For checking whether the function of 2/3/4 variables have local maxima, local minima, or saddle pts.

By second Derivative Test & it's extension for the f^ns of 3/4 variables

(I) For the function of 2 variables $f(x,y)$ i.e. $f : \mathbb{R}^2 \to \mathbb{R}$

Discriminant or Hessian of f is $f_{xx} f_{yy} - f_{xy}^2 = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$ $(= H)$

(Here by second Der. Test, f & it's first & second par. der. are continuous $\therefore f_{xy} = f_{yx}$)

(i) $f_{xx} < 0$, $f_{xx} f_{yy} - f_{xy}^2 > 0$ i.e. $|H| > 0 \to$ Local Max.

(ii) $f_{xx} > 0$, $f_{xx} f_{yy} - f_{xy}^2 > 0$ i.e. $|H| > 0 \to$ Local Min.

(iii) $f_{xx} f_{yy} - f_{xy}^2 < 0$ i.e. $|H| < 0 \to$ Saddle pt.

(II) For the function of 3 variables $f(x,y,z)$ i.e. $f : \mathbb{R}^3 \to \mathbb{R}$

$$\text{Hessian of } f = \begin{vmatrix} \boxed{f_{xx}} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{vmatrix} = H$$

H_1, H_2, H

(i) $\underset{|H_1|}{f_{xx}} < 0$, $\underbrace{\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}}_{|H_2|} = f_{xx} f_{yy} - f_{xy}^2 > 0$, &

$$\begin{vmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{vmatrix} = |H| < 0 \to \text{Local Max.}$$

i.e. Alternate signs $-, +, -$

(ii) $\underset{|H_1|}{f_{xx}} > 0$, $\underbrace{\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}}_{|H_2|} = f_{xx} f_{yy} - f_{xy}^2 > 0$, &

$$\begin{vmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{vmatrix} = |H| > 0 \to \text{Local Min.}$$

i.e. All signs +ve.

(iii) $f_{xx} f_{yy} - f_{xy}^2 < 0 \to$ saddle pt.

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(III) For the function of 4 variables $f(x,y,z,w)$ i.e. $f:\mathbb{R}^4 \to \mathbb{R}$.

$$\text{Hessian of } f = \begin{vmatrix} f_{xx} & f_{xy} & f_{xz} & f_{xw} \\ f_{yx} & f_{yy} & f_{yz} & f_{yw} \\ f_{zx} & f_{zy} & f_{zz} & f_{zw} \\ f_{wx} & f_{wy} & f_{wz} & f_{ww} \end{vmatrix} = H$$

H_1, H_2, H_3

(i) $\underbrace{f_{xx}}_{|H_1|} < 0$, $\underbrace{\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}}_{|H_2|} > 0$, $\underbrace{\begin{vmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{vmatrix}}_{|H_3|} < 0$

& $|H| > 0 \to$ Local Max.

i.e. Alternate ~~sing~~ sign $-, +, -, +$

(ii) $\underbrace{f_{xx}}_{|H_1|} > 0$, $\underbrace{\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}}_{|H_2|} > 0$, $\underbrace{\begin{vmatrix} f_{xx} & f_{xy} & f_{xz} \\ f_{yx} & f_{yy} & f_{yz} \\ f_{zx} & f_{zy} & f_{zz} \end{vmatrix}}_{|H_3|} > 0$

& $|H| > 0 \to$ Local min.

i.e. All +ve signs.

(iii) $f_{xx} f_{yy} - f_{xy}^2 < 0 \to$ Saddle pt.

! Find all the local maxima, local minima & saddle pts of the f^{ns}

(1) $f(x,y,z) = x^2 - xy + y^2 + yz + z^2 - 2z$

(2) $f(x,y,z,w) = x^2 + y^2 + z^2 - w^2 + xy + zw$

x