Final Exam Review Exercise II

Square matrices play a special role in linear algebra. A number of the course core problems apply only to square matrices: determinants, matrix inverse, diagonalization. To a great extent, the key distriction between square and nonsquare matrices is a square matrix can be multiplied by itself and hence raised to a power.

Instructions. For each matrix at the bottom of this page perform the following list of operations.

(1) Denote the matrix A. (This is just giving the matrix a name.)

(2) Compute $|A|$.

(3) If possible, find the inverse of A, if possible.

(4) Find and factor the characteristic polynomial of A.

(5) If possible, diagonalize A.

(6) If A is symmetric find an orthogonal diagonalization.

(7) Find $\exp(At)$.

(8) Determine $\lim_{n\to\infty} A^n$, if possible.

$$\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 0 \\ 4 & 1 \end{bmatrix}, \qquad \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}, \qquad \begin{bmatrix} 4 & -1 \\ -1 & 4 \end{bmatrix},$$

$$\begin{bmatrix} 3 & 4 \\ 4 & 3 \end{bmatrix}, \qquad \begin{bmatrix} 3 & -4 \\ 4 & 3 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}, \qquad \begin{bmatrix} \frac{1}{2} & \frac{2}{3} \\ \frac{1}{2} & \frac{1}{3} \end{bmatrix},$$

$$\frac{1}{4}\begin{bmatrix} 1 & 2 & -3 \\ 1 & 2 & -1 \\ 3 & -2 & 3 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \frac{1}{2}\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \qquad \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$