Homework2 - Sets, Intro to number theory, propotitions

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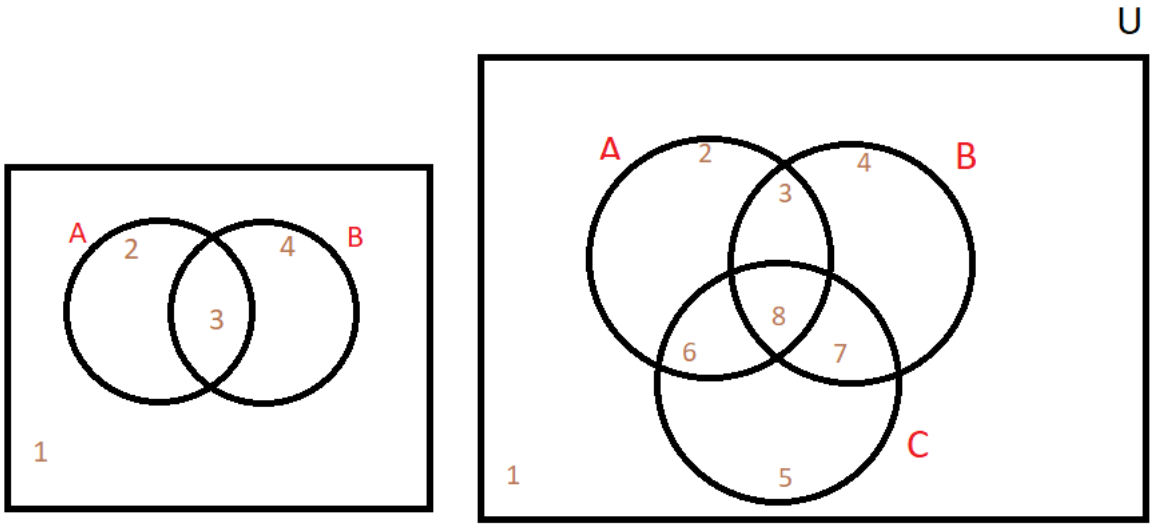
Sets

Standard Venn diagrams:



1. De Morgan for 3 variables: use a Venn diagram to explain the following properties:

$$\overline{A \cup B \cup C} = \overline{A} \cap \overline{B} \cap \overline{C}$$

$$\overline{A \cap B \cap C} = \overline{A} \cup \overline{B} \cup \overline{C}$$

2. In a Venn diagram of 2 sets, write what regions are included in each of the following sets. Describe the set using $\cup, \cap, \overline{\ }, A, B$

   (a) $\{t|(t \in A) \wedge (t \in B)\}$

   (b) $\{t|(t \in A) \vee (t \in B)\}$

   (c) $\{t|(t \in U) \wedge (\neg(t \in A))\}$

Divisors, modulo

Theorem (Quotient-Remainder Theorem)**.** Let $n \in \mathbb{Z}$, $d \in \mathbb{N}$.
Then there are (unique) integers q, r such that $n = dq + r$ and $0 \leq r < d$.
We define: n div d to returns q, and n mod d (or $n \,\%\, d$) to returns r.

1. Evaluate the following expressions. If not possible, write ”undefined”:

(a) $4 \mid 7$

(b) $4 \mid 12$

(c) $4 \mid -12$

(d) $12 \nmid 4$

(e) $0 \mid 12$

(f) $12 \mid 0$

(g) $\frac{4}{12}$

(h) $\frac{12}{4}$

(i) $\frac{0}{12}$

(j) $\frac{12}{0}$

2. For each of the following values of n and d find the integers q and r such that $n = dq + r$ and $0 \leq r < d$.

   (a) n = 28, d = 7

   (b) n = -28, d = 7

   (c) n = 31, d = 7

   (d) n = -31, d = 7

   (e) n = 0, d = 6

   (f) n = 5, d = 6

   (g) n = -5, d = 6

   (h) n = -10, d = 6

3. Evaluate:

   (a) 7 mod 5

   (b) -7 mod 5

   (c) 3 mod 5

   (d) -3 mod 5

   (e) 0 mod 5

   (f) 0 mod 3

   (g) 48 mod 12

   (h) 48 mod 11

   (i) -48 mod 11

4. Evaluate. If the statement is True, what is the integer q?

   (a) $6 \mid 42$

(b) $6 \mid 40$

(c) $39 \bmod 8 = 7$

(d) $41 \bmod 10 = 3$

(e) $41 \bmod 13 = 2$

Propositions

1. Write the truth table for $(p \vee q) \vee r$ and $p \vee (q \vee r)$. Is the connective $\vee$ assosiative? Explain
2. Is the connective $\vee$ commutative? Explain
3. Write the truth table for $(p \wedge q) \wedge r$ and $p \wedge (q \wedge r)$. Is the connective $\wedge$ assosiative? Explain
4. Is the connective $\wedge$ commutative? Explain
5. Write the truth table for $(p \rightarrow q) \rightarrow r$ and $p \rightarrow (q \rightarrow r)$. Is the connective $\rightarrow$ assosiative? Explain
6. Is the connective $\rightarrow$ commutative? Explain
7. We define the connectives $\oplus$ (read "xor", "exclusive or" or "ex-or") and $\leftrightarrow$ (read "if and only if", "iff"):

p	q	$p \oplus q$	$p \leftrightarrow q$
T	T	F	T
T	F	T	F
F	T	T	F
F	F	F	T

8. Write the truth table for $(p \oplus q) \oplus r$ and $p \oplus (q \oplus r)$. Is the connective $\oplus$ assosiative? Explain
9. Is the connective $\oplus$ commutative? Explain
10. Write an expression, logically equivalent to $p \oplus q$, using only the connectives $\neg, \vee, \wedge$
11. Write the truth table for $(p \leftrightarrow q) \leftrightarrow r$ and $p \leftrightarrow (q \leftrightarrow r)$. Is the connective $\leftrightarrow$ assosiative? Explain
12. Is the connective $\leftrightarrow$ commutative? Explain
13. Write an expression, logically equivalent to $p \leftrightarrow q$, using only the connectives $\neg, \vee, \wedge$
14. A function f takes n boolean arguments, and returns a boolean value. How many different inputs can f take? How many different such functions f exists? (consider logically equivalent functions as the same)