In partial fulfilment of the requirements of the course

Mathematical Modelling MATH F420

TOPIC: Irregular Warfare Model

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ABSTRACT

Mathematical models can be applied in a combat situation to study the dynamics of a war and henceforth predict the fate of such engagement. Military operations research and combat modeling use mathematical models to study and gain insights into a variety of military engagements.
With the help of Lanchester equations, which are a system of Ordinary differential equations, the mutual attritional dynamics of two opposing military forces have been analyzed which provide some insights into the outcomes of such exchanges.
In this paper, we review models that represent irregular warfare with the help of some basic and practical assumptions, a model which includes technological advancements in case of a long drawn war, emphasizing the importance of reinforcements in these types of operations as well as multilateral scenarios in which numerous armies are involved, where in the case of distribution of forces as per the war situation also has been accounted for.
Stability analysis which includes finding the fixed points, eigenvalues and simulation of graphs through python codes for the differential equations for various values of parameters have also been shown.

INTRODUCTION

One of the earliest and most important sets of models used for combat modeling is the Lanchester equations. They are a system of ordinary differential equations describing the mutual attrition that occurs continuously in time between two opposing forces engaged in violent confrontation. The equations involve state variables that represent the number of live combatants (or weapons) at any given time during the battle.

In this paper, we review recent applications to irregular warfare. One type of application is in which the situational awareness capabilities on both sides are asymmetric; one side can target better than the other and therefore gains an advantage. A second type of model relates to cases where the two sides of the conflict are profoundly different in terms of their force structures and their associated attritional dynamics. A third type of irregular conflict is when there are more than two sides competing for dominance. We call such conflicts multilateral conflicts. Each equation expresses the rate of change in one state variable as a function of other state variables. There are several models that differ in their underlying assumptions regarding the operational posture and/or the tactical situation. We describe the two most common models initially.

Lancaster's theory is used as a basis while incorporating required changes like introducing various other parameters to make the model more practical and real-life based and are used by military analysts to study force structure and accordingly plan courses of action.

Irregular warfare is also characterized by asymmetry in the mechanism that provides information, intelligence and situational awareness to the two sides.

ASSUMPTIONS

- We have assumed that the population is homogeneous, that is every soldier is equally likely to be killed. No such soldier has a different protection or special features.

- Visibility is relaxed in all situations, hence, there are no hidden soldiers.

- Injuries don't affect the effectiveness of the armies, that is a soldier is either dead or fighting.

- All soldiers have equal potential. Each one of them are trained equally and have similar skills, agility or capabilities.

- Natural deaths are negligible as compared to war casualties. The deaths due to extreme weather conditions, cardiac arrest have been considered negligible.

AIMED FIRE MODEL

In 1916, Frederick William Lanchester proposed using a set of ordinary differential equations to predict the reciprocal attrition of two fighting armies (ODE). Each of the ODEs captures the rate of reduction in a certain state variable as a function of the other state variables. These models were inspired by World War II air combat scenarios.

While, in general, each side may have several types of fighting combatant, each with different attrition rates, we assume here, for simplifying the initial exposition, that each of the two sides comprises a homogeneous force.

Let B = B(t) and R = R(t) be state variables indicating the sizes of the Blue and Red forces' remaining combatants at time t, respectively. The aimed-fire model depicts a combat scenario in which each combatant on the Blue (Red) side lowers the force on the Red (Blue) side at a set attrition rate. The aimed-fire model is defined as follows:

$$\frac{dB(t)}{dt} = -\alpha R$$
$$\frac{dR(t)}{dt} = -\epsilon B$$

By the separation of variables, one can obtain the state-equation: where B_0 and R_0 are the force sizes of Blue and Red, respectively, at the beginning of the battle.

$$\varepsilon(B_0^2 - B^2) = \alpha(R_0^2 - R^2)$$

In particular, we obtain the parity condition—the values of B_0, R_0,α and ε such that the battle ends with mutual annihilation.

$$\varepsilon B_0^2 = \alpha R_0^2$$

AREA FIRE MODEL

The second defines unaimed fire, in which the effect of one's fire is determined not only by the size of one's own surviving force, but also by the density of the opposing force's targets. Because the firing isn't targeted, the chances of hitting a target on the other side are proportional to the number of such targets in the vicinity. This is why this model is also known as the Area Fire model. In this example, the differential equations are:

$$\frac{dB(t)}{dt} = -\gamma RB$$

$$\frac{dR(t)}{dt} = -\delta RB$$

Here γ is the kill rate of R when R fires in the area containing B's Forces and δ is the kill rate of B when B fires in the area containing R's Forces.

LANCHESTER COMBINED EQUATIONS

There is hardly any war in practical scenarios based on only the area-fire model or only the aim fire model. A real war is likely to contain both area and aim fire model aspects. For e.g., a battle in which an army comprises both machine guns (area-fire) and snipers(aimed-fire).
The following equations represent the combined model:

$$\frac{dB(t)}{dt} = -\alpha R - \gamma RB$$

$$\frac{dR(t)}{dt} = -\varepsilon B - \delta RB$$

Where α = rate by which R is killing B in the aim-fire model
ε = rate by which B is killing R in the aim-fire model
γ = rate by which R is killing B when R is firing in the region containing B
δ = rate by which B is killing R when B is firing in the area having R

Stability Analysis:

$$\frac{dB(t)}{dt} = 0 \quad \text{and} \quad \frac{dR(t)}{dt} = 0$$

$$(\alpha+\gamma B)R=0 \quad \text{and} \quad (\varepsilon + \delta R)B = 0$$

Fixed points are (0,0) and $(-\frac{\alpha}{\gamma}, -\frac{\varepsilon}{\delta})$

Here $(-\frac{\alpha}{\gamma}, -\frac{\varepsilon}{\delta})$ is physically insignificant as R, B≥ 0 at all times t
Hence (0,0) is the only fixed point

Constructing Jacobian Matrix:

$$J = \begin{bmatrix} -\gamma R & -\alpha - \gamma B \\ -\varepsilon - \delta R & -\delta B \end{bmatrix}$$

$$\mathrm{J}(0,0) = \begin{bmatrix} 0 & -\alpha \\ -\varepsilon & 0 \end{bmatrix}$$

The characteristic equation of the above Jacobian is:

$$\lambda^2 - \alpha\varepsilon = 0$$

$$\lambda = \pm\sqrt{\alpha\varepsilon}$$

We can see that (0,0) will be a saddle point as we are getting 2 real eigenvalues, one positive and one negative.

INTRODUCING REINFORCEMENTS

In the previous model (aimed fire + unaimed fire), we got two fixed points (0,0) and $(-\frac{\alpha}{\gamma}, -\frac{\varepsilon}{\delta})$. However, since the population of an army can never be < 0 at any time t, we had to discontinue the model. Instead, we made some modifications by introducing reinforcements. In actual practical scenarios, the armies involved in the war keep on undergoing reinforcements and we wanted to show that through our model.

The reinforcement at the time t depends on only two factors:

1. The size of the opposing army at that time. For e.g., for army B, the reinforcement $\propto R$
2. The difference between the initial army size and the current army size. E.g., for army B, the reinforcement $\propto (B_0 - B)$

   Keeping these 2 points in mind, the equations popped out as:

$$\frac{dB(t)}{dt} = -\alpha R - \gamma BR + u_1 R(B_0 - B)$$

$$\frac{dR(t)}{dt} = -\varepsilon B - \delta BR + u_2 B(R_0 - R)$$

Where u_1 and u_2 are the reinforcement rates for B and R armies, respectively.

Stability Analysis:

$$\frac{dB(t)}{dt} = 0 \quad \text{and} \quad \frac{dR(t)}{dt} = 0$$

We get equations as:

$$R[\alpha + (\gamma + u_1)B - u_1 B_0] = 0$$
$$B[\varepsilon + (\delta + u_2)R - u_2 R_0] = 0$$

Now, if R=0, then either B=0 or $\varepsilon = u_2 R_0$
Now, if B=0, then either R= 0 or $\alpha = u_1 B_0$

If $R \neq B \neq 0$
$B^* = \frac{u_1 B_0 - \alpha}{\gamma + u_1}$ and $u_1 B_0 \geq \alpha$ is the condition for B^* to exist

$R^* = \frac{u_2 R_0 - \varepsilon}{\delta + u_2}$ and $u_2 R_0 \geq \varepsilon$ is the condition for R^* to exist

So the fixed points are (0,0) and (B^*, R^*)

Constructing the Jacobian Matrix:

$$J = \begin{bmatrix} -\gamma R - u_1 R & -\alpha - \gamma B + u_1(B_0 - B) \\ -\varepsilon - \delta R + u_2(R_0 - R) & -\delta B - u_2 B \end{bmatrix}$$

For (0,0) fixed point:

$$J(0,0) = \begin{bmatrix} 0 & -\alpha + u_1 B_0 \\ -\varepsilon + u_2 R_0 & 0 \end{bmatrix}$$

$\tau = 0$ and $\Delta = -(-\alpha + u_1 B_0)(-\varepsilon + u_2 R_0)$

Characteristic equation:

$$\lambda^2 - (u_1 B_0 - \alpha)(-\varepsilon + u_2 R_0) = 0$$

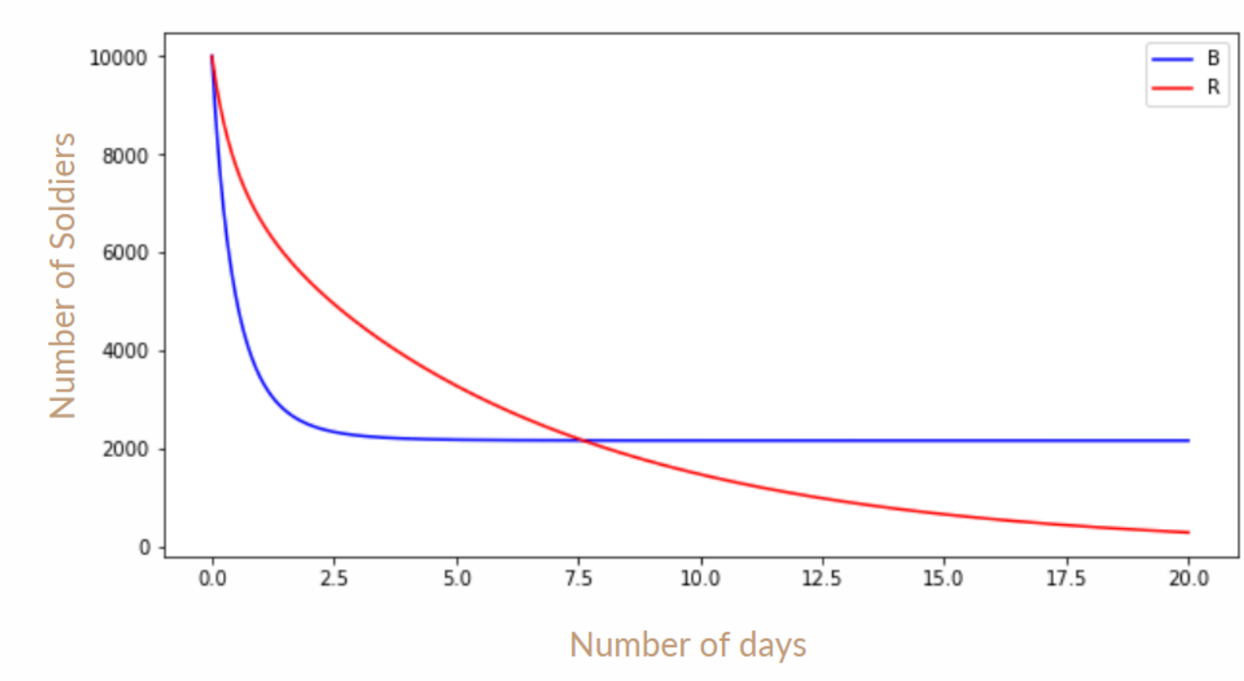
Eigenvalues: $\pm\sqrt{(-\alpha + u_1 B_0)(-\varepsilon + u_2 R_0)}$
Since we already have the conditions that $u_1 B_0 \geq \alpha$ and $u_2 R_0 \geq \varepsilon$, therefore, the eigenvalues are real, distinct and of opposite signs.
Therefore, (0,0) - saddle point

$$J(B^*, R^*) = \begin{bmatrix} -(\gamma + u_1)(\frac{u_2 R_0 - \varepsilon}{\delta + u_2}) & 0 \\ 0 & -(\delta + u_2)(\frac{u_1 B_0 - \alpha}{\gamma + u_1}) \end{bmatrix}$$

Eigenvalues:

λ_1 = -$(\gamma + u_1)(\frac{u_2 R_0 - \varepsilon}{\delta + u_2})$ < 0 due to the condition derived earlier
λ_2= $-(\delta + u_2)(\frac{u_1 B_0 - \alpha}{\gamma + u_1})$< 0 due to the condition derived earlier

Hence (B^*, R^*) is a stable fixed point.



STABILITY OF COMBINED EQUATIONS

First, consider the case of equal Ro and Bo for both armies, i.e, equal army sizes at the beginning of the war. For this case, under the following values of the parameters:

$$\alpha=0.3 \quad \gamma=0.00015 \quad \epsilon=0.7$$
$$\delta=0.000003 \quad \mu_1=0.00008 \quad \mu_2=0.00007$$

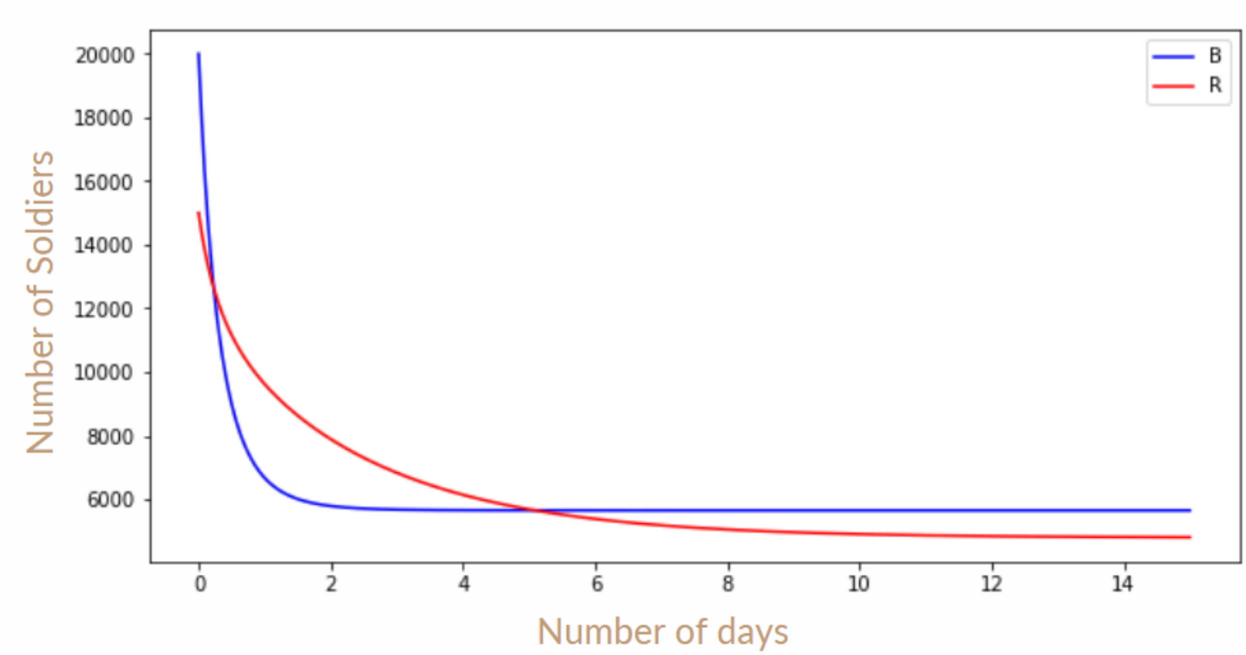
It can be clearly seen that the killing rate of blue is greater than red under aimed firing while under area firing, the killing rate of red is greater than blue. Coming to the reinforcement rates, it’s greater for red again. From the graph, it can be observed that for the initial period of time, size of the blue army becomes quite low but eventually in the long run it saturates to the value B= 2173.9 while it becomes 0 for R. A question one might ask is why the values of the parameters γ and δ are so low compared to ε and α.The reason behind that is because lanchester’s model considered that every soldier in the attacking army

has an equal range i.e he can target each and every soldier in the opposing force’s army. But in real life situations this is most likely to be false because a soldier standing at the rear of the army most likely cannot be targeted by the soldier standing in the rear of his army so to compensate for it and improve the feasibility of the model we have taken normalized firing rates.
The reinforcement terms are proportional to the product of army strength of both the armies and negatively proportional to RBo. In case the reinforcement rate is very high, the overall factor would become very large due to the proportionality, which is clearly not possible. Hence, μ_1 and μ_2 are normalized reinforcement rates, with such less values.
It can also be seen that the fixed points obtained from the stability analysis of the model:

$$(\mathrm{B}^*, R^*) = \left(\frac{\mu_1 B_o - \alpha}{\gamma + \mu_1}, \frac{\mu_2 R_0 - \epsilon}{\delta + \mu_2}\right)$$

gives the same point (B*, R*) = (2173.9, 0) for above mentioned values of the parameters, which depicts that eventually both armies’ sizes converge to the fixed point (B*, R*) found by the stability analysis of the model, which was expected as the fixed point (B*, R*) came out to be a stable fixed point.

Here, $\mu_2 R_o$ = ϵ and therefore R* comes down to 0 and consequently loses the war.

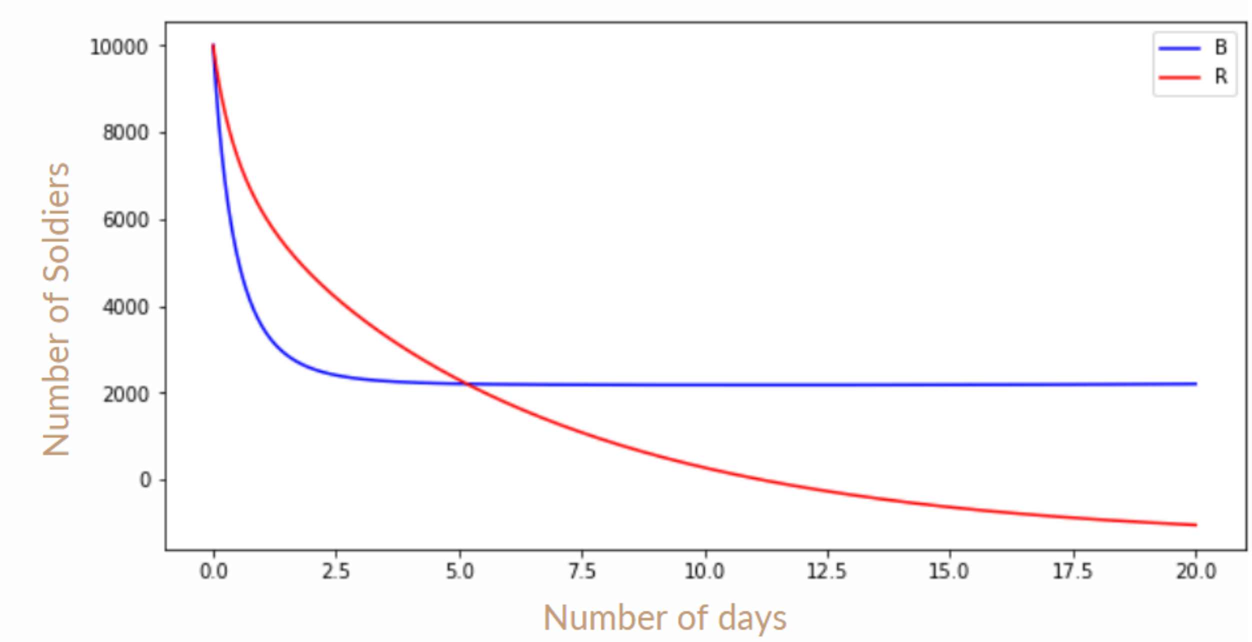


This time consider unequal sizes of the armies in the beginning of the war, 20000 for blue and 15000 for red, along with the following values of parameters:

$$\alpha=0.3 \quad \gamma=0.00015 \quad \epsilon=0.7$$
$$\delta=0.000003 \quad \mu_1=0.00008 \quad \mu_2=0.00007$$

It can be clearly seen that the killing rate of blue is greater than red under aimed firing while under area firing, the killing rate of red is greater than blue. Coming to the reinforcement rates, it's greater for red again. From the graph It can be seen that for the initial period of time size of the blue army becomes quite low but eventually in the long run it saturates to the value B*= 5652.2. For Red, they started with less army size, for a short period they had greater army size than blue , but at the end the value saturates to 4794.5, slightly less than blue, resulting in no army winning the war. Same above mentioned values are obtained for (B*,R*) when calculated using the formula obtained earlier:

$$(\mathrm{B}^*, R^*) = (\frac{\mu_1 B_o - \alpha}{\gamma + \mu_1}, \frac{\mu_2 R_0 - \epsilon}{\delta + \mu_2})$$



For this graph consider equal army sizes at the beginning of the war, with the following values of parameters:

α =0.3 γ=0.00015 ϵ =0.8

δ=0.000003 μ_1 =0.00008 μ_2=0.00007

It can be clearly seen that the killing rate of blue is greater than red under aimed firing while under area firing, the killing rate of red is greater than blue. Coming to the reinforcement rates, it's greater for red again.This time the value of firing rate of blue under aimed firing has been increased to 0.8, such that $\mu_2 R_0$ ϵ becomes less than ,
which makes the final value of size of red army negative (-1369.8), which is physically not feasible. While for blue the value decreases initially and saturates to 2173.9 over time.

Again,same above mentioned values are obtained for (B*,R*) when calculated using the formula obtained earlier:

$$(\mathrm{B}^*, R^*) = \left(\frac{\mu_1 B_o - \alpha}{\gamma + \mu_1}, \frac{\mu_2 R_0 - \epsilon}{\delta + \mu_2}\right)$$

A question one might ask is why the values of the parameters γ and δ are so low.

The reason behind that is because lanchester's model considered that every soldier in the attacking army has an equal range i.e he can target each and every soldier in the opposing force's army. But in real life situations this is most likely to be false because a soldier standing at the rear of the army most likely cannot be targeted by the soldier standing in the rear of his army so to compensate for it and improve the feasibility of the model we have taken normalized firing rates.

MULTILATERAL CONFLICT

Legacy Lanchester equations, as previously stated, effectively model the attrition between two opposed forces. They catch a force-on-force confrontation in a duel. Recent and certain historical battles, on the other hand, involve more than two opposing forces. Bosnian Civil War (Croatia, Bosnia and Herzegovina, Serbia, NATO), Iraq Civil War (Coalition Forces, Sunni Militia, Shia Militia), and, most recently, Syria Civil War (Assad Regime Forces, Free Syrian Army, Hezbollah, Kurds, Russia, Turkey) are only a few examples of multilateral violent wars.

Neither army is compelled to make a choice during the engagement. In a multi-person fight, however, each army must decide how to distribute his or her strength among the other opponents in order to optimize his or her chances of winning. This decision, which affects all other players, results in a prescriptive paradigm in which each of the n players (n > 2) must dynamically distribute their existing power among their n - 1 opponents. While the results are universal, the focus in this report will only be on the n = 3 scenario.

In order to model such a scenario Lanchester proposed that each army distribute their army strength into two proportions that remain constant throughout the war.

Let, α_{ij} denote the proportion of the army directed by ‘i’ towards ‘j’, For example α_{RB} denotes the proportion of the army Red directs towards Blue and so on.

Then, the total force blue faces becomes $\alpha_{RB}.R$ from Red and $\alpha_{GB}.G$ from Green. Then, if $\rho_{B(G)}$ denotes attrition/killing rate of Red against Blue(Green), $\beta_{R(G)}$ denotes attrition/killing rate of Blue against Red(Green) and $\gamma_{R(B)}$ denotes attrition/killing rate of Green against Red(Blue), then Red Army is reducing the Blue's army by the term $\alpha_{RB}.R.\rho_B$ and Green Army is reducing the Blue's army by the term $\alpha_{GB}.G.\gamma_B$. Therefore, Blue's army is being destroyed by the combined effect of both the Red's and Green's armies and similarly the implication holds for the rate of decrease of Green's and Red's armies as well.

Therefore, the Lanchester direct-fire model for three armies (Blue, Red, and Green) is as follows:

$$\frac{dB(t)}{dt} = -\alpha_{RB}\rho_B R(t) - \alpha_{GB}\gamma_B G(t)$$

$$\frac{dR(t)}{dt} = -\alpha_{BR}\beta_R B(t) - \alpha_{GR}\gamma_R G(t)$$

$$\frac{dG(t)}{dt} = -\alpha_{BG}\beta_G B(t) - \alpha_{RG}\rho_G R(t)$$

Where,

$$\alpha_{BR} + \alpha_{BG} = \alpha_{GR} + \alpha_{GB} = \alpha_{RB} + \alpha_{RG} = 1$$

Multilateral Conflicts with variable proportion

The Multilateral Conflict model given by Lanchester was under the assumption that the proportion in which each army divides its forces against other armies is decided at the starting of the war and remains constant throughout the war i.e. α_{ij} is predefined before the war and is constant throughout the war.

But, clearly in the real world it is possible that this proportion may change continuously during the war and more often than not this is the case in real-war scenarios. Therefore, for better utility of the model this assumption needs to be dropped.

The model prepared by us aimed to solve this limitation of the Lanchester's equations.

Assuming, each army has information of the attrition/killing rates of other armies against them, a reasonable way for each army to distribute its army size towards another army would be in the ratio of opposing force experienced due to that particular army upon the total opposing force against the army.

Consider the Red army for instance. The opposing force of Blue army against Red would be $\beta_R B$ and similarly the opposing force of Green army against Red would be $\gamma_R G$. Therefore, the total opposing force faced by Red would be $\beta_R B + \gamma_R G$.

So, Red may choose to direct $\left(\frac{\beta_R B}{\beta_R B+\gamma_R G}\right)$ proportion of its army against Blue and $\left(\frac{\gamma_R G}{\gamma_R G+\beta_R B}\right)$proportion of its army against Green.

Therefore, we get :

$$\boldsymbol{\alpha}_{RB} = \left(\frac{\beta_R B}{\beta_R B+\gamma_R G}\right)$$

$$\boldsymbol{\alpha}_{RG} = \left(\frac{\gamma_R G}{\gamma_R G+\beta_R B}\right)$$

and similarly,

$$\boldsymbol{\alpha}_{BR} = \left(\frac{\rho_B R}{\rho_B R+\gamma_B G}\right) \qquad \boldsymbol{\alpha}_{GR} = \left(\frac{\rho_G R}{\rho_G R+\beta_G B}\right)$$

$$\boldsymbol{\alpha}_{BG} = \left(\frac{\gamma_B G}{\gamma_B G+\rho_B R}\right) \qquad \boldsymbol{\alpha}_{GB} = \left(\frac{\beta_G B}{\beta_G B+\rho_G R}\right)$$

It can also be noted that the conditions

$$\alpha_{BR} + \alpha_{BG} = \alpha_{GR} + \alpha_{GB} = \alpha_{RB} + \alpha_{RG} = 1$$

still hold.

Therefore, the final equations become:

$$\frac{dB(t)}{dt} = -(\frac{\beta_R B}{\beta_R B+\gamma_R G})\rho_B R - (\frac{\beta_G B}{\beta_G B+\rho_G R})\gamma_B G$$

$$\frac{dR(t)}{dt} = -(\frac{\rho_B R}{\rho_B R+\gamma_B G})\beta_R B - (\frac{\rho_G R}{\rho_G R+\beta_G B})\gamma_R G$$

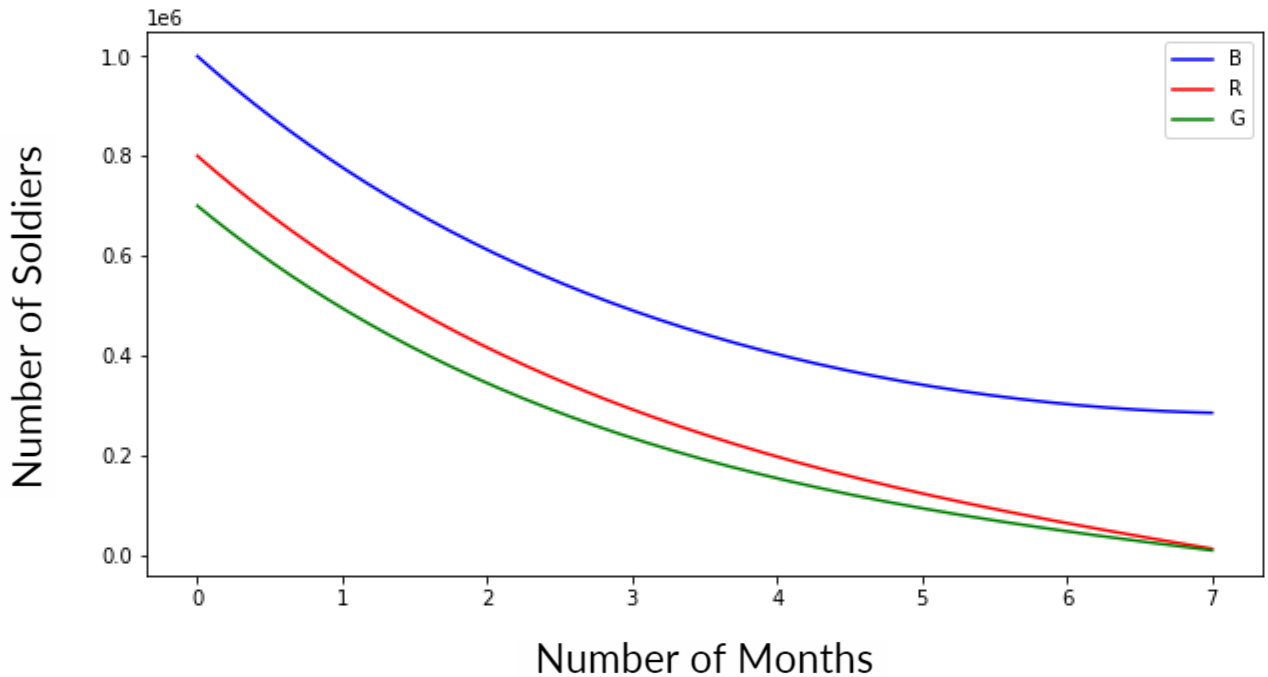
$$\frac{dG(t)}{dt} = -(\frac{\gamma_B G}{\gamma_B G+\rho_B R})\beta_G B - (\frac{\gamma_R G}{\gamma_R G+\beta_R B})\rho_G R$$

Computer Simulations for Multilateral conflict with Variable Proportion

The graphs can be plotted for the various values of killing rates of R, B and G to deduce the outcome of the war and observe the decline in the army strength for each army, which can be used in making appropriate strategies, plan of action, etc.

First, consider the case where killing rates of all the armies against each other is same, say 0.3

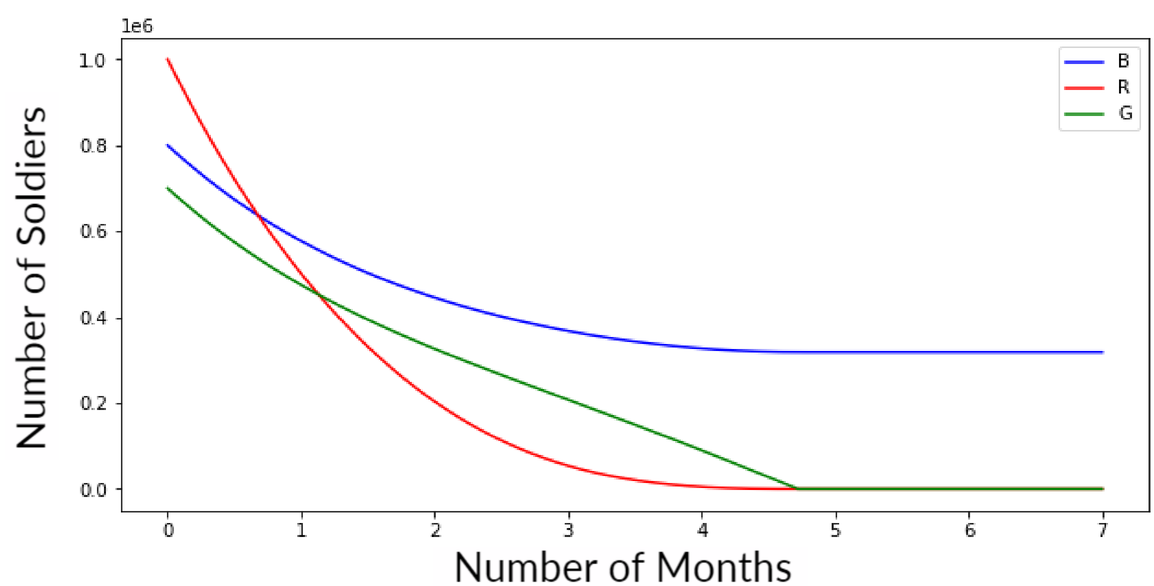
i.e. $\boldsymbol{\rho}_B = \boldsymbol{\rho}_G = \boldsymbol{\beta}_R = \boldsymbol{\beta}_G = \boldsymbol{\gamma}_R = \boldsymbol{\gamma}_B = 0.3$



Here, as the initial army size of Blue was largest followed by Red and then Green and since all the killing rates were equal, Green army is annihilated first followed by Red and Blue remains the ultimate winner

For the killing rates as:

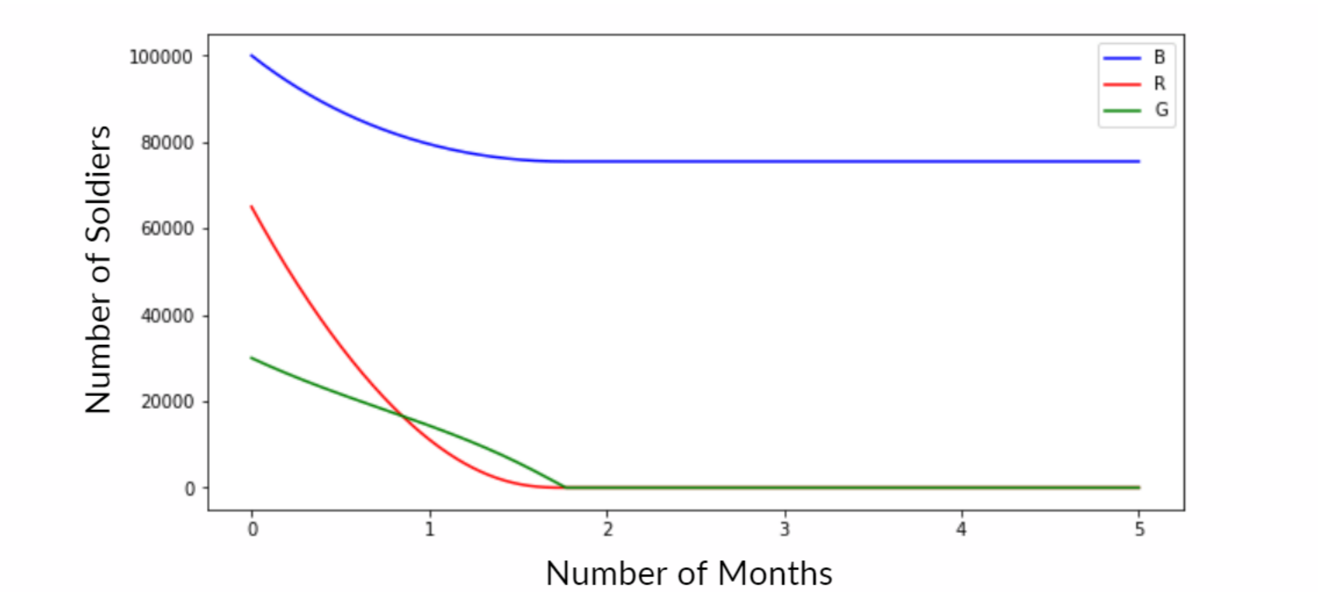
$\boldsymbol{\rho}_B$ =0.35, $\boldsymbol{\rho}_G$ =0.45, $\boldsymbol{\beta}_R$ =0.75, $\boldsymbol{\beta}_G$ = 0.4, $\boldsymbol{\gamma}_R$ =0.55, $\boldsymbol{\gamma}_B$ = 0.25



Even though, initial army size of Red was largest, since the killing rates of both Blue and Green against Red are comparatively high, Red loses first.

For killing Rates as:

$\boldsymbol{\rho}_B = 0.6$, $\boldsymbol{\rho}_G = 0.4$, $\boldsymbol{\beta}_R = 0.8$, $\boldsymbol{\beta}_G = 0.3$, $\boldsymbol{\gamma}_R = 0.7$, $\boldsymbol{\gamma}_B = 0.55$



AIMED FIRE MODEL WITH TECHNOLOGICAL ADVANCEMENTS

Lanchester's Aimed fire model

$$\frac{dB(t)}{dt} = -\alpha R$$

$$\frac{dR(t)}{dt} = -\epsilon B$$

Lanchester assumed for an aimed fire model that the attrition rates of both the armies remained constant throughout the war. This can be considered a reasonable assumption in case of a short term war but for long drawn wars, consider the World War which lasted for years, this assumption is not valid. Even history suggests that the maximum developments in war technologies have occured during wars. Most of the modern technologies had been given birth during wars.
To incorporate this point, we tried to formulate a modified model in which the attrition rates are a function of time. Hence the varying attrition rates depict the technological advancements happening over time.

$$\frac{dB(t)}{dt} = -\alpha(t)R$$

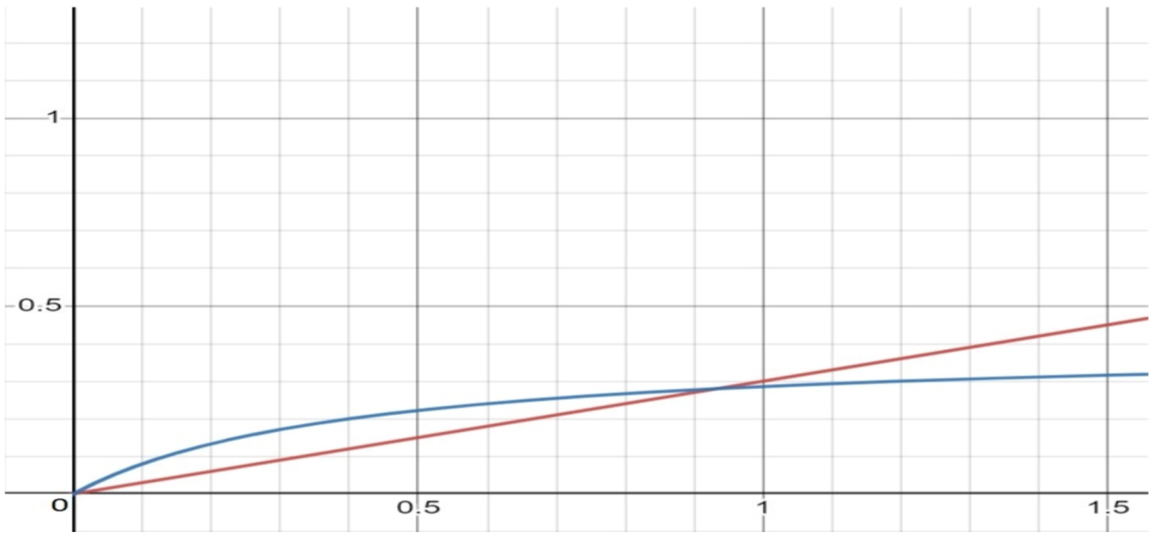
$$\frac{dR(t)}{dt} = -\epsilon(t)B$$

Two cases considered - one in which the attrition rate increases linearly with time while the other increases at a high rate upto a certain time and then starts saturating.

$$\alpha(t) = \lambda_1 t$$

$$\epsilon(t) = \left(\frac{\lambda_2 t}{\lambda_2 + t}\right)$$

where λ1 and λ2 are constants.



λ_1 = 0.3 λ_2 = 0.4

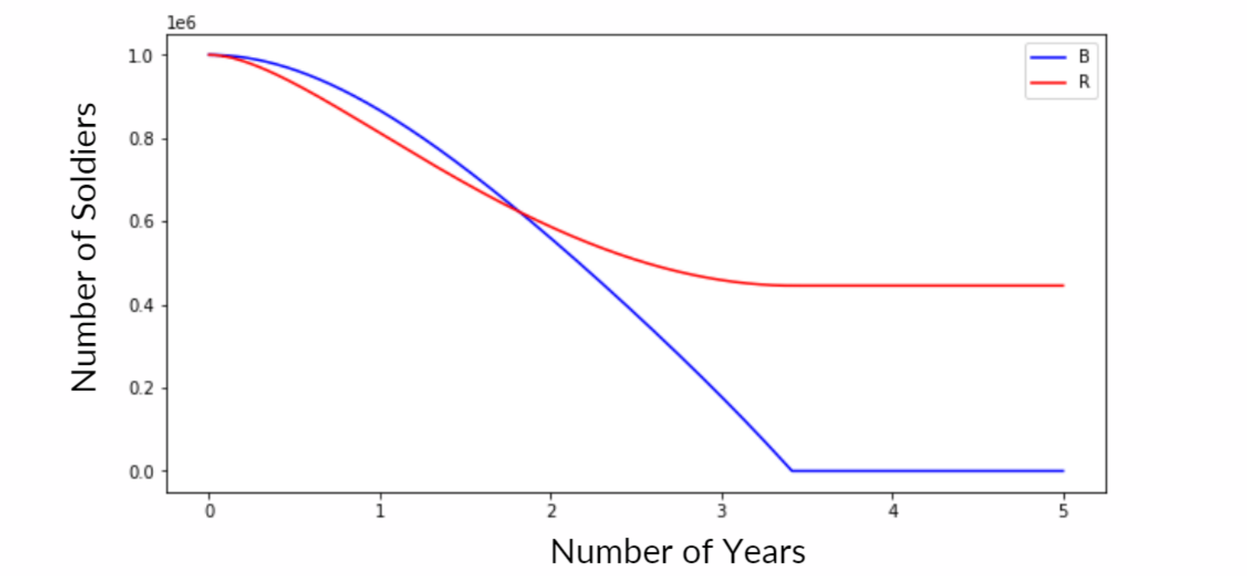
The graph above shows how the killing rates α, ε Of Blue and Red team respectively change with time. As evident from the graph, ε first increases faster than α but then saturates eventually.

The modified Lanchester Equations will be :-

$$\frac{dB(t)}{dt} = -(\lambda_1 t)R$$

$$\frac{dR(t)}{dt} = -\left(\frac{\lambda_2 t}{\lambda_2 + t}\right)B$$

When we plot these equation using python we get the following graph:



For, λ_1 = 0.3 λ_2 = 0.4

The initial army size of both the armies is the same. Initially, the killing rate of B is greater than that of R and hence, as depicted in the above graph, initially B wins the war. But after a particular point, the killing rate of B saturates but that of R is still increasing linearly with time. Even though the army of Red is less than that of Blue at that point (as visible in graph), Red starts overtaking Blue after that point and eventually wins the war.

We can use the above results to check for various other parameters. We can modify α(t) and ε(t) for different functions based on the situations and the data available to us and therefore model the equations and predict the fate of the war.

This model will be valid only for the long drawn war since the change in killing rates in a short term war is negligible.

CONCLUSION

From the various models we have developed and discusses, it can be seen that the fate of wars can be predicted and analysed. Recent conflict situations are not regular in the sense that they are profoundly asymmetric, they increasingly rely on data and information, they are affected by the behavior of civilians and they may involve more than two adversaries.

The equation for reinforcements is a very critical one by which many situations of war can be studied depending upon various values for different parameters. Taking into account the basic assumptions, we can observe how one parameter can dominate over the other which can inturn affect the whole outcome of the war, for example, if reinforcements are enormous in number then it may happen that the attrition rates play a secondary effect. A long drawn war could also be accounted for by this equation.

The model for multilateral conflicts considers the case of multiple armies being involved in the war. It not only considers the occasion of equal distribution of the armies but also the case of distribution of forces based on the current situation of the war in which the troops have to be deployed based on the threat they will be receiving from the other side.

This model has been designed such that for any values of the various parameters an outcome of that particular situation can be analysed through graphs and hence could prove to be a very useful tool which could be a success if used appropriately.

FUTURE ASPECTS

This model can further adapt numerous situations and/or variables with reasonable assumptions.
The case of ambush could be built upon where questions like what happens if some portion of the force is visible to the other side while the rest of the force remains concealed? How does the level of situational awareness regarding the opponent’s targets affect the outcome of the battle? could be answered.
In this model the reinforcements are constant in nature. This can be modified with a sight complexity by involving variable reinforcements.
The next stage is to consider what causes the flows. We might, for example, find from expert military opinion that the replenishment rate depends on the need to build up missile stocks in the field to some target level and the supply line capacity as affected by enemy air interdiction effort.

References

1. Research Paper 1
2. Research Paper 2
3. Python Codes for Graphs