Homework 5 - Rules of Inference, Quantifiers, Proofs

CS241

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Rules of Inference

Section 1.4. exercises: 5, 11, 14, 15, 22, 30, 33, 34, 35

Quantifiers

1. Let A, B be sets. What relation between A and B is defined by:
   (a) $\forall x.\, (x \in A) \rightarrow (x \in B)$
   (b) $[\forall x.\, (x \in A) \rightarrow (x \in B)] \;\wedge\; [\forall x.\, (x \in B) \rightarrow (x \in A)]$

Java assignment

2. In this assignment we compute the statements

$$\begin{gathered} \forall x. \forall y.\, P(x, y) \\ \exists x. \exists y.\, P(x, y) \\ \forall x. \exists y.\, P(x, y) \\ \exists x. \forall y.\, P(x, y) \end{gathered}$$

for a given finite domain and a given predicate P. Complete the TO DO parts in the java files attached to this assingment.

- There are 3 functions to implement.
- Currently all functions return true as a stub, change this code.
- Each of these functions should compute the statement mentioned above it.
- The fourth implementation is given as an example.
- You can use a helper function as seen in the code examples.
- Similar pseudocode can be found in examples 1.5.7, 1.6.5, 1.6.8.
- Code sould be able to work properly for all arrays A, B and function P.
- A java file that does not compile will receive no credit.
- Make your code is as readable as possible.

Proofs

3. Prove / disprove the statements from the previous assignment:

   (a) in $\mathbb{Z} : \forall x.\forall y.\ x - y = 7$

   (b) in $\mathbb{Z} : \exists x.\exists y.\ x - y = 7$

   (c) in $\mathbb{Z} : \forall x.\exists y.\ x - y = 7$

   (d) in $\mathbb{Z} : \exists x.\forall y.\ x - y = 7$

   (e) in $\mathbb{Z} : \forall x.\exists y.\ xy = 7$

   (f) in $\mathbb{Q} : \forall x.\exists y.\ xy = 7$

   (g) in $\mathbb{Q} : \forall x.\exists y.\ (x \neq 0) \to (xy = 7)$

   (h) in $\mathbb{Q} - \{0\} : \forall x.\exists y.\ xy = 7$

   (i) in $\mathbb{Z} : \forall y.\exists x.\ x > y$

   (j) in $\mathbb{Z} : \exists y.\forall x.\ x > y$

4. Prove / disprove the statements in $D = \{2, 6\}$. Use the definitions of $|$ (divides) and % (mod):

   (a) $\forall x.\ 2|x$

   (b) $\forall x.\ 3|x$

   (c) $\exists x.\ 5|x$

   (d) $\exists x.\ x \,\%\, 5 = 2$

5. Prove / disprove the statements, using the definitions of $|$ (divides) .
   We define the predicate DV with respect to $D = \{2, 3\} \times \{6, 9, 11\}$: $DV(a, b)$ is TRUE iff $a|b$

   (a) $\forall x.\forall y.\ DV(x, y)$

   (b) $\exists y.\forall x.\ DV(x, y)$

   (c) $\exists y.\forall x.\ \neg DV(x, y)$