College of Engineering Pune

(An Autonomous Institute of Govt. of Maharashtra)

(MA-20001) Ordinary Differential Equations and Multivariate Calculus

Program : S.Y.B.Tech. Sem. III (All Branches) — Academic Year : 2022-23

Examination : End Semester Examination — Maximum Marks : 60

Date : 29/01/2023 — Time : 02.30 pm to 05.30 pm

Student MIS Number : | | | | | | | | | |

Instructions :

1. Write your MIS Number on Question Paper.
2. Writing anything on question paper is not allowed.
3. Mobile phones and programmable calculators are strictly prohibited.
4. Exchange/Sharing of stationery, calculator etc. is not allowed.
5. Figures to the right indicate the course outcomes and maximum marks.
6. Unless otherwise mentioned symbols and notations have their usual standard meanings.
7. Any essential result, formula or theorem assumed for answering questions must be clearly stated.

Q.1 **Attempt the following:**

(i) Solve $x\,\dfrac{dy}{dx} = (y-x)^3 + y$. (CO2)[1.5]

(ii) Say true or false and justify your answer : The integrating factor of a differential equation is unique. (CO4)[1]

(iii) Find the Laplace transform of $f(t) = e^{-t}\sinh 4t$. (CO2)[1]

(iv) Find the Laplace transform of $f(t) = sin^4 t$. (CO3)[2]

OR

Find the Laplace transform of $f(t) = te^{-t}cost$. (CO3)[2]

(v) Find the inverse Laplace transform of $\displaystyle\sum_{k=1}^{4} \frac{(k+1)^2}{s+k^2}$ (CO5)[2]

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1.2 19 to 33

(vi) Find the domain and the range of the function $f(x,y) = 4 - \sqrt{x^2+y^2}$. Sketch the surface $z = f(x,y)$ and the level curve $f(x,y) = -3$. (CO3)[3]

(vii) State two path test for non-existence of limit of a function of two variables. Find the following limit, if it exists:

$$\lim_{(x,y)\to(0,0)} \left(\frac{-x}{\sqrt{x^2+y^2}}\right)$$ (CO1, CO3)[3]

(viii) Let $f(x,y) = \dfrac{x^2-y^2}{x^2+y^2}$ for $(x,y) \neq (0,0)$. Is it possible to define $f(0,0)$ in a way that makes f continuous at the origin ? Why ? (CO3)[1.5]

Q.2 Attempt the following:

(i) State giving reason whether the functions $\cos^2 x, \sin^2 x$ and $\cos 2x$ are linearly independent or dependent. (CO2)[1]

(ii) Using the method of reduction of order, determine $y_2(x)$, the other linearly independent solution of $(1+x^2)y'' - 2xy' + 2y = 0$, if $y_1(x) = x$ is one solution. (CO3)[1.5]

(iii) Using appropriate properties /theorems of Laplace transforms, evaluate

$$\int_0^{\infty} e^{-\sqrt{3}t} \left\{\frac{\sin t}{t}\right\} dt$$ (CO3)[2]

(iv) Find the inverse Laplace transform of $\tan^{-1}\left(\frac{2}{s^2}\right)$. (CO4)[3]

OR

Find the inverse Laplace transform of $\ln\left(\frac{s^2+s-6}{s^2+s+1}\right)$. (CO4)[3]

(v) If the partial derivatives f_x, f_y of a function $f(x,y)$ exist over an open region R then the function $f(x,y)$ is differentiable at every point of R. State true or false using appropriate result. (CO1)[2]

(vi) Calculate $\lim_{(x,y)\to(1,1)} f(x,y)$, if it exists, where (CO3)[3]

$$f(x,y) = \begin{cases} x & , \quad xy \neq 1 \\ x^2+y^2 & , \quad xy = 1 \end{cases}$$

(vii) If $z = f(x, y)$, $x = g(t, s)$ and $y = h(t, s)$ then draw the branch/tree diagram and write the chain rule for $\dfrac{\partial z}{\partial t}$. (CO2)[1.5]

(viii) Find $\dfrac{\partial f}{\partial x}$ at $(-2, 1)$ if $f(x, y) = \tan^{-1}\left(\frac{y}{x}\right)$. (CO2)[1]

Q.3 **Attempt the following:**

(i) A force of 400N stretches a spring 2 meters. A mass of 50 kg is attached to the end of the spring and is initially released from the equilibrium position with an upward velocity of 10m/s. Find the equation of motion. Also determine the natural frequency, period and amplitude. (CO4)[3]

OR

A large tank is initially filled with $100L$ of brine in which $1kg$ of salt is dissolved. Brine containing $0.5kg$ of salt *per Liter* is pumped into the tank at a rate of $6L/min$. The well-mixed brine is pumped out of the tank at a slower rate of $4L/min$. Assuming that the tank does not overflow, find the amount of salt in the tank after t minutes. Give your answer to the nearest gram. (CO4) [3]

(ii) Use properties of Laplace transform to find :

a) $\mathcal{L}\left\{f(t)\delta(t) + \sin(5t - 5)u(t - 1)\right\}$. (CO2)[2]

b) $\mathcal{L}^{-1}\left\{e^{-3s}\frac{s+1}{s^2+2s+2}\right\}$. (CO3)[2]

c) Find $\mathcal{L}\left\{f(t) = t^2;\ \ 0 < t < 2\right\}$, $f(t)$ is periodic with period 2. (CO2)[2]

(iii) Prove: If $F(x, y)$ is differentiable and the equation $F(x, y) = 0$ defines y implicitly as a differentiable function of x then, at any point where $F_y \neq 0$,

$$\frac{dy}{dx} = -\frac{F_x}{F_y}.$$

Hence find $\frac{dy}{dx}$ at the point $(0, \ln 2)$ if $xe^y + \sin(xy) + y = \ln 2$. (CO3)[3]

(iv) Let $T(x, y, z) = x^2 + 2y^2 + 2z^2$ be a function which gives the temperature at any point in space. Let $P = (1, 1, 1)$. Find : (CO3)[3]

a) grad T at the point P,

b) the directional derivative of T at the point P in the direction of $\bar{v} = 2\hat{j} + \hat{j} + 2\hat{k}$,

c) In which direction should you go to get the most rapid decrease in T at the point P ? What is the directional derivative in this direction ?

Q.4 **Attempt the following:**

(a) Solve **any one**: (CO3)[2.5]

(i) Find the solution of $(D^3 - 2D^2 - 9D + 18I)y = e^{2x}$ using the method of undetermined coefficients.

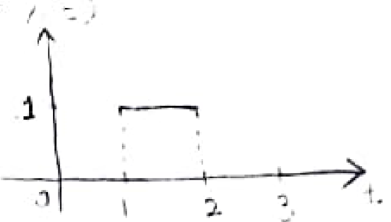
(ii) If $y'' + p(x)y' + q(x)y = r(x)$ then prove that $y_p = -y_1 \int \frac{y_2 r}{W} dx + y_2 \int \frac{y_1 r}{W} dx$ is a solution where y_1 and y_2 are linearly independent solutions of $y'' + p(x)y' + q(x)y = 0$ and $W = y_1 y_2' - y_2 y_1'$.

(b) Solve the following:

(i) Find Laplace transform of the convolution of $f(t)$ and $g(t)$ where $f(t) = \cos \omega t$ and

$g(t) = e^{-at}$ (CO2)[1]

(ii) Determine the response of the damped mass spring system under a square wave modeled by the equation $y'' + 3y' + 2y = r(t)$ where $r(t)$ is as shown below:



and the initial conditions $y(0) = y'(0) = 0$ (CO5)[4]

(c) Solve **any three**: (CO3)[7.5]

(i) Find all local maxima, minima and saddle points for the function $f(x, y) = e^{2x} \cos y$.

(ii) Find the coldest and the hottest point(s) on a circular plate $x^2 + y^2 \leq 1$ if the temperature at any point (x, y) is given by $T(x, y) = x^2 + 2y^2 - x$.

(iii) Find the points lying on the curve $x^2 + xy + y^2 = 1$ in the xy plane that are closest and farthest from the origin.

(iv) Discuss the local extrema at $(0, 0)$ for different values of k for the function $f(x, y) = x^2 + kxy + y^2$.