ICS # Sheet 9 Arnav Singh

9.1

A	B	C_{in}	$A \dot{\vee} B$	$(A \dot{\vee} B) \dot{\vee} C_{in}$	$(A \wedge B) \vee (C_{in} \wedge A \dot{\vee} B)$
				S	C_{out}
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	1	1	0
0	1	1	1	0	1
1	0	0	1	1	0
1	0	1	1	0	1
1	1	0	0	0	1
1	1	1	0	1	1

a) $S_{DNF} = (\neg A \wedge \neg B \wedge C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})$

$C_{out\,DNF} = (\neg A \wedge B \wedge C) \vee (A \wedge \neg B \wedge C) \vee (A \wedge B \wedge \neg C_{in}) \vee (A \wedge B \wedge C)$

b) $Sum_{CNF} = (A \vee B \vee C_{in}) \wedge (A \vee \neg B \vee \neg C_{in}) \wedge (\neg A \vee B \vee \neg C_{in}) \wedge (\neg A \vee \neg B \vee C_{in})$

$C_{out\,CNF} = (A \vee B \vee C_{in}) \wedge (A \vee B \vee \neg C_{in}) \wedge (A \vee \neg B \vee C_{in}) \wedge (\neg A \vee B \vee C_{in})$

c)

c) Sum $(A \vee B \vee C_{in})$

$$= \neg\neg\big((\neg A \wedge \neg B \wedge C_{in}) \vee (\neg A \wedge B \wedge \neg C_{in}) \vee (A \wedge \neg B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})\big)$$

$$= \big(\neg\,(\neg(\neg A \wedge \neg B \wedge C_{in}) \wedge \neg\neg(\neg A \wedge B \wedge \neg C_{in}) \wedge \neg(A \wedge \neg B \wedge \neg C_{in}) \wedge \neg(A \wedge B \wedge C_{in})\big)$$

$$= \boxed{(\neg A \uparrow B \uparrow C_{in}) \uparrow (\neg A \uparrow B \uparrow \neg C_{in}) \uparrow (A \uparrow \neg B \uparrow \neg C_{in}) \uparrow (A \uparrow B \uparrow C_{in})}$$

Cout:

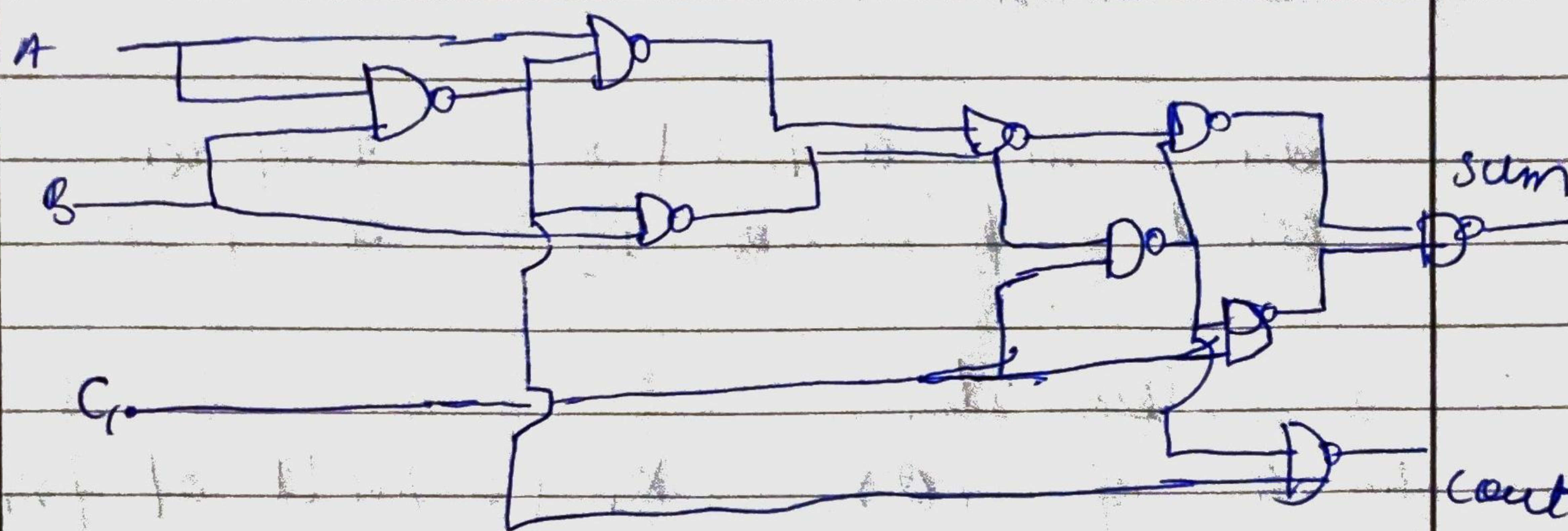
$$(\neg A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in})$$

$$= \neg\neg\big((\neg A \wedge B \wedge C_{in}) \vee (A \wedge \neg B \wedge C_{in}) \vee (A \wedge B \wedge \neg C_{in}) \vee (A \wedge B \wedge C_{in}\big)$$

$$= \neg(\neg A \wedge B \wedge C_{in}) \uparrow \neg(A \wedge \neg B \wedge C_{in}) \uparrow \neg(A \wedge B \wedge \neg C_{in}) \uparrow \neg(A \wedge B \wedge C_{in})$$

Cout: $\boxed{= (\neg A \uparrow B \uparrow C_{in}) \uparrow (A \uparrow \neg B \uparrow C_{in}) \uparrow (A \uparrow B \uparrow \neg C_{in}) \uparrow (A \uparrow B \uparrow C_{in})}$

d)



9.2
a)

Base step with 1 one element

foldr op e [a] = a op e ✓
foldl op e [a] = a op e ✓

Wa for the n^{th} element

foldr op e $[a_1 \ldots a_n]$ = foldl op e $[a_1 \ldots a_n]$

To prove $n+1^{the}$ element is also true
take LHS

↳ foldr op e $[a_1 \ldots a_n, a_{n+1}]$
$= a_1 \text{ op } (a_2 \text{ op } (\ldots \ldots a_n \text{ op } (e \text{ op } a_{n+1})))$

$= a_1 \text{ op } (a_2 \text{ op } (a_n \text{ op } e)) \text{ op } a_{n+1}$

= Assumption

= foldl op e $[a_1 \ldots a_n]$ op a_{n+1}

= foldl op e $[a_1 \ldots a_n, a_{n+1}]$

b) Base case:

foldr op1 e [a] = a op$_1$ e
foldl op2 e [a] = e op$_2$ a

With n elements
foldr op1 e [a_1 ... a_n] = foldl op2 e [a_1 ... a_n]

Now n+1 elements
LHS foldr op1 [a_1 ... a_{n+1}]
= a_1 (op1 (a_2 op1 ... (a_n op1 (a_{n+1} op1 e))))
= a_1 (op1 (a_2 op1 ... (a_n op1 (e op2 a_{n+1}))))
= $\underbrace{a_1 \text{ op1} (a_2 \text{ op1} (a_n \text{ op1 } e)))}$ op 2 a_{n+1}
using Assumption = foldl op2 e [a_1 ... a_n] op 2 a_{n+1}
= foldl op 2 e [a_1 ... a_n, a_{n+1}]
Hence Proved

c) Base case:

$$foldr\ op\ a\ [a] = a\ op\ a$$
$$foldl\ op'\ revers\ [a] = a\ op\ a$$

N elements,

$$foldr\ op\ [x_1 \ldots x_n] = foldl\ op'\ [x_n \ldots x_1]$$

Now n+1 elements

taking LHS

$$= x_1\ op\ (x_2\ op \cdots (x_n\ op(x_{n+1}\ op\ a)))$$

$$= x_1\ op\ (x_2\ op \cdots (x_n\ op\ (a\ op'\ x_{n+1})))$$

$$= \underbrace{x_1\ op\ (x_2\ op\ (x_n\ op\ a)}_{\text{Assumption}}\ op'\ x_{n+1}$$

$$= foldl\ op'\ a[x_n \ldots x_1]\ op'\ x_{n+1}$$

Hence,

$$fold\ op'\ a\ [x_{n+1}, x_n \cdots x_1]$$

Hence proved.