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Homework - 1

1) a) Wiedemann - Franz Law states that for metals at not-too low temperatures, the ratio of thermal conductivity is proportional to temperature.

$$\frac{K}{\sigma T} = L$$

$\hookrightarrow$ Lorenz number.

$\rightarrow$ In classical Drude picture: $L = \frac{3}{2}\left(\frac{k_B}{e}\right)^2$

b) Electrical conductivity $\sigma = \frac{ne^2\tau}{m}$

Thermal conductivity $K = \frac{1}{3} n c_v \bar{v}^2 \tau$

c_v = heat capacity per electron

$\bar{v}^2$ = mean squared speed.

For classical electrons,

$$c_v = \frac{3}{2} k_B \, ; \quad \frac{1}{2} m \bar{v}^2 = \frac{3}{2} k_B T$$

$$\bar{v}^2 = \frac{3 k_B T}{m}$$

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$$K = \frac{1}{3} n \left(\frac{3}{2}k_B\right)\left(\frac{3k_BT}{m}\right)\tau$$

$$\frac{K}{\sigma} = \frac{\frac{3nk_B^2T\tau}{2m}}{\frac{ne^2\tau}{m}} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2 T$$

2) $$\left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)\Psi = -i\hbar\frac{\partial\Psi}{\partial t} = E\Psi$$

$\Rightarrow$ For ~~indep~~ time-independent potentials,

$$\Psi = \psi(x)\,\varpi(t)$$

$$\left(\frac{-\hbar^2}{2m}\nabla^2 + V\right)\psi(x)\varpi(t) = -i\hbar\frac{\partial}{\partial t}\left(\psi(x)\varpi(t)\right)$$

$$= E\psi(x)\varpi(t)$$

$$-i\hbar\,\cancel{\psi(x)}\frac{\partial}{\partial t}(\varpi(t)) = E\cancel{\psi(x)}\varpi(t)$$

$$\varpi(t) = ~~B~~\,~~A~~\,e^{-\frac{E}{\hbar}t}$$

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$$-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x) = E\psi(x)$$

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

$$k = \frac{\sqrt{2mE}}{\hbar} \Rightarrow E = \frac{\hbar^2 k^2}{2m}$$

a) ~~If~~ $V=0$ for $x<0$ & $V=P$ for $x>0$

For $x<0$ → free particle

For $x>0$ →

$$\psi(x) = Ae^{ik_2 x} + Be^{-ik_2 x}$$

$$k_2 = \frac{\sqrt{2m(E-P)}}{\hbar}$$

Here, if $E>P \Rightarrow k_2 = \text{real}$ → $\psi(x)$ is oscillating.

⇒ There is partial transmission & partial reflection at ~~x>0~~ $x=0$ boundary.

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if $E < P \Rightarrow k_2 =$ complex

$\Psi(x) =$ exponential decay

$= Ae^{-k_2 x}$.

↳ The electron has a chance to 'tunnel' but cannot propogate freely.

This is the basic idea of quantum tunneling.

3) From hydrogenic spectrum,

$$E_n = \frac{-m_e e^4}{2(4\pi\epsilon_0)^2 \hbar^2} \times \frac{1}{n^2}$$

$$E_n = \frac{-(9.1\times10^{-31})(1.67\times10^{-19})^4}{2\times(4\pi\times8.85\times10^{-12})^2(1.054\times10^{-34})}$$

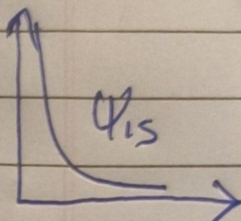
$E_n = -13.6\,eV$

→ Bohr radius: $a_0 = \dfrac{4\pi\epsilon_0 \hbar^2}{m_e e^2}$

$= 5.29 \times 10^{-11} m = \underline{\underline{0.529 \text{Å}}}$

The normalized 1s function:

$$\psi_{100}(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}$$

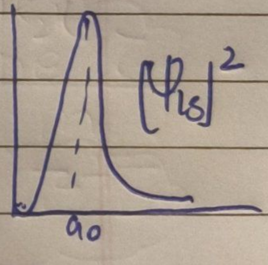


The radial probability density

$$P(r) = 4\pi r^2 |\psi_{100}(r)|^2$$

$$= 4\cancel{\pi} r^2 \times \frac{1}{\cancel{\pi} a_0^3} e^{-2r/a_0}$$

$$\boxed{P(r) = \frac{4r^2}{a_0^3} e^{-2r/a_0}}$$



→ Maximum probability at extremum

$$\frac{dP}{dr} = +\frac{8r}{a_0^3} e^{-2r/a_0} - \frac{8r^2 e^{-2r/a_0}}{a_0^4} = 0$$

$$\frac{\cancel{8r}}{a_0^{\cancel{3}}} \cancel{e^{-2r/a_0}} = \frac{\cancel{8} r^{\cancel{2}}}{a_0^{\cancel{4}}} \cancel{e^{-2r/a_0}}$$

$$\boxed{r = a_0}$$

★ So the most probable radius ★ $r = a_0$

→ Most of the electrons are found at Bohr radius.