Lecture 4

Bulk Modulus K:

$$K = \frac{\sigma_m}{\Delta} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}} = \frac{-p}{\Delta} = \frac{1}{\beta}$$

β = compressibility

Elastic & shear modulii of common metals.

Identities b/w E, K, G:

$$\tau_{xy} = G\gamma_{xy}\ ; \quad \tau_{yz} = G\gamma_{yz}\ ; \quad \tau_{xz} = G\gamma_{xz}$$

$$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu(\sigma_y + \sigma_z)\right)$$

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2\nu}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$\Delta = \left(\frac{1-2\nu}{E}\right) 3\sigma_m$$

$$K = \frac{\sigma_m}{\Delta} = \frac{E}{3(1-2\nu)}$$

$$G = \frac{E}{2(1+\nu)}$$

Strain Energy:

Load deformation: $U = \frac{1}{2} P\delta$

$$dU = \frac{1}{2} P\,du = \frac{1}{2}(\sigma A)(\varepsilon_x dx)$$

$$= \frac{1}{2}(\sigma_x \varepsilon_x)(A\,dx)$$

$$\boxed{U_0 = \frac{1}{2}\sigma_x \varepsilon_x}$$

strain energy per unit volume.

area under $\sigma - \varepsilon$ curve

☆ $$\frac{\partial U_0}{\partial \varepsilon_x} = \sigma_x$$

For pure shear; strain energy/vol is given by

$$U_0 = \frac{1}{2}\tau_{xy}\gamma_{xy} = \frac{1}{2}\frac{\tau_{xy}^2}{G} = \frac{1}{2} G\gamma_{xy}^2$$

Strain energy in 3 dimension!

$$U_0 = \frac{1}{2}\left(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{xz}\gamma_{xz} + \tau_{yz}\gamma_{yz}\right)$$

$$U_0 = \frac{1}{2E}\left(\sigma_x^2 + \sigma_y^2 + \sigma_z^2\right) - \frac{\nu}{E}\left(\sigma_x\sigma_y + \sigma_y\sigma_z + \sigma_x\sigma_z\right)$$

$$+ \frac{1}{2G}\left(\tau_{xy}^2 + \tau_{xz}^2 + \tau_{yz}^2\right)$$

Q) Consider plane stress condition $\{\sigma_3 = 0\}$

$$\varepsilon_x = \frac{1}{E}\left(\sigma_x - \nu\left(\sigma_y + \cancel{\sigma_z}^{\,0}\right)\right)$$

$$\varepsilon_y = \frac{1}{E}\left(\sigma_y - \nu\left(\sigma_x + \cancel{\sigma_z}^{\,0}\right)\right)$$

$$\varepsilon_z = \frac{1}{E}\left(\cancel{\sigma_z}^{\,0} - \nu\left(\sigma_x + \sigma_y\right)\right)$$

$$\sigma_x - \nu\sigma_y = E\varepsilon_x$$

$$\sigma_y - \nu\sigma_x = E\varepsilon_y$$

$$\nu(\sigma_x + \sigma_y) = -E\varepsilon_z$$

$$\sigma_x + \sigma_y = E\left(\varepsilon_x + \varepsilon_y - \varepsilon_z\right) = \frac{-E\varepsilon_z}{\nu}$$

$$\sigma_y = E\left(\varepsilon_x + \varepsilon_y - \varepsilon_z\right) - \sigma_x$$

$$\sigma_x - \nu\left(E(\varepsilon_x+\varepsilon_y-\varepsilon_z) - \sigma_x\right) = E\varepsilon_x$$

$$(1+\nu)\sigma_x = E\varepsilon_x + \nu E(\varepsilon_x+\varepsilon_y-\varepsilon_z)$$

$$\sigma_x = \frac{E\left((1+\nu)\varepsilon_x + \nu(\varepsilon_y-\varepsilon_z)\right)}{1+\nu}$$

$$\sigma_x = E\left(\varepsilon_x + \frac{\nu}{1+\nu}(\varepsilon_y-\varepsilon_z)\right)$$

$$\sigma_y = E(\varepsilon_x+\varepsilon_y-\varepsilon_z) - \sigma_x$$

$$= E(\varepsilon_x+\varepsilon_y-\varepsilon_z) - E\varepsilon_x - \frac{E\nu}{1+\nu}(\varepsilon_y-\varepsilon_z)$$

$$= E\varepsilon_y - E\varepsilon_z - \frac{E\nu}{1+\nu}(\varepsilon_y-\varepsilon_z)$$

$$= E(\varepsilon_y-\varepsilon_z)\left[1 - \frac{\nu}{1+\nu}\right] = \frac{E(\varepsilon_y-\varepsilon_z)}{1+\nu}$$

$$\sigma_x+\sigma_y = E\left(\varepsilon_x + \frac{\nu}{1+\nu}(\varepsilon_y-\varepsilon_z)\right) + E\frac{(\varepsilon_y-\varepsilon_z)}{1+\nu}$$

$$= E\left[\varepsilon_x + \varepsilon_y - \varepsilon_z\right] = -\frac{E\varepsilon_z}{\nu}$$

$$\nu\varepsilon_x + \nu\varepsilon_y - \nu\varepsilon_z = -\varepsilon_z$$

$$\nu(\varepsilon_x+\varepsilon_y) = (\nu-1)\varepsilon_z$$

$$\varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x+\varepsilon_y)$$

$$\varepsilon_x+\varepsilon_y-\varepsilon_z = (\varepsilon_x+\varepsilon_y)\left(1+\frac{\nu}{1-\nu}\right)$$

$$= \frac{\varepsilon_x+\varepsilon_y}{1-\nu}$$

$$\sigma_x+\sigma_y = \frac{E(\varepsilon_x+\varepsilon_y)}{1-\nu}$$

$$\sigma_y = \frac{E(\varepsilon_x+\varepsilon_y)}{1-\nu} - \sigma_x$$

$$\sigma_x - \nu\left[\frac{E(\varepsilon_x+\varepsilon_y)}{1-\nu} - \sigma_x\right] = E\varepsilon_x$$

$$\sigma_x(1+\nu) = E\varepsilon_x + \frac{\nu}{1-\nu}E(\varepsilon_x+\varepsilon_y)$$

$$\sigma_x = \frac{E}{(1+\nu)}\left[\varepsilon_x\left(1+\frac{\nu}{1-\nu}\right)+\frac{\nu}{1-\nu}\varepsilon_y\right]$$

$$= \frac{E}{(1+\nu)}\left[\frac{\varepsilon_x+\nu\varepsilon_y}{1-\nu}\right]$$

$$\sigma_x = \frac{E(\varepsilon_x+\nu\varepsilon_y)}{1-\nu^2}$$

$$\sigma_y = \frac{E(\varepsilon_x+\varepsilon_y)}{1-\nu} - \frac{E(\varepsilon_x+\nu\varepsilon_y)}{(1-\nu)(1+\nu)}$$

$$= \frac{E}{1-\nu}\left[\frac{(1+\nu)(\varepsilon_x+\varepsilon_y) - \varepsilon_x - \nu\varepsilon_y}{1+\nu}\right]$$

$$= \frac{E}{1-\nu}\left[\cancel{\varepsilon_x}+\varepsilon_y+\nu\varepsilon_x+\cancel{\nu\varepsilon_y} - \cancel{\varepsilon_x} - \cancel{\nu\varepsilon_y}\right]$$

$$\sigma_y = \frac{E(\varepsilon_y+\nu\varepsilon_x)}{(1-\nu)(1+\nu)} = \frac{E(\varepsilon_y+\nu\varepsilon_x)}{1-\nu^2}$$

$$\sigma_x = \frac{E(\varepsilon_x+\nu\varepsilon_y)}{1-\nu^2}$$

$$\varepsilon_z = \frac{-\nu}{1-\nu}(\varepsilon_x+\varepsilon_y)$$