Tutorial-6

3) The Navier-Stokes theorem:

$$\rho\left(\cancelto{0}{\frac{\partial \underset{\sim}{v}}{\partial t}} + \underset{\sim}{v}\cdot\nabla\underset{\sim}{v}\right) = -\underset{\sim}{\nabla} P + \mu\nabla^2\underset{\sim}{v} + \rho\underset{\sim}{g}$$

Steady State

Case 1: Inertial term vanishes $\underset{\sim}{v}\cdot\underset{\sim}{\nabla}\underset{\sim}{v} = 0$

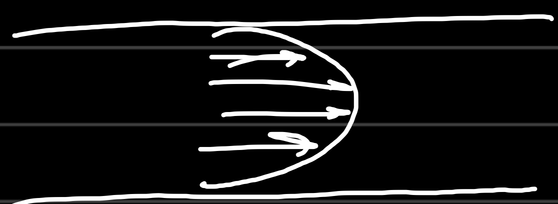
$$\therefore\; -\underset{\sim}{\nabla} P + \mu\nabla^2\underset{\sim}{v} + \rho\underset{\sim}{g} = 0$$

Case 2: Viscosity vanishes:

$$\rho\,\underset{\sim}{v}\cdot\underset{\sim}{\nabla}\underset{\sim}{v} = -\underset{\sim}{\nabla} P + \rho\underset{\sim}{g}$$

= dynamic pressure
{combine g with P}

For unidirectional flow $\underset{\sim}{v}\cdot\underset{\sim}{\nabla}\underset{\sim}{v} = 0$



$$\frac{V_x}{V_c} \sim \left(1 - \frac{y^2}{H^2}\right)$$

V_c → Velocity scale factor.

$$V_c \propto \frac{\Delta P}{L}\left(\frac{1}{\mu}\right) \qquad \frac{kg \times \cancel{m}}{m^{3\cancel{2}}\, s^2} = \mu \times \frac{1}{\cancel{m}\, s}$$

$$\frac{kg\,\cancel{m}}{s^2 \times m^2 \times \cancel{m}} \times \frac{\cancel{ms}}{\cancel{kg}} = \frac{1}{ms} \qquad \mu = \frac{kg}{ms}$$

$$V_c \propto \left(\frac{\Delta P}{L} - \rho g\right) H^2 \longrightarrow \text{velocity scale factor.}$$

In Case2:- $\longrightarrow$ solvable for constant velocity profile away from boundary

mass flow rate $\int_0^L dz \; \rho \int_{-H}^{H} v_x \, dy$

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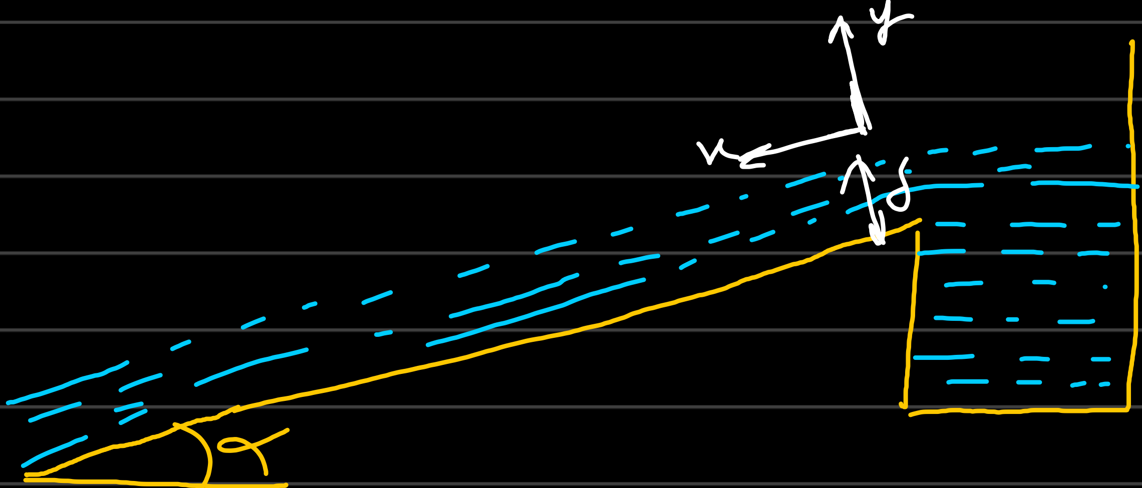


Equation of Motion: $\rho\left(\frac{\partial \underset{\sim}{v}}{\partial t} + \underset{\sim}{v}\cdot\nabla \underset{\sim}{v}\right) = -\underset{\sim}{\nabla} P + \mu \nabla^2 v + \rho g$

$$-\underset{\sim}{\nabla} P + \mu \nabla^2 \underset{\sim}{v} + \rho \underset{\sim}{g} = 0$$

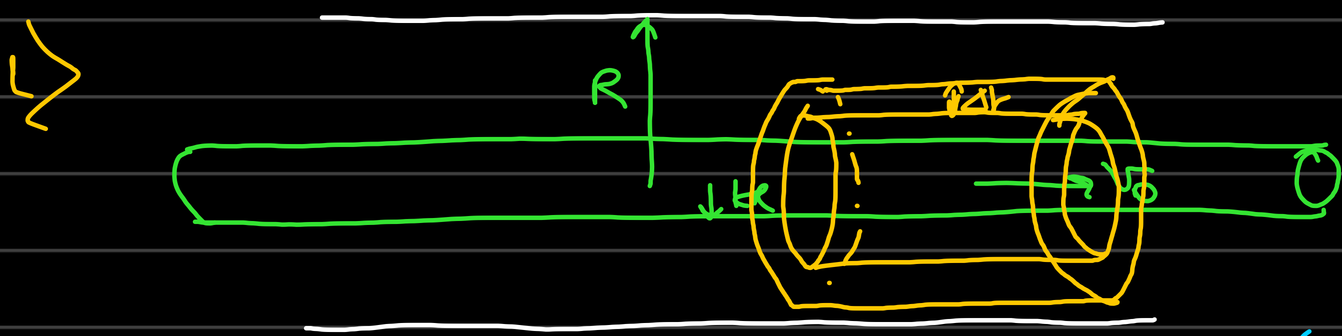
$$-\nabla_i P + \mu \nabla^2 v_i + \rho g_i = 0$$

$$-\frac{\partial P}{\partial x} \cancel{+ \mu \frac{\partial^2 v}{\partial x^2} + \rho g_x = 0}$$

$$-\underbrace{\frac{\partial P}{\partial x}}_{\text{freely balance.}} \rightarrow 0 + \mu \left[\underbrace{\frac{\partial^2}{\partial x^2}}_{\text{mass conservation.}} \rightarrow 0 + \frac{\partial^2}{\partial y^2} + \underbrace{\frac{\partial^2}{\partial z^2}}_{\text{infinite}} \right] v_x + \rho g_x = 0$$

$$+\mu \frac{\partial^2 v_x}{\partial y^2} + \rho g_x = 0$$

→ In y-direction $\quad -\frac{\partial P}{\partial y} + \rho g_y = 0$



$$\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})$$

$$\phi_{iz} = \rho v_i v_z - p\delta_{iz} + \tau_{iz}$$