COEP Technological University Pune

Department of Mathematics

(MA- 20001) - Ordinary Differential Equations and Multivariate Calculus

S.Y. B.Tech. Semester III (All Branches)

Tutorial 1 (AY: 2023-24)

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1. Explain the basic concepts ordinary and partial differential equations (ODEs, PDEs), order, degree, initial value problems (IVPs). Give examples.
2. Define general solution, particular solution and singular solution of an ordinary differential equation and explain the difference between them with an example.
3. What is a linear ODE? Why is it easier to solve than a nonlinear ODE?
4. What is an exact ODE?
5. Explain the idea of an integrating factor. Give two examples.
6. Can an ODE sometimes be solved by several methods? Give two examples.
7. Find an ODE for the straight lines through the origin.
8. Verify that the given function is a solution of corresponding differential equation. (a, b are arbitrary constants.)
   (a) $y = a\cos\pi x + b\sin\pi x, \quad y'' + \pi^2 y = 0$
   (b) $y = 5e^{-2x} + 2x^2 + 2x + 1, \quad y' + 2y = 4(x+1)^2$
   (c) $y = e^{x^2}\displaystyle\int_0^x e^{-t^2}dt, \quad y' = 2xy + 1$
   (d) $y = -\sin x + ax^2 + bx + c, \quad y''' = \cos x$
   (e) $x^2 + y^2 = 1, \quad x + yy' = 0$
9. Obtain the general solution (or particular solution) of the following differential equations.
   (a) $y^{(3)} = e^{-0.2x}$
   (b) $y' = 2\sec 2y$
   (c) $\dfrac{dr}{dt} = -2tr; \quad r(0) = r_0$
   (d) $x' = \cos(x+y)$
   (e) $x\,dy - y\,dx = x\sqrt{x^2+y^2}dx\,; \quad y(1) = 1$
   (f) $y' = \dfrac{y}{x} + x\sin\left(\dfrac{y}{x}\right)$
10. Model the RC-circuit in Figure 1. Find a general solution when R, C, E are any constants.

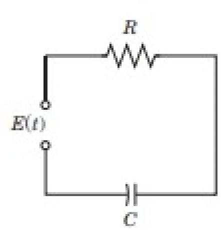

Figure:1

11. Model the RL-circuit in Figure 2. Find a general solution when R, L, E are any constants.

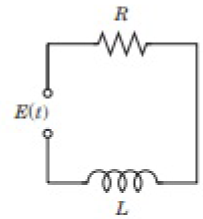

Figure:2

12. If in a culture of yeast the rate of growth $y'(t)$ is proportional to the amount $y(t)$ present at time t, and if $y(t)$ doubles in 1 day, how much yeast can be expected after 3 days at the same rate of growth ? After 1 week ?
(Ans: $8\,y_0$, $128\,y_0$)

13. If the growth rate of the amount of yeast at any time t is proportional to the amount present at that time and doubles in 1 week, how much yeast can be expected after 2 weeks? After 4 weeks?

14. Experiments show the rate of inversion of cane sugar in dilute solution is proportional to the the concentration $y(t)$ of unaltered sugar. Let the concentration be $\frac{1}{100}$ at t = 0 and $\frac{1}{300}$ at $t = 4hrs$. Find $y(t)$. (Ans: $0.01\,e^{-0.275t}$)

15. A thermometer, reading 10^0C, is brought into a room whose temperature is 23^0C. Two minutes later the thermometer reading is 18^0C. How long will it take until the reading is 22.8^0C. (Ans : 8.73 mins.)

16. A body originally at 80^0C cools down to 60^0C in 20 minutes, the temperature of the air being 40^0C. What will be the temperature of the body after 40 minutes from the original ? (Ans: 50^0C)

17. A tank contains 1000 gal of water in which initially 100 lb of salt is dissolved. Brine runs in at a rate of 10 gal min, and each gallon contains 5 lb of dissoved salt. The mixture in the tank is kept uniform by stirring. Brine runs out at 10 gal min. Find the amount of salt in the tank at any time t.

18. A tank contains 5000 liters of fresh water. Salt water which contains 100 gms of salt per liter flows into it at the rate of 10 liters per minute and the mixture kept uniform by stirring, runs out at the same rate. When will the tank contain 200, 000 gms of salt?

19. A tank contains 1000 gallons of water in which 200 lb of salt, are dissolved. Fifty gallons of brine, each containing $(1 + \cos t)lb$ of dissolved salt, runs into the tank per minute, the mixture kept uniform by stirring, runs out at the same rate. Find the amount of salt $y(t)$ in the tank at any time t. (Ans: $y(t) = 1000 + 2.494\,\cos t + 49.88\,\sin t - 802.5\,e^{-0.05t}$)

20. Explain the idea of an integrating factor. Give two examples.

21. State the necessary and sufficient condition of exactness for $M(x, y)\,dx + N(x, y)\,dy = 0$.

22. Let $M(x, y)$ and $N(x, y)$ be continuous and have continuous first partial derivatives. If $M(x, y)\,dx + N(x, y)\,dy = 0$ is exact then prove that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

23. If $\dfrac{Q_x - P_y}{P}$ is a function of y alone, say $\phi(y)$, then $e^{\int \phi(y)\,dy}$ is an integrating factor of a differential equation $P(x,y)\,dx + Q(x,y)\,dy = 0$.

24. Verify that $\dfrac{y+1}{x^4}$ is an integrating factor of $3(y+1)\,dx = 2x\,dy$, and solve the ODE.

25. Find the constant $\mathbf{n}$ such that $y^{\mathbf{n}}$ is an integrating factor of the differential equation : $y\,(2x^2y + e^x)\,dx - (e^x + y^3)\,dy = 0$.

26. Under what conditions for the constants a, b, p, q is $(ax+by)\,dx+(px+qy)\,dy = 0$ exact? Solve the exact ODE.

27. Find the value of α such that $e^{\alpha\, y^2}$ is an integrating factor of the differential equation: $\left(e^{\frac{-y^2}{2}} - xy\right)\,dy - dx = 0$.

28. Test for exactness. If exact, solve. If not, then find the integrating factor and solve. Also, if an initial condition is given, find the corresponding particular solution.

    (a) $x^3 dx + y^3 dy = 0$
    (b) $\left(\dfrac{\cos y}{x+3}\right)\,dx - \left(\sin y\,\ln(5x+15) - \dfrac{1}{y}\right)\,dy = 0$
    (c) $e^x(\cos y\;dx - \sin y\;dy) = 0$
    (d) $3x(xy-2)\,dx + (x^3+2y)\,dy = 0$
    (e) $(\cos x\cos y - \cot x)\,dx - \sin x\,\sin y\,dy = 0$
    (f) $(y^2\,e^{xy^2} + 4x^3)\,dx + (2xy\;e^{xy^2} - 3y^2)\,dy = 0$
    (g) $(\sin x\;\cosh\;y)\,dx - (\cos x\;\sinh\;y)\,dy = 0$
    (h) $e^{-y}\,dx + e^{-x}(1 - e^{-y})dy = 0$
    (i) $e^{3\theta}(dr + 3\,r\,d\theta) = 0$
    (j) $(2xy\,dx + dy)\,e^{x^2} = 0,\;\; y(0) = 2$
    (k) $(2x\;\ln x - xy)\,dy + 2y\,dx = 0$
    (l) $(1+\ln(xy)\;)\,dx + \left(1 + \dfrac{x}{y}\right)\,dy = 0$

29. Solve the following linear/ nonlinear differential equations:

    (a) $x(1-x^2)\,\dfrac{dy}{dx} + (2x^2-1)\,y = x^3$
    (b) $e^{-y}\;\sec^2 y\,dy = dx + x\,dy$
    (c) $\dfrac{dy}{dx} = x^3\,y^3 - xy$

30. Find the orthogonal trajectories of the family of curves:

    (a) $y = \sqrt{x + c}$

    (b) $y^2 = \dfrac{x^3}{c - x}$

    (c) $x^2 + y^2 = 2ax$

31. Show that the family of curves $x^2 + 4y^2 = c_1$ and $y = c_2 x^4$ are orthogonal to each other.

32. Show that the family of parabolas $y^2 = 4c(x + c)$ is **self orthogonal**.

Please report any mistakes in the problems and/or answers given here.