Homework 5 - Arnav Kucheriya

Section 1.4:

Exercise 5

$$(p \lor r) \to q$$

$$\neg q$$

$$\therefore \neg p \land \neg r$$

Exercise 11

$$p \to (r \lor q)$$

$$r \to \neg q$$

$$\therefore p \to r$$

Exercise 14

$$p \to r$$

$$r \to q$$

$$p$$

$$\therefore q$$

Exercise 15

$$(p \lor q) \to (r \lor s)$$

$$p$$

$$\neg r$$

$$\therefore s$$

Exercise 22

$$p \land \neg p$$

$$\therefore q$$

Exercise 30

1. **Hypothesis**:
   - If there is gas in the car, then I will go to the store.
   - If I go to the store, then I will get a soda.
   - There is gas in the car.
2. **Applying hypothetical syllogism**:

$$g \to s$$

$$s \to d$$
$$g$$
$$\therefore d$$

Exercise 33 (Modus Tollens)

$$p \to q$$
$$\neg q$$
$$\therefore \neg p$$

Exercise 34 (Addition)

$$p$$
$$\therefore p \lor q$$

Exercise 35 (Simplification)

$$p \land q$$
$$\therefore p$$

Quantifiers Problem Set

Problem 1

(a)
The given statement:

$$\forall x\ (x \in A \to x \in B)$$

This defines the subset relation:

$$A \subseteq B$$

(b)
The given statement:

$$[\forall x\ (x \in A \to x \in B)] \land [\forall x\ (x \in B \to x \in A)]$$

This defines the set equality:

$$A = B$$

Problem 3 (Proofs in Integer and Rational Numbers)

(a) **False**:

$$\forall x, y \in \mathbb{Z}, \quad x - y = 7$$

This is not true for all integers (x, y), e.g.,

$$1 - 2 = -1 \neq 7$$

(b) **True**:

$$\exists x, y \in \mathbb{Z}, \quad x - y = 7$$

Example:

$$x = 10, y = 3 \Rightarrow 10 - 3 = 7$$

(c) **True**:
For any (x), we can choose:

$$y = x - 7$$

such that:

$$x - y = 7$$

(d) **False**:
If:

$$\exists x \forall y, \quad x - y = 7$$

then some (x) must work for all (y), which is impossible.

(e) **False**:
For all (x \in \mathbb{Z}), (x = 0) does not satisfy:

$$x \cdot y = 7$$

for any integer (y).

(f) **False**:
In rationals:

$$\forall x \in \mathbb{Q}, \exists y \in \mathbb{Q}, \quad x \cdot y = 7$$

(x = 0) does not work.

(g) **True**:
If (x \neq 0), then:

$$y = \frac{7}{x}$$

works.

(h) **True**:
In (\mathbb{Q} - {0}), the equation:

$$x \cdot y = 7$$

is always solvable for some (y).

(i) **True**:
For any integer (y), we can find:

$$x = y + 1$$

such that:

$$x > y$$

(j) **False**:
There is no single integer (y) such that all integers (x) are greater than (y).