Tutorial-3

1) $\phi(\underset{\sim}{r}, t)$

- $\underset{\sim}{\nabla} \wedge \phi = \frac{\partial}{\partial x_i} \underset{\sim}{\delta_i} \wedge \phi_j \Rightarrow$ DNE

→ cannot cross pdt a scalar.

- $\underset{\sim}{\nabla} \cdot \underset{\sim}{\nabla} \phi = \frac{\partial}{\partial x_i} \underset{\sim}{\delta_i} \cdot \frac{\partial \phi(x, t)}{\partial x_j}$

$$= \frac{\partial}{\partial x_i} \underset{\sim}{\delta_i} \cdot \left[\phi'(\underset{\sim}{r}, t) \frac{\partial r \delta_j}{\partial x_j} \right]$$

$$= \frac{\partial}{\partial x_i} \left[\phi'(\underset{\sim}{r}, t) \frac{\partial x}{\partial x_j} \right] \delta_{ij}$$

$$= \frac{\partial}{\partial x_i} \left[\phi'(\underset{\sim}{r}, t) \frac{\partial x}{\partial x_i} \right]$$

$$= \frac{\partial \phi'(x, t)}{\partial x_i} \cdot \frac{\partial x}{\partial x_i} + \phi'(x, t) \frac{\partial^2 x}{\partial x_i}$$

$$= \phi''(\underset{\sim}{r}, t) \cdot \left(\frac{\partial x}{\partial x_i}\right)^2 + \phi'(\underset{\sim}{r}, t) \frac{\partial^2 x}{\partial x_i}$$

$$\underset{\sim}{\nabla} \wedge \underset{\sim}{\nabla} \phi$$

$$\frac{\partial}{\partial x_i} \underset{\sim}{\delta_j} \wedge \frac{\partial \phi}{\partial x_j} \delta_j = \varepsilon_{ijk} \frac{\partial^2 \phi}{\partial x_i \partial x_j} \underset{\sim}{\delta_k}$$

$$= \varepsilon_{1jk} \frac{\partial^2 \phi}{\partial x_1 \partial x_j} + \varepsilon_{2jk} \frac{\partial^2 \phi}{\partial x_2 \partial x_j} + \varepsilon_{3jk} \frac{\partial^2 \phi}{\partial x_3 \partial x_j}$$

$$= \overset{0}{\cancel{\varepsilon_{11k}}} \frac{\partial^2 \phi}{\partial x_1 \partial x_1} + \varepsilon_{12k} \frac{\partial^2 \phi}{\partial x_1 \partial x_2} + \varepsilon_{13k} \frac{\partial^2 \phi}{\partial x_1 \partial x_3}$$

$$\varepsilon_{21k} \frac{\partial^2 \phi}{\partial x_1 \partial x_1} + \overset{0}{\cancel{\varepsilon_{22k}}} \frac{\partial^2 \phi}{\partial x_2 \partial x_2} + \varepsilon_{23k} \frac{\partial^2 \phi}{\partial x_2 \partial x_3}$$

$$\varepsilon_{31k} \frac{\partial^2 \phi}{\partial x_3^2 \partial x_1} + \varepsilon_{32k} \frac{\partial^2 \phi}{\partial x_3 \partial x_2} + \overset{0}{\cancel{\varepsilon_{33k}}} \frac{\partial^2 \phi}{\partial x_3 \partial x_3}$$

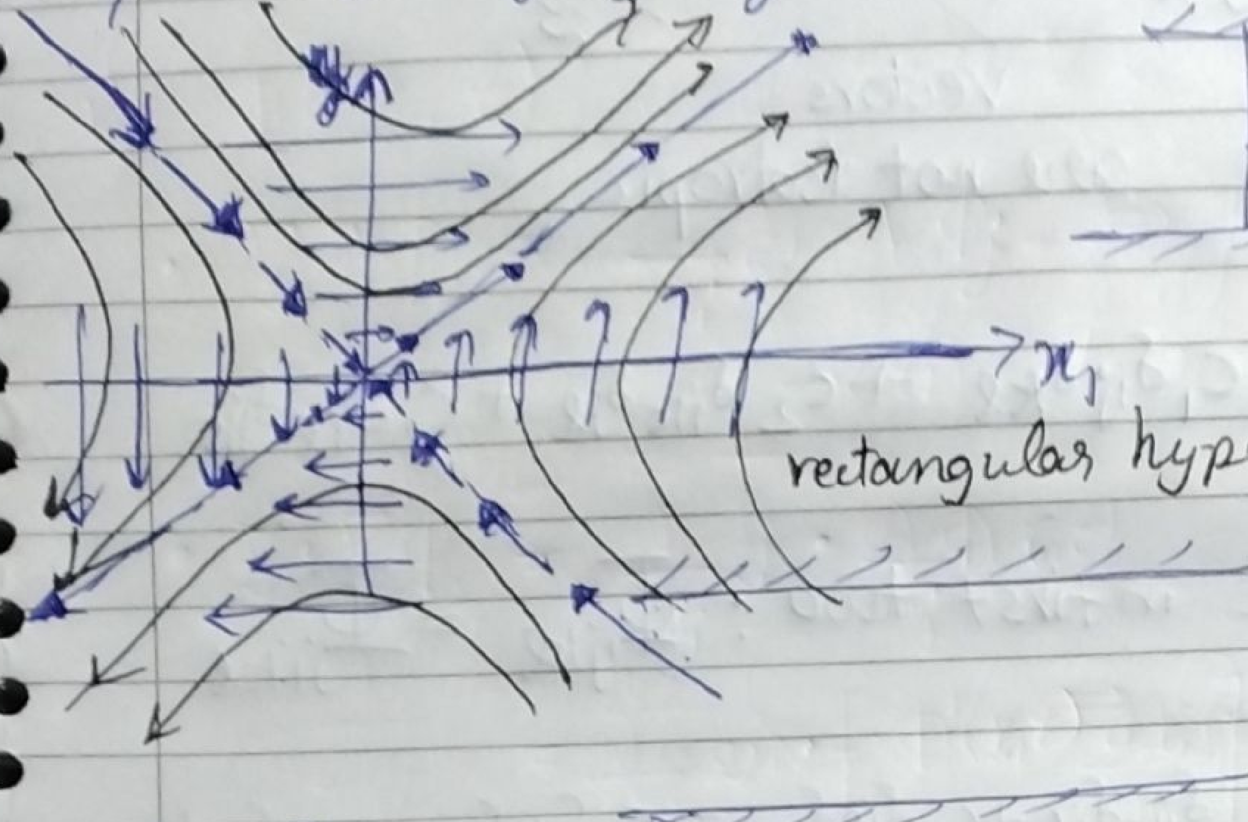
$$= \varepsilon_{123} \frac{\partial^2 \phi}{\partial x_1 \partial x_2} \underset{\sim}{\delta_3} + \varepsilon_{132} \frac{\partial^2 \phi}{\partial x_1 \partial x_3} \underset{\sim}{\delta_2}$$

$$+ \varepsilon_{213} \frac{\partial^2 \phi}{\partial x_1 \partial x_2} \underset{\sim}{\delta_3} + \varepsilon_{231} \frac{\partial^2 \phi}{\partial x_2 \partial x_3} \underset{\sim}{\delta_1}$$

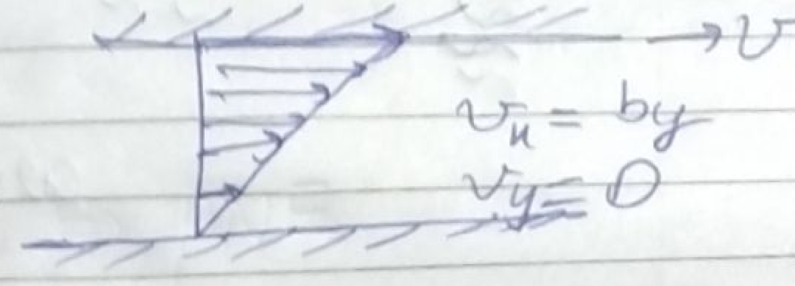
$$+ \varepsilon_{312} \frac{\partial^2 \phi}{\partial x_1 \partial x_3} \underset{\sim}{\delta_3} + \varepsilon_{321} \frac{\partial^2 \phi}{\partial x_3 \partial x_2} \underset{\sim}{\delta_1}$$

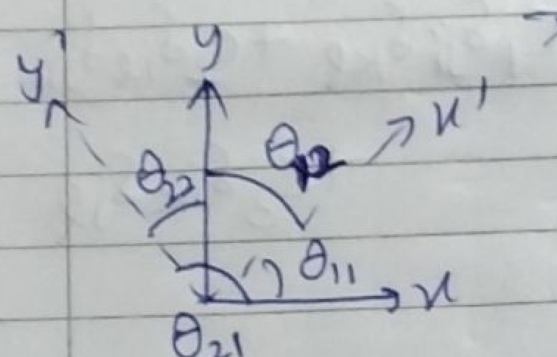
$$= \underline{\underline{0}}$$

4) $v_x = by \;;\; v_y = bx$



rectangular hyperbola.





$$\underset{\sim}{v}' = \underset{\approx}{L}^t \cdot \underset{\sim}{v}$$

$$\downarrow$$

$$L_{ij} v_i$$

$$\begin{bmatrix} \cos\theta_{11} & \cos\theta_{12} \\ \cos\theta_{21} & \cos\theta_{22} \end{bmatrix}_{2\times2} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}_{2\times2}$$

2) $$\underset{\approx\approx}{\Phi} = \underset{\sim}{\delta}\,\underset{\sim}{\delta} + \cancel{\underset{\sim}{a}\,\underset{\sim}{b}\,\underset{\sim}{c}\,\underset{\sim}{d}} + \cancel{\underset{\sim}{a}\,\underset{\approx}{\varepsilon}}$$

vectors
are not isotropic

$$\Phi_{ijkl} = c_1\delta_{ij}\delta_{kl} + c_2\delta_{ik}\delta_{jl} + c_3\delta_{il}\delta_{jk}$$

→ If symmetric in first two : $\Phi_{ijkl} = \Phi_{jikl}$

$$c_1\delta_{ij}\delta_{kl} + c_2\delta_{ik}\delta_{jl} + c_3\delta_{il}\delta_{jk} = c_1\delta_{ji}\delta_{kl} + c_2\delta_{il}\delta_{jk} + c_3\delta_{ik}\delta_{jl}$$

$\therefore$ $\underline{\underline{c_2 = c_3}}$

→ Reduced to 2 constants.

3) i) $\underset{\approx}{\tau} \propto \underset{\approx}{\nabla v}$

$$\underset{\approx}{\tau} = \underset{\approx\approx}{\mu} : (\underset{\approx}{\nabla v})$$ most general way

↳ isotropic 4^{th} order tensor.

$$\tau_{ij} = \mu_{ijkl}\nabla_l v_k$$

$$\tau_{ij} = \tau_{ji} \quad \text{(symmetric)}$$

$$\mu_{ijkl} \nabla_l v_k = \mu_{jikl} \nabla_l u_k$$

symmetric isotropic $\rightarrow$ 2 constants.

$$c_1 \delta_{ij} \delta_{kl} \nabla_l V_k + c_2 (\delta_{il} \delta_{jk} + \delta_{ik} \delta_{jl}) \nabla_l v_k$$

$$\tau_{ij} = c_1 \delta_{ij} \nabla_l V_k + c_2 (\nabla_i v_j + \nabla_j v_k)$$

$$\tau_{ij} = c_1 \underset{\approx}{\delta} (\underset{\sim}{\nabla} \cdot \underset{\sim}{v}) + c_2 (\underset{\sim}{\nabla} \underset{\sim}{v} + \underset{\sim}{\nabla} \underset{\sim}{v}^T)$$