Arnav Kucheriya CS 241 - Homework 1

Part 1

Question 1: Determine True, False, or Syntax Error

Expression	Result	Explanation
$A \subseteq B$	False	$B = \{2, 3\}$, but $A = \{1, 2, 3\}$, and $1 \in A$ but not in B
$B \subseteq A$	True	All elements of B (2, 3) are in A
$A \subseteq A$	True	A set is always a subset of itself
$2 \subset B$	Syntax error	"2" is an element, not a set
$A \subset A$	False	A set cannot be a proper subset of itself
$B \subset A$	True	B is a proper subset of A as $B \subseteq A$ and $A \neq B$
$B \in A$	False	$B = \{2, 3\}$ is not an element of A
$B \in C$	True	$C = \{A, B\}$, and B is an element of C
$2 \in 2$	Syntax error	An element cannot contain itself
$(A, B) \in C$	False	(A, B) is not an element of C
$(1, 3) \in A \times B$	True	
$(1, 3) \subseteq A \times B$	False	$(1, 3)$ is a pair, not a set of pairs
$(1, 3) \in C$	False	$(1, 3)$ is not an element of C
$0 \in E$	False	$E = \{\{0, 4\}\}$, and 0 is not directly an element of E

Question 2: Set Elements

a) $B \times A$

$$\{(2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)\}$$

b) $A \times D$

$$\{(1, 'a'), (2, 'a'), (3, 'a')\}$$

c) $\emptyset \times B$

$$\emptyset \text{ (empty set)}$$

d) $C \times C$

$$\{(A, A), (A, B), (B, A), (B, B)\}$$

e) $E \times B$

$$\{\{0, 4\}, 2\}, \{\{0, 4\}, 3\}$$

f) $P(E)$

$$\{\emptyset, \{\{0, 4\}\}\}$$

g) $P(A)$

$$\{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

h) $P(\emptyset)$

$$\{\emptyset\}$$

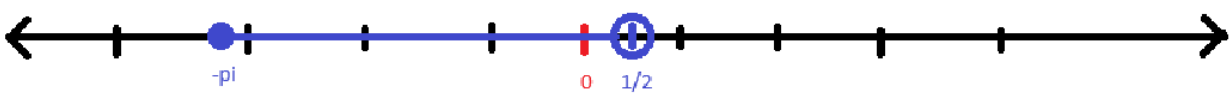
Question 3: Set Conditions

Expression	Solution
$\{x \mid x - 5 = 0\}$	$\{5\}$
$\{x \mid x^2 - 5 = 0\}$	$\{\sqrt{5}, -\sqrt{5}\}$
$\{(x, y) \mid x \in \mathbb{N}, x < 5, y = 0\}$	$\{(1, 0), (2, 0), (3, 0), (4, 0)\}$
$\{4x \mid x \in \mathbb{Z}, -1 \leq x \leq 1\}$	$\{-4, 0, 4\}$
$\{a + b \mid a \in \mathbb{N}, a < 3, b \in \{5, 6\}\}$	$\{6, 7, 8\}$

Question 4: Interval Notation

$t|t \in R$, AND $-\pi \leq t \leq 1/2$

The subset is represented as an interval starting at $-\pi$ (inclusive) and going up to $\frac{1}{2}$ (exclusive).



Question 5: Set Relations

Expression	Result	Explanation
$2 \in P(B)$	True	2 is an element of the powerset of B
$2 \in P(B)$	False	2 is not a subset of B
$2 \subseteq P(B)$	True	2 is a subset of the powerset of B
$2 \subseteq P(B)$	Syntax error	2 is an element, not a set

Question 6: Set Cardinality

a) $E = \{\{0, 4\}\}$

$|E| = 1$

b) $C = \{\{1, 2, 3\}, \{2, 3\}\}$

$|C| = 2$

c) $|B| = |C|$

True, since $|B| = 2$ and $|C| = 2$

d) $|B| = |E|$

False, since $|B| = 2$ and $|E| = 1$

e) $|B = D|$

False

Question 7: Number Sets

Theorem: $\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}$

Definition of Proper Subset

For sets X and Y, $X \subset Y$ means:

1. Every element of X is in Y ($X \subseteq Y$)
2. $X \neq Y$ (Y has at least one element not in X)

Natural Numbers ⊂ Integers

- $\mathbb{N} = \{1, 2, 3, 4, \ldots\}$

- $\mathbb{Z} = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$
- Every natural number is an integer
- $\mathbb{Z}$ contains negatives and zero, not in $\mathbb{N}$

Integers ⊂ Rational Numbers

- $\mathbb{Q} = \{\frac{a}{b} \mid a, b \in \mathbb{Z}, b \neq 0\}$
- Every integer z can be written as $\frac{z}{1}$
- $\mathbb{Q}$ contains non-integer fractions

Rational Numbers ⊂ Real Numbers

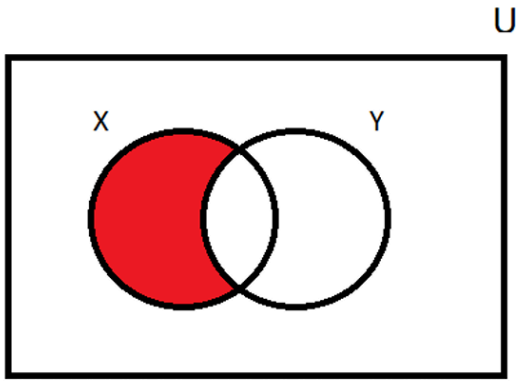
- $\mathbb{R}$ = all numbers on the number line
- Includes rational and irrational numbers (e.g., π, $\sqrt{2}$, e)

Question 8: Set Operations

8. Definition (Set Difference): Let U be the universal set, and let X, Y be sets. We define the difference set between X and Y:

$$X - Y = X \setminus Y = \{t \mid t \in U \quad \text{AND} \quad t \in X \quad \text{AND} \quad t \notin Y\}$$

The red region in this Venn Diagrams is the difference between X and Y:



Apply the above definition to the following sets and write the members of each set, where:

$$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\},\ A = \{1, 2, 3\},\ B = \{3, 4, 5\},\ C = \{7, 8\}$$

Operation	Result
$A - B$	$\{1, 2\}$
$B - A$	$\{4, 5\}$
$C - A$	$\{7, 8\}$
$U - A$	$\{4, 5, 6, 7, 8, 9, 10\}$
$U - (A \cup B)$	$\{6, 7, 8, 9, 10\}$

Operation	Result
$U - \overline{A}$	$A = \{1, 2, 3\}$
$B - \emptyset$	$B = \{3, 4, 5\}$

Question 9: Set Intersection

Given:

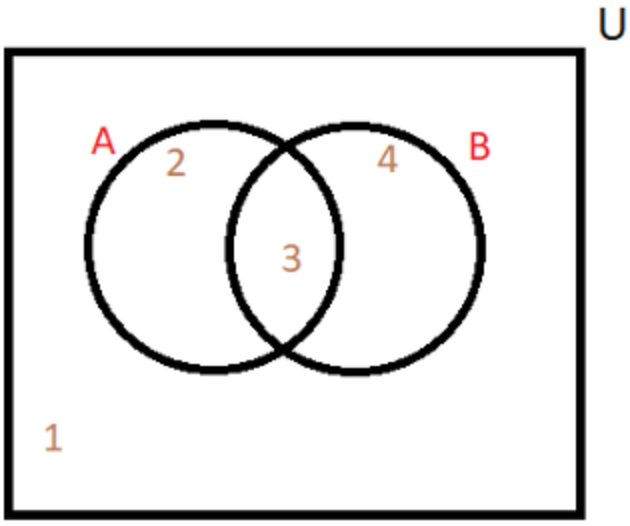
- $|U| = 20$
- $|K| = 7$
- $|K - L| = 10$
- $|L - K| = 5$

Using inclusion-exclusion:
$|K| = |K - L| + |K \cap L|$
$7 = 10 + |K \cap L|$
Therefore, $|K \cap L| = -3$

Venn Diagrams

1. Two-Set Diagram Elements:

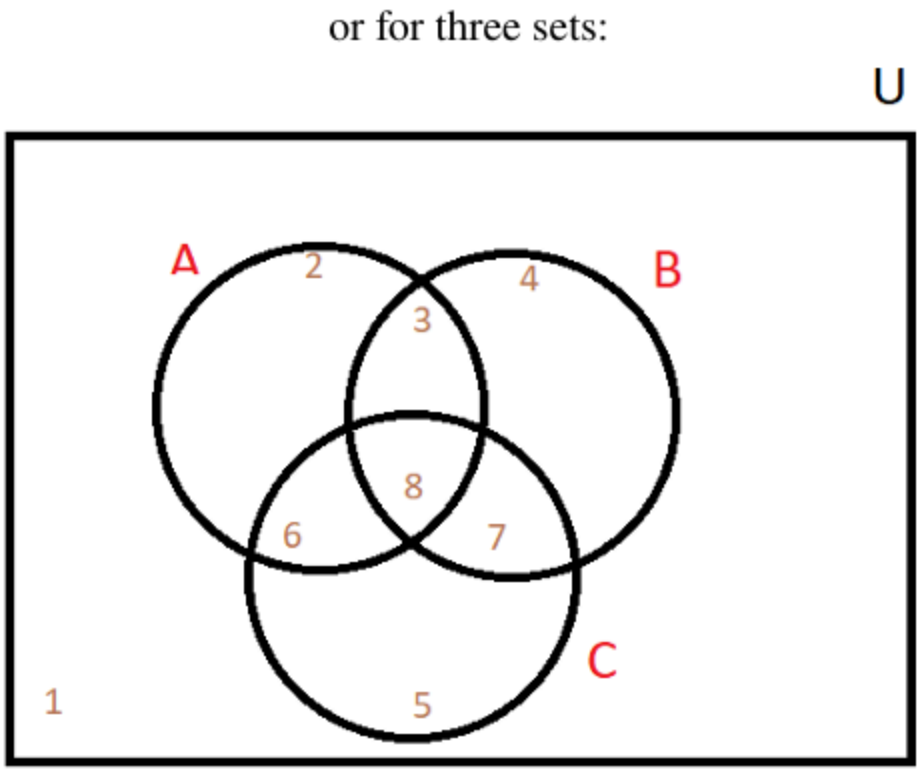
For two sets:



$U = \{a, b, c, d, e, f, g, h, i, j\}$
$A = \{a, b, g, d\}$
$B = \{b, d, f, h, j\}$

- $A \cap B = \{b, d\}$
- $A - B = \{a, g\}$
- $B - A = \{f, h, j\}$
- Outside both = $\{c, e, i\}$

2. Three-Set Diagram Regions:



1. Outside all sets: $\{c, e, i\}$
2. A only: $\{a\}$
3. B only: $\{f, j\}$
4. C only: $\emptyset$
5. $A \cap C$ (not B): $\{g\}$
6. $B \cap C$ (not A): $\{h\}$
7. $A \cap B$ (not C): $\{b\}$
8. $A \cap B \cap C$: $\{d\}$

3. Shading regions of the Venn Diagram:

(a) U: Entire diagram is shaded.

(b) $A \cap B$: Only $A \cap B$ region is shaded.

(c) $A \cap \overline{B}$: A only region is shaded.

(d) $A \cup B$: A and B are shaded.

(e) $B \cap \emptyset$: No region is shaded.

(f) $(A \cup B) - (A \cap B)$: A only and B only regions are shaded.

(g) $\overline{A} \cup \overline{B}$: Outside A and B is shaded.

(h) $A \cup U$: Entire diagram is shaded.

(i) $A \cap \overline{B}$: A only is shaded.

(j) $A - B$: A only is shaded.

(k) $B - A$: B only is shaded.

(l) $B \cap A$: $A \cap B$ is shaded.

(m) $(A - B) \cup (B - A)$: A only and B only are shaded.

(n) $\overline{U}$: No region is shaded.

Part 2: Section 1.1 Exercises

1.

$A \cup B$ where:

- $A = 1, 4, 7, 10$
- $B = 1, 2, 3, 4, 5$ **Solution:** $A \cup B = 1, 2, 3, 4, 5, 7, 10$

2.

$B \cap C$ where:

- $B = 1, 2, 3, 4, 5$
- $C = 2, 4, 6, 8$ **Solution:** $B \cap C = 2, 4$

3.

$A - B$ where:

- $A = 1, 4, 7, 10$
- $B = 1, 2, 3, 4, 5$ **Solution:** $A - B = 7, 10$

5.

$\overline{A}$ where:

- Universe $U = 1, 2, 3, \ldots, 10$
- $A = 1, 4, 7, 10$ **Solution:** $\overline{A} = 2, 3, 5, 6, 8, 9$

8.

$A \cup \emptyset$ **Solution:** $A \cup \emptyset = A = 1, 4, 7, 10$

10.

$A \cup U$ **Solution:** $A \cup U = U = 1, 2, 3, \ldots, 10$

11.

$B \cap U$ where $B = 1, 2, 3, 4, 5$ **Solution:** $B \cap U = B = 1, 2, 3, 4, 5$

12.

$A \cap (B \cup C)$ where:

- $A = 1, 4, 7, 10$
- $B \cup C = 1, 2, 3, 4, 5, 6, 8$ **Solution:** $A \cap (B \cup C) = 1, 4$

24.

$X \cap Y$ where:

- $X = 1, 2, 3, 4, 5$
- $Y = 2n \mid n \in \mathbb{Z}^+ = 2, 4, 6, \ldots$ **Solution:** $X \cap Y = 2, 4$

28.

What is the cardinality of $\emptyset$? **Solution:** $|\emptyset| = 0$

29.

What is the cardinality of 0? **Solution:** $|0| = 1$

34.

$A = 1, 2, 3$, $B = n \mid n \in \mathbb{Z}^+ \text{ and } n^2 < 10$ **Solution:** $A = B = 1, 2, 3$

43.

Determine if sets are equal: $x \mid x \in \mathbb{R} \text{ and } 0 < x \leq 2$, $1, 2$ **Solution:** The sets are not equal because $x \mid x \in \mathbb{R} \text{ and } 0 < x \leq 2$ contains all real numbers in $(0, 2]$, not just $1, 2$

56.

$B \cap (C \cup A)$ where:

- $B = 1, 2, 3, 4, 5$
- $C = 2, 4, 6, 8$
- $A = 1, 4, 7, 10$
- $C \cup A = 1, 2, 4, 6, 7, 8, 10$ **Solution:** $B \cap (C \cup A) = 1, 2, 4$

97.

$A \cap B = A$

- $A = 1, 2, 3$
- $B = 1, 2, 3, 4, 5$ **Condition:** $A \subseteq B$

98.

$A \cup B = A$ **Condition:** $B \subseteq A$

99.

$A \cap B = \emptyset$ **Condition:** A and B are disjoint sets

100.

$A \cap \overline{B} = B$ **Condition:** $B = A$

101.

$A \Delta B$ where:

- $A = 1, 2, 3$
- $B = 2, 3, 4, 5$ **Solution:** $A \Delta B = (A \cup B) - (A \cap B) = 1, 4, 5$