Tutorial-5

1) $x = r\sin\theta\cos\phi$; $y = r\sin\theta\sin\phi$; $z = r\cos\theta$

$$e_i = \frac{1}{h_i}\frac{\partial r}{\partial q_i} \qquad r = x\,\underset{\sim}{\delta_x} + y\,\underset{\sim}{\delta_y} + z\,\underset{\sim}{\delta_z}$$

$$(h_i)^2 = \left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2$$

$$h_r^2 = \sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta = 1$$

$$\underline{\underline{h_r = 1}}$$

Similarly: $h_\theta = r \qquad h_z = r\sin\theta$

$$\underset{\sim}{r} = r\sin\theta\cos\phi\,\underset{\sim}{\delta_x} + r\sin\theta\sin\phi\,\underset{\sim}{\delta_y} + r\cos\theta\,\underset{\sim}{\delta_z}$$

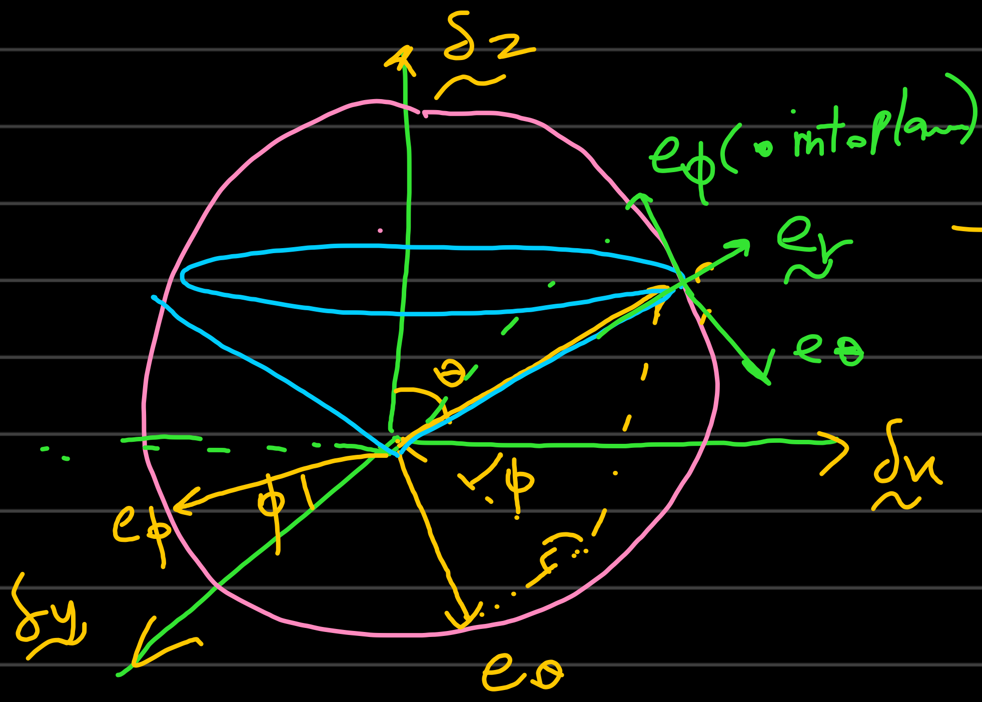
$$\underset{\sim}{e_r} = \frac{1}{h_r}\frac{\partial \underset{\sim}{r}}{\partial r} = \left(\sin\theta\cos\phi\,\underset{\sim}{\delta_x} + \sin\theta\sin\phi\,\underset{\sim}{\delta_y} + \cos\theta\,\underset{\sim}{\delta_z}\right)$$

$$\underset{\sim}{e_\theta} = \frac{1}{h_\theta}\frac{\partial \underset{\sim}{r}}{\partial \theta} = \frac{1}{r}\left(r\cos\theta\cos\phi\,\underset{\sim}{\delta_x} + r\cos\theta\sin\phi\,\underset{\sim}{\delta_y} - r\sin\theta\,\underset{\sim}{\delta_z}\right)$$

$$= \cos\theta\cos\phi\,\underset{\sim}{\delta_x} + \cos\theta\sin\phi\,\underset{\sim}{\delta_y} - r\sin\theta\,\underset{\sim}{\delta_z}$$

$$\underset{\sim}{e_\phi} = \frac{1}{h_\phi}\frac{\partial \underset{\sim}{r}}{\partial \phi} = \frac{1}{r\sin\theta}\left(-r\sin\theta\sin\phi\,\underset{\sim}{\delta_x} + r\sin\theta\cos\phi\,\underset{\sim}{\delta_y}\right)$$

$$= \sin\phi\, \underset{\sim}{\delta_x} + \cos\phi\, \underset{\sim}{\delta_y}$$



→ longitudinal variation in e_θ

→ The displacement vector in spherical coordinate system. $\boxed{\underset{\sim}{r} = r\underset{\sim}{e_r}}$

→ The displacement vector in cylindrical coordinates: $\underset{\sim}{r} = r\underset{\sim}{e_r} + z\underset{\sim}{\delta_z}$

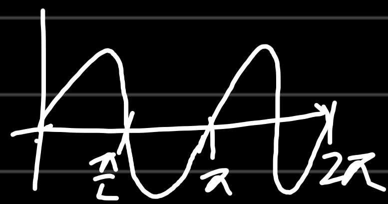
v) solid angle differential: $\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta\, \underset{\sim}{e_r}$

$$\underset{\sim}{\delta_x} \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta\, d\theta (\sin\theta\cos\phi)$$

$$= \cancel{\underset{\sim}{\delta_x} \int_0^{2\pi} \cos\phi} \int_0^{\pi} \sin^2\theta\, d\theta$$

$$\underset{\sim}{\delta}_z \int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \, (\cos\theta)$$

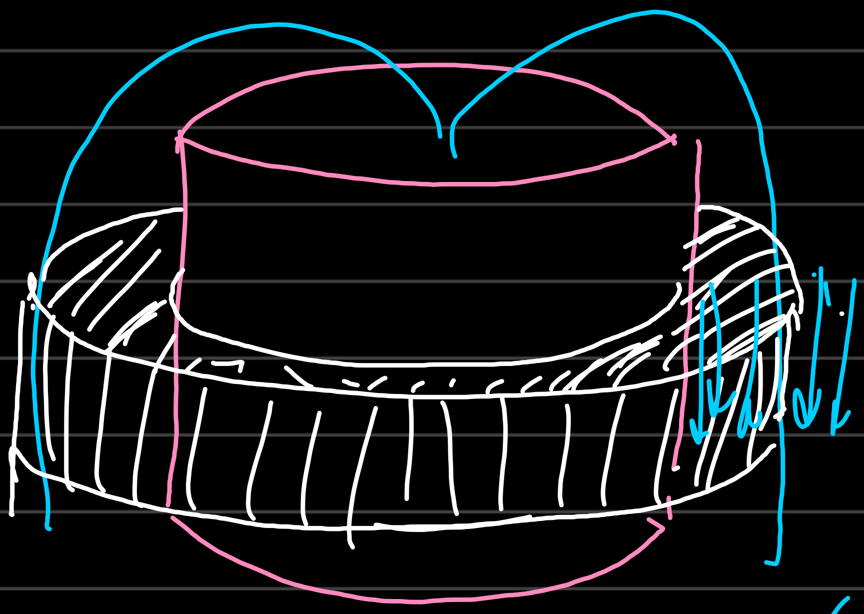
$$= \int_0^{2\pi} \frac{d\phi}{2} \int_0^{\pi} \cancelto{0}{\sin 2\theta \, d\theta}$$



ii) $\displaystyle\int_0^{2\pi} d\phi \int_0^{\pi} \sin\theta \, d\theta \; \underset{\sim}{e}_r \underset{\sim}{e}_r$

$$\underset{\sim}{e}_r \underset{\sim}{e}_r = \begin{bmatrix} (\sin\theta\cos\phi)^2 & \sin^2\theta\cos\phi\sin\phi & \sin\theta\cos\theta\cos\phi \\ \vdots & (\sin\theta\sin\phi)^2 & \sin\theta\cos\theta\sin\phi \\ \vdots \; \ldots & \ldots & (\cos\theta)^2 \end{bmatrix}$$

a)



$v_z = 0$ at R
(no slip)

$\tau_z = 0$
at $r\,R$
($\mu_{air} < \mu_{fluid}$)