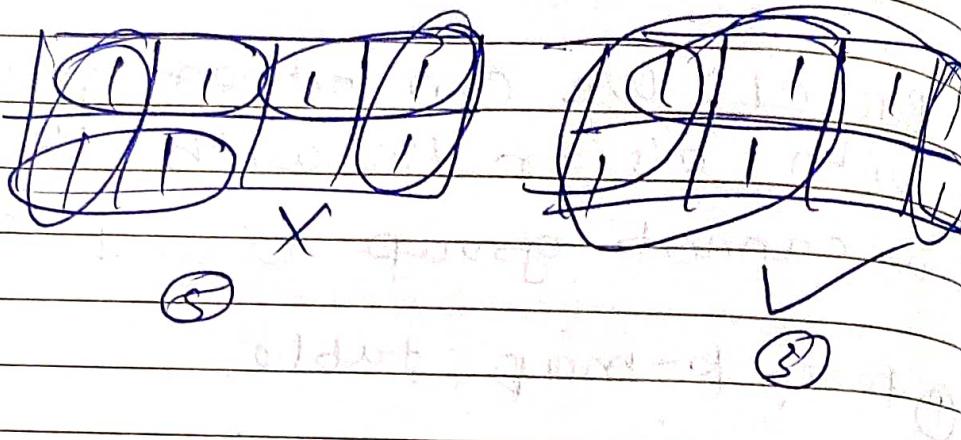
Rule 03: groups as cells having number of $= 2^n$



Rule 04 : No diagonal groups.



Rule 05 : Each group should be as large as possible



Rule 06 : corner grouping



corner group 2 group 1 group

available space

group go 2 group 1 group

Q. question OR gate

X	Y	F	B	B
0	0	0		
0	1	1	A	0 1
1	0	1	A	1 2
1	1	1		

For SOP consider 1

$$= \underline{A'B + AB} + \underline{AB' + AB}$$

$$= A + B.$$

		$\bar{Y}\bar{Z}$	$\bar{Y}Z$	YZ	$Y\bar{Z}$	
ex:	X	1	0	1	1	2
	X	0	0	0	0	1

common
is
only

$$\begin{array}{cccc|c}
 X & Y & Z & F & \\
 \hline
 0 & 0 & 0 & 0 & 1 \\
 1 & 0 & 0 & 1 & 1 \\
 2 & 0 & 1 & 0 & 1 \\
 3 & 0 & 1 & 1 & 1 \\
 4 & 1 & 0 & 0 & 0 \\
 5 & 1 & 0 & 1 & 0 \\
 6 & 0 & 1 & 0 & 1 \\
 7 & 1 & 1 & 1 & 1
 \end{array}
 = \bar{X}\bar{Y}\bar{Z}$$

$$\begin{aligned}
 &+ \bar{X}\bar{Y}Z \\
 &+ \bar{X}Y\bar{Z} \\
 &+ \bar{X}Y\bar{Z} \\
 &+ XY\bar{Z} \\
 &+ XY\bar{Z} \\
 &+ XY\bar{Z} \\
 &+ XY\bar{Z} \\
 &+ XY\bar{Z}
 \end{aligned}$$

$$x' = (\bar{Y}\bar{Z} + \bar{Y}Z + Y\bar{Z} + XY\bar{Z})$$

$$x' = (Y' + Y)(\bar{Z} + Z)$$

$$x' = Y'$$

sum of products

Q. minimize $f(A, B, C, D) = \sum m(0, 1, 1, 2, 3, 4, 7, 8, 9)$

	CD	CD	CD	CD	CD	CD
AB	1	1	1	1	1	X
ABC	1	0	1	1	3	2
AB	1	1	1	1	1	0
AB	1	1	1	1	1	1
AB	1	1	1	1	1	1
$\sum A + B$	1	1	1	1	1	1
$\sum A + B$	1	1	1	1	1	1

$$\bar{C}D = A + B$$

	CD	CD	CD	CD	CD	CD
AB	1	1	1	1	1	X
ABC	1	0	1	1	3	2
AB	1	1	1	1	1	1
AB	1	1	1	1	1	1
AB	1	1	1	1	1	1
$\sum A + B$	1	1	1	1	1	1
$\sum A + B$	1	1	1	1	1	1

$$\text{square} = \bar{B}D$$

$$Q. \quad \bar{A}\bar{B} \quad CD \mid CD, CD$$

	CD	CD	CD	CD	CD	CD
AB	1	1	1	1	1	X
ABC	1	0	1	1	3	2
AB	1	1	1	1	1	1
AB	1	1	1	1	1	1
AB	1	1	1	1	1	1
$\sum A + B$	1	1	1	1	1	1
$\sum A + B$	1	1	1	1	1	1

$$\sum B + \bar{B}$$

$$f(A, B) = \sum m(0, 1, 3)$$

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G.S Pattern

- ① Exp O/P in Boolean form
- ② Write in SOP/POS form
- ③ Expand/write in canon form
- ④ What are minterms/max terms of
 $f = \bar{A} + B$
- ⑤ What are EPI/PI/RPI &

A	B	C	Y
0	0	0	1 - $A'B'C'$
1	0	1	1 - $A'B'C$
2	0	1	1 - $A'B'C'$
3	0	1	1 - $A'BC$
4	1	0	0
5	1	0	1 - ABC
6	1	1	0
7	1	1	0

~~canonical f~~ = $A'B'C' + A'B'C + A'BC' + ABC$
~~canoncial f~~ = $+ ABC'$

$$\min = A'B' + A'B + AB'C$$

$$= A' + AB'C$$

$$= A' + B'C$$

d 1 s 2

4 5 7 6

map 8 9 10 11 12 13 14 15 16

8 9 11 10

using K map

$\bar{B}C - \bar{B}C + BC + \bar{B}\bar{C}$

\bar{A}	0	1	1	1	1	$\rightarrow \bar{A}$
A	0	1	0	0	0	$\rightarrow A$
	4	5	7	8	6	

1 2 3 4 5 6 7 8 9 10

simplified

$$\bar{A}'\bar{B} + \bar{B}C + A'BC + A'B'C + ABC + A'B'C'$$

$$(A'B + A'B')(C + C') + (ABC + A'B'C)$$

$$\cancel{ABC + A'B'C + ABC' + A'B'C'}$$

$$\cancel{ABC + A'B'C + ABC' + A'B'C'}$$

$$= A'B'C' + A'B'C + A'B'C' + A'B'C$$

Using K-map,

	$\bar{B}C$	$B\bar{C}$	$\bar{B}\bar{C}$	BC	
\bar{A}	1	1	1	1	$A' + BC'$
A	0	0	0	0	$\bar{A}' + \bar{B}C$
	1	1	1	1	$\bar{A}' + \bar{B}C$

$$f = \bar{A}\bar{B}C + A\bar{B}C + \bar{A}B\bar{C} + \bar{A}\bar{B}\bar{C}$$

$$+ A\bar{B}C$$

~~f = $\sum m(0, 1, 2, 3, 5)$~~

BOP

POS $\bar{B}H\bar{C}$ $B+\bar{C}$ $\bar{B}+\bar{C}$ $\bar{B}+C$

	\bar{A}	1	0	1	1	0	1	$\bar{A}' + C$
\bar{A}	1	1	0	1	1	0	1	$\bar{A}' + \bar{B}$
A	0	0	1	0	0	1	0	$\bar{A}' + C$
	1	1	0	1	1	0	1	$\bar{A}' + C$

$$\text{POS} = \underline{(\bar{A} + \bar{B})(A' + C)}$$

$$\text{SOP for } O = AB + A'C$$

$$= (A' + B')(A' + C)$$

$\bar{B}+C$ $B+\bar{C}$ $\bar{A}+\bar{C}$ $\bar{B}+\bar{C}$

	0	1	3	2	
A	0	1	0	1	$\pi_{m(0, 1, 3)}$
A	0	1	0	1	$\pi_{m(0, 1, 3)}$
	0	1	0	1	$\pi_{m(0, 1, 3)}$

another e.g.

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$\bar{Y} + Z$	$\bar{Y} \bar{Z}$	$\bar{Y} + \bar{Z}$	$\bar{Y} \bar{Z}$
$\bar{Y} Z$	$\bar{Y} Z$	$Y \bar{Z}$	$Y \bar{Z}$
$w \bar{x} \bar{y} \bar{z}$	0	1	0 3 2
$w + \bar{x}$	$\bar{w} x$	0 9	5 7 6
$\bar{w} + \bar{x}$	$w x$	0	1 2 3 4
$\bar{w} + x$	$\bar{w} \bar{x}$	18 19 15 14	
		8 9 11 10	

$$\text{SOP for } O = X \bar{Y} \bar{Z} + \bar{w} Y Z$$

$$\text{POS} = (X' + Y + Z)(w + Y' + Z')$$

$$\sum(0, 2, 8, 14, 18, 10, 11)$$

$$\bar{C}D \bar{C}D CD C\bar{D}$$

$$\begin{array}{|c|c|c|c|c|} \hline AB & D_1 & 1 & 3 & (1) \\ \hline \bar{A}B & 4 & (1) & 2 & 6 \\ \hline AB & 1 & 1 & 1 & 1 \\ \hline \end{array}$$

SOP =

$$BD + B'D'$$

$$\begin{array}{|c|c|c|c|c|} \hline AB & D_1 & 1 & 1 & (1) \\ \hline \end{array}$$

Prime Implicants

Essential

Redundant

	1	0	1	1	1	1		
C.E.P This is one is covered by one group	1	0	1	1	1	1		
1	1	0	1	1	1	1		
1	1	1	0	1	1	1		
1	1	1	1	0	1	1		
1	1	1	1	1	0	1		
1	1	1	1	1	1	0		
1	1	1	1	1	1	1	0	
1	1	1	1	1	1	1	1	0

$$\text{No. of implicants / minterms in SOP} = 7$$

$$\text{No. of prime implicants} = 6$$

(max no. of groups)

$$\text{No. of essential prime implicants}$$

= 2

$$\text{No. of redundant prime implicants}$$

= 2

1	1	0	1	1	1	1
1	1	0	1	1	1	1
1	1	0	1	1	1	1
1	1	0	1	1	1	1

$$\text{Implicants} = 6$$

$$\text{per P.I.} = 6$$

$$\text{E.P.I.} = 0$$

$$\text{RPI} = 6$$

Prime Implicant

Number of minterms = 8

	1	1	1	1
	1	0	0	1
	1	1	1	0
	1	1	0	1

P.I - $\overline{AB} \overline{C} \overline{D}$

	1	1	1	1
	1	1	1	1
	1	1	0	1
	1	0	1	1

S.P.I - $\overline{A} \overline{B} \overline{C} \overline{D}$

R.P.F - 5

A B F

0 0 0

1 0 1

2 1 0

3 1 1

$$f(SOP) = A'B + AB'$$

$$f(0,0) =$$

$$f(0,1) =$$

$$f(1,0) =$$

$$f(1,1) =$$

$$f = 902 \text{ minterms}$$

don't care

$$\theta = A' + B'$$

$$= A + B$$

$$f = (1 \ 2 \ X \ 3)$$

$$= f(0,0) + f(0,1)$$

$$f = f(1,0) + f(1,1)$$

$$f = f(A, B)$$

Ansible (Point group only $\rightarrow X$)

$$f = \sum m(1, 2, 3) + d(0, 4)$$

A = $\overline{AB} \overline{C} \overline{D}$

B = $\overline{A} \overline{B} \overline{C} \overline{D}$

C = $\overline{A} \overline{B} C \overline{D}$

D = $\overline{A} \overline{B} \overline{C} D$

don't care

9011 (9) Gates: ... (max. 2)

- building block as digital design

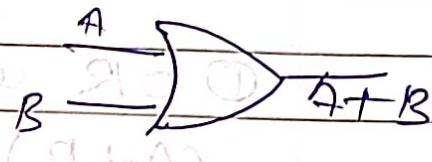
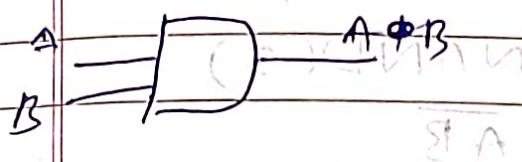
gates \rightarrow flip flops \rightarrow registers \rightarrow storage

① AND $A \cdot B$

$$\begin{array}{ccc} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{array}$$

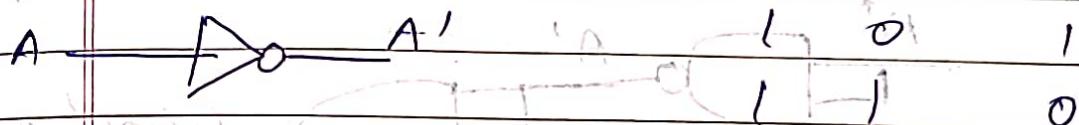
② OR $A + B$

$$\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 1 \end{array}$$



③ NOT

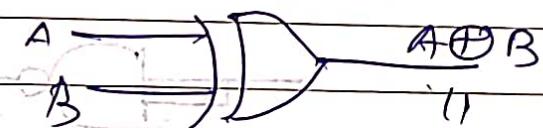
$$\begin{array}{ccc} 0 & 1 & B + A = (B \oplus A) \\ 1 & 0 & 0 \end{array}$$



④ XOR $A \oplus B$

$$\begin{array}{ccc} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$$

⑤ XNOR $A \ominus B$

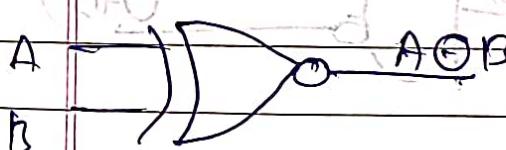


$$\begin{array}{ccc} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{array}$$

Note:- $\downarrow A'B + AB'$

$$\textcircled{1} \quad \text{odd 1's} = 1$$

$$\text{so } 1, 1, 1 = 1$$



$$\textcircled{2} \quad x \oplus 1 = \bar{x} \quad \star$$

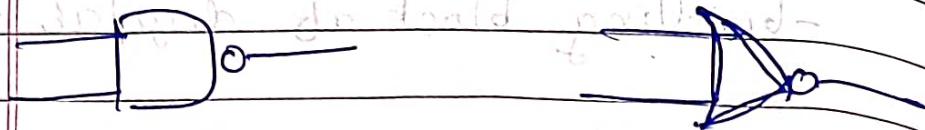
$$x \oplus 0 = x$$

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⑥ NAND

⑦ NOR



$$\text{Logic: } \textcircled{6} \text{ } O \text{ } \oplus \text{ } (A \cdot B)' \text{ } \text{and } A \oplus B = O \text{ } \text{output}$$

$$0 \quad 1 \quad 0 \quad 1$$

$$0 \quad 1 \quad 1 \quad 0$$

$$1 \quad 1 \quad 0 \quad 0$$

$$1 \quad 0 \quad 0 \quad 1$$

$$1 \quad 0 \quad 1 \quad 0$$

$$0 \quad 1 \quad 0 \quad 0$$

$$1 \quad 1 \quad 1 \quad 1$$

$$0 \quad 1 \quad 1 \quad 1$$

Used to make all other gates.

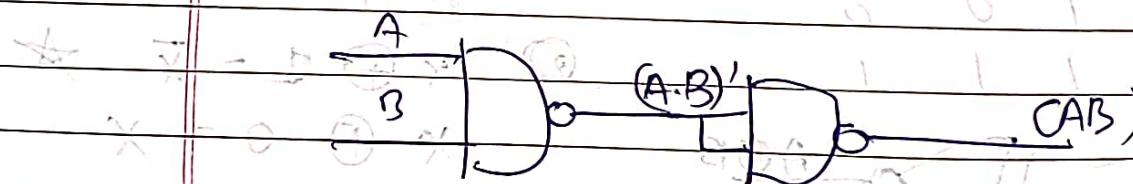
NOR using NAND(1) $D = \overline{A \cdot B}$

① OR using NAND(3)

$$(A + B) = \overline{A \cdot B}$$

$$(\overline{A \cdot B}) = A + B$$

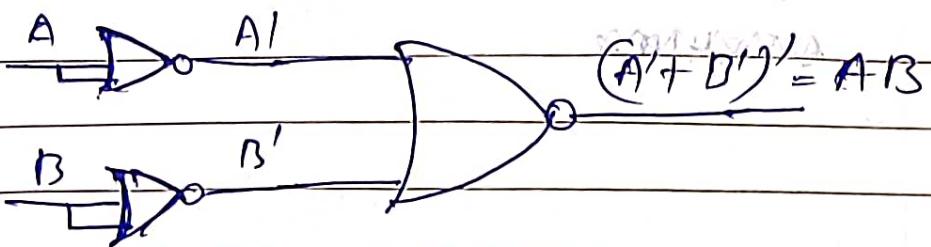
② AND using NAND(2)



③ NOR using NAND AND using NOR (C2)

$$\overline{(\overline{A} + \overline{B})} = AB$$

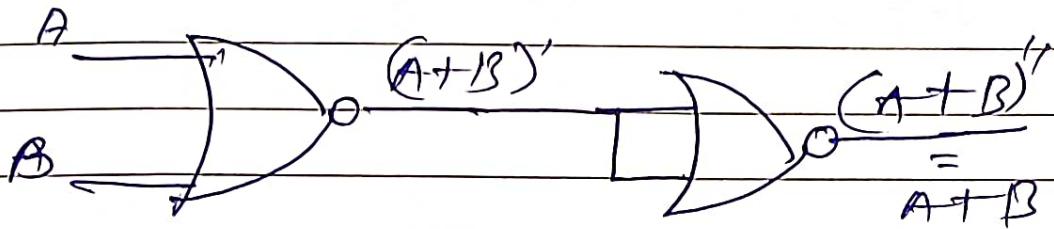
$$(\overline{A} + \overline{B}) = A \cdot B$$



④ OR using NOR C2)

$$A + B$$

$$(A + B)'$$



OR

⑤ NAND

$$\overline{A} \cdot \overline{B}$$

$$\overline{A} \cdot \overline{B} = \overline{A + B}$$

in hand

in hand

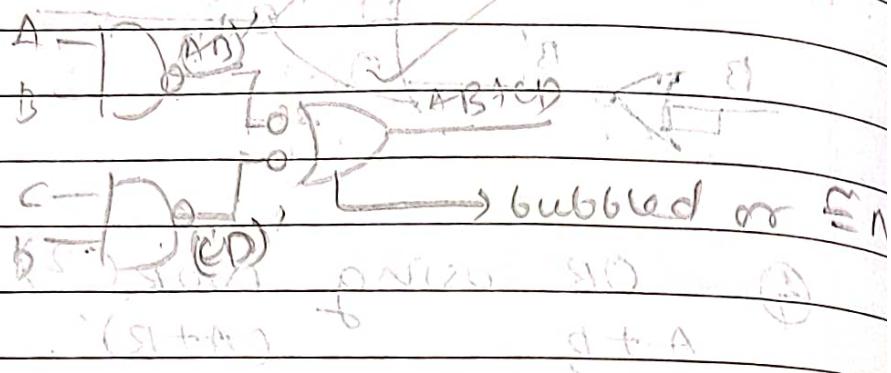
AND \equiv NAND

Q. How many NAND gates for this ex
 $(A+B)'$

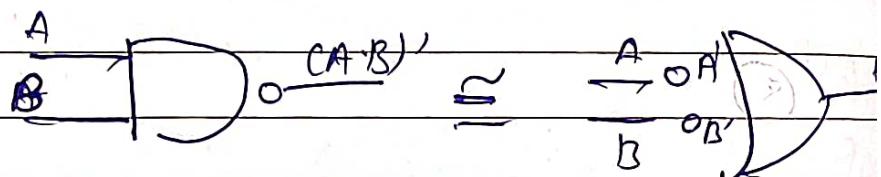
We change to SOP - Sum of prod.
 Then $SOP = (B+A)$

$AB + BC + CD \rightarrow 3$ Nand gate

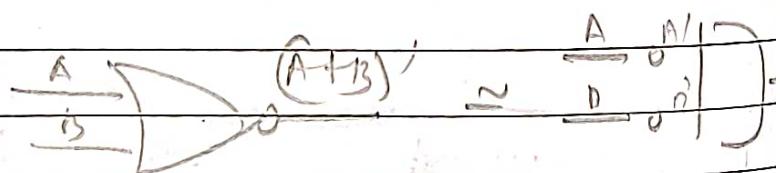
~~AND~~



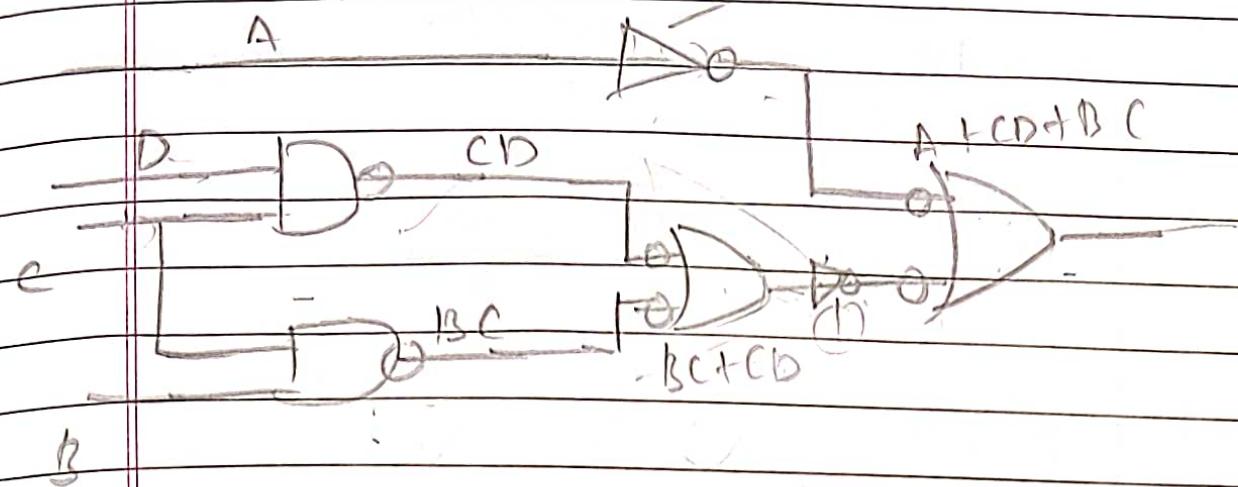
Note



NAND \equiv B OR



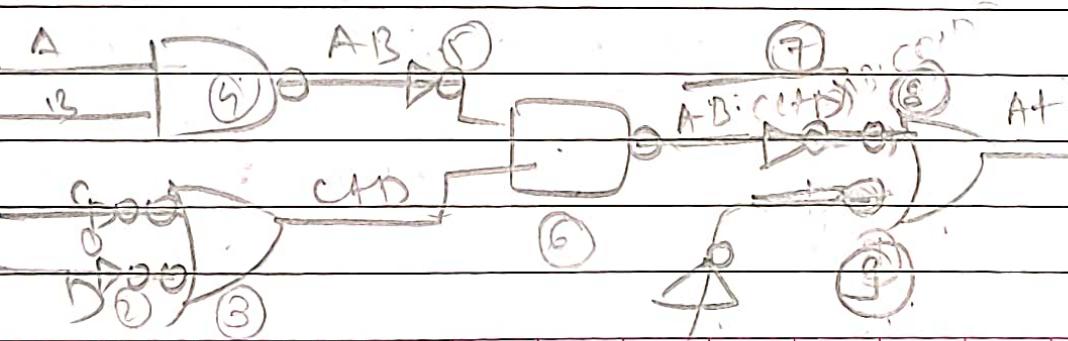
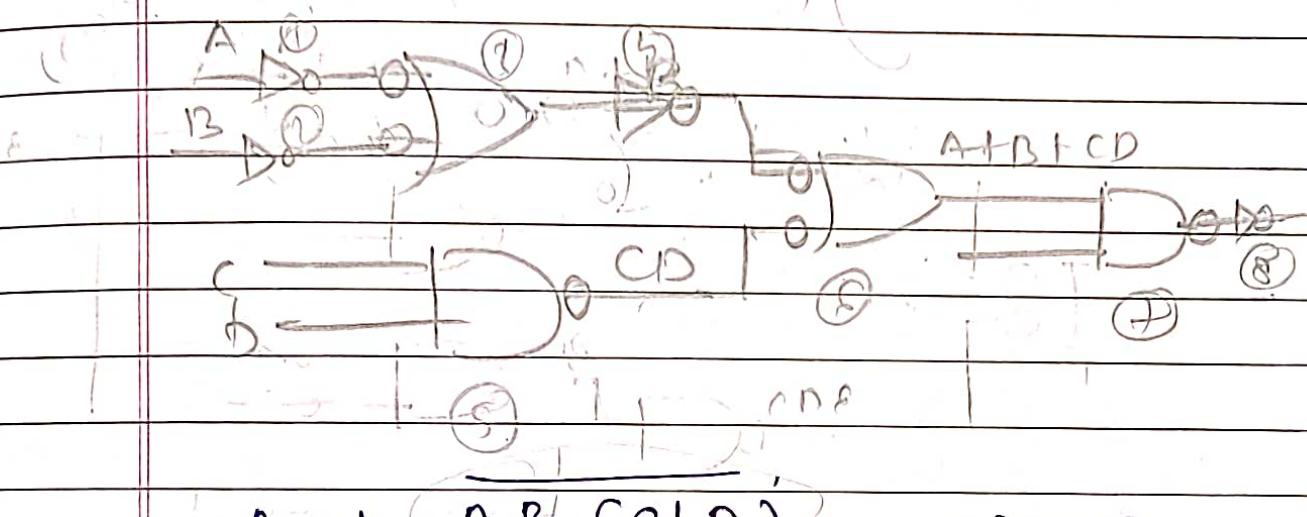
using 3 input AND gate
 $A + CD + BC$ - ^{com}_{input}
 - ^{input}_{device}



Using nand gate design

$$(A + B + CD)E \rightarrow \text{POS}$$

$$AE + BE + CDE$$



BCD - ina

BCD - Binary Coded Decimal
→ 6 binary

21 has → 10101010 + Q + P

Div 10101010

register 2010

2 4

0010 0201

0110 0601

5 5

A B C D

A B C D

12 0 0 0 0

1 0 0 0 0

3 0 0 0 0

4 0 0 0 0

5 0 0 0 0

6 0 0 0 0

7 0 0 0 0

8 0 0 0 0

9 0 0 0 0

10 0 0 0 0

11 0 0 0 0

12 0 0 0 0

13 0 0 0 0

14 0 0 0 0

15 0 0 0 0

16 0 0 0 0

17 0 0 0 0

18 0 0 0 0

19 0 0 0 0

20 0 0 0 0

21 0 0 0 0

22 0 0 0 0

23 0 0 0 0

24 0 0 0 0

25 0 0 0 0

26 0 0 0 0

27 0 0 0 0

28 0 0 0 0

29 0 0 0 0

30 0 0 0 0

31 0 0 0 0

32 0 0 0 0

33 0 0 0 0

34 0 0 0 0

35 0 0 0 0

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37 0 0 0 0

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121 0 0 0 0

122 0 0 0 0

123 0 0 0 0

124 0 0 0 0

125 0 0 0 0

126 0 0 0 0

127 0 0 0 0

128 0 0 0 0

129 0 0 0 0

130 0 0 0 0

131 0 0 0 0

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134 0 0 0 0

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136 0 0 0 0

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140 0 0 0 0

141 0 0 0 0

142 0 0 0 0

143 0 0 0 0

144 0 0 0 0

145 0 0 0 0

146 0 0 0 0

147 0 0 0 0

148 0 0 0 0

149 0 0 0 0

150 0 0 0 0

151 0 0 0 0

152 0 0 0 0

153 0 0 0 0

154 0 0 0 0

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161 0 0 0 0

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168 0 0 0 0

169 0 0 0 0

170 0 0 0 0

171 0 0 0 0

172 0 0 0 0

173 0 0 0 0

174 0 0 0 0

175 0 0 0 0

176 0 0 0 0

177 0 0 0 0

178 0 0 0 0

179 0 0 0 0

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193 0 0 0 0

194 0 0 0 0

195 0 0 0 0

196 0 0 0 0

197 0 0 0 0

198 0 0 0 0

199 0 0 0 0

200 0 0 0 0

201 0 0 0 0

202 0 0 0 0

203 0 0 0 0

204 0 0 0 0

205 0 0 0 0

206 0 0 0 0

207 0 0 0 0

208 0 0 0 0

209 0 0 0 0

210 0 0 0 0

211 0 0 0 0

212 0 0 0 0

213 0 0 0 0

214 0 0 0 0

215 0 0 0 0

216 0 0 0 0

217 0 0 0 0

218 0 0 0 0

219 0 0 0 0

220 0 0 0 0

221 0 0 0 0

222 0 0 0 0

223 0 0 0 0

224 0 0 0 0

225 0 0 0 0

226 0 0 0 0

227 0 0 0 0

228 0 0 0 0

229 0 0 0 0

230 0 0 0 0

231 0 0 0 0

232 0 0 0 0

233 0 0 0 0

234 0 0 0 0

235 0 0 0 0

236 0 0 0 0

237

$$\begin{array}{r} 1010 \\ 10 + 0110 \\ \hline 0000 \end{array}$$

$$\begin{array}{r} 0101 \\ 0100 \\ \hline 0110 \end{array} \quad \begin{array}{l} (10) \\ (6) \end{array}$$

$$1010 \rightarrow 0000$$

ans.
not carry

Grey Code

Binary code -

Too much change in digits from one decimal to next

why Grey Code

from one decimal to next

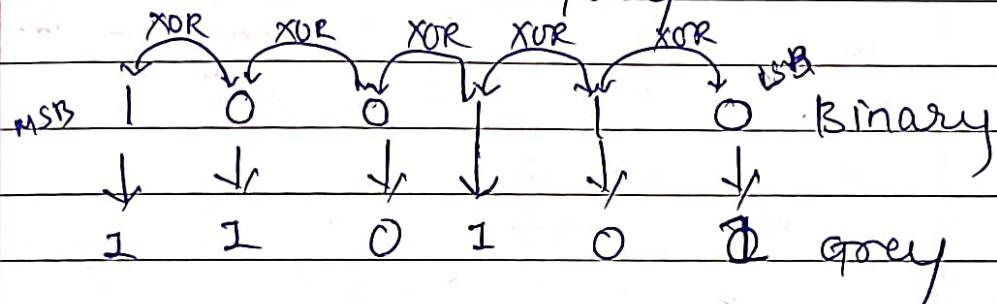
e.g. $3 \rightarrow 4$ then only ^{one} bit changes max only one

Question

Bin to Grey

Grey to Bin ka circuit

* Bin to Grey

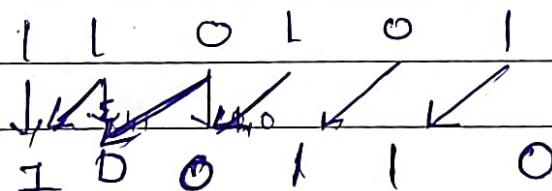


0010 bin

\downarrow

01# grey

* Grey to Bin



Bin \rightarrow Gray

	w	x	y	A	B	C
0	0	0	0	0	0	0
1	0	0	1	0	0	1
2	0	1	0	0	1	1
3	0	1	1	0	1	0
4	1	0	0	1	1	0
5	1	0	1	1	1	1
6	1	1	0	0	0	1
7	1	1	1	1	0	0

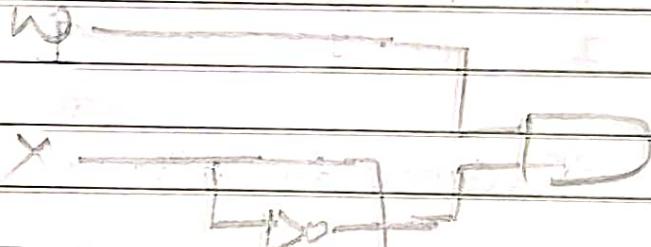
for A $\bar{xy} \bar{xy} \bar{xy} \bar{xy}$

\bar{w}	0	1	2	3
w	0	1	5	7

$$A = w.$$

$$\text{sum } B = \bar{w}x + w\bar{x} = x \oplus w$$

$$\text{sum } C = \bar{y}y + y\bar{y} = x \oplus y$$



convert BCD to Excess 3, and
design a circuit

BCD	Excess 3
0 0 0 0	0 0 0 0
1 0 0 0	1 0 0 1
2 0 0 1	1 0 1 0
3 0 0 1	1 0 1 1
4 0 1 0	1 1 0 0
5 0 1 1	1 1 0 1
6 0 1 1	1 1 1 0
7 0 1 1	1 1 1 1
8 1 0 0 0	1 0 0 1
9 1 0 0 1	1 0 1 0

$X = \overline{A} \overline{B} \overline{C} \overline{D}$ $Y = \overline{A} \overline{B} C D$

$Z = A \overline{B} \overline{C} \overline{D}$

$\overline{A} \overline{B}$		1	1	1	1
$\overline{A} B$	1	1	1	1	0

$\overline{A} B$	1	1	1	1	0
$A \overline{B}$	0	1	1	1	1

$\overline{B} \overline{A} = \overline{B} \overline{B}$	X	1	X	1	X
$B \overline{A} = \overline{B} \overline{B}$	1	1	1	1	1

$\overline{A} \overline{D}$	1	1	1	X	Y
$A \overline{D}$	0	1	1	1	0

$$X = \overline{A} \overline{B} \overline{C} + \overline{B} \overline{C} \overline{D} + \overline{A} \overline{B} \overline{C}$$

$$Y =$$

Unit 2, Sequential Circuits and

Sequence ↓
Combination
consider previous state ↓
no previous state
- considers only input

Combinational circuit :-

* Half Adder - to add two bits

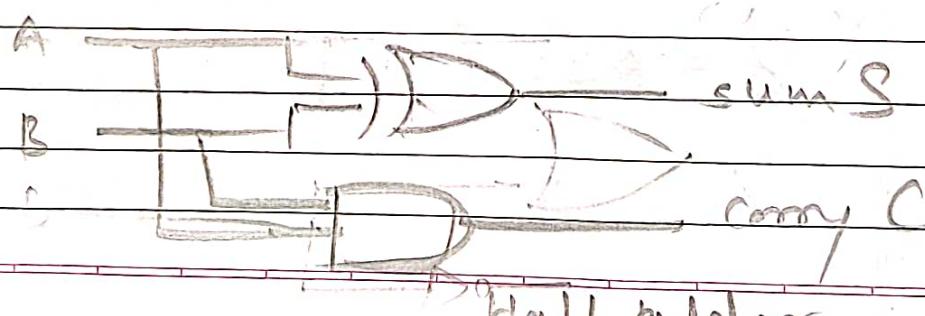
A	B	S	C	I	S	D
0	0 0 0 1 0 0 1	0 1 1 0 0 0	0	1	0	0
0	1 1 0 1 1 0 1	0 0 0 0 1 1 0	0	0	1	0
1	0 0 1 1 1 0 1	1 0 0 0 1 1 0	1	0	1	0
1	0 1 1 0 0 1 1	1 1 0 0 1 1 0	1	1	0	0

For sum

$$\text{sum} = A\bar{B} + \bar{A}B$$

$$= A \oplus B$$

For carry



* Full Adder → to add two bits and one carry.

	A	B	C	S	C
0	0	0	0	0	0
1	0	0	1	1	0
2	0	1	0	1	0
3	0	1	1	0	1
4	1	0	0	1	0
5	1	0	1	0	1
6	1	1	0	0	1
7	1	1	1	1	1

for sum

\overline{BC} \overline{BC} \overline{BC} \overline{BC}

A	0	1	3	10
A	1	1	5	10

$A \oplus B \oplus C$

$$S = A\overline{B}\overline{C} + \overline{A}\overline{B}C + AB\overline{C} + \overline{A}BC$$

~~$(\overline{A}CA\overline{C} + \overline{A}C) + B\overline{C},$
 $- B\overline{C}A\overline{C})$~~

$\overline{A}\overline{B}\overline{C}$	0	1	10	2
$\overline{A}\overline{B}C$	0	1	10	2

$\overline{A}B\overline{C}$	1	1	10	2
$A\overline{B}\overline{C}$	1	1	10	2

$A\overline{B}C$	1	1	10	2
ABC	1	1	10	2

$A\overline{B}C$	1	1	10	2
$A\overline{B}C$	1	1	10	2

Remember Full Adder circuit diagram

Question no. ab AND or OR and XOR gates to add

(8) 3 bits:

$$\begin{array}{r} 010 \\ 011 \\ \hline \end{array}$$

3 full adder

1 full adder - how many?
memorize

* Parallel Adder:

↓ more than one pair of bits
→ When bits are added simultaneously
always propagate carry

* Ripple Adder - Parallel + full adder

$$\begin{array}{r} 0101 \\ 0100 \\ \hline \end{array} \quad \begin{array}{l} \text{Carry = 0} \\ \text{full adder} \\ \text{Carry = 0} \end{array}$$

$$0101 + 0100 + 0101 = \text{full adder 2} \\ 0101 + 0100 + 0101 \\ \hline \quad \begin{array}{l} \text{Carry = 0} \\ \text{Carry = 0} \end{array}$$

g. Carry propagation - time to generate

(1) optime = 5 sec to add 5 bits
sptime = 10 sec. worst case to add

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80 sec

$$\text{total optime} = \text{optime} * n(\text{bits})$$

$$\text{total optime} = 10 \text{ sec}$$

5x4

$$\text{so worst delay} = 20 \text{ sec.}$$

$$cn-1) * \text{optime} = (5-1) * 5$$

$$= 3 * 5$$

$$= 15 \text{ sec}$$

for carry to
reach to last

in last instead of carry time takes
worst case time i.e. sp time (C)
($\text{if sp time} > \text{optime}$)

$$\therefore \text{worst delay} = 15 \text{ sec} + 10 \text{ sec}$$

$$= 25 \text{ sec.}$$

formula = $\max(\text{sp}, \text{cp})$

+ $(n-1) * \text{cp}$

↓
bits.

③ ~~eg. XOR - 5 min/sec~~ AND ~~3 min/sec~~ - 4 min/sec

T delay time = 20

sum = no. of gates * delay time of gate

$$\text{optime} = 7-sec$$

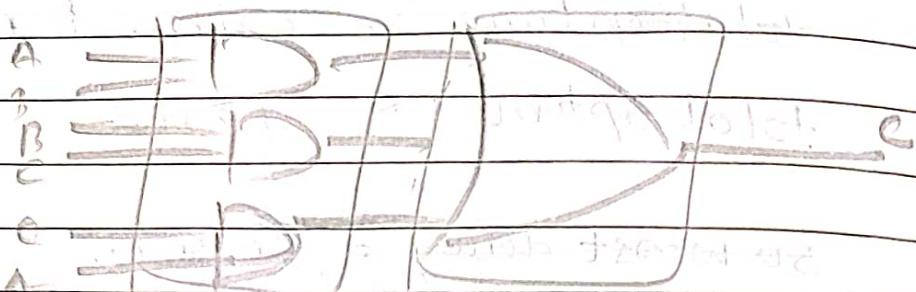
$$\text{worst delay} = (3 \times 7) + 3$$

$$= 21 + 3 = 24$$

optime = AND + OR time
sptime = NOR time

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$\Rightarrow S(A) = \text{parallel}$
 $S(B) = \text{AND}$

optime = 5 sec.

Half subtractor Full subtractor

$$\begin{array}{r} A \\ - B \\ \hline \end{array} \quad \begin{array}{r} D \\ - C \\ \hline \end{array}$$

Difference Borrow

0	0	0	0
0	1	1	1
0	0	0	0

(P + Q) NOR = 0

$$P^2(1-Q) +$$

↓
= 2L10

$$Q^2P$$

↓
= 2L00

∴ P + Q = P + Q + P + Q = P + Q

∴ P + Q = P + Q

∴ P + Q = P + Q

g) 16 bits - ripple carry adder
 16 identical full adder
 $cptime = 12\text{ns}$ of each FA
 $sptime = 15\text{ns}$ of each FA

$$= (16-1) * 12 + 15 = 15$$

$$\Rightarrow 15 * 12 + 15 = 195$$

$$= 15 * 13$$

$$= 195$$

g) $\oplus \text{XOR} = 20 \text{ AND} = 15 \text{ OR} = 10$

answer

$$cptime = 15 + 10 = 25\text{nsec}$$

$$sptime = 20\text{ ns.}$$

$$\text{Total delay cp time} = 16 * 25 = 400\text{ ns.}$$

total worst case delay time

~~$$= 20 + (15) * 25$$~~

$$= 20 + 375$$

$$= 395\text{ ns.}$$

answ
195ns

book

* Carry lookahead adder

$$G_i^o = A_1 P B_1^o \quad \begin{matrix} C_3 \\ A_3 \\ B_3 \end{matrix} \quad \begin{matrix} C_2 \\ A_2 \\ B_2 \end{matrix} \quad \begin{matrix} C_1 \\ A_1 \\ B_1 \end{matrix}$$

$$P_i^o = A_i \oplus B_i^o$$

$$G_2^o = G_0 P_0 + G_1 =$$

$$C_2 = G_0 P_0 + G_1 + G_2$$

$$C_3 = G_0 P_0 P_1 P_2 + G_1 P_1 P_2 + G_2$$

$$O_3 = C_3 = G_0 P_0 P_1 P_2 P_3 + G_1 P_1 P_2 P_3 + G_2 P_3 + G_3$$

$$+ G_1 P_2 P_3 + G_2 P_3 + G_3$$

$O_3 = 011$ = English

$$0110010 = 75 * 21 = \text{mid} \quad \text{and} \quad \text{and} \quad \text{and}$$

$$75 * (21) + 0110010 =$$

$$750 + 0110010 =$$

$$750 + 0110010 =$$

