1.1 Problem-Solving Tips

- To verify that two sets A and B are equal, written $A = B$, show that for every x, if $x \in A$, then $x \in B$, and if $x \in B$, then $x \in A$.
- To verify that two sets A and B are *not* equal, written $A \neq B$, find at least one element that is in A but not in B, or find at least one element that is in B but not in A. One or the other conditions suffices; you need not (and may not be able to) show both conditions.
- To verify that A is a subset of B, written $A \subseteq B$, show that for every x, if $x \in A$, then $x \in B$. Notice that if A is a subset of B, it is possible that $A = B$.
- To verify that A is *not* a subset of B, find at least one element that is in A but not in B.
- To verify that A is a proper subset of B, written $A \subset B$, verify that A is a subset of B as described previously, and that $A \neq B$, that is, that there is at least one element that is in B but not in A.
- To visualize relationships among sets, use a Venn diagram. A Venn diagram can suggest whether a statement about sets is true or false.
- A set of elements is determined by its members; order is irrelevant. On the other hand, ordered pairs and n-tuples take order into account.

1.1 Review Exercises

[†]**1.** What is a set?

2. What is set notation?

3. Describe the sets $\mathbf{Z}$, $\mathbf{Q}$, $\mathbf{R}$, $\mathbf{Z}^+$, $\mathbf{Q}^+$, $\mathbf{R}^+$, $\mathbf{Z}^-$, $\mathbf{Q}^-$, $\mathbf{R}^-$, $\mathbf{Z}^{nonneg}$, $\mathbf{Q}^{nonneg}$, and $\mathbf{R}^{nonneg}$, and give two examples of members of each set.

4. If X is a finite set, what is $|X|$?

5. How do we denote *x is an element of the set X*?

6. How do we denote *x is not an element of the set X*?

7. How do we denote the empty set?

8. Define *set X is equal to set Y*. How do we denote *X is equal to Y*?

9. Explain a method of verifying that sets X and Y are equal.

10. Explain a method of verifying that sets X and Y are *not* equal.

11. Define *X is a subset of Y*. How do we denote *X is a subset of Y*?

12. Explain a method of verifying that X is a subset of Y.

13. Explain a method of verifying that X is *not* a subset of Y.

14. Define *X is a proper subset of Y*. How do we denote *X is a proper subset of Y*?

15. Explain a method of verifying that X is a proper subset of Y.

16. What is the power set of X? How is it denoted?

17. Define *X union Y*. How is the union of X and Y denoted?

18. If $\mathcal{S}$ is a family of sets, how do we define the union of $\mathcal{S}$? How is the union denoted?

19. Define *X intersect Y*. How is the intersection of X and Y denoted?

20. If $\mathcal{S}$ is a family of sets, how do we define the intersection of $\mathcal{S}$? How is the intersection denoted?

21. Define *X and Y are disjoint sets*.

22. What is a pairwise disjoint family of sets?

23. Define the *difference* of sets X and Y. How is the difference denoted?

24. What is a universal set?

25. What is the complement of the set X? How is it denoted?

26. What is a Venn diagram?

27. Draw a Venn diagram of three sets and identify the set represented by each region.

[†]Exercise numbers in color indicate that a hint or solution appears at the back of the book in the section following the References.

28. State the associative laws for sets.

29. State the commutative laws for sets.

30. State the distributive laws for sets.

31. State the identity laws for sets.

32. State the complement laws for sets.

33. State the idempotent laws for sets.

34. State the bound laws for sets.

35. State the absorption laws for sets.

36. State the involution law for sets.

37. State the 0/1 laws for sets.

38. State De Morgan's laws for sets.

39. What is a partition of a set X?

40. Define the *Cartesian product* of sets X and Y. How is this Cartesian product denoted?

41. Define the *Cartesian product* of the sets $X_1, X_2, \ldots, X_n$. How is this Cartesian product denoted?

1.1 Exercises

In Exercises 1–16, let the universe be the set $U = \{1, 2, 3, \ldots, 10\}$. *Let* $A = \{1, 4, 7, 10\}$, $B = \{1, 2, 3, 4, 5\}$, *and* $C = \{2, 4, 6, 8\}$. *List the elements of each set.*

1. $A \cup B$

2. $B \cap C$

3. $A - B$

4. $B - A$

5. $\overline{A}$

6. $U - C$

7. $\overline{U}$

8. $A \cup \varnothing$

9. $B \cap \varnothing$

10. $A \cup U$

11. $B \cap U$

12. $A \cap (B \cup C)$

13. $\overline{B} \cap (C - A)$

14. $(A \cap B) - C$

15. $\overline{A \cap B} \cup C$

16. $(A \cup B) - (C - B)$

In Exercises 17–27, let the universe be the set $\mathbf{Z}^+$. *Let* $X = \{1, 2, 3, 4, 5\}$ *and let* Y *be the set of positive, even integers. In set-builder notation,* $Y = \{2n \mid n \in \mathbf{Z}^+\}$. *In Exercises 18–27, give a mathematical notation for the set by listing the elements if the set is finite, by using set-builder notation if the set is infinite, or by using a predefined set such as* $\varnothing$.

17. Describe $\overline{Y}$ in words.

18. $\overline{X}$

19. $\overline{Y}$

20. $X \cap Y$

21. $X \cup Y$

22. $\overline{X} \cap Y$

23. $\overline{X} \cup Y$

24. $X \cap \overline{Y}$

25. $X \cup \overline{Y}$

26. $\overline{X} \cap \overline{Y}$

27. $\overline{X} \cup \overline{Y}$

28. What is the cardinality of $\varnothing$?

29. What is the cardinality of $\{\varnothing\}$?

30. What is the cardinality of $\{a, b, a, c\}$?

31. What is the cardinality of $\{\{a\}, \{a, b\}, \{a, c\}, a, b\}$?

In Exercises 32–35, show, as in Examples 1.1.2 and 1.1.3, that $A = B$.

32. $A = \{3, 2, 1\}, \ B = \{1, 2, 3\}$

33. $C = \{1, 2, 3\}, \ D = \{2, 3, 4\}, \ A = \{2, 3\}, \ B = C \cap D$

34. $A = \{1, 2, 3\}, \ B = \{n \mid n \in \mathbf{Z}^+ \text{ and } n^2 < 10\}$

35. $A = \{x \mid x^2 - 4x + 4 = 1\}, \ B = \{1, 3\}$

In Exercises 36–39, show, as in Example 1.1.4, that $A \neq B$.

36. $A = \{1, 2, 3\}, \ B = \varnothing$

37. $A = \{1, 2\}, \ B = \{x \mid x^3 - 2x^2 - x + 2 = 0\}$

38. $A = \{1, 3, 5\}, \ B = \{n \mid n \in \mathbf{Z}^+ \text{ and } n^2 - 1 \leq n\}$

39. $B = \{1, 2, 3, 4\}, \ C = \{2, 4, 6, 8\}, \ A = B \cap C$

In Exercises 40–43, determine whether each pair of sets is equal.

40. $\{1, 2, 2, 3\}, \{1, 2, 3\}$

41. $\{1, 1, 3\}, \{3, 3, 1\}$

42. $\{x \mid x^2 + x = 2\}, \{1, -1\}$

43. $\{x \mid x \in \mathbf{R} \text{ and } 0 < x \leq 2\}, \{1, 2\}$

In Exercises 44–47, show, as in Examples 1.1.5 and 1.1.6, that $A \subseteq B$.

44. $A = \{1, 2\}, \ B = \{3, 2, 1\}$

45. $A = \{1, 2\}, \ B = \{x \mid x^3 - 6x^2 + 11x = 6\}$

46. $A = \{1\} \times \{1, 2\}, \ B = \{1\} \times \{1, 2, 3\}$

47. $A = \{2n \mid n \in \mathbf{Z}^+\}, \ B = \{n \mid n \in \mathbf{Z}^+\}$

In Exercises 48–51, show, as in Example 1.1.9, that A is not a subset of B.

48. $A = \{1, 2, 3\}, \ B = \{1, 2\}$

49. $A = \{x \mid x^3 - 2x^2 - x + 2 = 0\}, \ B = \{1, 2\}$

50. $A = \{1, 2, 3, 4\}, \ C = \{5, 6, 7, 8\}, \ B = \{n \mid n \in A \text{ and } n + m = 8 \text{ for some } m \in C\}$

51. $A = \{1, 2, 3\}, \ B = \varnothing$

In Exercises 52–59, draw a Venn diagram and shade the given set.

52. $A \cap \overline{B}$

53. $\overline{A} - B$

54. $B \cup (B - A)$

55. $(A \cup B) - B$

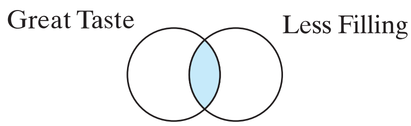
56. $B \cap \overline{(C \cup A)}$

57. $(\overline{A} \cup B) \cap (\overline{C} - A)$

58. $((C \cap A) - \overline{(B - A)}) \cap C$

59. $(B - \overline{C}) \cup ((B - \overline{A}) \cap (C \cup B))$

60. A television commercial for a popular beverage showed the following Venn diagram



What does the shaded area represent?

Exercises 61–65 refer to a group of 191 students, of which 10 are taking French, business, and music; 36 are taking French and business; 20 are taking French and music; 18 are taking business and music; 65 are taking French; 76 are taking business; and 63 are taking music.

61. How many are taking French and music but not business?

62. How many are taking business and neither French nor music?

63. How many are taking French or business (or both)?

64. How many are taking music or French (or both) but not business?

65. How many are taking none of the three subjects?

66. A television poll of 151 persons found that 68 watched "Law and Disorder"; 61 watched "25"; 52 watched "The Tenors"; 16 watched both "Law and Disorder" and "25"; 25 watched both "Law and Disorder" and "The Tenors"; 19 watched both "25" and "The Tenors"; and 26 watched none of these shows. How many persons watched all three shows?

67. In a group of students, each student is taking a mathematics course or a computer science course or both. One-fifth of those taking a mathematics course are also taking a computer science course, and one-eighth of those taking a computer science course are also taking a mathematics course. Are more than one-third of the students taking a mathematics course?

In Exercises 68–71, let $X = \{1, 2\}$ *and* $Y = \{a, b, c\}$*. List the elements in each set.*

68. $X \times Y$

69. $Y \times X$

70. $X \times X$

71. $Y \times Y$

In Exercises 72–75, let $X = \{1, 2\}$, $Y = \{a\}$, *and* $Z = \{\alpha, \beta\}$*. List the elements of each set.*

72. $X \times Y \times Z$

73. $X \times Y \times Y$

74. $X \times X \times X$

75. $Y \times X \times Y \times Z$

In Exercises 76–82, give a geometric description of each set in words. Consider the elements of the sets to be coordinates. For example, $\mathbf{R} \times \mathbf{Z}$ *is the set* $\{(x, n) \mid x \in \mathbf{R} \text{ and } n \in \mathbf{Z}\}$*. Interpreting the ordered pairs* (x, n) *as coordinates in the plane, the graph of all such ordered pairs is the set of all parallel horizontal lines spaced one unit apart, one of which passes through* (0,0).

76. $\mathbf{R} \times \mathbf{R}$

77. $\mathbf{Z} \times \mathbf{R}$

78. $\mathbf{R} \times \mathbf{Z}^{nonneg}$

79. $\mathbf{Z} \times \mathbf{Z}$

80. $\mathbf{R} \times \mathbf{R} \times \mathbf{R}$

81. $\mathbf{R} \times \mathbf{R} \times \mathbf{Z}$

82. $\mathbf{R} \times \mathbf{Z} \times \mathbf{Z}$

In Exercises 83–86, list all partitions of the set.

83. $\{1\}$

84. $\{1, 2\}$

85. $\{a, b, c\}$

86. $\{a, b, c, d\}$

In Exercises 87–92, answer true or false.

87. $\{x\} \subseteq \{x\}$

88. $\{x\} \in \{x\}$

89. $\{x\} \in \{x, \{x\}\}$

90. $\{x\} \subseteq \{x, \{x\}\}$

91. $\{2\} \subseteq \mathcal{P}(\{1, 2\})$

92. $\{2\} \in \mathcal{P}(\{1, 2\})$

93. List the members of $\mathcal{P}(\{a, b\})$. Which are proper subsets of $\{a, b\}$?

94. List the members of $\mathcal{P}(\{a, b, c, d\})$. Which are proper subsets of $\{a, b, c, d\}$?

95. If X has 10 members, how many members does $\mathcal{P}(X)$ have? How many proper subsets does X have?

96. If X has n members, how many proper subsets does X have?

In Exercises 97–100, what relation must hold between sets A and B in order for the given condition to be true?

97. $A \cap B = A$

98. $A \cup B = A$

99. $\overline{A} \cap U = \varnothing$

100. $\overline{A \cap B} = \overline{B}$

The symmetric difference *of two sets A and B is the set*

$$A \triangle B = (A \cup B) - (A \cap B).$$

101. If $A = \{1, 2, 3\}$ and $B = \{2, 3, 4, 5\}$, find $A \triangle B$.

102. Describe the symmetric difference of sets A and B in words.

103. Given a universe U, describe $A \triangle A$, $A \triangle \overline{A}$, $U \triangle A$, and $\varnothing \triangle A$.

104. Let C be a circle and let $\mathcal{D}$ be the set of all diameters of C. What is $\cap \mathcal{D}$? (Here, by "diameter" we mean a line segment through the center of the circle with its endpoints on the circumference of the circle.)

[†]★**105.** Let P denote the set of integers greater than 1. For $i \geq 2$, define

$$X_i = \{ik \mid k \in P\}.$$

Describe $P - \bigcup_{i=2}^{\infty} X_i$.

[†]A starred exercise indicates a problem of above-average difficulty.