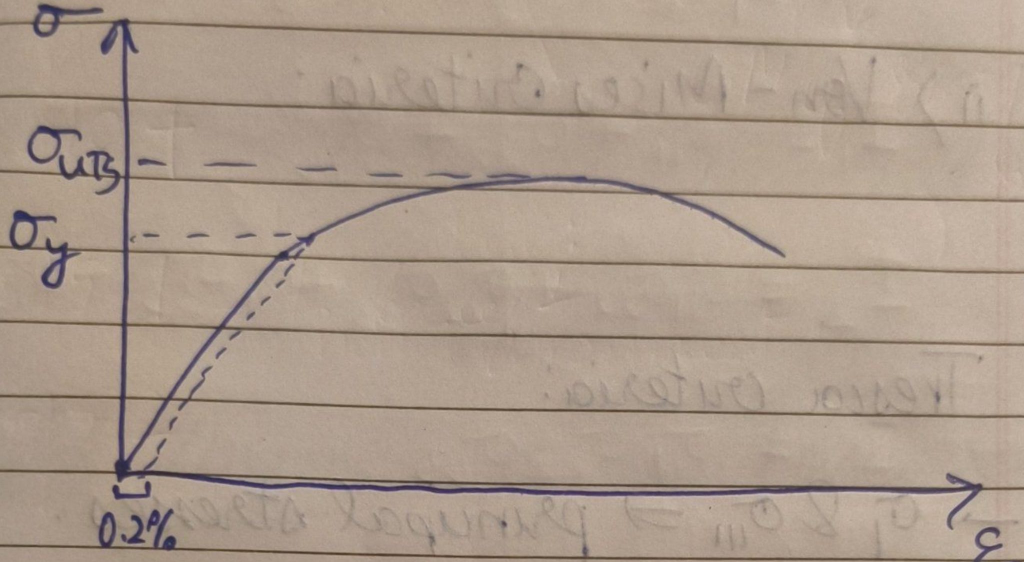
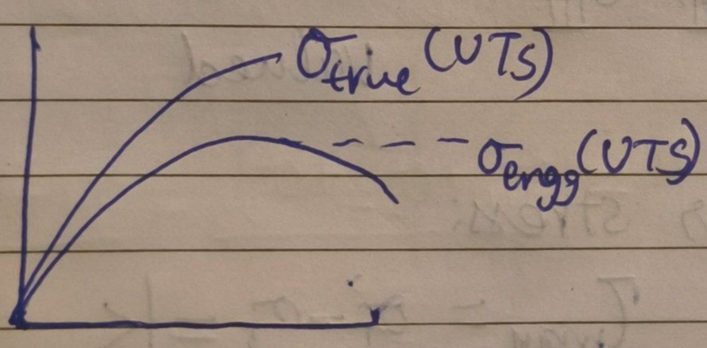
MLL251

Mechanical Behaviours of Materials:

Engineering Stress-Strain Curve:



$$\sigma = \frac{P}{A_0}(e+1) \neq (s+1)$$



Yield Criteria:

i) Plastic Deformation:

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i) Tresca: (Max shear stress)

$$\tau_{max} = \frac{\sigma_{I} - \sigma_{III}}{2} = K$$

ii) Von-Mises Criteria:

Tresca criteria:

→ σ_{I} & σ_{III} ⇒ principal stresses.

$$\sigma_0 = \sigma_{I} = \sigma_{III}$$

$\sigma_{I} > \sigma_{II} > \sigma_{III}$ → ~~extrem~~ extreme valued

Max shear stress:

$$\tau_{max} = \frac{\sigma_{I} - \sigma_{3}}{2} = K$$

⇒ determine K from simple tensile test

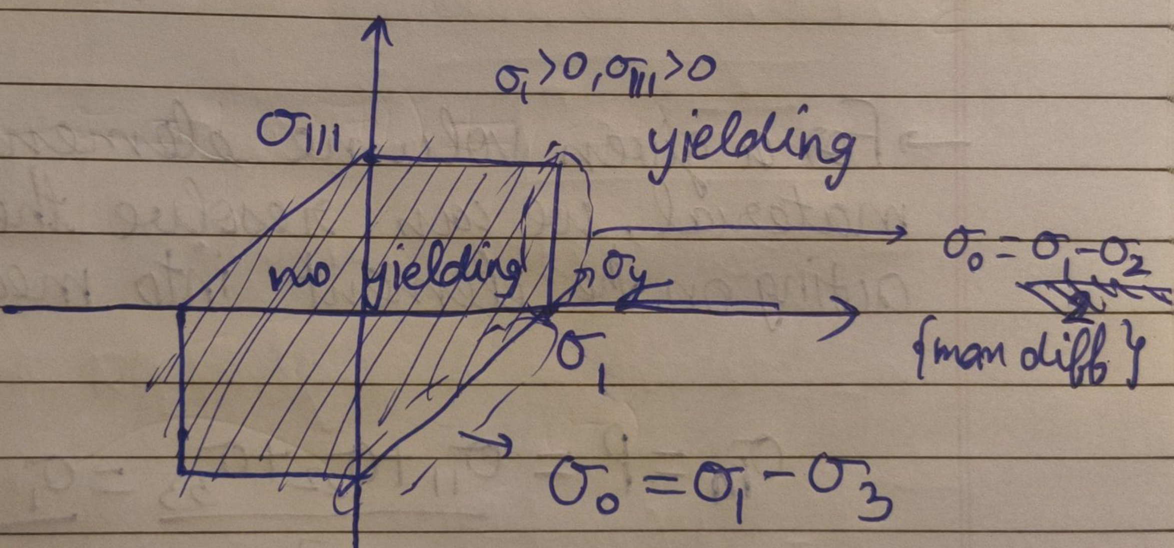
→ In uniaxial tension ($\sigma_2 = \sigma_3 = 0$)

$\sigma_1 = \sigma_0$ σ_1 = yield stress.

$k = \frac{\sigma_0}{2}$

→ Tresca criterion is equivalent to saying

Yield Locus: $\sigma_{II} = 0$ {assumption}



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Von-Mises Criteria:

~~$\sigma^3 - (\sigma_{11}$~~

$$\sigma^3 - I_1\sigma^2 + I_2\sigma - I_3 = 0$$

$$I_1 = \sigma_{11} + \sigma_{22} + \sigma_{33}$$

$$I_2 = \sigma_{11}\sigma_{22} + \sigma_{22}\sigma_{33} + \sigma_{11}\sigma_{33} - \sigma_{12}^2 - \sigma_{23}^2 - \sigma_{31}^2$$

$$I_3 = \sigma_{11}\sigma_{22}\sigma_{33}$$

→ For a given volume element of a material, we can resolve the stresses acting on the element into mean stress

$$\sigma_m = P = \frac{\sigma_{11} + \sigma_{22} + \sigma_{33}}{3} = \frac{\sigma_{I} + \sigma_{II} + \sigma_{III}}{3} = \frac{I_1}{3}$$

$$\underline{\sigma} = \begin{bmatrix} \sigma_{11} & \tau_{12} & \tau_{13} \\ & \sigma_{22} & \tau_{23} \\ & & \sigma_{33} \end{bmatrix} = \begin{bmatrix} \sigma_m & & \\ & \sigma_m & \\ & & \sigma_m \end{bmatrix} + \begin{bmatrix} \sigma_{11}-\sigma_m & & \\ & \sigma_{22}-\sigma_m & \\ & & \sigma_{33}-\sigma_m \end{bmatrix}$$

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→ solving determinant for stress deviator.

$$\cancel{\sigma_1'^3 - J_1 \sigma_1'^2 + J_2 \sigma_2}$$

$$\sigma'^3 - J_1 \sigma'^2 + J_2 \sigma' - J_3$$

$$J_1 = (\sigma_{11} - \sigma_m) + (\sigma_{22} - \sigma_m) + (\sigma_{33} - \sigma_m)$$

The Von-Mises criteria:

$$J_2 = k^2$$

→ In simple uniaxial tensile test:

$$\sigma_{II} = \sigma_{III} = 0$$

$$\sigma_I = \sigma_0$$

$\sigma_I, \sigma_{II}, \sigma_{III}$ principal stresses of Deviatoric

$$J_2 = \frac{1}{6}\left[(\sigma_I - \sigma_{II})^2 + (\sigma_{II} - \sigma_{III})^2 + (\sigma_{III} - \sigma_I)^2\right]$$

$$= \frac{1}{6}\left[(\sigma_0)^2 + (-\sigma_0)^2\right] = \frac{\sigma_0^2}{3}$$

$$k^2 = \frac{\sigma_0^2}{3} = \frac{YS^2}{3}$$

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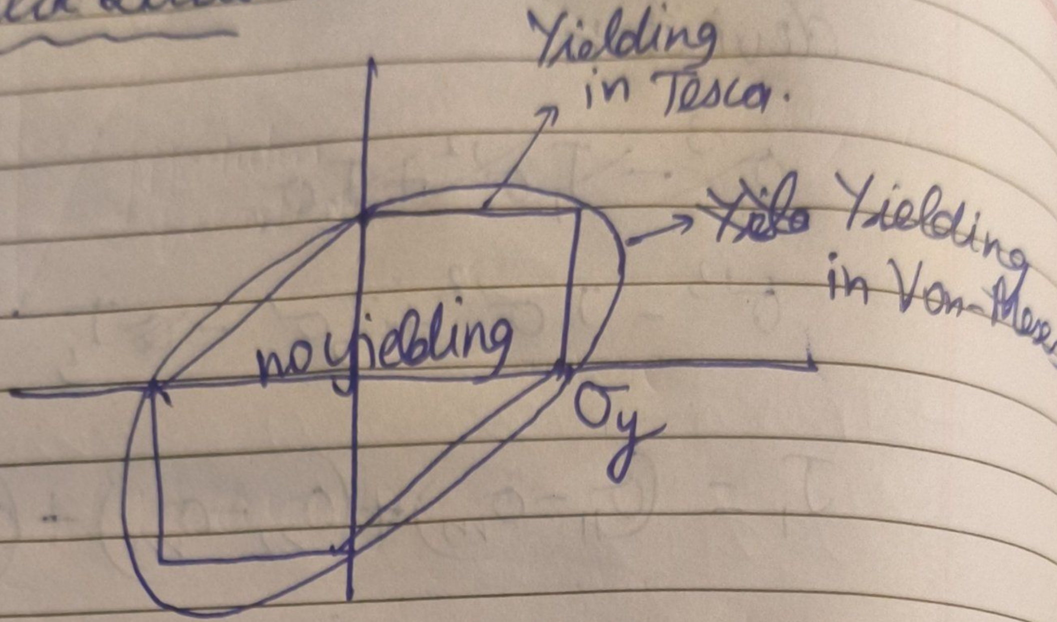
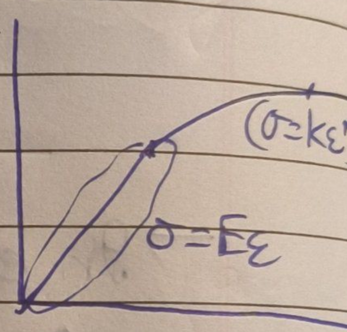
Yield Laws:



Strain Hardening:

$\boxed{\sigma = K\varepsilon^n}$ → strength coefficient.



→ after elastic limit.

n = strain hardening exponent.

n = 0 perfectly plastic

$$n = \frac{d(\log\sigma)}{d(\log\varepsilon)} = \frac{\varepsilon}{\sigma}\frac{d\sigma}{d\varepsilon}$$

n = 1 ⇒

Strain hardening rate:

$$\frac{d\sigma}{d\varepsilon} = n\frac{\sigma}{\varepsilon}$$

$\boxed{n = \varepsilon_0}$

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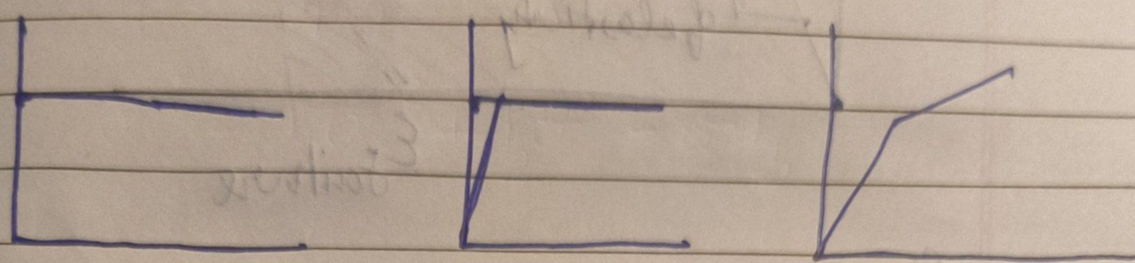
Strain Hardening $\Rightarrow \sigma \uparrow$ as $\varepsilon_{plastic} \uparrow$

$\sigma(\varepsilon_{plastic})$

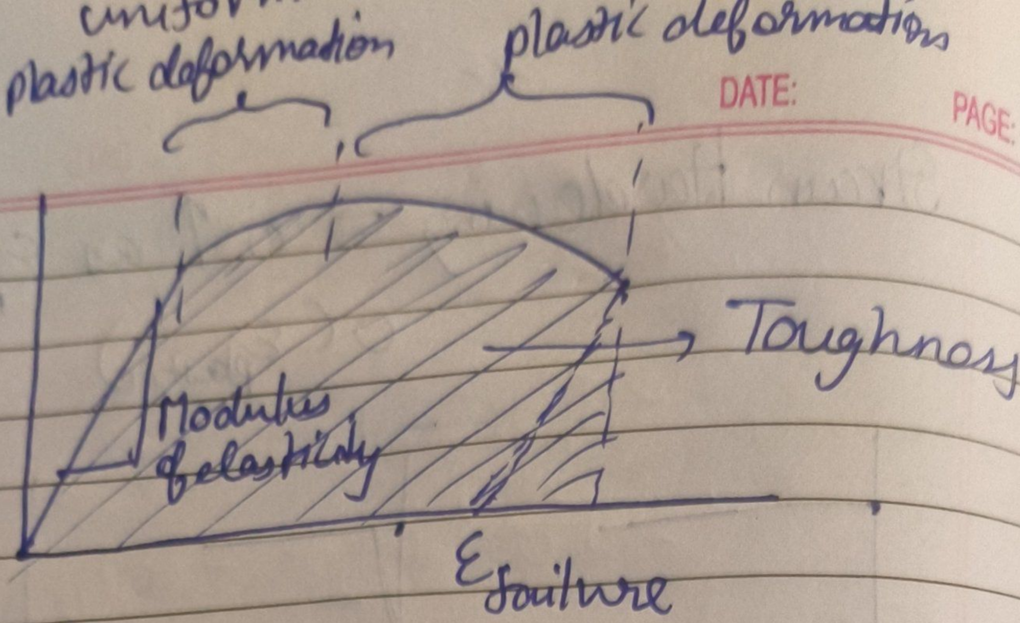


$n=0$	$n=0$	$n=1$
Rigid perfectly plastic	Rigid perfectly plastic with elastic region	linear strain hardening.

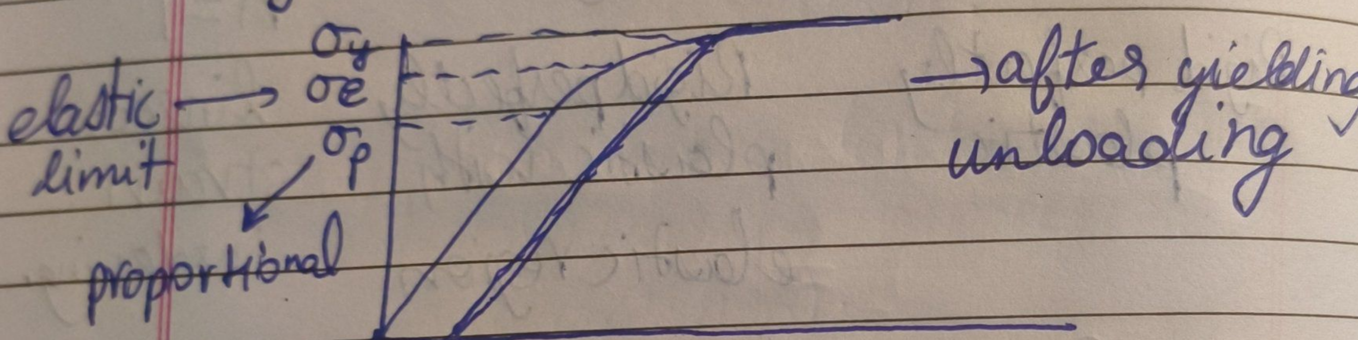
<u>$n = 0.1 - 0.5$ for most metals.</u>

☆ Strain Hardening always occurs for metals.

Eg: materials that are perfectly elastic perfectly plastic.



Physical meaning:



Strain hardening: increase strength by deforming the material.

→ process to promote the metal harder and stronger due to plastic deformation.

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Strain hardening: dislocation interaction

→ dislocations generated with plastic deformation.

→ dislocations will interact and become pinned or tangled:
↳ dislocation forest.

→ This blocks further movement of the dislocation. & promote the strength.

* commonly called work-hardening or cold working

→ plastic deformation occurs at a temperature.

1062°C → MP of Cu

1510°C → MP of Fe

661°C → MP of Al