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7.1a. Sheet 7 — Arnav Singh

$(\neg A \rightarrow \neg B)$

A	B	$\neg A$	$\neg B$	$(\neg A \rightarrow B)$	$(A \rightarrow \neg B)$	$\neg(\neg A \rightarrow B)$	$\neg(A \rightarrow \neg B)$	$\neg A \rightarrow \neg B$
0	0	1	1	0	1	1	0	1
0	1	1	0	1	1	0	0	0
1	0	0	1	1	1	0	0	1
1	1	0	0	1	0	0	1	1

A	B	$A \cup B$	$A \cap B$
0	0	0	0
0	1	1	0
1	0	1	0
1	1	1	1

From the tables it is evident that the OR expression and $(\neg A \rightarrow B)$ are the same, therefore we can conclude OR is proved.
Now we see that the AND Expression is equivalent to $\neg(A \rightarrow \neg B)$, therefore AND is satisfied

Since AND and OR are satisfied all boolean operators can be satisfied, Hence is universal.

T.2

E = A ∧ B

P	Q	R	S	$(\neg P \vee Q)$	$\neg Q \vee R$	$(\neg R \vee S)$	$\neg S \vee P$	A	B	E
0	0	0	0	1	1	1	1	1	1	1
0	0	0	1	1	1	1	0	1	0	0
0	0	1	0	1	1	0	1	1	0	0
0	0	1	1	1	1	1	0	1	0	0
0	1	0	0	1	0	1	1	0	1	0
0	1	0	1	1	0	1	0	0	0	0
0	1	1	0	1	1	0	1	1	0	0
0	1	1	1	1	1	1	0	1	0	0
1	0	0	0	0	1	1	1	0	1	0
1	0	0	1	0	1	1	1	0	1	0
1	0	1	0	0	1	0	1	0	0	0
1	0	1	1	0	1	1	1	0	1	0
1	1	0	0	1	0	1	1	0	1	0
1	1	0	1	1	0	1	1	0	1	0
1	1	1	0	1	1	0	1	1	0	0
1	1	1	1	1	1	1	1	1	1	1

a) 2 since two lines with 1's ∴ only 2 implications.
If 0 take it as not

b) DNF = $\underbrace{(\neg P \wedge \neg Q \wedge \neg R \wedge \neg S)}_{1^{st}\ \text{line}} \vee \underbrace{(P \wedge Q \wedge R \wedge S)}_{\text{last line}}$

c) $(\neg P \vee Q) \wedge (\neg Q \vee R) \wedge (\neg R \vee S) \wedge (\neg S \vee P)$

These two first

$(\neg P \wedge \neg Q) \vee (\cancel{\neg Q \wedge Q}) \vee (\neg P \wedge R) \vee (Q \wedge R)$

$(\neg P \wedge \neg Q) \vee (\neg P \wedge R) \vee (Q \wedge R)$

Now including $\wedge (\neg R \vee S)$ in above expression

$[(\neg P \wedge \neg Q) \vee (\neg P \wedge R) \vee (Q \wedge R)] \vee (\neg R \vee S)$

$= (\neg P \wedge Q \wedge \neg R) \vee (\neg P \wedge \cancel{R \wedge \neg R}) \vee (Q \wedge \cancel{R \wedge \neg R})$

$(\neg P \wedge \neg Q \wedge S) \vee (\neg P \wedge R \wedge S) \vee (Q \wedge R \wedge S)$

$R \wedge \neg R = 0$

$\therefore \neg P \cdot 0 = 0$

$\therefore$ cancels out

$= (\neg P \wedge Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge S) \vee (\neg P \wedge R \wedge S)$

$\vee (Q \wedge R \wedge S)$

Now including $\wedge (\neg S \vee P)$ in expression

we get,

$= [(\neg P \wedge Q \wedge \neg R) \vee (\neg P \wedge \neg Q \wedge S) \vee (\neg P \wedge R \wedge S)$

$\vee (Q \wedge R \wedge S)] \wedge (\neg S \vee P)$

$= (\neg P \wedge Q \wedge \neg R \wedge \neg S) \vee (\neg P \wedge \neg Q \wedge \cancel{S \wedge \neg S}) \vee (\neg P \wedge \cancel{R \wedge S \wedge \neg S})$

$\vee (Q \wedge R \wedge \cancel{S \wedge \neg S})$

cancel out $\because S \wedge \neg S = 0$

hence anything multiplied by 0 is 0.

cancel out, 0's.

$\cdot (\neg P \wedge Q \wedge \cancel{\neg R} \wedge P) \vee (\neg P \wedge \cancel{\neg Q} \wedge S \wedge P) \vee (\neg P \wedge \cancel{R} \wedge S \wedge P)$

$\vee (Q \wedge R \wedge S \wedge P)$

become 0 cuz $(\neg P \wedge P) = 0$

We are now left with 2 expressions highlighted

$(\neg P \wedge \neg Q \wedge \neg R \wedge \neg S) \vee (P \wedge Q \wedge R \wedge S)$