COEP Technological University

(COEP Tech)

A Unitary Public University of Government of Maharashtra

(MA-20001) Ordinary Differential Equations and Multivariate Calculus

Program : S.Y.B.Tech. Sem. III

Examination : Test 1 (SET-1)

Date : 17/09/2023

Academic Year : 2023-24

Maximum Marks : 20

Time : 11.00 am - 12.00 noon

Branch:

Student MIS Number :

Name and Signature of the Invigilator: ______________________

Q.1	Q.2	Q.3	Q.4	Total	Signature

Attempt All the Questions.

Question [I](5 marks)

(1) Write true or false and justify your answer: $y''' + yy'' - 2y\sin x = 0$ is a homogeneous linear differential equation. (1 mrks each) [CO2][2]

Answer: False. $\because$ given D.E is homo. Non-linear D.E, as yy'' occurs in the D.E.

(2) Find the linear differential equation whose linearly independent solutions are $1, \sin 3x, \cos 3x$. [CO3][3] (½ mrks each steps)

Detailed Answer: Above are sol^{ns} of Homo LDE with const. coeffs whose sol^{ns} are of the form $y(x) = e^{\lambda x}$, where

$\therefore \lambda = 0, \pm 3i$

$\lambda(\lambda + 3i)(\lambda - 3i) = 0$, $e^{\lambda x} \neq 0$.

$\lambda(\lambda^2 + 9) = 0$.

$\therefore \lambda^3 + 9\lambda = 0$

$\lambda^3 e^{\lambda x} + 9\lambda e^{\lambda x} = 0$.

$\therefore \boxed{y''' + 9y' = 0.}$

Question [II](5 marks)

(1) Find orthogonal trajectories of the family of curves $4x^2 + 9y^2 = k$, where k is a constant. [CO2][2]

Detailed Answer:

$4x^2 + 9y^2 = K$

$8x + 18yy' = 0$ — ½m

$y' \to -\frac{1}{y'}$ — ½m

$4x + 9y\left(-\frac{1}{y'}\right) = 0$

$4\frac{dy}{y} = 9\frac{dx}{x}$

$4\ln y = 9\ln x + \ln c$

$\boxed{y^4 = cx^9}$ — 1m

(2) Solve the differential equation $y'' = 1 + (y')^2$. [CO3][2]

Detailed Answer:

$y'' = 1 + (y')^2$ — (I)

let $y' = z$, $y'' = z'$ } — $\frac{1}{2}$m

$\therefore$ (I) beomes, $z' = 1 + z^2$ — $\frac{1}{2}$m

$\therefore \frac{dz}{1+z^2} = dx$

$\therefore \tan^{-1}(z) = x + c$

$\therefore z = \tan(x+c)$ — $\frac{1}{2}$m

$\therefore \frac{dy}{dx} = \tan(x+c)$

$dy = \tan(x+c)\, dx$

$\boxed{y = \log_e|\sec(x+c)| + K}$ — $\frac{1}{2}$m

(3) Fill in the blank. The set $\{x^2, \frac{1}{x^2}, 0\}$ is linearly .dep... (independent / dependent) over $(0, \infty)$.

[CO1][1]

Question [III](5 marks)

(1) Let $M(x,y)$ and $N(x,y)$ be continuous and have continuous first partial derivatives. If $M(x,y)\,dx + N(x,y)\,dy = 0$ is an exact differential equation then prove that $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$. [CO4][2]

Detailed Answer:

Since $M(x,y)dx + N(x,y)dy = 0$ is an exact DE, we have

$M dx + N dy = du$ — (I) $(\frac{1}{2})$ for some f^n u of x & y. $(\frac{1}{2})$

But $du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$ — (II)

(As u has cont. partial derivatives)

Comparing (I) & (II), we get $M = \frac{\partial u}{\partial x}$ & $N = \frac{\partial u}{\partial y}$

$\Rightarrow \frac{\partial M}{\partial y} = \frac{\partial^2 u}{\partial y \partial x}$ &

$\frac{\partial N}{\partial x} = \frac{\partial^2 u}{\partial x \partial y}$ — $(\frac{1}{2})$

As u is continuous,

$\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ — $(\frac{1}{2})$

(2) Solve $\frac{dy}{dx} = \frac{y(2y^3+x)}{6x^2}$. [CO3][3]

Detailed Answer:

$$6x^2 \frac{dy}{dx} = 2y^4 + xy$$

$$\therefore 6x^2 \frac{dy}{dx} - xy = 2y^4$$

$$\Rightarrow \frac{dy}{dx} - \frac{1}{6x} y = \frac{1}{3x^2} y^4 \quad -\left(\tfrac{1}{2}\right)$$

which is Bernoulli's Eq^n.

Divide by y^4,

$$y^{-4} \frac{dy}{dx} - \frac{1}{6x} y^{-3} = \frac{1}{3x^2} \quad -\left(\tfrac{1}{2}\right)$$

put $y^{-3} = t$

$$\therefore -\frac{1}{3}\frac{dt}{dx} - \frac{1}{6x} t = \frac{1}{3x^2}$$

$$\Rightarrow \frac{dt}{dx} + \frac{1}{2x} t = -\frac{1}{x^2} \quad -\left(\tfrac{1}{2}\right)$$

which is a LDE.

$$\therefore IF = e^{\int \frac{1}{2x} dx} = \sqrt{x} \quad -\left(\tfrac{1}{2}\right)$$

$$\therefore \text{G.S. is } t\sqrt{x} = \int \sqrt{x}\left(-\frac{1}{x^2}\right) dx + c \quad -\left(\tfrac{1}{2}\right)$$

$$\Rightarrow y^{-3}\sqrt{x} = -\int x^{-3/2} dx + c$$

$$\Rightarrow y^{-3}\sqrt{x} = \frac{2}{\sqrt{x}} + c$$

$$\Rightarrow \sqrt{x} = y^3 \frac{2}{\sqrt{x}} + cy^3 \quad -\left(\tfrac{1}{2}\right)$$

Question [IV](5 marks)

(1) Solve: $(16D^4 + 24\,D^2 + 9\,I)\,y = 0$ [CO3][2]

Detailed Answer:

Given eq^n is 4^{th} order HLDE with constant coefficients.

$$16y^{(iv)} + 24y'' + 9y = 0 \quad \rightarrow \left(\tfrac{1}{2}\right)$$

It's AE is

$$16\lambda^4 + 24\lambda^2 + 9 = 0$$

$$\Rightarrow (4\lambda^2+3)(4\lambda^2+3) = 0$$

$$\Rightarrow \lambda^2 = -\frac{3}{4}, \quad \lambda^2 = -\frac{3}{4}$$

$$\Rightarrow \lambda = \pm\frac{\sqrt{3}}{2} i, \ \pm\frac{\sqrt{3}}{2} i \quad -\left(\tfrac{1}{2}\right)$$

(repetated complex roots)

$$\therefore y_1(x) = \cos\left(\frac{\sqrt{3}}{2}x\right),$$

$$y_2(x) = \sin\left(\frac{\sqrt{3}}{2}x\right),$$

$$y_3(x) = x\cos\left(\frac{\sqrt{3}}{2}x\right)$$

$$y_4(x) = x\sin\left(\frac{\sqrt{3}}{2}x\right)$$

are linearly independent sol^{n}s

$\therefore$ General sol^n of given ODE is.

$$y(x) = c_1\cos\left(\frac{\sqrt{3}}{2}x\right) + c_2\sin\left(\frac{\sqrt{3}}{2}x\right) + c_3 x\cos\left(\frac{\sqrt{3}}{2}x\right) + c_4 x\sin\left(\frac{\sqrt{3}}{2}x\right) \quad -(1)$$

where c_1, c_2, c_3 and c_4 are arbitary consts.

(2) Find the current in the RL circuit with a 4-ohm resistor, a 0.1-henry inductor, and a 20-volt e.m.f, if initial current in the circuit is zero. [CO5][3]

Detailed Answer:

Given data $R = 4$, $L = 0.1$, $E = 20$

$L\frac{dI}{dt} + RI = E(t)$, $I(0) = 0$

$0.1\frac{dI}{dt} + 4I = 20$, $I(0) = 0$

$\frac{dI}{dt} + 40I = 200$, $I(0) = 0$ — (1)

→ (1) which is first order linear ODE with Initial condition (IC)

soln of (1) is.

$I\,(I.F) = \int 200\,(I.F)\,dt + c$. — (2)

But Integrating factor $\boxed{(I.F) = e^{40t}}$ — $(\frac{1}{2})$

∴ By (2)

$I \cdot e^{40t} = \int 200\, e^{40t}\, dt + c$. $(\frac{1}{2})$

$I \cdot e^{40t} = \frac{20\cancel{0}}{4\cancel{0}} e^{40t}\ c$

$I \cdot e^{40t} = 5\, e^{40t} + c$

$I(t) = 5 + c\, e^{-40t}$

But $\boxed{I(0) = 0}$ — $(\frac{1}{2})$

∴ $0 = 5 + c \Rightarrow c = -5$

∴ $I(t) = 5 - 5\, e^{-40t}$

$\boxed{I(t) = 5(1 - e^{-40t})}$.

— $(\frac{1}{2})$

ROUGH WORK (Will Not Be Assessed)