College of Engineering Pune

(An Autonomous Institute of Govt. of Maharashtra)

(MA-20001) Ordinary Differential Equations and Multivariate Calculus

Program : S.Y.B.Tech. Sem. III (All Branches) | Academic Year : 2022-23

Examination : Re-End Semester Examination | Maximum Marks : 60

Date : 17/02/2023 | Time : 10.00 am to 01.00 pm

Student MIS Number :

Instructions :

1. Write your MIS Number on Question Paper.
2. Writing anything on question paper is not allowed.
3. Mobile phones and programmable calculators are strictly prohibited.
4. Exchange/Sharing of stationery, calculator etc. is not allowed.
5. Figures to the right indicate the course outcomes and maximum marks.
6. Unless otherwise mentioned symbols and notations have their usual standard meanings.
7. Any essential result, formula or theorem assumed for answering questions must be clearly stated.

Q.1 Attempt the following:

(i) Solve $y' + \frac{y}{x} - \sqrt{y} = 0$. (CO3)[2.5]

(ii) Find Laplace transform of $\sin 3t$. Hence evaluate the Laplace transform of $t^2 \sin 3t$. (CO3)[2]

(iii) Find the inverse Laplace transform of $\frac{1}{s^4 - s^2}$. (CO3)[3]

(iv) For the function $f(x, y) = \sqrt{9 - x^2 - y^2}$, find it's domain and range. Describe it's level curves. Find the boundary of the domain, if any. State if the domain is open, closed or neither or both, bounded or unbounded. (CO3)[3]

(v) Find $\lim_{\substack{(x,y)\to(2,2) \\ x+y\neq 4}} \left(\frac{x + y - 4}{\sqrt{x + y} - 2} \right)$. (CO3)[2]

(vi) Define continuity of a function $f(x, y)$ at a point (x_0, y_0).

At what points is the function $f(x, y, z) = \dfrac{1}{4 - \sqrt{x^2 + y^2 + z^2 - 9}}$ continuous ? (CO4)[2.5]

Q.2 **Attempt the following:**

(i) Find the general solution of $y''' + 4y' = 0$. (CO3)[1.5]

(ii) Functions $\tan x, \cot x$ and 0 are linearly dependent. State true or false giving reason. (CO1)[1]

(iii) Using appropriate properties/theorems of Laplace transforms, evaluate $\displaystyle\int_0^{\infty} e^{-3t} \{t \cos t\}\, dt$ (CO3)[2]

(iv) Find the inverse Laplace transform of $\cot^{-1}\left(\dfrac{2}{s^2}\right)$. (CO4)[3]

(v) If $u(x, y) = \ln(x^3 + y^3 - x^2 y - xy^2)$ then show that $u_x + u_y = \dfrac{2}{x + y}$. (CO2)[3]

(vi) Find x_u, y_v if x and y are functions of u and v given by $u = x^2 - y^2, v = x^2 - y$ (CO3)[2.5]

(vii) Find the rate at which the area of a rectangle is changing at a given instant when the length of the rectangle is 4 ft and it's width is 3 ft and they are increasing at the rate of 1.5 ft/sec and 0.5 ft/sec respectively. (CO4)[2]

Q.3 **Attempt the following:**

(i) A spring of mass of 2 kg has natural length 0.5 m. A force of 25.6 N is required to maintain it stretched to a length of 0.7 m. If the spring is stretched to a length of 0.7 m and then released with initial velocity 0, find the position of the mass at any time t. Also determine the frequency, period and amplitude. (CO4)[3]

OR

A decaying e.m.f. $E = 200e^{-5t}$ is connected in series with a 20 ohm resistor and 0.01 farad capacitor. Find the charge and current at any time assuming $Q = 0$ at $t = 0$. Show that the charge reaches a maximum, calculate it and find the time when it is reached. (CO4)[3]

(ii) Consider $f(t) = t[1 - u(t-1)] + e^t[u(t-1) - u(t-2)]$,

(a) write this as a piecewise function and sketch the graph.(CO1)[1]

(b) compute it's Laplace transform. (CO2)[2]

(iii) Use properties of Laplace transform to find :

$$\mathcal{L}^{-1}\left\{\left(1 + e^{-2\pi s+2\pi}\right)\frac{s-1}{s^2 - 2s + 2}\right\}$$

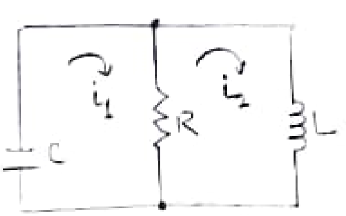
(CO2)[3]

(iv) Prove: If $F(x, y, z)$ is differentiable and the equation $F(x, y, z) = 0$ defines z implicitly as a differentiable function of x and y then, at any point where $F_z \neq 0$, $\frac{\partial z}{\partial x} = -\frac{F_x}{F_z}$ and $\frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$ (CO4)[3]

(v) Consider the function $f(x, y) = x^2y^3 - y^4$. Find the directional derivative of $f(x, y)$ at the point $P = (2, 1)$ in the direction given by angle $\theta = \frac{\pi}{4}$. (CO3)[3]

Q.4 Attempt the following:

(i) Find the currents i_1, i_2 at any time t in the following circuit by setting up a system of ordinary differential equations and solving them by using matrices: (CO3)[2.5]



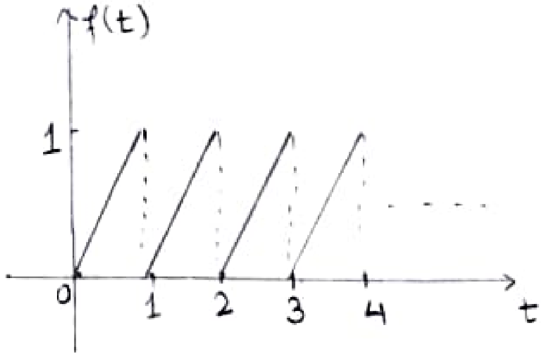
$R = 1\Omega$, $L = 1.25$ Henry, $C = 0.2$ Farad

Given that $i_1(0) = 1$ amp $= i_2(0)$

(ii) Attempt **any two**:

(a) Solve : $y(t) + 2e^t \int_0^t y(\tau)e^{t-\tau}d\tau = te^t$. (CO3)[2.5]

(b) Find Laplace transform of the saw tooth wave shown in the figure below: (CO5)[2.5]



(c) Define Dirac Delta function $\delta(t-a), a \geq 0$ and evaluate it's Laplace transform. (CO3)[2.5]

(iii) Say True or False: If (a, b) is an interior point in the domain of $f(x, y)$ such that f_y does not exist at (a, b) then (a, b) is a critical point. (CO1)[1]

(iv) Say True or False. Justify your answer: $f(x, y) = xy$ has a saddle point at $(0, 0)$. (CO2)[2]

(v) Find all critical point(s) of the function $f(x, y) = x^3 + 3xy + y^3$ and discuss if it is a point of local maximum, local minimum or a saddle point. (CO3)[2]

(vi) Find dimensions of the rectangle with largest perimeter that can be inscribed in the ellipse $9x^2 + 4y^2 = 36$. What is the largest perimeter? (CO5)[2.5]