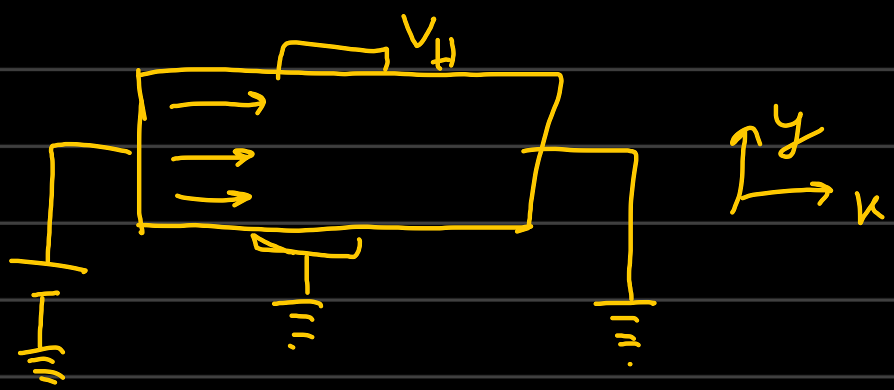
Lecture 3

MLL253

$$\sigma = (N_e)\left(\frac{e\tau}{m}\right)$$

$\tau \rightarrow \mu$ {mobility}

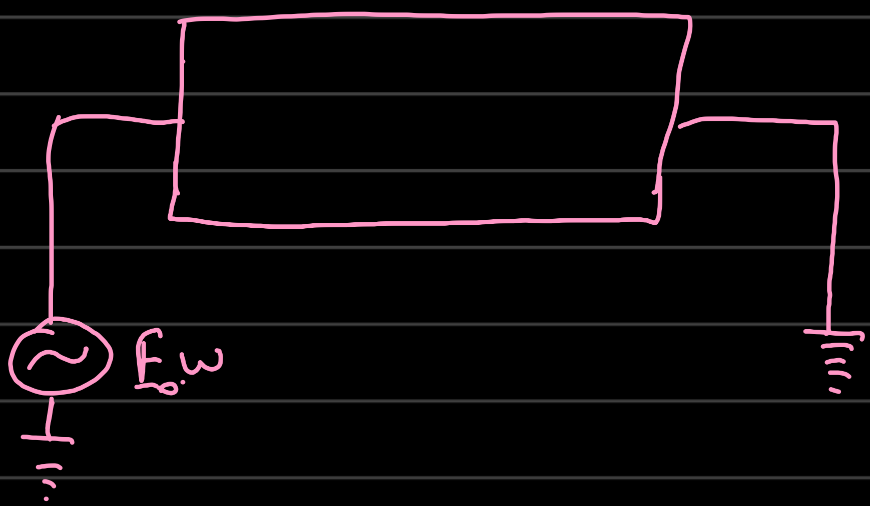
$N_e \downarrow$ no. of e^-



→ V_H transverse potential exists but no transverse current.

$$E_y = R_H \cdot J_x \cdot B \qquad R_H = \frac{1}{N_e \cdot e}$$ {Hall coefficient}

AC field:



$$E(t) = E_0 e^{-i\omega t}$$

$$V_d = V_d e^{-i\omega t}$$ → assumed to be in phase with $E(t)$

Writing Newton's Equation: $F = ma$

$$\frac{dV_d}{dt} = \frac{eE(t)}{m} - \frac{V_d(t)}{\tau}$$

$$-V_d(i\omega)e^{-i\omega t} = \frac{e}{m}E_0 e^{-i\omega t} - \frac{V_d e^{-i\omega t}}{\tau}$$

$$V_d(t)\left[\frac{1}{\tau} - i\omega\right] = \frac{e}{m}E(t)$$

$$E(t) = \frac{m}{e}\left[\frac{1}{\tau} - i\omega\right] V_d(t)$$

$$V_d(t) = \frac{e\tau}{m} \cdot \left(\frac{1}{1 - i\omega\tau}\right) E(t)$$

$$(N_e e) V_d(t) = \frac{e\tau}{m} \cdot \left(\frac{1}{1 - i\omega\tau}\right) E(t) (N_e e)$$

$$J(t) = \frac{\sigma_0 E(t)}{(1 - i\omega\tau)}$$

$$\boxed{\sigma(t) = \frac{\sigma_0}{(1 - i\omega\tau)}}$$

→ When frequency increases continuously

↳ electrons start radiating EM radiations when operated at RF frequencies.

↳ Maxwell Equations:

$$\boxed{\begin{aligned} \vec{\nabla} \times \vec{B} &= \mu_0 J + \varepsilon_0 \mu_0 \frac{\partial E}{\partial t} \\ \vec{\nabla} \times \vec{E} &= -\frac{\partial B}{\partial t} \end{aligned}}$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = -\frac{\partial}{\partial t}(\vec{\nabla} \times \vec{B})$$

$$-\nabla^2 \cdot \underline{E} = -\frac{\partial}{\partial t}\left(\mu_0 J + \varepsilon_0 \mu_0 \frac{\partial \underline{E}}{\partial t}\right)$$

$$E = E_0 e^{-i\omega t} e^{-ikx}$$

$$= E_0 e^{-i(kx+\omega t)}$$

{time dependence & space dependence}

$$\nabla^2 E = (-ik)^2 \cdot E_0 e^{-ikt} e^{-i\omega t}$$

$$\nabla^2 E = -k^2 E$$

$$\boxed{\left(k = \frac{2\pi}{\lambda}\right) \text{ spatial frequency}}$$

$$K = \frac{2\pi}{\lambda} \quad ; \quad \omega = \frac{2\pi}{t}$$

$$-\frac{\partial}{\partial t}\left(\mu_0 J + \varepsilon_0 \mu_0 \frac{\partial E}{\partial t}\right) = -\mu_0 \frac{\partial J}{\partial t} - \varepsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

$$= -\mu_0 \frac{\partial}{\partial t}\left(\sigma(\omega) E(t)\right) - \varepsilon_0 \omega_0 \cdot (-i\omega)^2 E$$

$$= -\mu_0 \sigma (-i\omega) E - \varepsilon_0 \mu_0 (i^2 \omega^2) E$$

$$= \mu_0 \sigma i \omega E + \varepsilon_0 \mu_0 E \omega^2$$

$$= \omega^2 \mu_0 \varepsilon_0 E \left(1 + \frac{i\sigma}{\omega \varepsilon_0}\right)$$

$$\text{RHS} = \frac{\omega^2}{c^2}\left(1 + \frac{i\sigma(\omega)}{\omega \varepsilon_0}\right) E$$

$$K^2 E = \frac{\omega^2}{c^2}\left(1 + \frac{\sigma_0}{(1 - i\omega\tau)\omega \varepsilon_0}\right) E$$

$$\boxed{K^2 = \frac{\omega^2}{c^2}\left(1 + \frac{\sigma_0}{(1 - i\omega\tau)\omega \varepsilon_0}\right)}$$

for $\omega\tau \gg 1$

$$\boxed{K^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\sigma_0}{\omega^2 \varepsilon_0 \tau}\right) \text{ for } \omega\tau \gg 1}$$

$$k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

$$\boxed{\omega_p^2 = \frac{\sigma_0}{\varepsilon_0 \tau} = \frac{N_e e^2}{m \varepsilon_0}}$$ plasma frequency

If $\omega < \omega_p$; $k^2 = -ve$

↳ implies $k = ia$

then we will have,

$$E = E_0 e^{-ikx}$$

$$E = E_0 e^{ax}$$ → decaying field :-

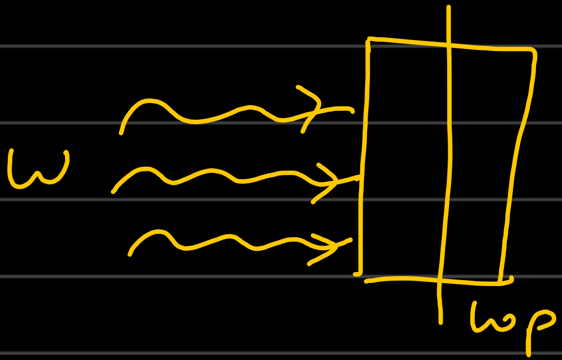
* Assume; $N_e = 1 \times 10^{24}\ m^{-3}$

$m = 9.1 \times 10^{-31}$

$\varepsilon_r = 1$

$$f = \frac{\omega}{2\pi} = 2.2 \times 10^{15}\ Hz$$

$$\lambda \approx 136\ nm$$



for metals
$\omega_p > \omega_{blue}$

for insulators
$\omega_p < \omega_{red}$

$\omega_{red} < \omega_{blue}$

→ For light to propogate $\omega > \omega_p$

If $\omega < \omega_p$ → a decaying E implies opacity to radiation.