Homework1 - Sets

January 22, 2025

Let $A = \{1, 2, 3\}$, $B = \{2, 3\}$, $C = \{A, B\}$, $D = \{'a'\}$, $E = \{\ \{0, 4\}\ \}$

1. For each of the following, determine if the statement is True, False, or has a syntax error (undefined):

   (a) $A \subseteq B$

   (b) $B \subseteq A$

   (c) $A \subseteq A$

   (d) $2 \subset B$

   (e) $A \subset A$

   (f) $B \subset A$

   (g) $B \in A$

   (h) $B \in C$

   (i) $2 \in 2$

   (j) $(A, B) \in C$

   (k) $(1, 3) \in A \times B$

   (l) $(1, 3) \subseteq A \times B$

   (m) $(1, 3) \in C$

   (n) $0 \in E$

2. Explicitly write all elements in the following sets:

   (a) $B \times A$

   (b) $A \times D$

   (c) $\emptyset \times B$

   (d) $C \times C$

   (e) $E \times B$

   (f) $\mathcal{P}(E)$ (the powerset of E)

   (g) $\mathcal{P}(A)$

   (h) $\mathcal{P}(\emptyset)$

3. Explicitly write all elements in the following sets:

   (a) $\{x \mid x - 5 = 0\}$

   (b) $\{x \mid x^2 - 5 = 0\}$

(c) $\{(x, y) \mid x \in \mathbb{N} \quad \text{AND} \quad x < 5 \quad \text{AND} \quad y = 0\}$

(d) $\{4x \mid x \in \mathbb{Z} \quad \text{AND} \quad -1 \leq x \leq 1\}$

(e) $\{a + b \mid a \in \mathbb{N} \quad \text{AND} \quad a < 3 \quad \text{AND} \quad b \in \{5, 6\}\}$

4. Draw the following subset of $\mathbb{R}$ on the number line: $\{t \mid t \in \mathbb{R} \quad \text{AND} \quad -\pi \leq t < \frac{1}{2}\}$

5. For each of the following, determine if the statement is True, False, or has a syntax error (undefined):

   (a) $\{2\} \in \mathcal{P}(B)$

   (b) $2 \in \mathcal{P}(B)$

   (c) $\{2\} \subseteq \mathcal{P}(B)$

   (d) $2 \subseteq \mathcal{P}(B)$

6. Evaluate the following expressions if possible, or write "undefined":

   (a) $|E|$

   (b) $|C|$

   (c) $|B| = |C|$

   (d) $|B| = |E|$

   (e) $|B = D|$

7. Reminder: Let X,Y be sets. We can say that $X \subset Y$ if $X \subseteq Y$ AND $X \neq Y$
   That is; $X \subset Y$ if all members of X are in Y, and Y has at least one element that is not in X.

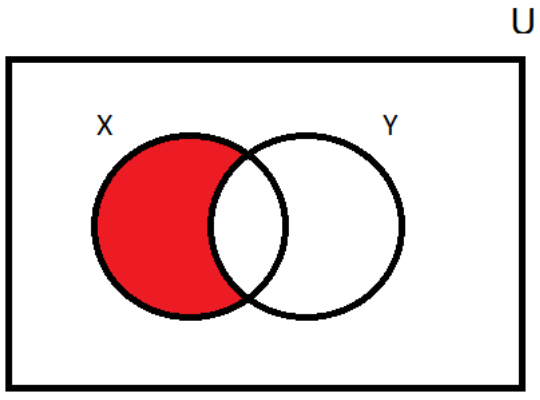
   Explain every step of the following statement, make sure to show examples

   $$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$$

8. Definition (Set Difference): Let U be the universal set, and let X, Y be sets. We define the difference set between X and Y:

   $$X - Y = X \setminus Y = \{t \mid t \in U \quad \text{AND} \quad t \in X \quad \text{AND} \quad t \notin Y\}$$

   The red region in this Venn Diagrams is the difference between X and Y:



   Apply the above definition to the following sets and write the members of each set, where:

   $$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\},\ A = \{1, 2, 3\},\ B = \{3, 4, 5\},\ C = \{7, 8\}$$

(a) $A - B =$

(b) $B - A =$

(c) $C - A =$

(d) $U - A =$

(e) $U - (A \cup B) =$

(f) $U - \overline{A} =$

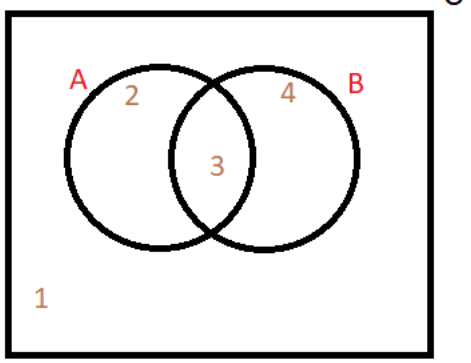
(g) $B - \emptyset =$

9. Let U be the universe, $K \subset U$, $L \subset U$. It is known that $|U| = 20, |\overline{K}| = 7, |K - L| = 10, |L - K| = 5$. What is $|K \cap L|$?

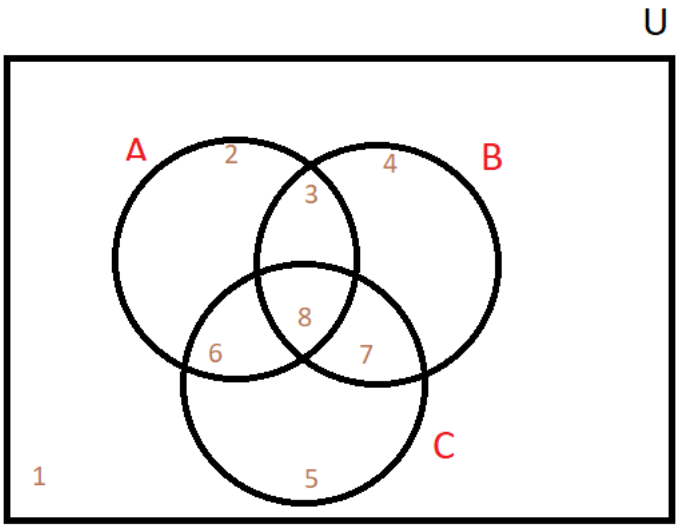
Venn diagrams

For the next questions, use the standard Venn diagram:

For two sets:



or for three sets:



1. Let $U = \{a, b, c, d, e, f, g, h, i, j\}$, $A = \{a, b, g, d\}$, $B = \{b, d, f, h, j\}$.
   Write every element of U in the correct region in the two set Venn diagram

2. Let $U = \{a, b, c, d, e, f, g, h, i, j\}$, $A = \{a, b, g, d\}$, $B = \{b, d, f, h, j\}$, $C = \{d, h, g\}$.
   Write every element of U in the correct region in the three set Venn diagram

3. What regions are shaded in the two set Venn diagram for each of the following expressions?

   (a) U

   (b) $\overline{A \cap B}$

   (c) $A \cap \overline{B}$

(d) $\overline{A \cup B}$

(e) $B \cap \emptyset$

(f) $(A \cup B) - (A \cap B)$

(g) $\overline{A} \cup \overline{B}$

(h) $A \cup U$

(i) $\overline{A} \cap \overline{B}$

(j) $A - B$

(k) $B - A$

(l) $B \cap \overline{A}$

(m) $(A - B) \cup (B - A)$

(n) $\overline{U}$

Find the pairs of equivalent expressions and write the equations, in example:

$$\overline{U} = B \cap \emptyset$$