Lecture 23

Dielectrics & Insulators

Polarization:

$$P = \alpha E \qquad P = \frac{N(Ze)^2}{\beta} E \qquad \beta = \frac{(Ze)^2}{4\pi\varepsilon r^3}$$

→ for changing electric field $E(\omega)$

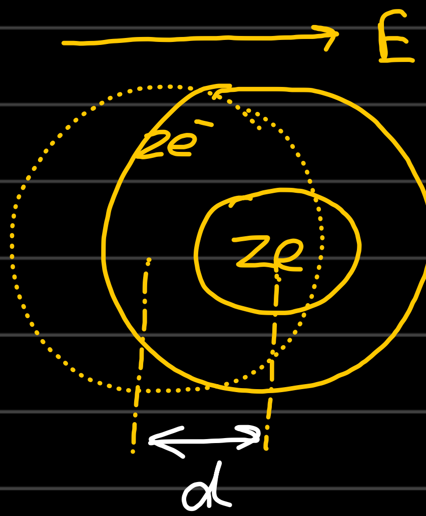
$$\frac{dP}{dt} = -\frac{P - \alpha E(\omega)}{\tau}$$

$$\alpha(\omega) = \frac{\alpha(0)}{1 + j\omega\tau}$$

$$P = \alpha E \;;\; P = \chi \varepsilon_0 E \Rightarrow \varepsilon_r = 1 + \chi$$

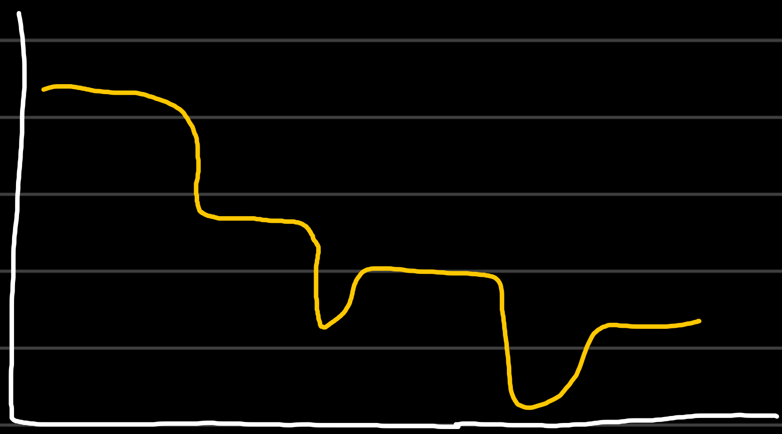
$$\varepsilon_r = \varepsilon_r' - j\,\varepsilon_r''$$

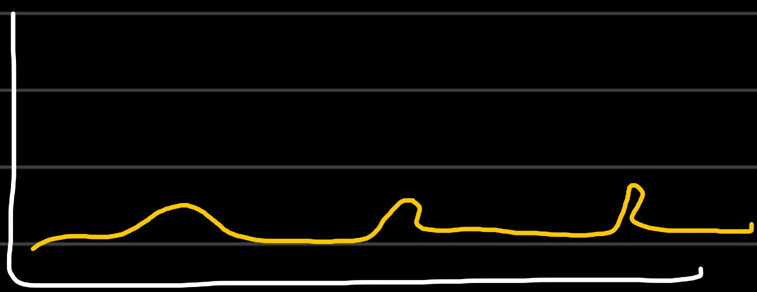
↳ At DC and low frequencies, complex permittivity is the real part.



Loss tangent ⇒ $\tan\delta = \dfrac{\varepsilon_r''}{\varepsilon_r'}$

→ For a typical crystal:



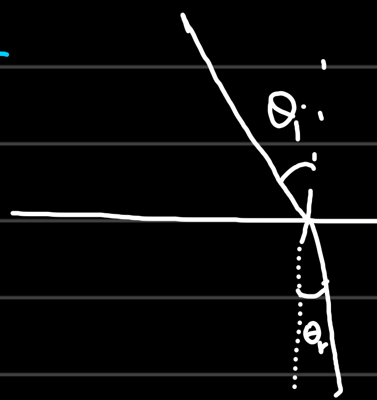


Dielectric and Refractive Indices:

Maxwell's Equation (3rd)

$$\nabla \times H = \frac{\varepsilon}{c}\frac{\partial E}{\partial t} + \frac{4\pi\sigma}{c}E$$

$\nabla \times H \rightarrow$ magnetic field intensity



Snell's Law

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3\times10^8\ m/s$$ speed of light in free air

Vector Identity:

$$\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E$$

Wave Equation of Light:

$$\nabla^2 E = \frac{\varepsilon\mu}{c^2}\frac{\partial^2 E}{\partial t^2}$$

Solution for 3D space:

$$\dot{E}(r,t) = E_0 \exp i(q\cdot r - wt)$$

When travelling in a new medium

$$E'(r,t) = E_0 \exp i(q'\cdot r - wt)$$

$$q' = q_0(\tilde{n})$$

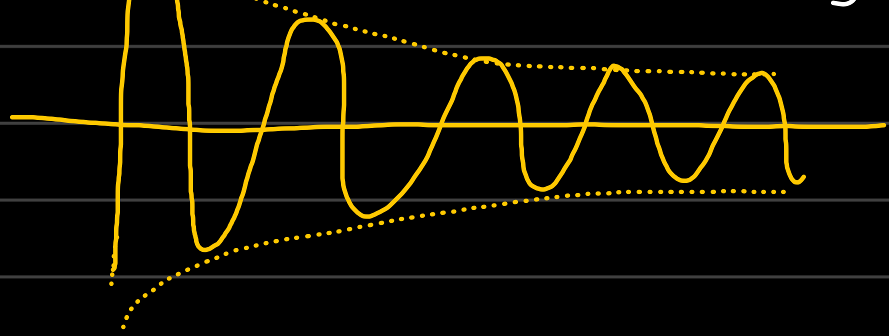
$$q' = q_0(n + iK) \qquad \left\{ \lambda = \frac{\lambda_0}{\tilde{n}} \right\}$$

$$\tilde{n} = n + iK$$

↳ complex refractive index.

$$E'(r,t) = E_0 \underbrace{e^{-K q_0 r}}_{\substack{\text{damping} \\ \{\text{real part}\}}} \; e^{\overbrace{i(n q_0 r - \omega t)}} $$

$$\underbrace{\qquad\qquad}_{\substack{\text{oscillating} \\ \{\text{img part}\}}}$$



→ The reflectivity according to Fresnel eqns.

$$R = \left| \frac{\tilde{n}_1 \cos\theta_i - \tilde{n}_2 \cos\theta_t}{\tilde{n}_1 \cos\theta_i + \tilde{n}_2 \cos\theta_t} \right|^2$$

Absorption in medium:

$$I = E'^2 = I_0 \exp(-2K q_0 r)$$

$$E = E_0 e^{-i(q r - \omega t)}$$

$$\boxed{E' = E_0 e^{-k q_0 r} e^{-i(q_0 n r - \omega t)}}$$

$$E' = E' e^{-i(q' r - \omega t)}$$

$$q' = q_0 (n + ik)$$

Characterization penetration depth W.

$$W = \frac{\lambda_0}{4\pi k}$$

depth at which

$$I = \frac{I_0}{e}$$ {reduces $1/e$ times}

Substiting E' back into wave eqn.

↳ we obtain: $\boxed{\tilde{n}^2 = \varepsilon}$

$\tilde{n}$ → refractive index

ε → dielectric constant.

$$\tilde{n}^2 = n^2 - k^2 - 2ink = \varepsilon_r' - i\varepsilon_r''$$

$$\varepsilon_r' = n^2 - k^2 \qquad \varepsilon_r'' = 2nk$$

$$n^2 = \frac{1}{2}\left(\sqrt{\varepsilon_1^2 + \varepsilon_2^2} + \varepsilon_1\right)$$

$$K^2 = \frac{1}{2}\left(\sqrt{\varepsilon_1^2 + \varepsilon_2^2} - \varepsilon_1\right)$$

→ at small frequencies ω is small.

$$\boxed{\varepsilon_2 = \frac{\sigma}{\varepsilon_0 \omega}} \cong 10^5 \gg \varepsilon_1$$

leaving $\tilde{n}^2 = K^2$

→ Hagen - Ruben relation: $\boxed{R = 1 - \frac{2}{n}}$