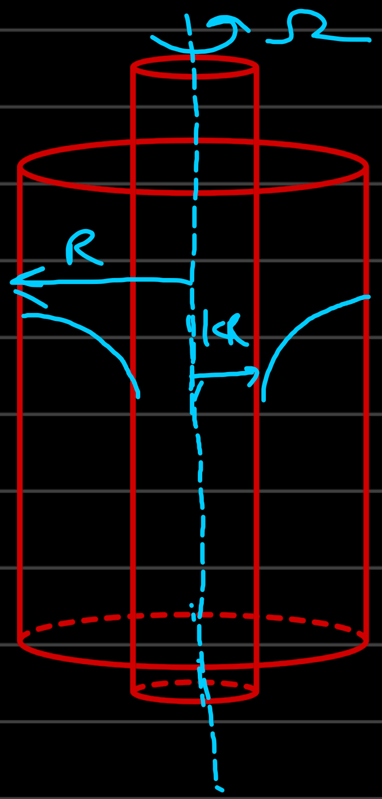
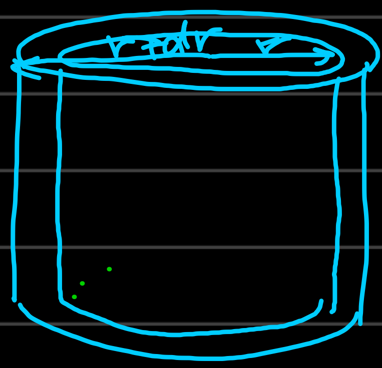
Tutorial-7

3)



$v_\phi = f(r) \qquad v_z = 0$

$v_r = 0$



$$0 = \cancel{\frac{1}{r}\frac{dp}{d\theta}}^{\,0} + \mu\left\{ \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta) \right) \right\}$$

radially symmetric problem $\rightarrow$ no supply of pressure gradient

$$\frac{C_1}{\mu} = \frac{1}{r}\frac{\partial}{\partial r} r v_\theta)$$

$$\frac{C_1 r^2}{2\mu} + C_2 = r v_\theta$$

$$\boxed{v_\theta = \frac{C_1 r}{2\mu} + \frac{C_2}{r}}$$

$$v_\theta(\kappa R) = \frac{C_1 \kappa R}{2\mu} + \frac{C_2}{\kappa R} = \Omega \kappa R$$

$$V_\theta(R) = \frac{c_1 R}{2\mu} + \frac{c_2}{R} = 0$$

$$c_2 = \frac{-c_1 R^2}{2\mu}$$

$$\frac{c_1 k \cancel{R}}{2\mu} - \frac{c_1 \cancel{R}}{2k\mu} = \Omega k \cancel{R}$$

$$c_1 = \frac{2\Omega k^2 \mu}{k^2-1}$$

$$\frac{c_1}{2}\frac{(k^2-1)}{k\mu} = \Omega k$$

$$v_\theta = \frac{k^2 \Omega r}{(k^2-1)} - \frac{k^2 R^2 \Omega}{(k^2-1)}$$

$$V_\theta = \frac{k^2\Omega}{(k^2-1)}\left(r - \frac{R^2}{r}\right) = \frac{k^2\Omega}{(k^2-1)r}(r^2-R^2)$$

→ r-direction momentum balance:

$$\frac{\rho V_\theta^2}{r} = \frac{-\partial P}{\partial r} \qquad P =$$

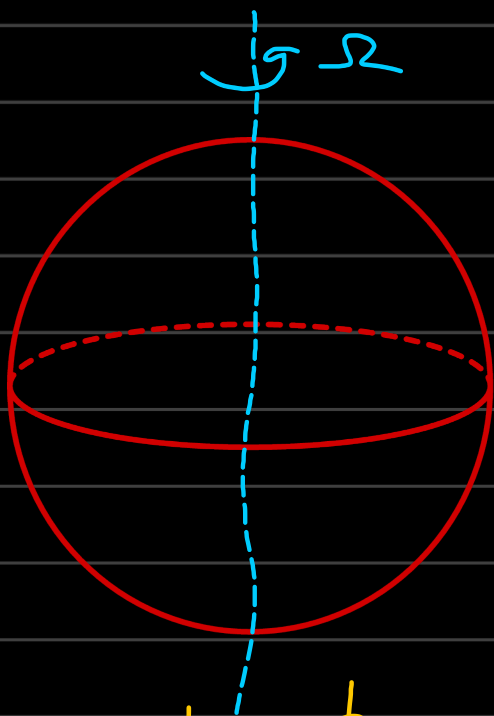
$$-\frac{\partial P}{\partial z} + \rho g = 0 \qquad \{z\text{-direction}\}$$

$$P = \rho g z$$

$$\frac{\rho}{r}\frac{k^4\Omega^2(r^2-R^2)^2}{(k^2-1)^2 r^2} = -\frac{\partial P}{\partial r}$$

$$\frac{k^4\Omega^4(r^4+R^4-2r^2R^4)}{(k^2-1)^2(\cancel{r}^5} = -\frac{\partial P}{\partial r}$$

Q2)



$v_r = 0$

$v_\theta = 0$

$v_\phi = f(r,\theta)$

Spherical Coordinates ϕ momentum balance.

$$0 = \cancel{-\frac{1}{r\sin\theta}\frac{\partial P}{\partial \phi}}^{\,0\ \text{boundary driven flow.}} + \mu\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial v_\phi}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \theta}\left(\frac{1}{\sin\theta}\frac{\partial(v_\phi\sin\theta)}{\partial\theta}\right)\right]$$

Linear PDE need to be solved.

Boundary Conditions: $v_\phi = \Omega R\sin\theta$ (surface)

$v_\phi = 0 \quad (r\to\infty)$

→ Considering Seperation Variable

$$v_\phi = f(r)\,\boxed{g(\theta)} \longrightarrow \sin\theta$$

$$+ \frac{1}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial(v_\phi \sin\theta)}{\partial\theta}\right)\Bigg]$$

$$= \frac{f(r)}{r^2}\frac{\partial}{\partial\theta}\left(\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}(\sin^2\theta)\right)\Bigg] = -2\sin\theta\frac{f(r)}{r^2}$$

$$\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}(f(r))\right) - 2\frac{f(r)}{r^2} = 0}$$

Both dimensions are same.

Assume $f(r) = r^m$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 m r^{m-1}\right) - 2r^{m-2} = 0$$

$$m(m-1)r^{m-2} - 2r^{m-2} = 0$$

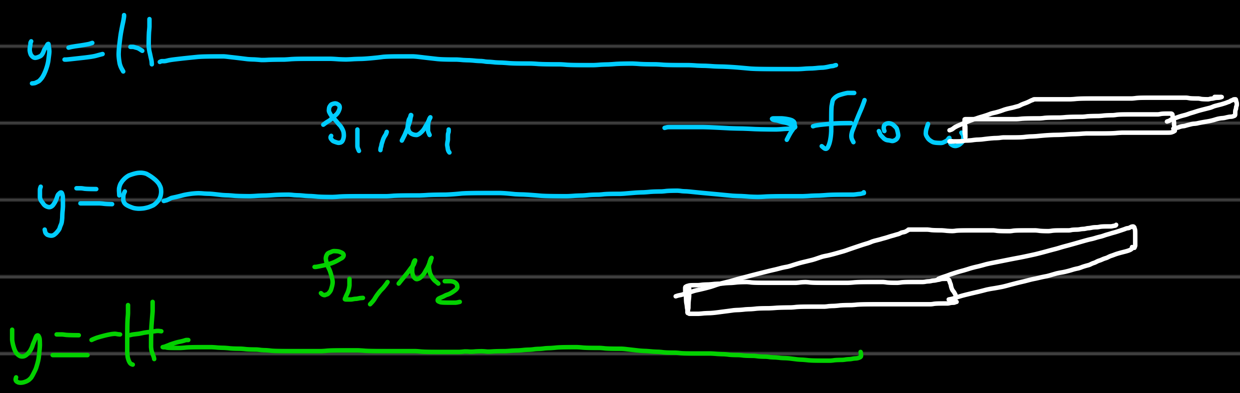
$$m^2 - m - 2 = 0$$

$$m = 2 \quad m = -1$$

$$v_\phi = r^m \sin\theta$$

$$= r^2\sin\theta, \text{ or } \frac{\sin\theta}{r}$$

Q3



$$-\frac{\partial P}{\partial x} + \mu \frac{\partial^2 v_x}{\partial y^2} = 0$$

$$\frac{\partial v_x}{\partial y} = \frac{P_0 y}{\mu} + c_1$$

$$v_x = \frac{P_0 y^2}{2\mu} + c_1 y + c_2$$

Boundary conditions: $v_x(-H_2) = 0$