# Homework 11 - Arnav Kucheriya

## Section 4.3 Questions

## Question 3

Expression:

$$2^n + 1$$

Using theta notation:

$$\Theta(2^n)$$

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## Question 5

Expression:
$2\log n + 4n + 3n\log n$

The dominant term is $3n\log n$.
Therefore:

$$\Theta(n\log n)$$

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## Question 7

Expression:

$$2 + 4 + 6 + \cdots + 2n$$

This is an arithmetic sequence:

$$\text{Sum} = n(n+1)$$

So the asymptotic bound is:

$$\Theta(n^2)$$

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## Question 34

Statement:

$$n! = O(n^n)$$

Justification:

$$n! = 1 \cdot 2 \cdot \dots \cdot n \leq n^n$$

Hence,

$$n! \in O(n^n)$$

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# Question 35

Statement:

$$2^n = O(n!)$$

Proof: For large enough $n$,

$$2^n < n!$$

So,

$$2^n \in O(n!)$$

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# Question 57

Expression:

$$(\log 2n)^2$$

Since $\log 2n = \log n + \log 2 = \log n + 1$,
we get:

$$(\log 2n)^2 = (\log n + 1)^2 = \Theta((\log n)^2)$$

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## (a) Prove $3^n \in \omega(2^n)$

$3^n \in \Omega(2^n)$

We compute:

$$\lim_{n\to\infty} \frac{3^n}{2^n} = \left(\frac{3}{2}\right)^n = \infty$$

Hence,

$$3^n \in \Omega(2^n)$$

### $2^n \notin \Omega(3^n)$

Compute:

$$\lim_{n\to\infty} \frac{2^n}{3^n} = \left(\frac{2}{3}\right)^n = 0$$

So,

$$2^n \notin \Omega(3^n)$$

Hence,

$$3^n \in \omega(2^n)$$

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