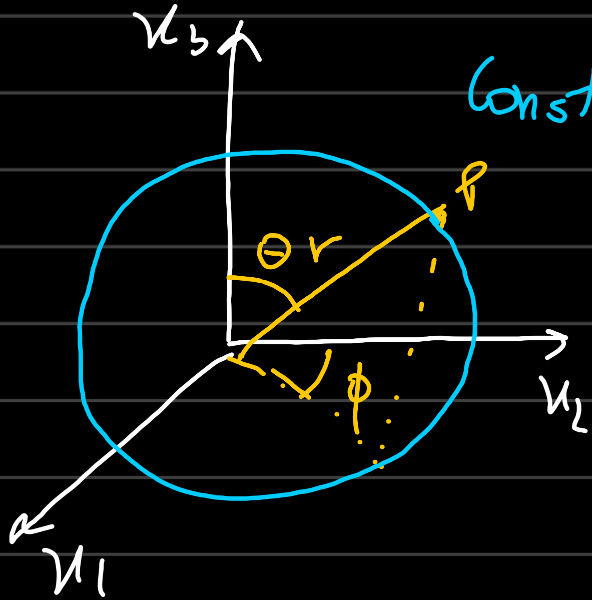
Tutorial-4

2) $x = r\sin\theta\cos\phi$; $y = r\sin\theta\sin\phi$; $z = r\cos\theta$

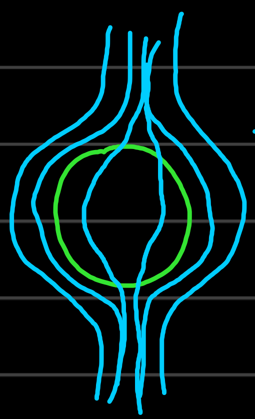


Constant coordinate surface.

r = sphere

θ = cone, circle

ϕ = plane with x_3



{axi-symmetry}

→ symmetry experienced due to constant ϕ.

$$\rightarrow \quad (h_i)^2 = \left(\frac{\partial x}{\partial q_i}\right)^2 + \left(\frac{\partial y}{\partial q_i}\right)^2 + \left(\frac{\partial z}{\partial q_i}\right)^2$$

$$(h_r)^2 = \left(\frac{\partial x}{\partial r}\right)^2 + \left(\frac{\partial y}{\partial r}\right)^2 + \left(\frac{\partial z}{\partial r}\right)^2$$

$= \sin^2\theta\cos^2\phi + \sin^2\theta\sin^2\phi + \cos^2\theta$

$h_r = 1$

$h_\theta = \sqrt{(r\cos\theta\cos\phi)^2 + (r\cos\theta\sin\phi)^2 + (r\sin\theta)^2}$

$h_\theta = \sqrt{r^2\cos^2\theta + r^2\sin^2\theta} = r^2$

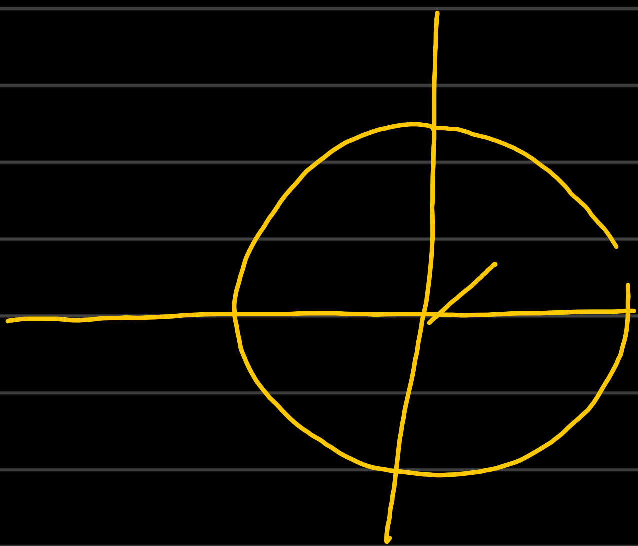
$\therefore \quad h_\theta = r$

$h_\phi = (r\sin\theta\sin\phi)^2 + (r\sin\theta\cos\phi)^2 + 0$

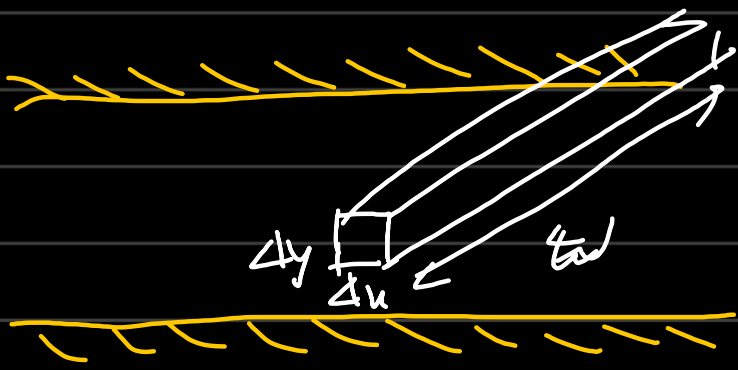
$= r^2\sin^2\theta$

$h_\phi = r\sin\theta$

★ $h_r = 1, \quad h_\theta = r, \quad h_\phi = r\sin\theta$



1)



$P = -P_0 x$

y-momentum balance:

$$\phi_{iy} = \Phi_{xy} + \Phi_{yy} + \Phi_{zy}$$

$$= w\Delta y \left[\Phi_{xy} \Big|_{IN-OUT} \right] + w\Delta x \, \phi_{yy} \Big|_{IN-OUT}$$

$$+ \Delta x \Delta y \, \phi_{zy} \Big|_{IN-OUT} + \rho g_y \Delta x \Delta y w$$

$$\underset{\sim}{\Phi} = \rho \underset{\sim}{v}\underset{\sim}{v} + \underset{\sim}{\tau}$$

$$\phi_{zy} = \rho \cancelto{0}{v_z} v_y + \tau_{zy} \qquad \phi_{zy} = 0$$

$$= p\delta_{zy} - \mu\left(\frac{\partial v_y}{\partial z} + \frac{\partial v_z}{\partial y}\right) = 0$$

$$\phi_{yx} = \rho \cancelto{0}{v_y} v_x + \rho \cancelto{0}{\delta_{yx}} - \mu\left(\frac{\partial v_x}{\partial y} + \frac{\partial v_y}{\partial x}\right)$$

$$\phi_{yx} = -\mu \frac{\partial v_x}{\partial y}$$

$$\Phi_{yy} = \rho v_y v_y + \tau_{yy}$$

$$= \rho \cancel{v_y v_y}^{0} + p\,\delta_{yy} - \mu\left(\cancel{\frac{dv_y}{dy}}^{0} + \cancel{\frac{dv_y}{dy}}^{0}\right)$$

$$\Phi_{yy} = p$$

$$\underbrace{w\Delta y\left(-\mu\frac{dv_x}{dy}\right)}_{\text{IN-OUT}} + \underbrace{w\Delta x\,(p)}_{\text{IN-OUT}}$$

$$+ \rho g_y \Delta x \Delta y \Delta z = 0$$

$$\frac{1}{\Delta x}\left(-\mu\frac{dv_x}{dy}\right) + \frac{1}{\Delta y}(-p) + \rho g_y = 0$$

$$\cancel{\frac{\partial}{\partial x}\left(-\mu\frac{dv_x}{dy}\right)}^{0} + \frac{\partial}{\partial y}(-p) + \rho g_y = 0$$

From mass conservation $v_x(y)$

$$\frac{dp}{dy} = \rho g_y$$

$$\boxed{p = \rho g y}$$

Hydrostatic Balance