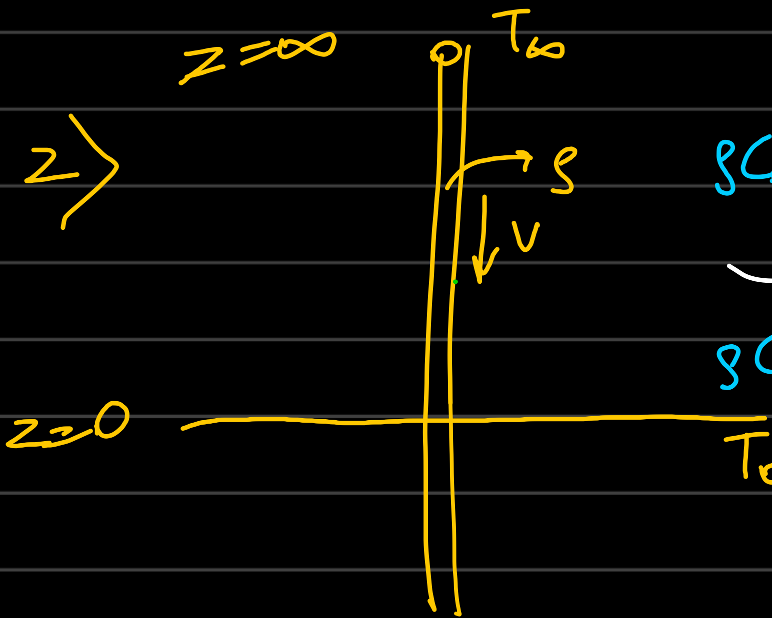
Tutorial-12

1) $$\rho \frac{D\hat{u}}{Dt} = -\underset{\sim}{\nabla}\cdot\underset{\sim}{q} - p(\underset{\sim}{\nabla}\cdot\underset{\sim}{v}) - \underset{\sim}{\tau}:\underset{\sim}{\nabla}\underset{\sim}{v}$$

$$d\hat{u} = \left.\frac{\partial\hat{u}}{\partial T}\right|_{V} dT + \left.\frac{d\hat{u}}{dV}\right|_{T} dV$$



$$\rho C_p \underbrace{\frac{DT}{Dt}} = -\underset{\sim}{\nabla}\cdot\underset{\sim}{q} - \underset{\sim}{\nabla}\cdot(\underset{\sim}{\tau}\cdot\underset{\sim}{v})$$

$$\rho C_p\left[\cancel{\frac{dT}{dt}}^{\,0} + \underbrace{v_r\cancel{\frac{dT}{dr}}^{\,0} + \frac{v_\theta}{r}\cancel{\frac{dT}{d\theta}}^{\,0}} + v_z\frac{dT}{dz}\right]$$

steady state

r << length
Ignore radial heat transport.
{sufficiently thin}

$$\rho C_p v_z \frac{dT}{dz} = -\underset{\sim}{\nabla}\cdot\underset{\sim}{q} - \underbrace{\underset{\sim}{\nabla}\cdot(\underset{\sim}{\tau}\cdot\underset{\sim}{v})}$$

$$\frac{\partial}{\partial x_i}\underset{\sim}{\delta_i}\cdot\{\tau_{pq} v_q \underset{\sim}{\delta_p}\}$$

$$\frac{\partial}{\partial x_i}\tau_{ij} v_j$$

$$\rho C_p v_z \frac{dT}{dz} = -\underset{\sim}{\nabla}\cdot\underset{\sim}{q} - \cancel{\frac{\partial}{\partial x_i}\tau_{iz} v_z}^{\,0}$$

no shearing effect
It is a rigid wire.
with constant velocity.

$$\rho C_p v_z \frac{dT}{dz} = -\underset{\sim}{\nabla}\cdot\underset{\sim}{q}$$

$$\rho C_p v_z \frac{dT}{dz} = k\frac{d^2T}{dz^2}$$

$$-\rho C_p v \frac{dT}{dz} = k\frac{d^2T}{dz^2}$$

$$\Theta = \frac{T-T_0}{T_\infty - T_0}$$

$$-\rho C_p v \frac{d\Theta}{dz} = k \frac{d^2\Theta}{dz^2}$$

Boundary Condition $z=0 : \Theta = 0$

$z=\infty : \Theta = 1$

Let $u = \frac{d\Theta}{dz}$

$$-\rho C_p v \, u = k \frac{du}{dz}$$

$$-Az = \ln u + c_1$$

$$u = Ce^{-Az}$$

$$\frac{d\Theta}{dz} = Ce^{-Az}$$

$$\Theta = -ACe^{-Az} + C_2$$

$$\boxed{\Theta = Ce^{-Az} + C_2}$$

$0 = C + C_2 \quad 1 = C_2$

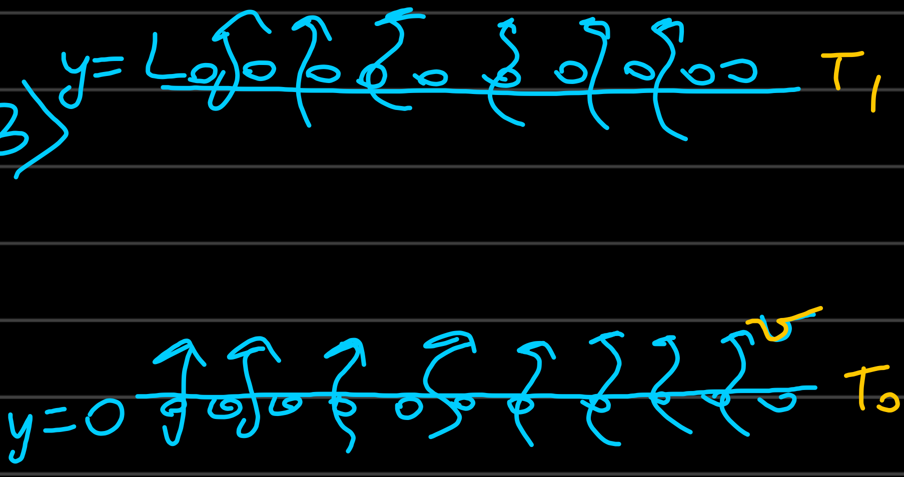
$$\Theta = 1 - e^{-Az}$$

$$\Theta - 1 = -e^{-Az}$$

$$\left\{\frac{T_\infty - T}{T_\infty - T_0}\right\} = \Theta'$$

$$\boxed{\Theta' = c' e^{-\frac{vz}{k}}}$$

3)



$$\rho C_p \frac{DT}{Dt} = -\nabla\cdot q - \nabla\cdot(\tau\cdot v)$$

$$\rho C_p\left(\frac{dT}{dt} + \cancel{v_x}^{\,0}\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + \cancel{v_z}^{\,0}\frac{\partial T}{\partial z}\right)$$

$$= -\nabla\cdot q - \cancel{\nabla\cdot(\tau_{ij}v_j)}^{\,0}$$

$v_y(x,y,z) = \text{const}$

$\frac{\partial v_x}{\partial x} = \frac{\partial v_y}{\partial y} = \frac{\partial v_z}{\partial z} = 0$

$$\rho C_p v_y \frac{dT}{dy} = +k\frac{d^2T}{dz^2}$$

$$\rho C_p v \frac{dT}{dy} = k\frac{d^2T}{dy^2}$$

$\Theta = \frac{T-T_1}{T_0-T_1}$

$\Theta = 1 \quad y=0$

$\Theta = 0 \quad y=L$

$$\rho C_p \frac{v}{L}\frac{d\Theta}{d\bar{y}} = \frac{k}{L^2}\frac{d^2\Theta}{d\bar{y}^2}$$

$$\frac{d\Theta}{d\bar{y}} = \frac{k}{\rho C_p L}\frac{d^2\Theta}{d\bar{y}^2}$$

$\boxed{\frac{k}{\rho C_p} = \text{Thermal Diffusity}}$

$$u = \frac{k}{\rho c_p L}\frac{du}{d\bar{y}} \Rightarrow \ln u + C_2 = \frac{\rho c_p L}{k}\bar{y}$$

$$u = Ce^{\frac{\rho c_p L \bar{y}}{k}}$$

$$\frac{d(H)}{dy} = Ce^{\frac{\rho c_p L \bar{y}}{k}}$$

$$(H) = \frac{C\rho c_p L}{k} e^{\frac{\rho c_p L \bar{y}}{k}} + C_2$$

$$(H) = c' e^{-\frac{\rho c_p L \bar{y}}{k}} + C_2 \qquad \bar{y} = \frac{y}{L}$$

$$1 = c' + C_2 \; ; \; 0 = c' e^{-\frac{\rho c_p L}{k}} + C_2$$

$$c' = 1 + c' e^{-\frac{\rho c_p L}{k}}$$

$$(H) =$$