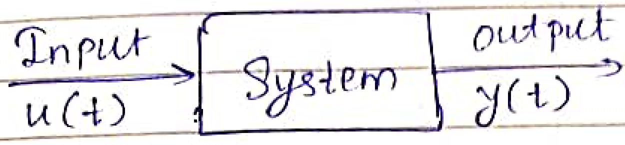
summing point

disturbance cannot be compensated.

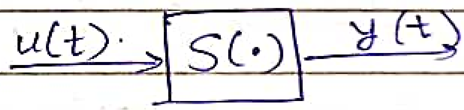
System:



Dynamic system: Time is the independent variable.
All variables that are associated with the system are f^n of time.

System :- (a collection of objects) / (a process) that is under study.

Mathematically, we can visualize a dynamic system as a mapping $S(\cdot)$ from $u(t)$ to $y(t)$.
i.e. $y(t) = S(u(t))$



Classification of dynamic systems:

SISO vs MIMO : SISO $\rightarrow$ Single input single output
MIMO $\rightarrow$ Multiple input multiple output

- SISO $\rightarrow$ $u(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$ $\quad t \in \mathbb{R}$
  $\rightarrow$ $y(\cdot) : \mathbb{R} \rightarrow \mathbb{R}$.
- MIMO $\rightarrow$ m inputs & p outputs, $m > 1, p > 1$
  $u(\cdot) : \mathbb{R} \rightarrow \mathbb{R}^m$
  $y(\cdot) : \mathbb{R} \rightarrow \mathbb{R}^p$

2) $\boxed{\text{Linear}}$ vs nonlinear:

Consider these two sets of i/p & o/p for a system $S(\cdot)$

① $u_1(t) \rightarrow \boxed{S(\cdot)} \rightarrow y_1(t)$

② $u_2(t) \rightarrow \boxed{S(\cdot)} \rightarrow y_2(t)$

The system $S(\cdot)$ is linear if

(i) $u_1(t)+u_2(t) \rightarrow \boxed{S(\cdot)} \rightarrow y_1(t)+y_2(t)$

(ii) $c u_1(t) \rightarrow \boxed{S(\cdot)} \rightarrow c y_1(t)$

$c \in \mathbb{R}$

or $c_1 u_1(t) + c_2 u_2(t) \rightarrow \boxed{S(\cdot)} \rightarrow c_1 y_1(t) + c_2 y_2(t)$

$c_1, c_2 \in \mathbb{R}$

(superposition)

) $\boxed{\text{Time invariant}}$ or Time varying:

A time invariant system is one ~~where~~ that provides the same o/p for the same i/p irrespective of when the input is given.

If $y(t) = S(u(t))$, then time invariance $\Rightarrow$

$$y(t-T) = S(u(t-T)) \quad \forall\, T \in \mathbb{R}.$$

Ex. time scale of interest needs to be noted.

[Causal] v/s Non-causal system:
A causal system is one where the o/p at any instant of time depends only ~~of~~ on past and current inputs.

Causal system is ~~a~~ Non-Anticipative.

Class of systems (under study):

SISO $\underbrace{\text{LINEAR TIME INVARIANT}}_{\text{LTI}}$ CAUSAL DYNAMIC SYSTEMS

Control: Making a system to behave as desired.

If we wish to achieve a desired angular speed, what is the input voltage that should be provided? (for DC motor)

Steps involved to solve:

I. Develop a mathematical representation for S(·)
→ Mathematical modelling of dynamic systems
1) Physics based
2) Empirical (Data driven)
3) Mixed approach.

II. Analyze the system response.

III. Design the controller.

Laplace Transform

- A system represented by a differential eq^n is difficult to model as a block diagram.
  Thus, with Laplace Transform, we can represent the input, output and system as seperate entities.
  Further, their interrelationship will be simply algebraic.

- Laplace transform is defined as

$$\mathcal{L}[f(t)] = F(s) = \int_{0^-}^{\infty} f(t) \cdot e^{-st} \, dt$$

where $s = \sigma + j\omega$, a complex variable.
Thus, knowing $f(t)$ and that the integral in the eq^n exists, we can find a function, $F(s)$, that is called the Laplace transform of $f(t)$.

- The notation for the lower limit means that even if $f(t)$ is discontinuous at $t=0$ we can start the integration prior to the discontinuity as long as the integral converges. Thus, we can find the laplace transform of impulse functions.

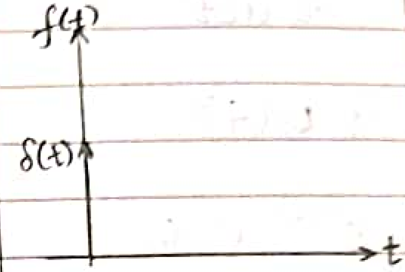
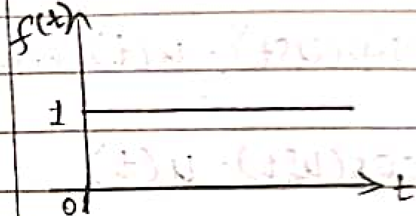
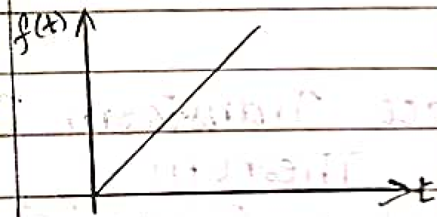
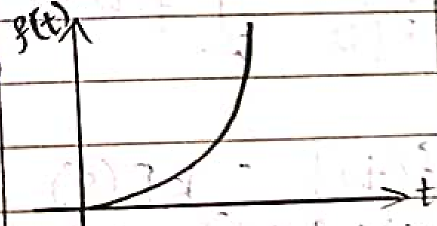
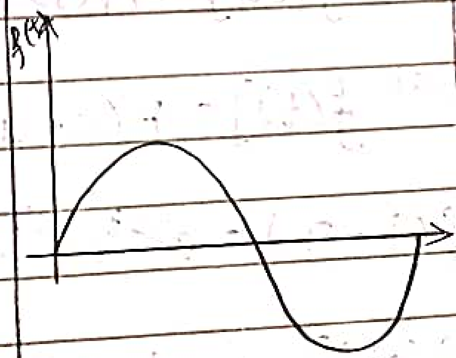
- The inverse Laplace transform which allows us to find $f(t)$ given $F(s)$, is,

$$\mathcal{L}^{-1}[F(s)] = \frac{1}{2\pi j} \int_{\sigma - j\infty}^{\sigma + j\infty} F(s) \cdot e^{st} \, ds = f(t) \cdot u(t)$$

where $u(t) = \begin{cases} 1 & \text{for } t > 0 \\ 0 & \text{for } t < 0 \end{cases}$

$u(t)$ is the unit step function.
Multiplication of $f(t)$ by $u(t)$ yields a time function tha
is zero for $t<0$.

Waveforms used in control systems.

Function	Description	Sketch	Use
$\delta(t)$	$\delta(t) = \infty$ for $0^- < t < 0^+$ $= 0$ elsewhere $\int_{0^-}^{0^+} \delta(t)\,dt = 1$.		Transient response Modeling
$u(t)$	$u(t) = 1$ for $t > 0$ $= 0$ for $t < 0$		Transient res Steady-state
$t \cdot u(t)$	$t \cdot u(t) = t$ for $t \geq 0$ $= 0$ ~~for~~ elsewhere		Steady-state
$\frac{1}{2}t^2 \cdot u(t)$	$\frac{1}{2}t^2 \cdot u(t) = \frac{1}{2}t^2$ for $t \geq 0$ $= 0$ elsewhere		Steady-st er
$\sin(\omega t)$			Transient r Modeling Steady-st error

$u(t)$ is the unit step function.
Multiplication of $f(t)$ by $u(t)$ yields a time function that is zero for $t<0$.

Waveforms used in control systems.

	Function	Description	Sketch	Use
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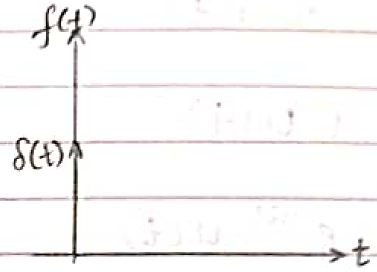
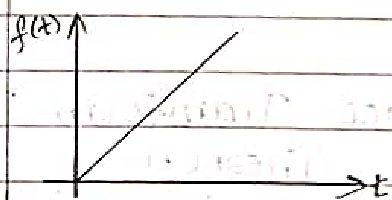
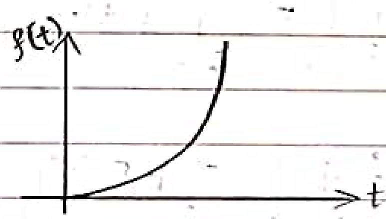
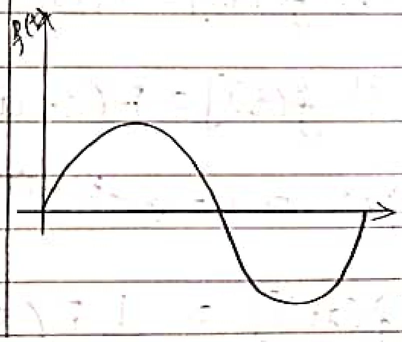
ule	$\delta(t)$	$\delta(t)=\infty$ for $0^-<t<0^+$ $=0$ elsewhere $\int_{0^-}^{0^+}\delta(t)dt=1$.		Transient response Modeling
p	$u(t)$	$u(t)=1$ for $t>0$ $=0$ for $t<0$		Transient response Steady-state error
amp	$t\cdot u(t)$	$t\cdot u(t)=t$ for $t\geq 0$ $=0$ ~~for~~ elsewhere		Steady-state error
ola	$\frac{1}{2}t^2\cdot u(t)$	$\frac{1}{2}t^2\cdot u(t)=\frac{1}{2}t^2$ for $t\geq 0$ $=0$ elsewhere		Steady-state error
u- dal	$\sin(\omega t)$			Transient response Modeling Steady-state error

Laplace Transform Table

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$t\cdot u(t)$	$\frac{1}{s^2}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{-at}\cdot u(t)$	$\frac{1}{s+a}$
$\sin(\omega t)\cdot u(t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos(\omega t)\cdot u(t)$	$\frac{s}{s^2+\omega^2}$

Laplace Transform Theorems

Theorem	Name
$\mathcal{L}[f(t)] = \int_{0^-}^{\infty} f(t)\cdot e^{-st}\,dt = F(s)$	Definition
$\mathcal{L}[kf(t)] = kF(s)$ $\mathcal{L}[f_1(t)+f_2(t)] = F_1(s) + F_2(s)$	Linearity theorem
$\mathcal{L}[e^{-at}\cdot f(t)] = F(s+a)$	Frequency shift theorem
$\mathcal{L}[f(t-T)] = \cancel{e^{-st}}\, e^{-sT}\cdot F(s)$	Time shift theorem.
$\mathcal{L}[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$	Scaling theorem

2-7 35, 36, 37, 41, 42, 43, 44 & 49

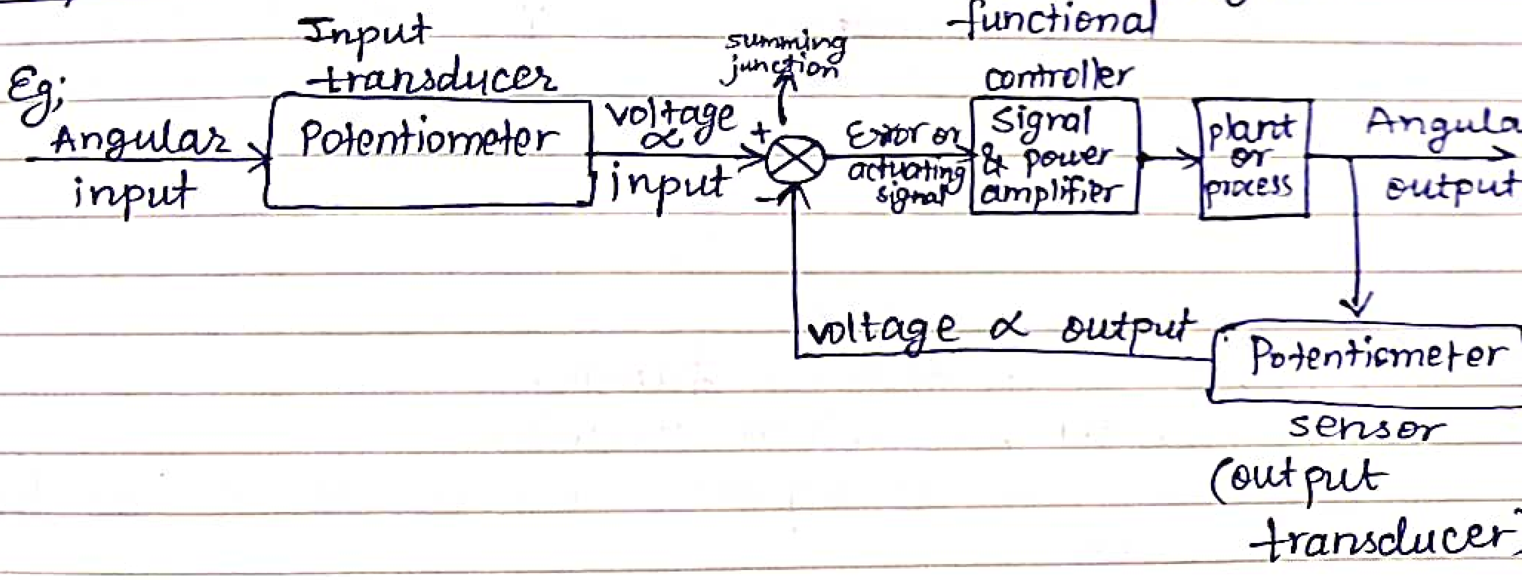
Theorem	Name
$\mathcal{L}\left[\frac{df}{dt}\right] = sF(s) - f(0^-)$	Differentiation theorem
$\mathcal{L}\left[\frac{d^2f}{dt^2}\right] = s^2F(s) - sf(0^-) - f'(0^-)$	
$\mathcal{L}\left[\frac{d^nf}{dt^n}\right] = s^nF(s) - \sum_{k=1}^{n} s^{n-k} f^{k-1}(0^-)$	
$\mathcal{L}\left[\int_{0^-}^{t} f(\tau)\, d\tau\right] = \frac{F(s)}{s}$	Integration theorem
$f(\infty) = \lim_{s\to 0} sF(s)$	Final value theorem
$f(0) = \lim_{s\to \infty} sF(s)$	Initial value theorem

The 6 stages of Design Process :

Step 1 : Having specifications - identifying numeric needs of system.

Step 2 : Representation in terms of functional block diagram

Eg;



PTO.

- ***Step 3*** : Schematic diagram.
- ***Step 4*** : Mathematical model
  (Electrical, mechanical, electromechanical)
- ***Step 5*** : Simplify the process in block diagram into a single bloc
  (just like this ↓)

  $i/p \longrightarrow \boxed{\text{process}} \longrightarrow o/p$

- ***Step 6*** : Analysis

☆ Transfer function.

For ~~an~~ a LTI system, transfer function is the ratio of Laplace transform of the output to the Laplace transform of the input with the initial conditions being zero.

Mathematically, if $U(s)$ is the Laplace transform of the input function and $Y(s)$ is the Laplace transform of the output, the transfer function $G(s)$ is given

$$\boxed{G(s) = \frac{Y(s)}{U(s)}}$$

Properties of transfer function.

1. ~~Transfer~~ Zero initial condition
2. It is same as Laplace transform of its impulse response.
3. Replacing 's' by $\frac{d}{dt}$ in the transfer function,

the differential equation can be obtained.
4. Poles and zeros can be obtained from the transfer function
5. Stability can be known
6. Can be applicable to linear system only.

Advantages of Transfer function:
1. It is a mathematical model and gain of the system.
2. ~~It is~~ Impulse response can be found.
(3., 4., 5. same as properties 3., 4., 5.)

Disadvantages of Transfer function:
1. Applicable to linear system only.
2. Not applicable if initial condition cannot be neglected.
3. Gives no information about actual structure of a physic[al] system

General form of transfer function of a system:

$$G(s) = \frac{Y(s)}{U(s)} = \frac{b_0 s^m + b_1 s^{m-1} + b_2 s^{m-2} + \dots + b_{m-1} s + b_m}{a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \dots + a_{n-1} s + a_n}$$

$$= \frac{K((s-z_1)(s-z_2)(s-z_3)\cdots(s-z_m))}{(s-p_1)(s-p_2)(s-p_3)\cdots(s-p_n)}.$$

n : order of the system

K : System gain or gain factor - A proportional value th[at] relates the magnitude of the input to that of the o[utput] signal at steady state.

$z_1, z_2, \dots, z_m$: Zeroes of the system.

$p_1, p_2, p_3, \dots, p_n$: Poles of the system.

$n \geq m$ because the system becomes non-casual and is not phy[sically] realizable if $n < m$.

// Poles, zeros, system gain K together characterise the I/O system dynamics.

- Poles : Roots of the denominator polynomial of the T.F.

(x) on cartesian coordinate system

Values of s at which the transfer f^n becomes unbounded

$$\lim_{s \to p_i} G(s) = \infty$$

- Zeros : Roots of the numerator polynomial of the T.F.

(o)

Values of s at which the T.F. becomes zero, i.e. vanishes.

$$\lim_{s \to z_i} G(s) = 0$$

☆ Laplace transforms can be solved using partial fraction method

Usually ~~Suppose~~, a system is represented by following dynamic eq^n

$$G(s) = \frac{Y(s)}{U(s)}$$

The factors of denominator, $U(s)$ is represented by following forms :
i. Unrepeated factors.
ii. Repeated factors
iii. Unrepeated complex factors

Unrepeated factors :

Q.1. Find laplace's inverse :

1) $\dfrac{s+6}{(s+1)(s+2)} = G(s)$ (say)

→ Let, $\dfrac{s+6}{(s+1)(s+2)} = \dfrac{k_1}{s+1} + \dfrac{k_2}{s+2}$ — (1)

Multiplying (1) by $(s+1)$ and put $s=-1$

$$\frac{5}{1} = k_1 \Rightarrow \boxed{k_1 = 5}$$

Multiplying ① by (s+2) & put s = -2,

$$\therefore \frac{4}{-1} = k_2$$

$$\therefore \boxed{k_2 = -4}$$

$$\therefore G(s) = \frac{5}{s+1} - \frac{4}{s+2}$$

$$\therefore G^{-1}(s) = 5e^{-t} - 4e^{-2t}$$

$$\frac{6}{(s+1)(s+2)(s+3)} = G(s) \text{ (say)}$$

Let, $$\frac{6}{(s+1)(s+2)(s+3)} = \frac{k_1}{s+1} + \frac{k_2}{s+2} + \frac{k_3}{s+3} \quad -(I)$$

mult. (I) by (s+1) & put s = -1

$$\therefore \frac{6}{1 \cancel{\times} \times 2} = k_1$$

$$\Rightarrow \boxed{k_1 = 3}$$

Mult. (I) by (s+2) & put s = -2

$$\therefore \frac{6}{(-1)(1)} = k_2 \Rightarrow \boxed{k_2 = -6}$$

Mult. (I) by (s+3) & put s = -3

$$\therefore \frac{6}{(-2)(-1)} = k_3 \Rightarrow \boxed{k_3 = +3}$$

$$\therefore G(s) = \frac{3}{s+1} - \frac{6}{s+2} + \frac{3}{s+3}$$

$$\therefore G^{-1}(s) = 3e^{-t} - 6e^{-2t} + 3e^{-3t}.$$

* When ~~d~~DC gain of a system, i.e. $\lim_{s\to 0} G(s)$, is not equal to 1, the o/p won't reach it's final value.

Unrepeated complex factors :-

Q. $\dfrac{50(s+3)}{(s+1)(s+2)(s^2+2s+5)} = G(s)$ (say)

→ $\therefore G(s) = \dfrac{A_1}{s+1} + \dfrac{A_2}{s+2} + \dfrac{A_3 s + A_4}{s^2+2s+5}$ — ①

Mult. ① by $(s+1)$ & put $s=-1$,

$\therefore \dfrac{50\times 2}{1\times 4} = A_1 \Rightarrow \boxed{A_1 = 25}$

Mult. ① by $(s+2)$ & put $s=-2$

$\therefore \dfrac{50\times 1}{(-1)(5)} = A_2 \Rightarrow \boxed{A_2 = -10}$

~~Mult ① by (~~

$\therefore ① \Rightarrow \dfrac{50(s+3)}{(s+1)(s+2)(s^2+2s+5)} = \dfrac{25}{s+1} - \dfrac{10}{s+2} + \dfrac{A_3 s + A_4}{s^2+2s+5}$

Put $s=1$ in ②

$\therefore \dfrac{50\times 4}{2\times 3\times 8} = \dfrac{25}{2} - \dfrac{10}{3} + \dfrac{A_3 + A_4}{8}$

$\therefore 6(A_3 + A_4) = 50\times 4 - 25\times 24 + 10\times 16$

$\therefore 6(A_3 + A_4) = 200 - 600 + 160$

$\boxed{A_3 + A_4 = -40}$ — ③

Put $s=0$ in ②

$\therefore \dfrac{50\times 3}{1\times 2\times 5} = 25 - 5 + \dfrac{A_4}{5}$

$\Rightarrow \dfrac{A_4}{5} = 15 - 20 \Rightarrow \boxed{A_4 = -25}$

$\therefore$ (3) $\Rightarrow$ $\boxed{r_2 = -1.5}$

$$\therefore G(s) = \frac{25}{s+1} - \frac{10}{s+2} - \frac{(15s+20)}{s^2+2s+5}$$

$$\cancel{G^{-1}(s)} = \cancel{25e}$$

$$G(s) = \frac{25}{s+1} - \frac{10}{s+2} - \frac{(15(s+1)+10)}{(s+1)^2+(2)^2}$$

$$= \frac{25}{s+1} - \frac{10}{s+2} - \frac{15(s+1)}{(s+1)^2+2^2} - \frac{\cancel{10}5\times 2}{(s+1)^2+(2)^2}$$