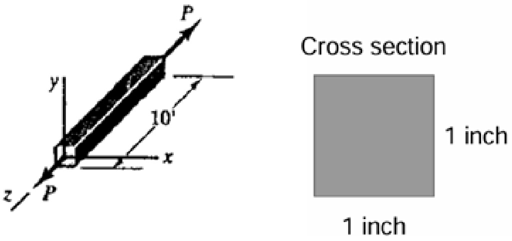
The First Quiz (Quiz 1)	Total marks 35 (Wt%=10)	09-09-25

1. Below is a bar with square cross-section loaded with a tensile stress P – if the change in length is 2' (inches), calculate the strains in x, y and z axes considering no volume change.



(5)

$\varepsilon_z = 2/10 = 0.2$
From volume constancy, $10*1*1 = 12*(1-\Delta x)(1-\Delta y)$
Considering $\Delta x=\Delta y$ (since square cross section)
$10=12\,(1-\Delta x)^2$; $\Delta x=0.0872$
$\varepsilon_x = \varepsilon_y = -0.0872$
You can verify: $\varepsilon_x + \varepsilon_y + \varepsilon_z = -0.872-0.872+0.2=0.02 \approx 0$

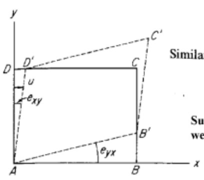
2. What, according to you would be the 2-D stress matrices for the following cases: a) pure shear, b) uniaxial compression, c) hydrostatic compression. (2 x 3=6)

a) $\sigma_{ij} = \begin{pmatrix} 0 & \tau & 0 \\ \tau & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ b) $\sigma_{ij} = \begin{pmatrix} -\sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ c) $\sigma_{ij} = \begin{pmatrix} \sigma & 0 & 0 \\ 0 & \sigma & 0 \\ 0 & 0 & \sigma \end{pmatrix}$

3. a) What is a displacement tensor – write down the tensor form.

$$e_{ij} = \begin{vmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{vmatrix}$$

b) Describe the components of the tensor.



Angular distortion of y-axis

$$e_{xy} = \frac{DD'}{DA} = \frac{\partial u}{\partial y}$$

Similarly, for the angular distortion of the x axis

$$e_{yx} = \frac{BB'}{AB} = \frac{\partial v}{\partial x}$$

Substituting the angular distortions for other axes, we get the total displacement tensor

displacement tensor

$$e_{ij} = \begin{vmatrix} e_{xx} & e_{xy} & e_{xz} \\ e_{yx} & e_{yy} & e_{yz} \\ e_{zx} & e_{zy} & e_{zz} \end{vmatrix} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} & \frac{\partial u}{\partial z} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} & \frac{\partial v}{\partial z} \\ \frac{\partial w}{\partial x} & \frac{\partial w}{\partial y} & \frac{\partial w}{\partial z} \end{vmatrix}$$

c) What are the different components of this tensor? And state the characteristics of the tensors

(2+3+2+3=10)

The displacement tensor can be written as

$$e_{ij} = \tfrac{1}{2}(e_{ij} + e_{ji}) + \tfrac{1}{2}(e_{ij} - e_{ji})$$

$$e_{ij} = \varepsilon_{ij} + \omega_{ij}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$$

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right)$$

$$\varepsilon_{ij} = \begin{vmatrix} \varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{yx} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zx} & \varepsilon_{zy} & \varepsilon_{zz} \end{vmatrix} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) & \frac{\partial v}{\partial y} & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right) & \frac{\partial w}{\partial z} \end{vmatrix} \tag{2-42}$$

$$\omega_{ij} = \begin{vmatrix} \omega_{xx} & \omega_{xy} & \omega_{xz} \\ \omega_{yx} & \omega_{yy} & \omega_{yz} \\ \omega_{zx} & \omega_{zy} & \omega_{zz} \end{vmatrix} = \begin{vmatrix} 0 & \frac{1}{2}\left(\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x}\right) & \frac{1}{2}\left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right) \\ \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) & 0 & \frac{1}{2}\left(\frac{\partial v}{\partial z} - \frac{\partial w}{\partial y}\right) \\ \frac{1}{2}\left(\frac{\partial w}{\partial x} - \frac{\partial u}{\partial z}\right) & \frac{1}{2}\left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right) & 0 \end{vmatrix}$$

Note that ε_{ij} is a symmetric tensor since $\varepsilon_{ij} = \varepsilon_{ji}$, that is, $\varepsilon_{xy} = \varepsilon_{xz}$, etc. ω_{ij} is an antisymmetric tensor since $\omega_{ij} = -\omega_{ji}$, that is, $\omega_{xy} = -\omega_{yx}$. If $\omega_{ij} = 0$, the deformation is said to be irrotational.

4. Objective questions (2x7 = 14)

a. True strain is additive – T/F? T

b. What are the stress invariants?

I_1: Sum of diagonal element	$I_1 = \sigma_{xx} + \sigma_{yy} + \sigma_{zz}$
I_2: Sum of determinants of minors	$I_2 = \begin{vmatrix} \sigma_{xx} & \sigma_{yx} \\ \sigma_{xy} & \sigma_{yy} \end{vmatrix} + \begin{vmatrix} \sigma_{yy} & \sigma_{zy} \\ \sigma_{yz} & \sigma_{zz} \end{vmatrix} + \begin{vmatrix} \sigma_{xx} & \sigma_{zx} \\ \sigma_{xz} & \sigma_{zz} \end{vmatrix}$
I_3: Determinant of tensor	$I_3 = \begin{vmatrix} \sigma_{xx} & \sigma_{yx} & \sigma_{zx} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{zy} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{vmatrix}$

c. How many slip systems does BCC material have and their types? *12* {110}<111> + *12* {112}<111> + *24* {123}<111> = 48

d. In edge dislocation, the burgers vector is parallel to the dislocation line – T/F? F

e. Formation of twins is the root cause for strain hardening in metals – T/F? F

f. In a wire drawing process, a copper rod (3.2 mm radius) is drawn through a die (2.50 mm radius). What is the force required to deform the copper? Is it sufficient to break the wire after it has bee formed? (given: yield strength is 150 MPa at 0% cold work and 390 MPa at 40% cold work).

*%cold work = $(A_f - A_o)/A_o$ *100% = 39% (~40%)*

*Force required to deform the wire, F = 150*pi*$(3.2)^2$ = 4825 N*

Stress on the wire after passing through the die = F/A_f = 4825/(pi$(2.5)^2$) = 246 MPa.*

This stress is less than the yield strength of Cu, 390 MPa at 40% elongation.

g. Zn and Ti have c/a ratio of 1.856 and 1.587 respectively – which one can slip on prismatic plane?

As Zn has a c/a ratio more than ideal value of 1.633, slip will occur on the ***basal*** *plane with low critical resolved shear stress. But for Ti with lesser than ideal c/a ratio, slip will occur* ***on prismatic planes i.e. {10-10}.***