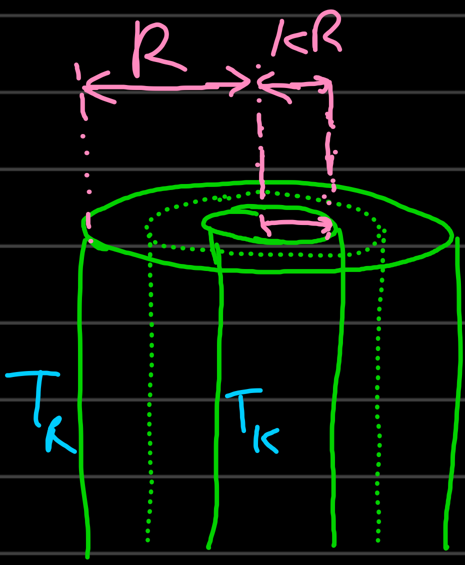
Tutorial - 11

3)



Energy shell balance in r direction.

$$2\pi r L\, e_r|_r - 2\pi r L\, e_r|_{r+\Delta r} = 0$$

$$\frac{1}{\Delta r}\left(e_r|_r - e_r|_{r+\Delta r}\right) = 0$$

$$\frac{1}{r}\frac{\partial}{\partial r}(r e_r) = 0$$

$$\boxed{e_r = \frac{C}{r}}$$

$$\underset{\sim}{e} = \left(\frac{1}{2}\rho v^2 + \rho \hat{H}\right)\underset{\sim}{v} + \underset{\approx}{\tau}\cdot\underset{\sim}{v} + \underset{\sim}{q}$$

$$e_r = \left(\frac{1}{2}\rho v^2 + \rho\hat{H}\right)v_r + \tau_{rr}v_r + \cancel{\tau_{r\theta}v_\theta}^{\,0} + \tau_{rz}v_z + q_r$$

2 way coupling of velocity & temperature.

↳ This is a $\underbrace{\text{free convection problem}}$:

Temp profile known, velocity profile known.

→ density is a function of Temperature.

→ fluid in contact with hot wall going up.
in contact with cold wall comes down.

$$\frac{1}{r}\frac{d}{dr}(r q_r) = 0$$

$$q_r = \frac{c_1}{r} \Rightarrow k\frac{dT}{dr} = -\frac{c_1}{r}$$

$$\Theta = \frac{T - T_k}{T_k - T_R} \qquad \Theta = -1 \text{ at } r = R$$

$$\Theta = 0 \text{ at } r = kR$$

$$\bar{r} = \frac{r}{R} \qquad \bar{r} = 1 \text{ at } r = R$$

$$\bar{r} = k \text{ at } r = kR$$

$$\frac{d\Theta}{d\bar{r}} = \frac{c}{\bar{r}} \Rightarrow \underline{\Theta = c\ln\bar{r}} + c_2$$

$$0 = c\ln k + c_2 \qquad -1 = c_2$$

$$c = \frac{-c_2}{\ln k} = \frac{1}{\ln k}$$

$$\Theta = \frac{\ln\bar{r}}{\ln k} - 1 = \underline{\frac{1}{\ln k}\ln\left(\frac{\bar{r}}{k}\right)}$$

$$\Theta + 1 = \frac{\ln\bar{r}}{\ln k}$$

$$\frac{T - T_R}{T_k - T_R} = \frac{\ln\bar{r}}{\ln k} \Rightarrow \frac{T_R - T}{T_R - T_k} = \frac{\ln\bar{r}}{\ln k}$$

$$\circledast\ \boxed{\Theta' = \frac{\ln\bar{r}}{\ln k}}$$

From Stokes Eqn:

$$= -\frac{\partial P}{\partial z} + \rho g_z + \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V_z}{\partial r}\right)$$

$\rho \downarrow$ function of temperature.

$$\rho(\Theta) = \rho_0\Big|_{\Theta=0} + \frac{\partial \rho}{\partial t}\Big|_{\Theta=0} \Theta + O(\Theta^2)$$

Coefficient of volume expansion: $\beta = \frac{1}{V}\frac{\partial V}{\partial T}$

$$\beta = \rho \frac{\partial (1/\rho)}{\partial T} = -\frac{1}{\rho}\frac{\partial \rho}{\partial T}$$

$$\frac{\partial \rho}{\partial T} = -\beta\rho \qquad \{\text{sensivity of density to temperature}\}$$

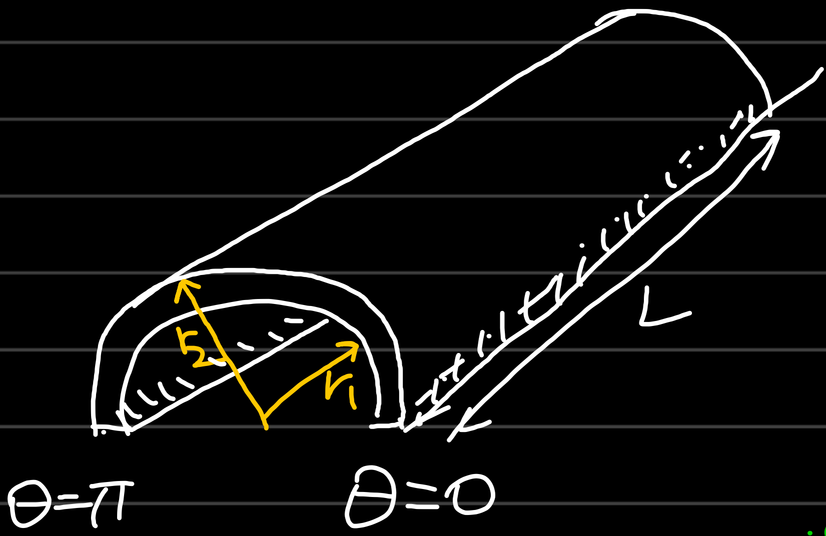
$$\rho(\Theta) = \rho_0 - \beta\rho\Theta$$

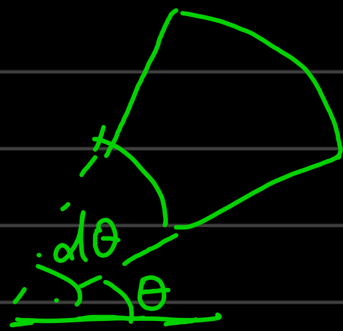
$$= \rho_0 - \beta\rho \frac{\ln \bar{r}}{\ln k}$$

$$0 = -\frac{\partial P}{\partial z} + \rho(\Theta) g + \mu \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V_z}{\partial r}\right)$$

$$0 = -\frac{\partial P}{\partial z} + \left(\rho_0 - \beta\rho \frac{\ln \bar{r}}{\ln k}\right) g + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r \frac{\partial V_z}{\partial r}\right)$$

2)





$$\frac{\Delta r L\, q_\theta|_\theta}{r\Delta\theta\Delta r L} - \frac{\Delta r L\, q_\theta|_{\theta+\Delta\theta}}{r\Delta\theta\Delta r L} = 0$$

$$\frac{1}{r\Delta\theta}\left[q_\theta|_\theta - q_\theta|_{\theta+\Delta\theta}\right] = 0$$

$$\frac{1}{r}\frac{\partial q}{\partial\theta} = 0$$

$$q = C$$

Total heat flow rate: $\int_0^L \int_{r_1}^{r_2} \partial z\, \partial r\, q_\theta$