

Anisotropic Elasticity:

$$\sigma_{k\ell} = \boxed{C_{ijkl}} \epsilon_{kl} \rightarrow \text{Stiffness tensor}$$

{the ratio of stress component}

$$\begin{aligned}\sigma_{nn} = & C_{nkkn} \epsilon_{kk} + C_{nky} \epsilon_{ny} + C_{nuz} \epsilon_{uz} \\ & + C_{nkyy} \epsilon_{yy} + C_{nyn} \epsilon_{yn} + C_{nuyz} \epsilon_{yz} \\ & + C_{nuzz} \epsilon_{zz} + C_{nuzn} \epsilon_{zn} + C_{nxzy} \epsilon_{zy}\end{aligned}$$

strain as a linear combination of stress compnt.

$$\epsilon \quad \boxed{\epsilon_{ny} = S_{nykl} \sigma_{kl}}$$

Compliance tensor

$$C_{nkkn} \neq \frac{1}{S_{nkkn}}$$

$$\begin{aligned}\epsilon_{kk} = & S_{nkkn} \sigma_{nn} + S_{nky} \sigma_{ny} + S_{nuz} \sigma_{uz} \\ & + S_{nkyk} \sigma_{yk} + S_{nkyy} \sigma_{yy} + S_{nuyz} \sigma_{yz} \\ & + S_{nuzn} \sigma_{zn} + S_{nuyz} \sigma_{zy} + S_{nuzz} \sigma_{zz}\end{aligned}$$

Stiffness Matrix: $C_{ijkl} = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}_{9 \times 9}$

↳ 81 elements

→ Possible simplifications:

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} \left. \begin{array}{l} \{\text{normal}\} \\ \{\text{shear}\} \end{array} \right\}$$

$9 \times 9 \longrightarrow 6 \times 6$

$$[\sigma] = [C][\varepsilon] = \underline{[C][S][\sigma]}$$

$$[C][S] = I \Rightarrow \underline{[C] = [S]^{-1}}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & \ddots & & & & \\ C_{31} & & \ddots & & & \\ C_{41} & & & \ddots & & \\ C_{51} & & & & \ddots & \\ C_{61} & & & & & \ddots \end{bmatrix}$$

→ Consider the symmetry of stiffness & compliance .

→ path-independent nature of linear elasticity

$$\omega = \frac{1}{2} \sigma \varepsilon = \frac{1}{2} E \varepsilon^2$$

$$\omega = \frac{1}{2} (\sigma_1 \varepsilon_1 + \sigma_2 \varepsilon_2 + \sigma_3 \varepsilon_3 + \dots)$$

linear combinations:

$$\sigma_i = \zeta_1 \varepsilon_1 + \zeta_2 \varepsilon_2 + \zeta_3 \varepsilon_3 + \zeta_4 \varepsilon_4 + \zeta_5 \varepsilon_5 + \zeta_6 \varepsilon_6$$

$$C_{12} = \frac{\partial \sigma_1}{\partial \varepsilon_2} = \frac{1}{\partial \varepsilon_2} \left(\frac{\partial \omega}{\partial \varepsilon_1} \right) = \quad \left. \begin{array}{l} \\ \end{array} \right\} C_{12} = C_{21}$$

→ reduces to 2 unique elements:

$$\begin{bmatrix} \zeta_{11} & \zeta_{12} & \zeta_{13} & \zeta_{14} & \zeta_{15} & \zeta_{16} \\ \zeta_{21} & \zeta_{22} & \zeta_{23} & \zeta_{24} & \zeta_{25} & \zeta_{26} \\ \zeta_{31} & \zeta_{32} & \zeta_{33} & \zeta_{34} & \zeta_{35} & \zeta_{36} \\ \zeta_{41} & \zeta_{42} & \zeta_{43} & \zeta_{44} & \zeta_{45} & \zeta_{46} \\ \zeta_{51} & \zeta_{52} & \zeta_{53} & \zeta_{54} & \zeta_{55} & \zeta_{56} \\ \zeta_{61} & \zeta_{62} & \zeta_{63} & \zeta_{64} & \zeta_{65} & \zeta_{66} \end{bmatrix}$$

→ From crystal structure symmetry:

Triclinic	⇒	21	No. of elastic constants
Monoclinic	⇒	13	
Orthorhombic	⇒	9	
Tetragonal	⇒	6	
Hexagonal	⇒	5	
Cubic	⇒	3	
Isotropic	⇒	2	

For Cubic Structures: C_{11}, C_{12}, C_{44}

required unique elastic constants.

→ Stiffness constants in units: GPa

→ Compliance constants in units: TPa⁻¹

$$(1/\text{TPa})$$

Giga
tera

$$C_{11} = \frac{S_{11} + S_{12}}{(S_{11} - S_{12})(S_{11} + 2S_{12})}$$

The elastic modulus in any direction.

$$\frac{1}{E} = S_{11} - 2 \left[(S_{11} - S_{12}) - \frac{1}{2} S_{44} \right] \left(l^2 m^2 + m^2 n^2 + l^2 n^2 \right)$$

l, m, n are direction cosines of the direction vector

$$[u_1 \ v_1 \ w_1] \wedge [u_2 \ v_2 \ w_2]$$

$$\cos \theta = \frac{u_1 u_2 + v_1 v_2 + w_1 w_2}{\sqrt{u_1^2 + v_1^2 + w_1^2} \sqrt{u_2^2 + v_2^2 + w_2^2}}$$

To find $E_{(u,v,w)}$: $\ell = [u_1 \ v_1 \ w_1] \wedge [100]$

$$m = [u \ v \ w] \wedge [010]$$

$$n = [u \ v \ w] \wedge [001]$$

→ For isotropic cubic structures:

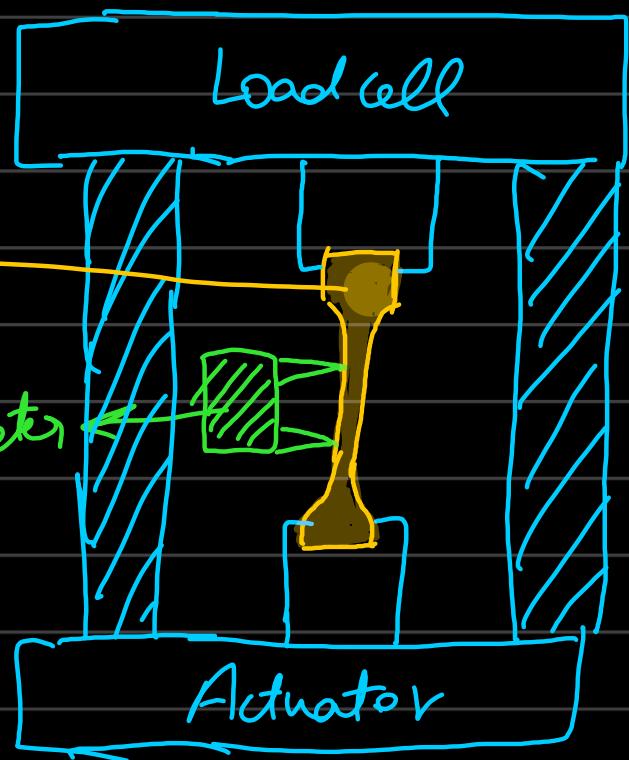
$$S_{11} = \frac{1}{E} ; \quad S_{12} = -\frac{15}{E} ; \quad S_{44} = \frac{1}{G}$$

Plasticity in Materials:

- Plastic deformation occurs due to dislocation slip or twinning
- ★ Hydrostatic stresses do not play a role in plastic deformation.
- Deviatoric stresses leads to plastic deformation $S_{ij} = \sigma_{ij} - \sigma_m$

Tensile Test:

Dog bone specimen
to calculate strain over gauge length.



BIS : Bureau of Indian Standards :