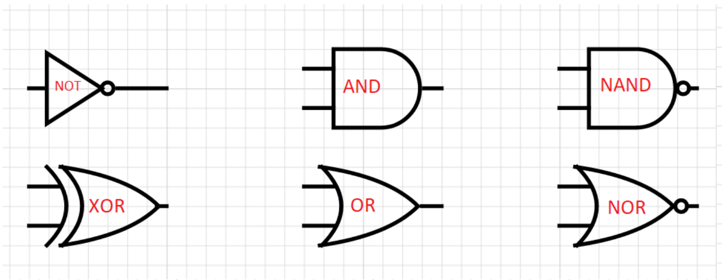
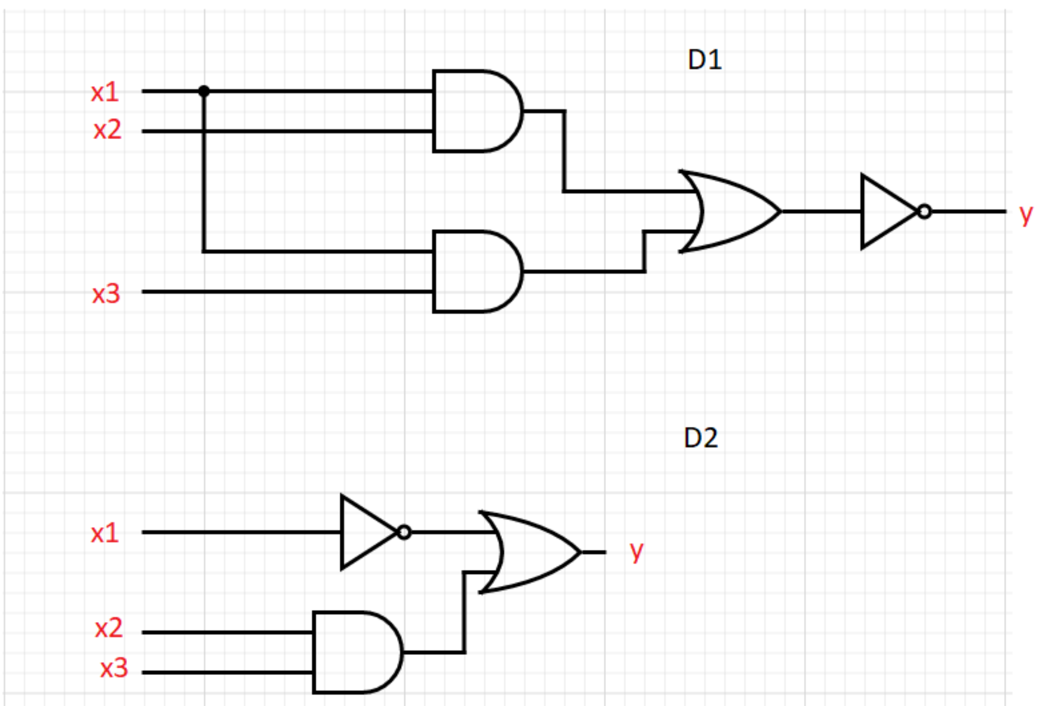
Homework 3 – Propositions

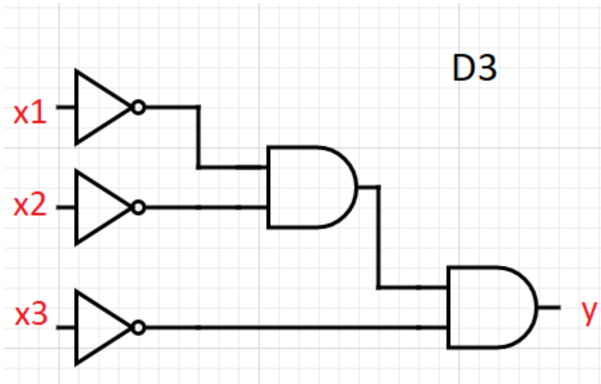
Combinatorial Circuits

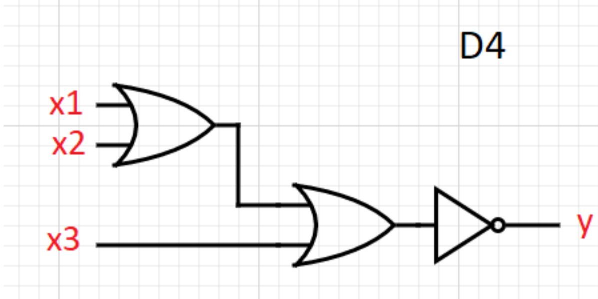
Consider the following logic gates:

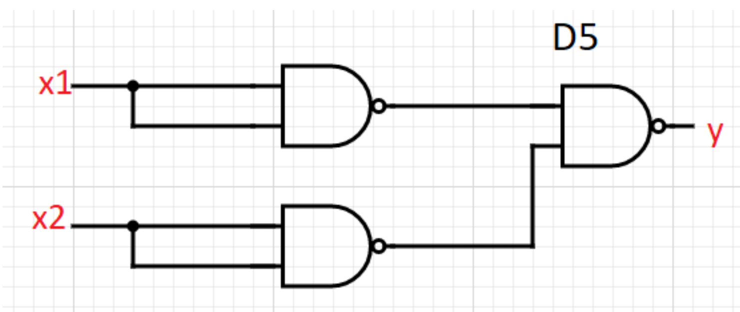


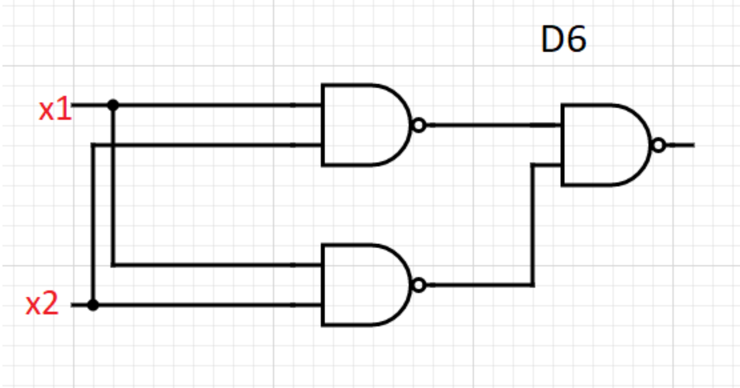
What expression is calculated in each diagram?

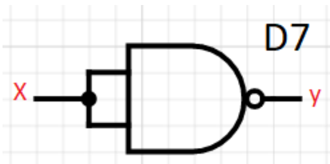
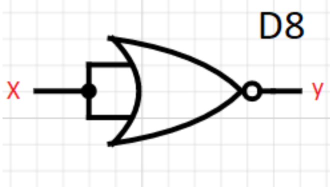
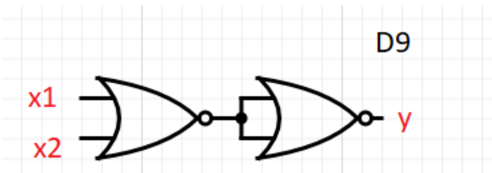
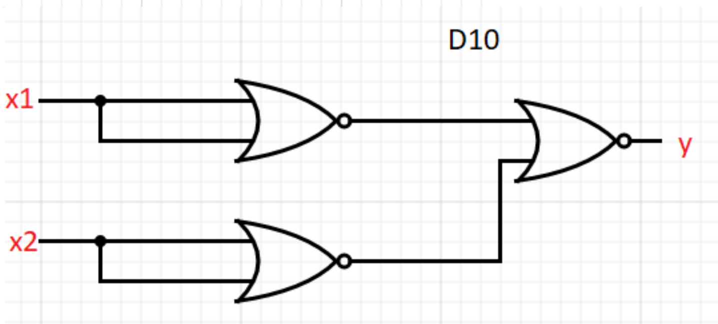


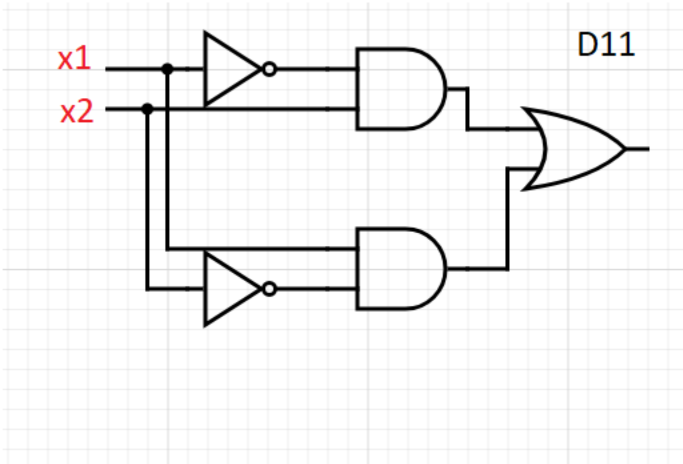

D3
x1
x2
x3
y

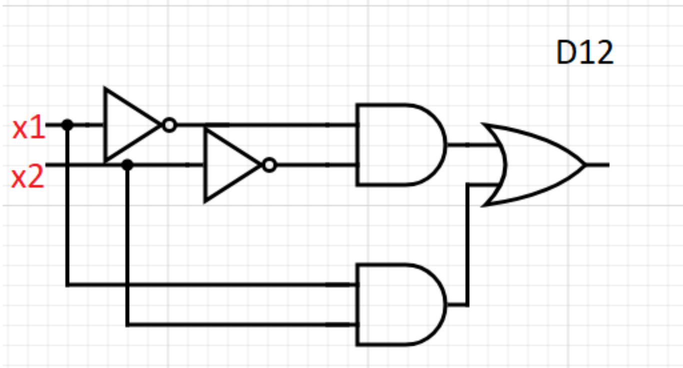

D4
x1
x2
x3
y


D5
x1
x2
y


D6
x1
x2


D7
X
y

D8
X
y

D9
x1
x2
y

D10
x1
x2
y


D11
x1
x2


D12
x1
x2

Connectives

1. Read the identities in Thm 1.1.22 parts (d), (h), (k). For every identity, explain why it is correct, using the regions in the standard Venn diagrams for 2 (or 3) sets.

2. For each of the identities in Thm 1.1.22 (a) - (k), write the identity in proposition notation:

   - Replace the set variables A, B, C with propositional variables p, q, r
   - $\wedge$ for $\cap$
   - $\vee$ for $\cup$
   - $\neg$ for $^{-}$
   - T for U ,
   - F for $\emptyset$.

3. We use the De Morgan law (twice) to show that De Morgan "works" for 3 variables:

$$\begin{aligned}
\neg(p_1 \vee p_2 \vee p3) &= \neg((p_1 \vee p_2) \vee p3) && \vee \text{ is associative} \\
&= \neg((p_1 \vee p_2) \vee p3) && \text{De Morgan over second } \vee \\
&= \neg(p_1 \vee p_2) \wedge \neg p3 && \text{De Morgan over first } \vee \\
&= (\neg p_1 \wedge \neg p_2) \wedge \neg p3 && \wedge \text{ is associative} \\
&= \neg p_1 \wedge \neg p_2 \wedge \neg p3
\end{aligned}$$

   Similarly, apply De Morgan law twice to show the following for all sets A, B, C:

   (a) $\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$
   (b) $\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$

4. Use the De Morgan laws to write a logically equivalent expression to the following, only using the symbols $\{\neg, \wedge\}$ Use the involution law (1.1.22 (i)) to simplify.

   (a) $\neg(p \vee q)$
   (b) $\neg(\neg p \vee \neg q)$
   (c) $\neg(p \vee \neg q)$
   (d) $p \vee q$
   (e) $p \vee \neg q$
   (f) $\neg p \vee \neg q$

5. Write a logically equivalent expression to the following, only using the symbols $\{\neg, \wedge, \vee\}$. You can use the identites from the previous questions

   (a) $p \rightarrow r$
   (b) $\neg(p \rightarrow r)$
   (c) $p \oplus r$
   (d) $p \oplus (r \wedge s)$
   (e) $p \leftrightarrow r$

Functional commpleteness

6. We saw that $\{\uparrow\}$ is functionally complete. Use the following table and equetions to show that $\{\downarrow\}$ is functionally complete.

   Definition (NOR). Let p, q be propositions. $p{\downarrow}q = \neg(p \vee q)$

p	q	$p \downarrow q$	$p \downarrow p$	$q \downarrow q$	$(p \downarrow q) \downarrow (p \downarrow q)$	$(p \downarrow p) \downarrow (q \downarrow q)$
T	T					
T	F					
F	T					
F	F					

   Show that we can express $\neg, \vee, \wedge$ using ONLY $\downarrow$

   - $\neg p \equiv$
   - $p \vee q \equiv$
   - $p \wedge q \equiv$

7. Show that $\{\neg, \vee\}$ is functionally complete. Show that we can express $\neg, \vee, \wedge$ using ONLY $\{\neg, \vee\}$:

   - $\neg p \equiv$
   - $p \vee q \equiv$
   - $p \wedge q \equiv$

   (Hint: De Morgan)

8. Show that $\{\neg, \rightarrow\}$ is functionally complete. Show that we can express $\neg, \vee, \wedge$ using ONLY $\{\neg, \rightarrow\}$:

   - $\neg p \equiv$
   - $p \vee q \equiv$
   - $p \wedge q \equiv$

9. Define $p \diamond q \equiv \neg(p \rightarrow q)$. Show that $\{\neg, \diamond\}$ is functionally complete.

Normal Forms

10. This quesrtion refers to a standard truth table of 3 variables .
    Write the following expressions only using $\{\neg, \vee, \wedge\}$. Negations ($\neg$) should only be in front of a single variable, and not in front of the parenthesis.

    (a) $R3$ is True in the 3rd line, False in all other lines

    (b) $R4$ is True in the 4th line, False in all other lines

    (c) $R4$ is True in the 8th line, False in all other lines

    (d) $Q3$ is False in the 3rd line, True in all other lines

(e) $Q4$ is False in the 4th line, True in all other lines

(f) $Q8$ is False in the 8th line, True in all other lines

11. Find the DNF of $p \oplus q$, $p \leftrightarrow q$, $\neg(p \rightarrow q)$

12. The proposition R_1 is given in the following truth table:

p	q	r	R_1	R_2	R_3
T	T	T	T		
T	T	F	F		
T	F	T	T		
T	F	F	F		
F	T	T	T		
F	T	F	F		
F	F	T	F		
F	F	F	F		

13. Write the DNF and CNF of R_1

14. It is known that

$$R_2 \equiv (p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (\neg p \wedge q \wedge r) \vee (\neg p \wedge \neg q \wedge \neg r)$$

Fill in the truth values of R_2 in the above table and find the CNF of R_2.

15. It is known that

$$R_3 \equiv (p \vee q \vee r) \wedge (p \vee q \vee \neg r) \wedge (\neg p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$$

Fill in the truth values of R_3 in the above table and find the DNF of R_3.