Review I

MATH 222, Feb 16, 2024

1. Consider the differential equation:

$$\frac{dy}{dt} = (1+y)(4-y^2).$$

(a) Draw a direction field. Clearly label your axes and make sure you choose a large enough range of y-values to demonstrate all the different types of behavior present in the system.
(b) Based on the direction field, determine the behavior of y as $t \to \infty$ and sketch the solution with initial condition $y(0) = 3$.

2. (a) Solve the initial value problem $t\ y' + 4y = 4t,\ y(1) = 2,\ t > 0$;
(b) Find the general solution of the equation $y' = x\ y^2(1+x^2)^{\frac{3}{2}}$. Write your answer in *explicit form*.
(c) Use Euler's Method to approximate $y(1)$ for the initial value problem (IVP) $y' = (1+x^2)y,\ y(1) = -1$. Use a time step $h = 0.25$.

3. A tank initially contains 100 gallons of pure water. A solution containing 1 lb of sugar per gallon is entering at a rate of 4 gal/min. A drain is opened at the bottom of the tank so that the well mixed solution is exiting the tank at a rate of 3 gal/min. Set up and solve an initial value problem to determine the mass of sugar at time t.

4. (a) Find the solution to the differential equation $y''(t)+2y'(t)+17y(t) = 0,\ y(0) = 1,\ y'(0) = 0$;
(b) Find the solution to the differential equation $y''(t) + 6y'(t) + 9y(t) = 0,\ y(0) = 0,\ y'(0) = -2$.

5. Use the method of reduction of order to find the general solution of the differential equation

$$t^2\ y'' + 2t\ y' - 6y = 0, \quad t > 0$$

knowing that $y_1(t) = t^2$ is a solution.