

Problem 0.1. (2 points) Find the reduced row-echelon form of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 2 \\ 7 & 8 & 9 & 3 \end{bmatrix}$$

$\text{rref}(A) =$

Problem 0.2. (2 points) Find the solution set of the linear system below in parametric vector form.

$$\begin{cases} x + 2y + 3z = 1 \\ 4x + 5y + 6z = 2 \\ 7x + 8y + 9z = 3 \end{cases}$$

solution
set

Problem 0.3. (2 points) Simplify the following matrix expression:

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{bmatrix} a & b \end{bmatrix} + \begin{bmatrix} e \\ f \\ g \end{bmatrix} \begin{bmatrix} e & f \end{bmatrix} \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

simplified
expression

Problem 0.4. (2 points) Find and simplify the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & a & 1 \end{bmatrix}$$

$$A^{-1} = \boxed{\phantom{\begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & a & 1 \end{bmatrix}}}$$

Problem 0.5. (2 points) Find the LU-factorization of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 2 \\ 7 & 8 & 9 & 3 \end{bmatrix}$$

$$L = \boxed{\phantom{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 7 & 0 & 1 & 0 \end{bmatrix}}} \quad U = \boxed{\phantom{\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -1 & -4 \end{bmatrix}}}$$

Problem 0.6. (2 points) Find the CR-factorization of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 2 \\ 7 & 8 & 9 & 3 \end{bmatrix}$$

$$C = \boxed{\phantom{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 1 & 0 & 0 \\ 7 & 0 & 1 & 0 \end{bmatrix}}} \quad R = \boxed{\phantom{\begin{bmatrix} 1 & 2 & 3 & 1 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & -1 & -4 \end{bmatrix}}}$$

Problem 0.7. (2 points) Showing/explaining your work, find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\det A = \boxed{}$$

Problem 0.11. (4 points) Diagonalize, if possible.

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 3 & -1 & -2 \end{bmatrix}$$

$A = SDS^{-1}$ where $S =$

and $D =$

Problem 0.12. (2 points) Find the least squares solutions of $\begin{bmatrix} 1 & -1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

$\mathbf{x} =$



Problem 1.1. (10 points) Find the QR-factorization of the matrix

$$A = \begin{bmatrix} 6 & -5 & 0 \\ 8 & 10 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$Q =$



$R =$



Problem 1.2. (10 points) Find an orthogonal diagonalization of the following matrix:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$A = UDU^T$ where $U =$



and $D =$



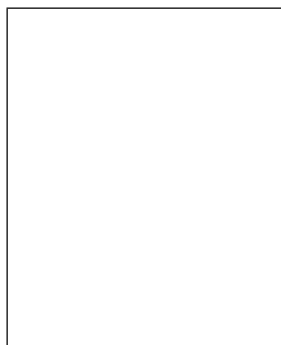
Problem 1.2. (10 points) Find the singular value decomposition (SVD) of the matrix:

$$A = \begin{bmatrix} 1 & 4 \\ 2 & -2 \\ 2 & 0 \end{bmatrix}$$

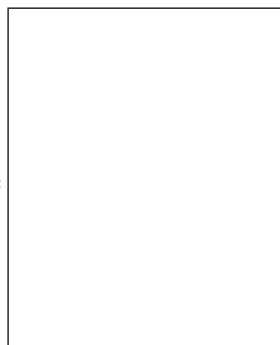
$$A = U\Sigma V^T$$

where

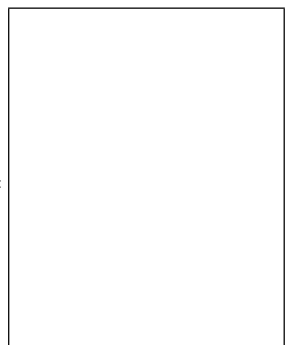
$$U =$$



$$\Sigma =$$



$$V =$$



Problem 2. (10 points) Evaluate the truth of each statement below. If the statement is true write T in the box preceding the statement. Otherwise, write F .

(a) ☐ The set of pivot columns of a matrix is linearly independent.

(b) ☐ If $\det(A) = 1$, then A is invertible.

(c) ☐ If $1 \in \sigma(A)$, then $A - I$ is singular.

(d) ☐ For an $n \times n$ matrix A with n singular values, $\|A\mathbf{x}\| \geq \sigma_n \|\mathbf{x}\|$.

(e) ☐ The normal equations $A^T A \mathbf{x} = A^T \mathbf{b}$ always have at least one solution.

Problem 3. (10 points) The built-in MATLAB function `null` returns a matrix having columns that form an orthonormal basis for the null space of the input matrix. For example,

```
>> null([0 1 0; 0 2 0; 0 3 0])
```

```
ans =
```

```
    0    -1
    0     0
    1     0
```

Provide the output expected from MATLAB for the given commands in the answer boxes:

```
>> A = [1 1 1 0; 1 1 1 0 ; 1 1 1 0; 0 0 0 0];
```

```
>> rref(A)
```

```
ans =
```

```
    1    1    1    0
    0    0    0    0
    0    0    0    0
    0    0    0    0
```

```
>> N=null(A);
```

```
>> A*N
```

```
ans =
```

```
>> transpose(N)*N           % transpose outputs the transpose of the input matrix
```

```
ans =
```