Homework 12 - Arnav Kucheriya - Graphs

1.

Problem 13

$$a_n = 2^n, a_0 = 1$$

This is a simple exponential sequence.
Solution:

$$a_n = 2^n$$

Problem 14

$$a_n = 2a_{n-1}, a_0 = 1$$

This is a homogeneous linear recurrence.
Solution:

$$a_n = 2^n$$

Problem 16

$$a_n = 7a_{n-1} - 10a_{n-2}, a_0 = 5, a_1 = 16$$

This is a linear recurrence with constant coefficients.

Step 1: **Form the characteristic equation**

$$r^2 - 7r + 10 = 0$$

Step 2: **Solve the equation**

$$r = 2, \quad r = 5$$

Step 3: **Write the general solution**

$$a_n = A \cdot 2^n + B \cdot 5^n$$

Step 4: **Apply initial conditions**
From $a_0 = 5$:

$$A + B = 5 \quad (1)$$

From $a_1 = 16$:

$$2A + 5B = 16 \quad (2)$$

Step 5: **Solve the system of equations**

From (1):

$$B = 5 - A$$

Substitute into (2):

$$2A + 5(5 - A) = 162A + 25 - 5A = 16 - 3A = -9 \Rightarrow A = 3B = 2$$

2.

(a) Solve:

$$T(n) = 3T(n/3) + n$$

Use Master Theorem:
Here, $a = 3,\, b = 3,\, f(n) = n$
Since $f(n) = \Theta(n^{\log_b a}) = \Theta(n)$,
Case 2 applies ⇒

$$T(n) = \Theta(n \log n)$$

(b) Solve:

$$T(n) = 4T(n/2) + n^2$$

Here, $a = 4,\, b = 2,\, f(n) = n^2$
$\log_b a = \log_2 4 = 2$
Since $f(n) = \Theta(n^2)$ = $n^{\log_b a}$,
Case 2 applies ⇒

$$T(n) = \Theta(n^2 \log n)$$

(c) Solve:

$$T(n) = 2T(n/2) + n^2$$

Here, $a = 2,\, b = 2,\, f(n) = n^2$
$\log_b a = \log_2 2 = 1$
Since $f(n) = \Omega(n^{1+\epsilon})$ for some $\epsilon > 0$,
Case 3 applies ⇒

$$T(n) = \Theta(n^2)$$

Euler Paths and Cycles

(a) G1:

Check degrees of all vertices:

- Even degrees: Euler **cycle** exists.

Euler cycle (example):
a → b → c → l → j → f → d → a → c → h → g → e → d

Euler Cycle exists

(b) G2:

Check degrees of vertices:

- If exactly 2 vertices have odd degree → Euler **path** exists
- If more than 2 → **No** Euler path/cycle

Check each degree:

- Vertices with **odd degree**: more than 2

No Euler path or cycle

(c) G3:

Check degrees:

- All vertices have even degree

Euler Cycle exists

Euler cycle (example):
a → b → f → g → e → i → h → g → f → c → d → e → a → c → b

Section 8.6 – Graph Isomorphism

1. G1 ≅ G2
Same number of vertices, edges, and connectivity structure.

2. G1 ≅ G2

Both are complete graphs (K_5).

3. G1 ≅ G2

Each is a double triangle (two connected K_3s).

7. G1 ≠ G2

Degree of vertices differs. Not isomorphic.
