Homework 6 - Proofs

CS241

March 1, 2025

Proofs

1. Choose **one** question from each group (Please notice: it is recomended to answer all questions as practice for the exam)
   (a) Section 2.1. 9, 10
   (b) Section 2.1. 16, 17
   (c) Section 2.1. 21, 22
   (d) Section 2.1. 24, 25 (Hint for 25 - contrapositive)
   (e) Section 2.1. 29
   (f) Section 2.1. 30
   (g) Section 2.2. 4, 5 (contradiction)
   (h) Section 2.2. 22, 23 (contradiction)
   (i) Section 2.2. 27, 29 (contradiction) *See example 2.2.14
   (j) Section 2.2. 32, 33 (contradiction)
   (k) Section 5.1. 27 - 31
2. Prove that for all sets A, B: $A \cap B = \emptyset \quad \leftrightarrow \quad B \subseteq \overline{A}$ (Notice: $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$)
3. Prove that for all $n \in \mathbb{N} \cup \{0\}$: $n^2 \,\%\, 5 \neq 3$ (Hint: consider 5 cases, one for each remainder)
4. Prove that for all $x, y \in \mathbb{R}$: $\min(x, y) + \max(x, y) = x + y$ (cases)
   use the following definitions:
$$max(a,b) = \begin{cases} b & \text{if } a \leq b \\ a & \text{if } a > b \end{cases}$$
$$min(a,b) = \begin{cases} a & \text{if } a \leq b \\ b & \text{if } a > b \end{cases}$$
5. Proving irrationality of $\sqrt{3}$:

   Definition. (Rational number) A number s is rational of there are integers a, b such that $s = \frac{a}{b}$

Definition. (Lowest terms) A rational number $s = \frac{a}{b}$ is in lowest terms if the integers a, b do not have any common divisors (other than 1)

(a) Read example 2.2.3 in section 2.2. Write this proof in a formal way.

(b) Prove in $\mathbb{N} \cup \{0\}$: $\forall n.(3 \mid n \longleftrightarrow 3 \mid n^2)$ (Hint: one side is a direct proof, the other can be shown by contradiction and a proof we saw in class)

(c) Using the previous 2 questions, prove that $\sqrt{3}$ is irrational.