Hydrostatic stress: $\sigma = \dfrac{\sigma_{xx} + \sigma_{yy} + \sigma_{zz}}{3}$
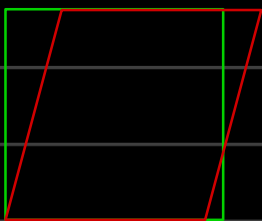
Deviatoric stress: $\boxed{\sigma'_{ij} = \sigma_{ij} - \dfrac{1}{3}\delta_{ij}\,\sigma_{kk}}$

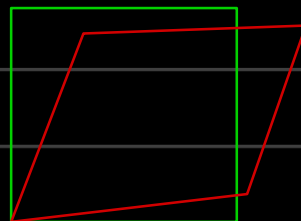Displacement field: $u(x, y, z)$

To determine strain: infinitismal disp.

Normal strain: $\epsilon_{xx} = \dfrac{\Delta du}{du}$ , $\epsilon_{yy} = \dfrac{\Delta dy}{dy}$

$$\epsilon_{zz} = \dfrac{\Delta dz}{dz}$$

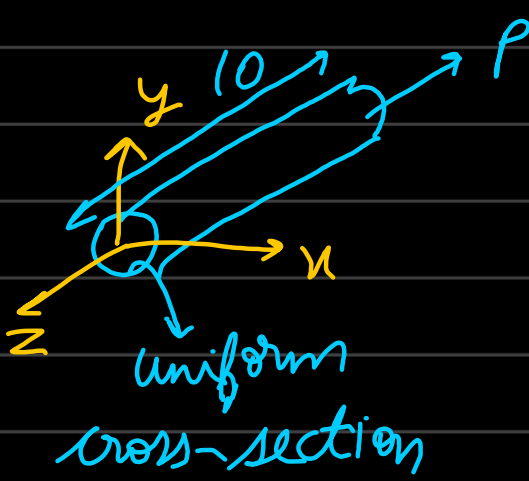Shear Strain: $\gamma_{xy} = 2\epsilon_{xy}$ ; $\gamma_{xz} = 2\epsilon_{xz}$



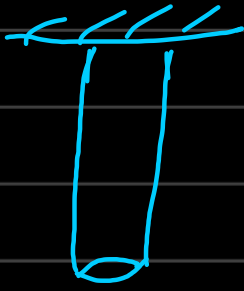Simple Shear          Pure Shear.

Q)

$$\underline{\underline{\sigma}} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \dfrac{P}{A} \end{bmatrix}$$

uniform cross-section

$A = 1\,inch^2$

Q)

$$W(z) = \gamma \frac{\pi D^2}{4} (L - z)$$

$z$

$L - z$

$W(z)$
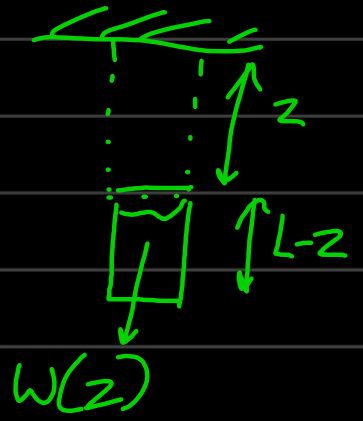
# Strain tensor:

↳ displacement coordinates : $(u, v, w)$

are function of coordinates $u, y, z$

$$u \equiv u(u, y, z) \quad v = v(u, y, z) \quad w = w(u, y, z)$$

$$u = \varepsilon_{uu} u + \varepsilon_{uy} y + \varepsilon_{uz} z$$

$$v = \varepsilon_{yu} u + \varepsilon_{yy} y + \varepsilon_{yz} z$$

$$z = \varepsilon_{zu} u + \varepsilon_{zy} y + \varepsilon_{zz} z$$

↳ Mechanical Metallurgy: John Deter.

Shear components: $\quad \varepsilon_{uy} = \dfrac{\partial u}{\partial y}$

$$\varepsilon_{yz} = \frac{\partial v}{\partial z} \quad \varepsilon_{yu} = \frac{\partial v}{\partial u}$$

$$\underline{\underline{\varepsilon}} = \begin{bmatrix} \varepsilon_{uu} & \varepsilon_{uy} & \varepsilon_{uz} \\ \varepsilon_{yu} & \varepsilon_{yy} & \varepsilon_{yz} \\ \varepsilon_{zu} & \varepsilon_{zy} & \varepsilon_{zz} \end{bmatrix}$$

$$\underline{\underline{e}} = \begin{bmatrix} \dfrac{\partial u}{\partial x} & \dfrac{\partial u}{\partial y} & \dfrac{\partial u}{\partial z} \\[2mm] \dfrac{\partial v}{\partial x} & \dfrac{\partial v}{\partial y} & \dfrac{\partial v}{\partial z} \\[2mm] \dfrac{\partial w}{\partial x} & \dfrac{\partial w}{\partial y} & \dfrac{\partial w}{\partial z} \end{bmatrix}$$

displacement tensor.

Pure shear: {No rotation} $\Rightarrow$ $\varepsilon_{xy} = \varepsilon_{yx}$

Pure Shear: {Rotation} $=$ $\varepsilon_{xy} = -\varepsilon_{yx}$

Simple Shear $=$ $\varepsilon_{xy} = \gamma$ $\varepsilon_{yx} = 0$

$$e_{ij} = \frac{1}{2}\left(e_{ij} + e_{ji}\right) + \frac{1}{2}\left(e_{ij} - e_{ji}\right)$$

$$= \underbrace{\varepsilon_{ij}}_{\text{Strain tensor}} + \underbrace{\omega_{ij}}_{\text{rotation tensor.}}$$

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial v_j}{\partial u_i}\right)$$

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial v_j}{\partial u_i}\right)$$

$$\varepsilon_{ij} = \text{symmetric matrix}$$

$$\omega_{ij} = \text{anti-symmetric matrix.}$$

$$\gamma_{ij} = 2\varepsilon_{ij} \quad \{\text{Engineering strain}\}$$

Volume strain: $\Rightarrow \dfrac{\Delta V}{V}$

$$\Delta = \frac{(1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z)\, dx\,dy\,dz - dx\,dy\,dz}{dx\,dy\,dz}$$

$$\Delta = (1+\varepsilon_x)(1+\varepsilon_y)(1+\varepsilon_z) - 1$$

$$\boxed{\Delta = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}}$$

$\{\text{ignoring } \varepsilon^2 \text{ and } \varepsilon^b \text{ too small}\}$

mean strain: $\varepsilon_m = \dfrac{\Delta}{3} = \dfrac{\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}}{3}$