Problem 0.1. (2 points) Find the reduced row-echelon form of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 2 \\ 7 & 8 & 9 & 3 \end{bmatrix}$$

rref(A) =

Problem 0.2. (2 points) Find the solution set of the linear system below in parametric vector form.

$$\begin{cases} x & + & 2y & + & 3z & = & 1 \\ 4x & + & 5y & + & 6z & = & 2 \\ 7x & + & 8y & + & 9z & = & 3 \end{cases}$$

solution set

Problem 0.3. (2 points) Simplify the following matrix expression:

$$\begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \left(\begin{bmatrix} a \\ b \\ c \end{bmatrix} \begin{bmatrix} a & b \end{bmatrix} + \begin{bmatrix} e \\ f \\ g \end{bmatrix} \begin{bmatrix} e & f \end{bmatrix} \right) \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

simplified expression

Problem 0.4. (2 points) Find and simplify the inverse of the matrix

$$A = \begin{bmatrix} 1 & 0 & 0 \\ a & 1 & 0 \\ 0 & a & 1 \end{bmatrix}$$

$$A^{-1} = \boxed{}$$

Problem 0.5. (2 points) Find the LU-factorization of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 2 \\ 7 & 8 & 9 & 3 \end{bmatrix}$$

$$L = \boxed{} \qquad U = \boxed{}$$

Problem 0.6. (2 points) Find the CR-factorization of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 4 & 5 & 6 & 2 \\ 7 & 8 & 9 & 3 \end{bmatrix}$$

$$C = \boxed{} \qquad R = \boxed{}$$

Problem 0.7. (2 points) Showing/explaining your work, find the determinant of the matrix

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\det A = \boxed{}$$

Problem 0.8. (2 points) If possible, diagonalize the matrix below.

$$A = \begin{bmatrix} 5 & -1 \\ 9 & -1 \end{bmatrix}$$

$A = SDS^{-1}$ where $S =$ $\boxed{}$ and $D =$ $\boxed{}$

Problem 0.9. (2 points) Showing your work, deterimine if the following set is linearly dependent or linearly independent.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}, \begin{bmatrix} 7 \\ 8 \\ 9 \end{bmatrix} \right\}$$

Circle one: | Linearly dependent | Linearly independent |

Problem 0.10. (4 points) Find bases for the four fundamental subspaces of the matrix

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 3 & 7 & 4 \end{bmatrix}$$

basis for Col(A) $\boxed{}$

basis for Row(A) $\boxed{}$

basis for Nul(A) $\boxed{}$

basis for Nul(A^T) $\boxed{}$

Problem 0.11. (4 points) Diagonalize, if possible.

$$A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 0 & -1 \\ 3 & -1 & -2 \end{bmatrix}$$

$A = SDS^{-1}$ where $S = \boxed{}$ and $D = \boxed{}$

Problem 0.12. (2 points) Find the least squares solutions of $\begin{bmatrix} 1 & -1 \\ 1 & -1 \\ 1 & -1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.

$\mathbf{x} = \boxed{}$

Problem 1.1. (10 points) Find the QR-factorization of the matrix

$$A = \begin{bmatrix} 6 & -5 & 0 \\ 8 & 10 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$Q = \boxed{}$ $R = \boxed{}$

Problem 1.2. (10 points) Find an orthogonal diagonalization of the following matrix:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$A = UDU^T$ where $U = \boxed{}$ and $D = \boxed{}$

Problem 1.2. (10 points) Find the singular value decomposition (SVD) of the matrix:

$$A = \begin{bmatrix} 1 & 4 \\ 2 & -2 \\ 2 & 0 \end{bmatrix}$$

$A = U\Sigma V^T$
where

$U = \boxed{}$ $\quad \Sigma = \boxed{}$ $\quad V = \boxed{}$

Problem 2. (10 points) Evaluate the truth of each statement below. If the statement is true write T in the box preceding the statement. Otherwise, write F.

(a) $\boxed{}$ The set of pivot columns of a matrix is linearly independent.

(b) $\boxed{}$ If $\det(A) = 1$, then A is invertible.

(c) $\boxed{}$ If $1 \in \sigma(A)$, then $A - I$ is singular.

(d) $\boxed{}$ For an $n \times n$ matrix A with n signular values, $||A\mathbf{x}|| \geq \sigma_n ||\mathbf{x}||$.

(e) $\boxed{}$ The normal equations $A^T A\mathbf{x} = A^T\mathbf{b}$ always have at least one solution.

Problem 3. (10 points) The built-in MATLAB function `null` returns a matrix having columns that form an orthonormal basis for the null space of the input matrix. For example,

```
>> null([0 1 0; 0 2 0; 0 3 0])

ans =
          0    -1
          0     0
          1     0
```

Provide the output expected from MATLAB for the given commands in the answer boxes:

```
>> A = [1 1 1 0; 1 1 1 0 ; 1 1 1 0; 0 0 0 0];
>> rref(A)

ans =
       1   1    1   0
       0   0    0   0
       0   0    0   0
       0   0    0   0

>> N=null(A);
>> A*N
```

ans =

```
>> transpose(N)*N       % transpose outputs the transpose of the input matrix
```

ans =