Tutorial-8

2) Continuity Equation: $\underset{\sim}{\nabla} \cdot \underset{\sim}{v} = 0$

$$\frac{1}{r}\frac{\partial}{\partial r}(r v_r) + \frac{1}{r}\cancelto{0}{\frac{\partial v_\theta}{\partial \theta}} + \cancelto{0}{\frac{\partial v_z}{\partial z}} = 0$$

$$\frac{v_r}{r} + \frac{\partial v_r}{\partial r} = 0 \qquad r v_r = \text{constant}$$

$$\frac{\partial v_r}{\partial r} = -\frac{v_r}{r} \qquad r v_r = f(\theta, z)$$

θ- momentum balance:

$$0 = -\frac{1}{r}\frac{\partial P}{\partial \theta} + \mu \times \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}$$

r- momentum balance:

$$\rho v_r \frac{\partial v_r}{\partial r} = -\frac{\partial P}{\partial r} + \mu\left[\cancelto{0}{\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial (r v_r)}{\partial r}\right)} + \cancelto{0}{\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}} + \frac{\partial^2 v_r}{\partial z^2}\right]$$

$$\Downarrow$$

$$-\frac{\rho v_r^2}{r} = -\frac{\partial P}{\partial r} + \mu \frac{\partial^2 v_r}{\partial z^2}$$

axisymmetry

$$\left\{ v_r = \frac{f(z)}{r} \right\}$$

$$-\frac{\rho f^2}{r^3} = -\frac{\partial P}{\partial r} + \mu \frac{f''(z)}{r}$$

$$-\int_{r_1}^{r_2} \frac{\rho f^3}{r^3} = -\int \partial P + \mu \int \frac{f''(z)}{r}$$

$$+\frac{\rho f^3}{2r^2}\Bigg|_{r_1}^{r_2} = P\Big|_{r_1}^{r_2} + \mu f''(z) \ln\left(\frac{r_2}{r_1}\right)$$

$$\cancelto{0}{\frac{\rho f^3}{2}\left[\frac{1}{r_2^2} - \frac{1}{r_1^2}\right]} = P\Big|_{r_1}^{r_2} + \mu f''(z) \ln\left(\frac{r_2}{r_1}\right)$$

No inertial forces in Stokes Law.
this term came from inertial force.

Reynolds no. : $\frac{\rho U L}{\mu} \rightarrow \frac{\rho U^2}{(\mu U / L)} = \frac{\text{inertial}}{\text{viscous}}$.

eg)