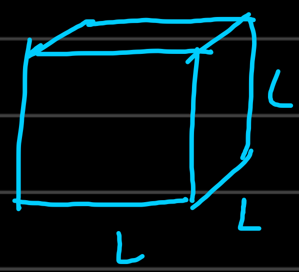
Lecture 13

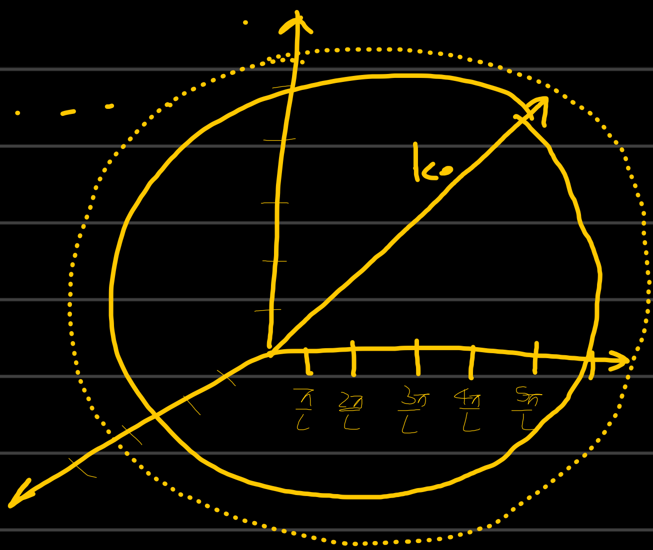
Particle in Cubical box: L^3



$$k_x = \frac{n\pi}{L}\ ;\ k_y = \frac{m\pi}{L}\ ;\ k_z = \frac{n\pi}{L}$$

reciprocal space:

$$k_x = 0, \frac{\pi}{L}, \frac{2\pi}{L}, \frac{3\pi}{L}, \ldots$$



$$D(E) = \frac{1}{V} \cdot \frac{dN}{dE}$$

$$dV = 4\pi k_0^2 \, dk$$

$$dN = \frac{4\pi k_0^2 \, dk}{\left(\frac{\pi}{L}\right)^3} = V \cdot \frac{4 k^2 \, dk}{\pi^2}$$

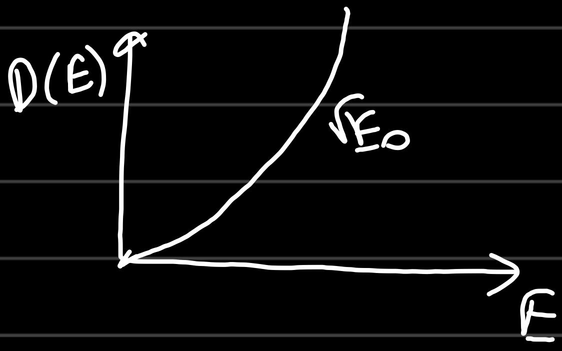
$$= V \cdot \frac{4}{\pi^2} k_0^{\cancel{2}} \times \frac{m\, dE}{\hbar^2 \cancel{k}}$$

$$E = \frac{\hbar^2 k^2}{2m}$$

$$\frac{1}{V}\frac{dN}{dE} = \frac{4km}{\hbar^2 \pi^2} = \frac{4m}{\pi^2 \hbar^2}\left\{\frac{\sqrt{2mE_0}}{\hbar}\right\}$$

real space

$$\underline{D(E) \propto \sqrt{E_0}}$$



$$D = C \cdot \sqrt{\frac{m^3}{\hbar^2}} \sqrt{E_0}$$

Q) How are the electrons distributed in a band at temp T. ?

at $T=0$;

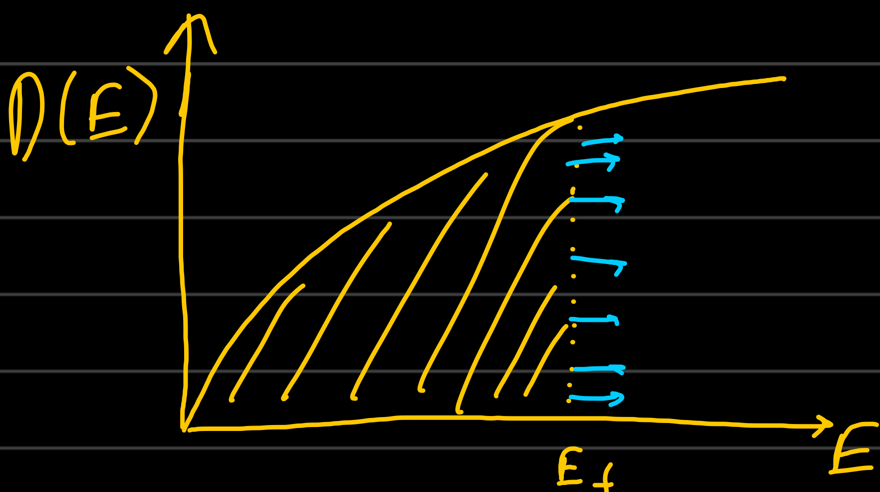
$$N = \int_0^{E_0} 2 \cdot D(E_0)\, dE$$

eg: If a metallic element cube of side L has $N_0 = 1 \times 10^{23}/cm^3$ electrons.

E_f = Fermi Level

↳ energy at which all the electrons are occupied at $T=0$.

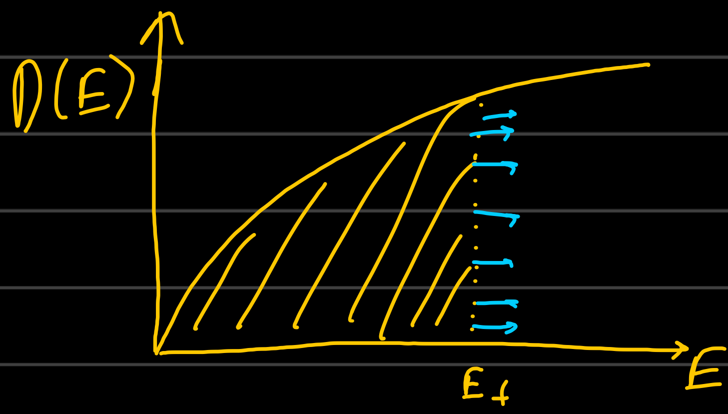
{The last occupied state at $T=0$}



at $T \neq 0$ $\Rightarrow$ electrons jump to higher energy states.

→ Some energy states above E_f will be occupied.

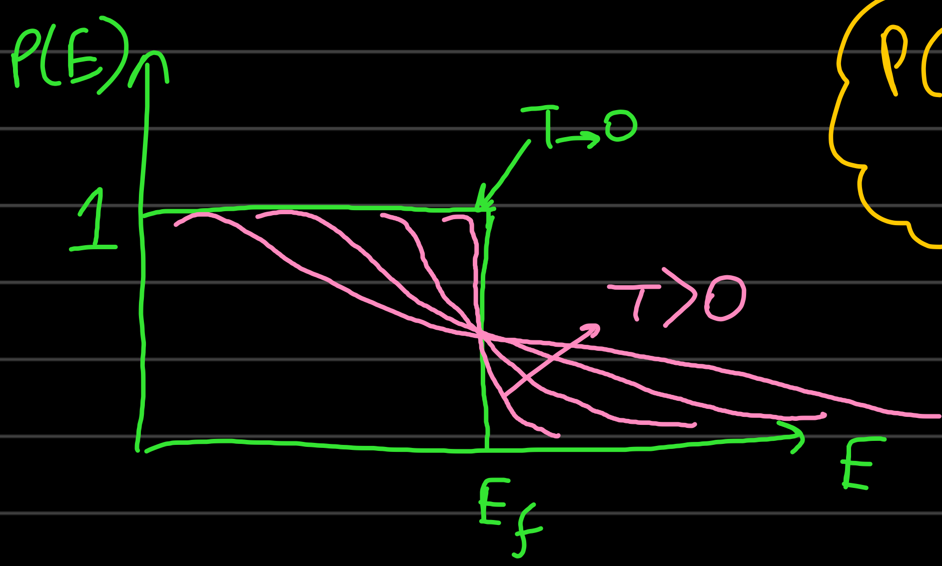
→ Some energy states below E_f will be unoccupied.



→ The Fermi-Dirac distribution.

$$\boxed{P(E) = \frac{1}{1+e^{(E-E_f)/k_BT}}}$$

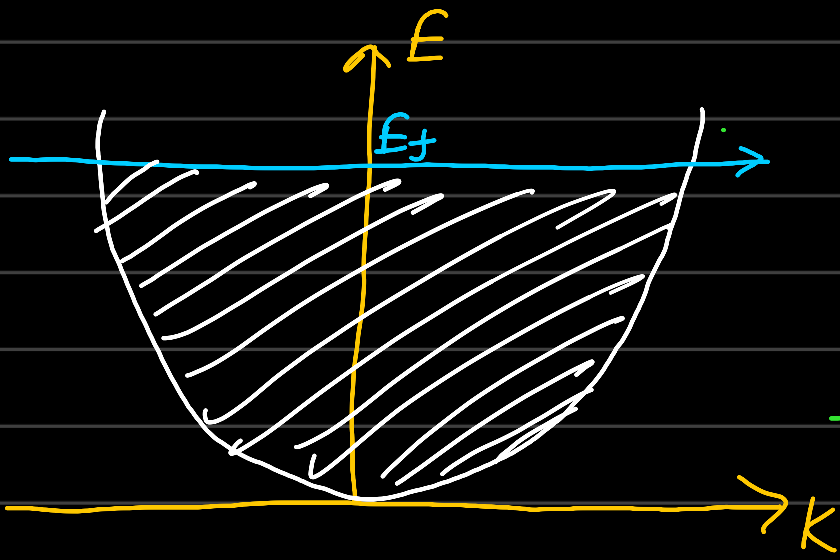
occupational probability of occupying energy level at any temperature T $\forall$ $T \neq 0$



$$\boxed{P(E=E_f) = \frac{1}{2}}$$

always

Quantum Conductivity:



$$E = \frac{\hbar^2 k^2}{2m} \quad p = \hbar k$$

$$E_f = \frac{\hbar^2 k_f^2}{2m} \; ; \; p_f = \hbar k_f$$

k_f = fermi wavevector

$\frac{\hbar k_f}{m} = v_f$ = fermi velocity.