Lecture 5

Electrons as Waves

$$\sigma_0 = \frac{N_e e^2 \tau}{m} \qquad \sigma(\omega) = \frac{\sigma_0}{1 - i\omega\tau}$$

$$E = E_0 e^{-i(kx + \omega t)} \qquad k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$$

$$k^2 = \frac{\omega^2}{c^2}\left(1 - \frac{\omega_p^2}{\omega^2}\right)$$ plasma frequency

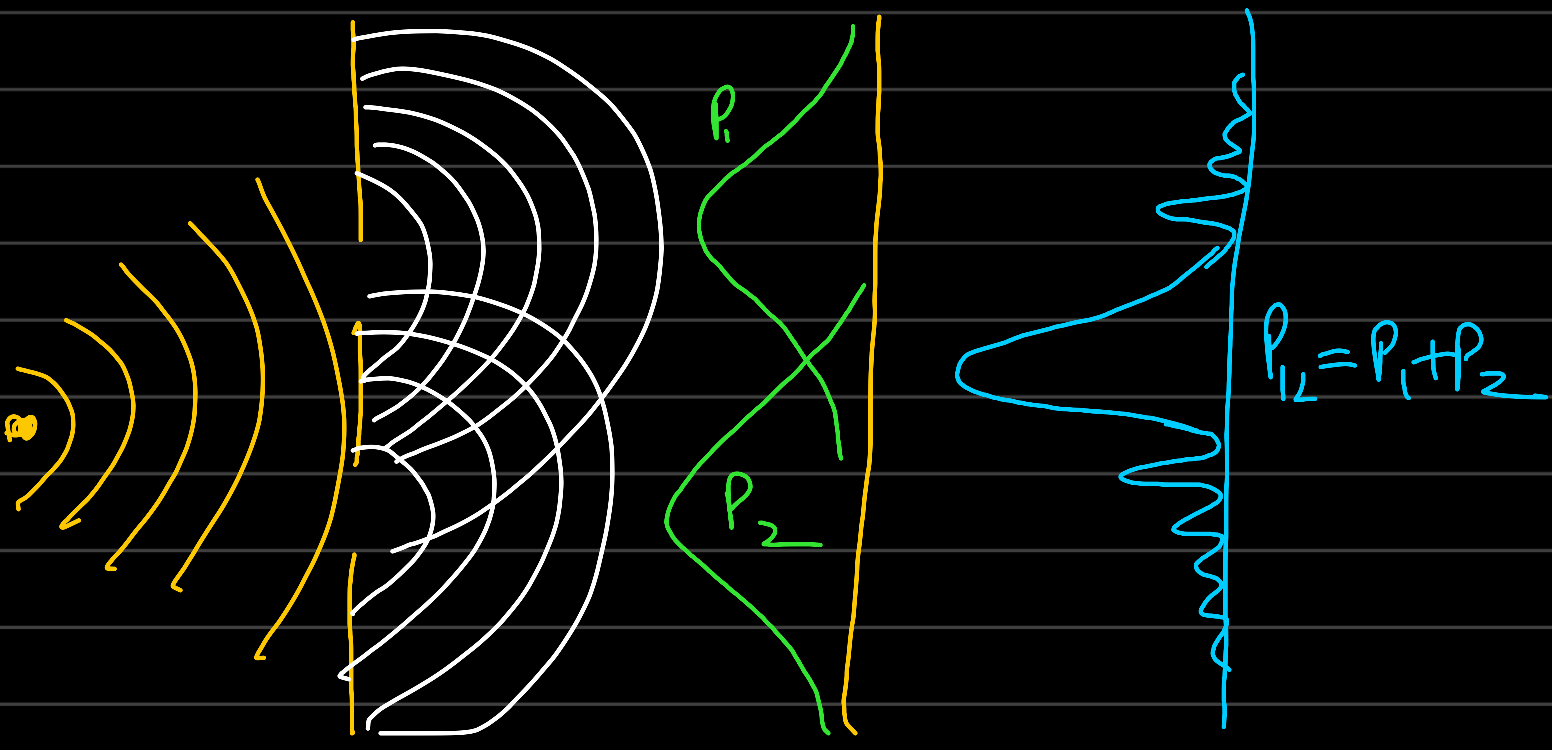
$$k^2 = \frac{\omega^2}{c^2}$$ { free space propagation }

$$\omega_p = \frac{N_e e^2}{m \varepsilon_0}$$

If $\omega < \omega_p$: attenuation

$\omega > \omega_p$: propagation of EM in metals.

$$\left\{ \begin{array}{c} k_B T = 26\,\text{meV} \\ T = 300\,\text{K} \end{array} \right\}$$



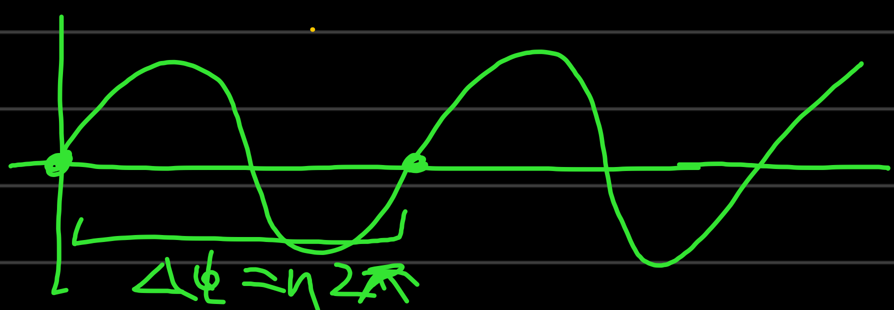
$P_1 = |h_1|^2$ $\quad$ $P_2 = |h_2|^2$ → complex {has a phase}

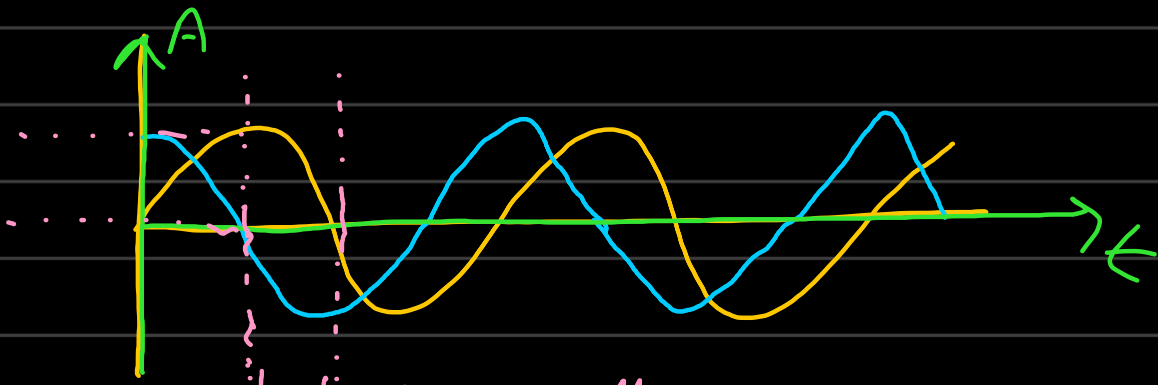
$$P_1 + P_2 = |h_1 + h_2|^2 \neq |h_1|^2 + |h_2|^2$$

$$P_1 + P_2 = |h_1|^2 + |h_2|^2 + 2|h_1||h_2|\cos\delta$$

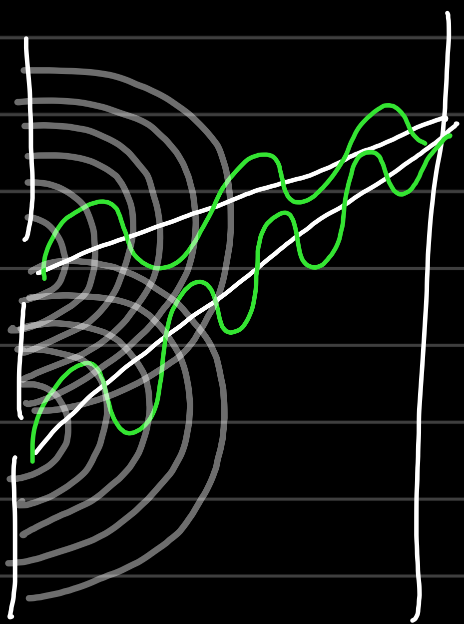
$h_1 = ce^{i\phi_1}$; $h_2 = ce^{i\phi_2}$ {not measurable quantity}

→ The intensity of wave measured is a function of phase difference.





phase difference $= \frac{\pi}{2}$



$$P = |h_1 + h_2|^2$$

amplitude wave 2

amplitude of wave 1

$P_1 = |h_1|^2$ { Measurable quantity P }

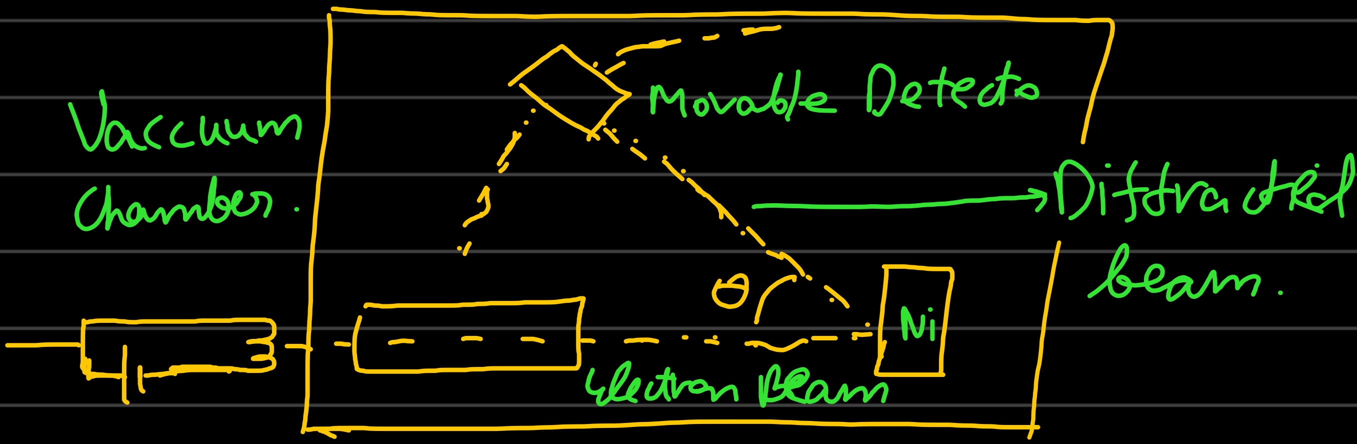
intensity $\propto$ (Amplitude)^2

$$P_1 = h_1^* h_1$$

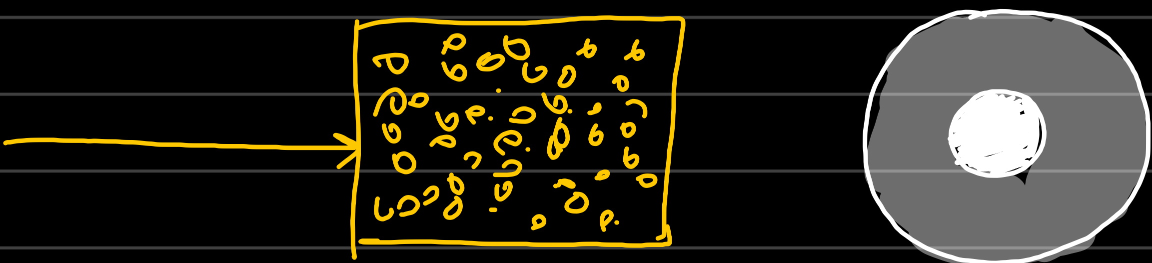
$$= c_1 e^{ik\phi} \cdot c_1 e^{-ik\phi} = c_1^2$$

Amplitude

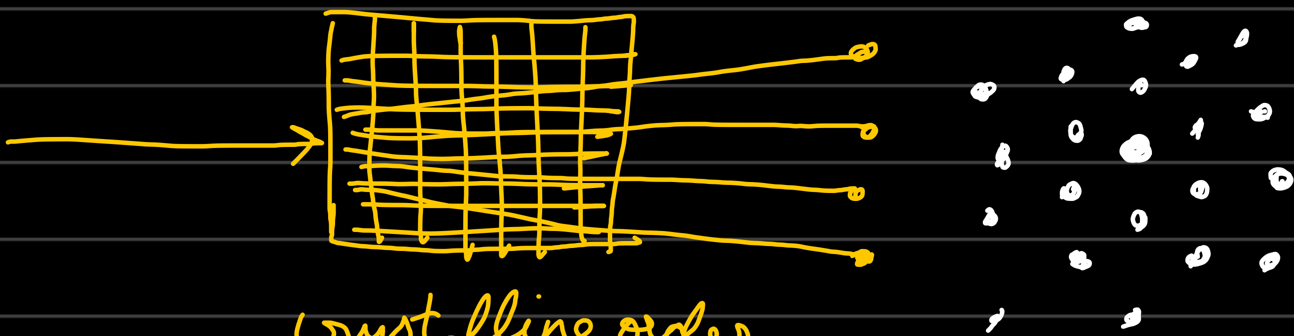
Davisson & Germer Experiment:



Typical TEM images.



amorphous
{no crystalline order}



crystalline order
{pure constructive & pure destructive interference at specific angles}

Crompton's Experiment: Duality of light

↳ Clear momentum transfer of electrons as a particle.

De Broglie:

↳ Any particle moving with a momentum P has a wavelength.

$$\boxed{\lambda = \frac{h}{p}}$$

Q> 50g ball, 20m/s

$$\lambda = \frac{6.67 \times 10^{-34}\ m}{1 kg\, m/s} \approx \text{insignificant}$$

Born Interpretation:

$$E_{(x,t)} = E_0 e^{-i(kx-\omega t)}$$

$$E(\underset{\sim}{r}, t) = E_0 e^{-i(k \cdot \underset{\sim}{r} - \omega t)}$$

$$\psi(x,t) = A e^{-i(kx-\omega t)}$$

$$\psi^* \psi = |\psi|^2 \Rightarrow \text{measurable quantity}$$

$|\psi|^2 \Rightarrow$ probability of finding particle at x, y, z at any time t.