CS 241 - Homework 2 - Arnav Kucheriya

Exercise 1.2:

Question 1: Determine if each sentence is a proposition and write its negation

1. $2 + 5 = 19$
   - Proposition: Yes
   - Negation: $2 + 5 \neq 19$
2. $6 + 9 = 15$
   - Proposition: Yes
   - Negation: $6 + 9 \neq 15$
3. $x + 9 = 15$
   - Proposition: No (contains variable x)
4. $\pi = 3.14$
   - Proposition: Yes
   - Negation: $\pi \neq 3.14$

Q13 - 16: Coin Flip Proposition:

13. "Ten heads were obtained"
    - Negation: "Fewer than ten heads were obtained"
14. "Some heads were obtained"
    - Negation: "No heads were obtained"
15. "Some heads and some tails were obtained"
    - Negation: "Only heads or only tails were obtained"
16. "At least one head was obtained"
    - Negation: "No heads were obtained"

Q20:

Given: $p = \text{false}$, $q = \text{true}$, $r = \text{false}$
Evaluate: $\neg p \wedge (q \wedge r)$

Solution:

- $\neg p = \text{true}$

- $q \wedge r = \text{false}$
- $\text{true} \wedge \text{false} = \text{false}$

Q26:

Truth table for $p \wedge \neg p$:

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

This is a **contradiction**.

Exercise 1.3:

Q30: $(p \wedge r) \leftrightarrow r$

Given: p is true, q is false, r unknown
Result: **True** (regardless of r)

Q31: $(q \wedge r) \leftrightarrow r$

Given: q is false, r unknown
Result: **Unknown**

Q54: $\neg q \rightarrow (r \wedge p)$

Given: q is false, p is true, r unknown
Result: **Unknown**

Q68: $P \rightarrow Q$ equivalence

- $P \rightarrow Q \equiv \neg P \vee Q$

Q70: Compound Implication

- $P \rightarrow (q \rightarrow r) \equiv P \rightarrow r$

Q73-76:

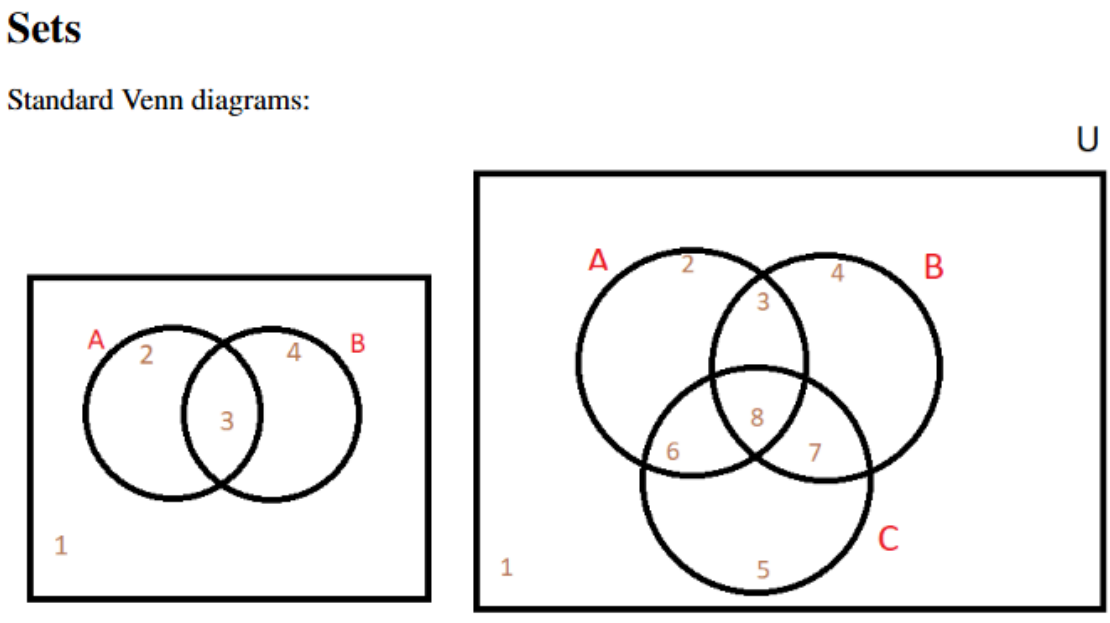
73. "Pat will use the treadmill or lift weights"
    - Original: $P \vee Q$

- Negation: $\neg P \wedge \neg Q$
- **Answer:** "Pat will neither use the treadmill nor lift weights."

74. "Dale is smart and funny"
    - Original: $P \wedge Q$
    - Negation: $\neg P \vee \neg Q$
    - **Answer:** "Dale is not smart or Dale is not funny."
75. "Shirley will either take the bus or catch a ride"
    - Original: $P \vee Q$
    - Negation: $\neg P \wedge \neg Q$
    - **Answer:** "Shirley will neither take the bus nor catch a ride to school."
76. "Red pepper and onions are required for chili"
    - Original: $P \wedge Q$
    - Negation: $\neg P \vee \neg Q$
    - **Answer:** "Red pepper is not required to make chili, or onions are not required to make chili."

Part 2:

Sets and Venn Diagrams:



1. De Morgan's Laws for 3 Variables

De Morgan's Laws are:

- $\neg(A \cup B \cup C) = \neg A \cap \neg B \cap \neg C$
- $\neg(A \cap B \cap C) = \neg A \cup \neg B \cup \neg C$

2. 2-Set Venn Diagram

a) $\{t | (t \in A) \wedge (t \in B)\}$

- Represents $A \cap B$ (region **3**)

b) $\{t | (t \in A) \vee (t \in B)\}$

- Represents $A \cup B$ (regions **2, 3, and 4**)

c) $\{t | (t \in U) \wedge (\neg(t \in A))\}$

- Represents $U - A$ (regions **1 and 4**)

Divisors and Modulo

1. Evaluating Divisibility Expressions

- $4 \mid 7 \rightarrow$ False
- $4 \mid 12 \rightarrow$ True
- $4 \mid -12 \rightarrow$ True
- $12 \nmid 4 \rightarrow$ True
- $0 \mid 12 \rightarrow$ False
- $12 \mid 0 \rightarrow$ True

2. Finding q, r such that $n = dq + r$, where $0 \leq r < d$

- $28 = 7(4) + 0 \rightarrow q = 4, r = 0$
- $-28 = 7(-4) + 0 \rightarrow q = -4, r = 0$
- $31 = 7(4) + 3 \rightarrow q = 4, r = 3$
- $-31 = 7(-5) + 4 \rightarrow q = -5, r = 4$
- $0 = 6(0) + 0 \rightarrow q = 0, r = 0$
- $5 = 6(0) + 5 \rightarrow q = 0, r = 5$
- $-5 = 6(-1) + 1 \rightarrow q = -1, r = 1$
- $-10 = 6(-2) + 2 \rightarrow q = -2, r = 2$

3. Modulo Calculations

- $7 \bmod 5 = 2$

- $-7 \bmod 5 = 3$
- $3 \bmod 5 = 3$
- $-3 \bmod 5 = 2$
- $0 \bmod 5 = 0$
- $0 \bmod 3 = 0$
- $48 \bmod 12 = 0$
- $48 \bmod 11 = 4$
- $-48 \bmod 11 = 7$

4. True/False Statements

- $6 \mid 42 \rightarrow$ True, $q = 7$
- $6 \mid 40 \rightarrow$ False
- $39 \bmod 8 = 7 \rightarrow$ True
- $41 \bmod 10 = 3 \rightarrow$ True
- $41 \bmod 13 = 2 \rightarrow$ True

Propositional Logic Problems

1. Truth Table for $(p \vee q) \vee r$ and $p \vee (q \vee r)$

p	q	r	$(p \vee q)$	$(p \vee q) \vee r$	$(q \vee r)$	$p \vee (q \vee r)$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

Answer: Yes, $\vee$ is associative as $(p \vee q) \vee r = p \vee (q \vee r)$ for all values.

2. Commutativity of $\vee$

Yes, $\vee$ is commutative because $p \vee q = q \vee p$ for all values of p and q.

3. Truth Table for $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$

p	q	r	$(p \wedge q)$	$(p \wedge q) \wedge r$	$(q \wedge r)$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	F	T	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Answer: Yes, $\wedge$ is associative.

4. Commutativity of $\wedge$

Yes, $\wedge$ is commutative because $p \wedge q = q \wedge p$ for all values.

5-6. Properties of Implication ($\rightarrow$)

- Not commutative: $p \rightarrow q \neq q \rightarrow p$
- Not associative: $(p \rightarrow q) \rightarrow r \neq p \rightarrow (q \rightarrow r)$

7-9. XOR ($\oplus$) Properties

Given truth table shows XOR and biconditional operations.

XOR ($\oplus$):

- Is commutative: $p \oplus q = q \oplus p$
- Is not associative: $(p \oplus q) \oplus r \neq p \oplus (q \oplus r)$

10. Logical Equivalence for XOR

$p \oplus q \equiv (p \vee q) \wedge \neg(p \wedge q)$

11-12. Biconditional ($\leftrightarrow$) Properties

- Is associative: $(p \leftrightarrow q) \leftrightarrow r = p \leftrightarrow (q \leftrightarrow r)$

- Is commutative: $p \leftrightarrow q = q \leftrightarrow p$

13. Logical Equivalence for Biconditional

$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$

14. Boolean Functions

For n boolean arguments:

- Number of different inputs: 2^n
- Number of different functions: 2^{2^n}