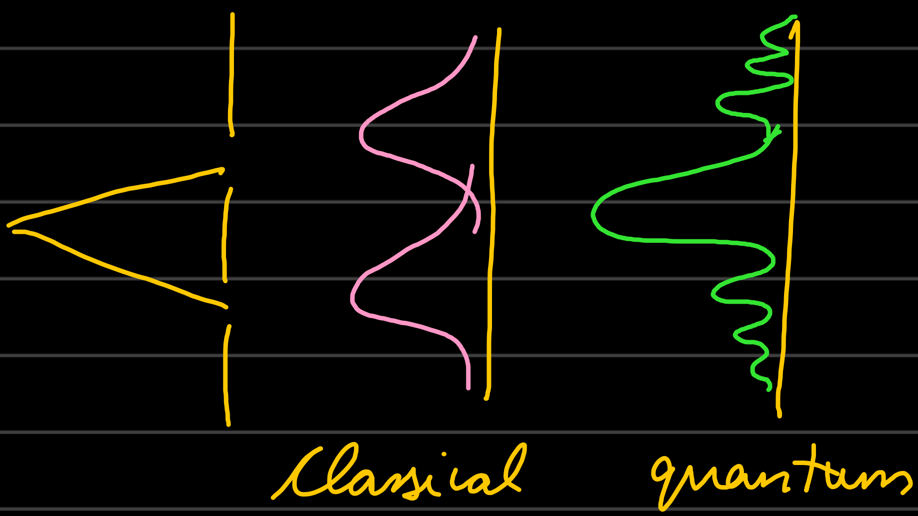
Lecture 6



$$E = E_0 e^{-i(kx+\omega t)}$$

$$k = \frac{2\pi}{\lambda} \quad \omega = \frac{2\pi}{T}$$

Born Interpretation:

$$\psi(r,t) \Rightarrow \quad \psi^* \psi = |\psi|^2 (x,y,z,t)$$

probability density of finding particle at (x,y,z) at t

probability $\Rightarrow |\psi|^2 dx\,dy\,dz$

Normalization condition: $\left.\int_{-\infty}^{\infty} \psi^* \psi \, d\underset{\sim}{x} \right|_t = 1$

(particle must exist somewhere)

* ψ must be continuous.
* $\frac{d\psi}{dx}$ is continuous

Schrodinger Equation:

$$-\frac{\hbar^2}{2m}\nabla^2 + V = E\psi = i\hbar\frac{\partial\psi}{\partial t}$$

$\psi(r,t) \Rightarrow$ soln to Schrodinger Equation.

↳ origin comes from the wave equation.

Components of S.E:

1) $-\frac{\hbar^2}{2m}\nabla^2 \rightarrow$ Kinetic Energy

2) $V \rightarrow$ Potential Energy

3) $E \rightarrow$ Energy Eigen value

4) $\psi \rightarrow$ Wave function / eigen vectors

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi = i\hbar\frac{\partial\psi}{\partial t}$$

→ Interacting force fields generate the potentials.

↳ eg: Electrostatic, Lorentz, Nuclear, etc.

* time independent force fields.

$$\boxed{\Psi(x,t) = \psi(x)\cdot \varpi(t)}$$

↳ possible when potential is time independent.

$$\left[-\frac{\hbar^2}{2m}\nabla^2 + V\right]\psi(x)\varpi(t) = i\hbar\frac{\partial}{\partial t}\left(\psi(x)\varpi(t)\right)$$

$$\varpi(t)\left[-\frac{\hbar^2\nabla^2}{2m} + V\right]\psi(x) = i\hbar\,\psi(x)\frac{\partial\varpi(t)}{\partial t} = E\Psi$$

↳ these are eigen expressions:

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi \qquad \text{Let soln: } \psi = Ae^{-i\omega t}$$

$$-i\hbar\, i\omega\,\psi = E\psi \Rightarrow E = -i^2\omega\hbar$$

$$\boxed{E = \hbar\omega}$$

$$\boxed{\omega = \frac{E}{\hbar}}$$ for time independent potential

$$\boxed{\varpi = Ae^{-\frac{iE}{\hbar}t}}$$

→ Ψ cannot be measured, we can measure only $|\Psi|^2 = \Psi^*\Psi$

$$\Psi^*\Psi = \psi^*\psi = e^{-\frac{iEt}{\hbar}} \cdot e^{\frac{iEt}{\hbar}}$$

$$= \underline{\underline{1}}$$

→ Probability does not depend on time.

↳ Free electron: $V=0$

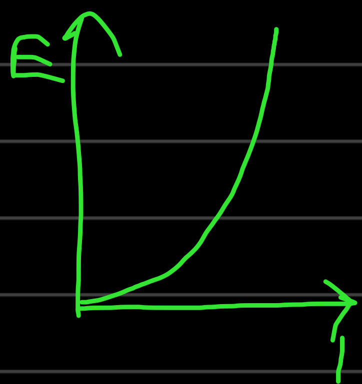
$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi \qquad \psi = Ae^{ikx} + Be^{-ikx}$$

$$\Psi(x,t) = Ae^{ikx} \cdot e^{-\frac{iEt}{\hbar}}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\left(Ae^{ikx}\right) = E\left(Ae^{ikx}\right)$$

$$-\frac{\hbar^2}{2m}(ik)(ik) = E$$

$$\boxed{E = \frac{k^2\hbar^2}{2m}}$$



dispersion relation

Normalization condition:

$$\int_{-\infty}^{\infty} \Psi^*\Psi \, dx = \int_{-\infty}^{\infty} A^*A \, dx = 1$$

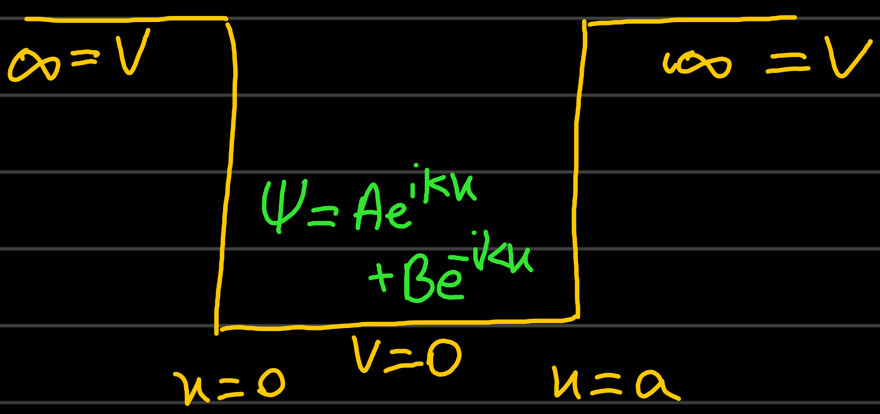
$$\int_{-\infty}^{\infty} |A|^2 dx = 1$$

$$\boxed{|A|^2 \int_{-\infty}^{\infty} dx = 1}$$

→ $\Psi(x,t) \Rightarrow$ plane travelling wave
↳ is not normalizable.

↳ As a result of Heisenberg uncertainity principle:
↳ If we have K to be finite, position x cannot be determined.

Case 2: Bounded Potential Well:



$$\Psi(x \leq 0) = \Psi(x \geq 0) = 0$$

$$\psi(x=0) = A+B=0 \Rightarrow B=-A$$

$$\psi(x) = A(e^{ikx} - e^{-ikx})$$

$$\psi(x) = 2Ai\sin kx$$

$$\psi(x=a) = 2Ai\sin ka = 0$$

$$ka = n\pi$$

$$k = \frac{n\pi}{a}$$

$n \rightarrow$ principal quantum number.

$n = 0, 1, 2, 3, 4, \ldots$

$\rightarrow$ k has become quantized, no longer continuous.

$$\psi(x) = 2iA\sin\left(\frac{n\pi x}{a}\right)$$

$$|\psi|^2 = \psi^*\psi = \left(-2iA\sin\left(\frac{n\pi x}{a}\right)\right)\left(2iA\sin\left(\frac{n\pi x}{a}\right)\right)$$

$$\boxed{|\psi|^2 = 4A^2\sin^2\left(\frac{n\pi x}{a}\right)}$$