

Anisotropic Elasticity:

$$\sigma_{kl} = \boxed{C_{ijkl}} \epsilon_{kl}$$

Stiffness tensor
{the ratio of stress component}

$$\begin{aligned}\sigma_{xx} = & C_{xxxx} \epsilon_{xx} + C_{xxxy} \epsilon_{xy} + C_{xxxz} \epsilon_{xz} \\ & + C_{xxyy} \epsilon_{yy} + C_{xyyx} \epsilon_{yx} + C_{xyyz} \epsilon_{yz} \\ & + C_{xxzz} \epsilon_{zz} + C_{xxzx} \epsilon_{zx} + C_{xxzy} \epsilon_{zy}\end{aligned}$$

strain as a linear combination of stress compnt.

$$\epsilon_{xy} = \boxed{S_{xykl}} \sigma_{kl}$$

Compliance tensor

$$C_{xxxx} \neq \frac{1}{S_{xxxx}}$$

$$\begin{aligned}\epsilon_{xx} = & S_{xxxx} \sigma_{xx} + S_{xxxy} \sigma_{xy} + S_{xxxz} \sigma_{xz} \\ & + S_{xxyx} \sigma_{yx} + S_{xxyy} \sigma_{yy} + S_{xxyz} \sigma_{yz} \\ & + S_{xxzx} \sigma_{zx} + S_{xxzy} \sigma_{zy} + S_{xxzz} \sigma_{zz}\end{aligned}$$

Stiffness Matrix: $C_{ijkl} = \begin{bmatrix} \vdots & \ddots & \vdots \\ \vdots & & \vdots \end{bmatrix}_{9 \times 9}$

↳ 81 elements

→ Possible simplifications:

$$\begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ & \sigma_{yy} & \\ & & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} \begin{matrix} \left. \vphantom{\begin{matrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{matrix}} \right\} \text{normal} \\ \left. \vphantom{\begin{matrix} \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{matrix}} \right\} \text{shear} \end{matrix}$$

$$9 \times 9 \longrightarrow 6 \times 6$$

$$[\sigma] = [C][\epsilon] = [C][S][\sigma]$$

$$[C][S] = \underline{\underline{I}} \Rightarrow \underline{\underline{[C] = [S]^{-1}}}$$

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & \cdot & & & & \\ C_{31} & & \cdot & & & \\ C_{41} & & & \cdot & & \\ C_{51} & & & & \cdot & \\ C_{61} & & & & & \cdot \end{bmatrix}$$

→ Consider the symmetry of stiffness & compliance.

→ path-independent nature of linear elasticity

$$W = \frac{1}{2} \sigma \epsilon = \frac{1}{2} E \epsilon^2$$

$$W = \frac{1}{2} (\sigma_1 \epsilon_1 + \sigma_2 \epsilon_2 + \sigma_3 \epsilon_3 + \dots)$$

linear combinations:

$$\sigma_1 = C_{11} \epsilon_1 + C_{12} \epsilon_2 + C_{13} \epsilon_3 + C_{14} \epsilon_4 + C_{15} \epsilon_5 + C_{16} \epsilon_6$$

$$\left. \begin{aligned} C_{12} &= \frac{\partial \sigma_1}{\partial \epsilon_2} = \frac{\partial}{\partial \epsilon_2} \left(\frac{\partial W}{\partial \epsilon_1} \right) = \\ C_{21} &= \frac{\partial \sigma_2}{\partial \epsilon_1} = \frac{\partial}{\partial \epsilon_1} \left(\frac{\partial W}{\partial \epsilon_2} \right) = \end{aligned} \right\} \underline{\underline{C_{12} = C_{21}}}$$

→ reduces to 21 unique elements:

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ & & C_{33} & C_{34} & C_{35} & C_{36} \\ & & & C_{44} & C_{45} & C_{46} \\ & & & & C_{55} & C_{56} \\ & & & & & C_{66} \end{bmatrix}$$

→ From crystal structure symmetry:

Triclinic	⇒	21	} No. of elastic constants
Monoclinic	⇒	13	
Orthorhombic	⇒	9	
Tetragonal	⇒	6	
Hexagonal	⇒	5	
Cubic	⇒	3	
Isotropic	⇒	2	

For Cubic Structures: C_{11}, C_{12}, C_{44}

required unique elastic constants.

→ Stiffness constants in units: GPa

→ Compliance constants in units: TPa⁻¹

(1/TPa)
GPa⁻¹

$$C_{11} = \frac{S_{11} + S_{12}}{(S_{11} - S_{12})(S_{11} + 2S_{12})}$$

→ The elastic modulus in any direction.

$$\frac{1}{E} = S_{11} - 2 \left[(S_{11} - S_{12}) - \frac{1}{2} S_{44} \right] (l^2 m^2 + m^2 n^2 + l^2 n^2)$$

l, m, n are direction cosines of the direction vector

$$[u_1, v_1, w_1] \wedge [u_2, v_2, w_2]$$

$$\cos \Theta = \left[\frac{u_1 u_2 + v_1 v_2 + w_1 w_2}{\sqrt{u_1^2 + v_1^2 + w_1^2} \sqrt{u_2^2 + v_2^2 + w_2^2}} \right]$$

To find $E_{(u,v,w)}$:

$$l = [u, v, w] \wedge [100]$$
$$m = [u, v, w] \wedge [010]$$
$$n = [u, v, w] \wedge [001]$$

→ For isotropic cubic structures:

$$S_{11} = \frac{1}{E} \quad ; \quad S_{12} = -\frac{\nu}{E} \quad ; \quad S_{44} = \frac{1}{G}$$

Plasticity in Materials:

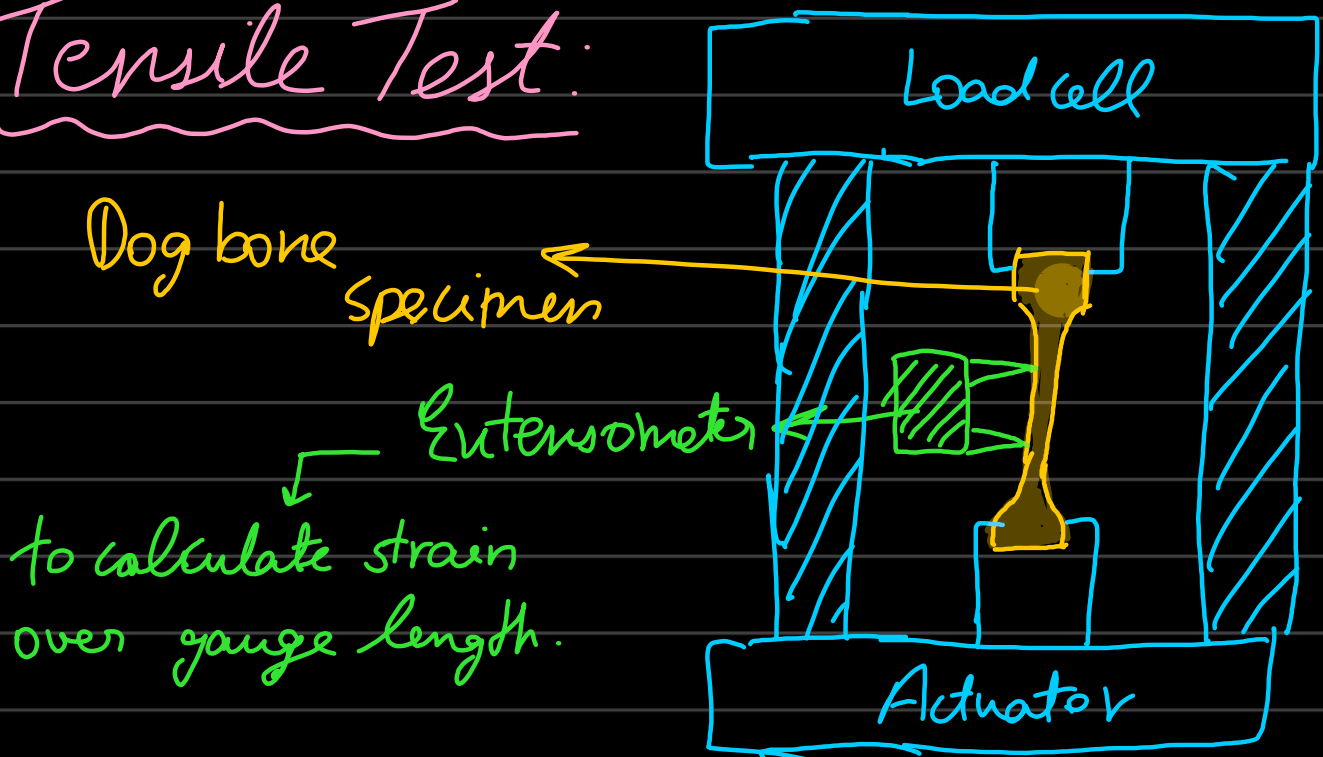
→ Plastic deformation occurs due to dislocation slip or twinning

★ Hydrostatic stresses do not play a role in plastic deformation.

→ Deviatoric stresses leads to plastic deformation

$$S_{ij} = \sigma_{ij} - \sigma_m$$

Tensile Test:



BIS: Bureau of Indian Standards: