

# Simple Harmonic Motion

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## Abstract

In this experiment, we looked at damped harmonic motion by analyzing a mass-spring system with varying masses (100g and 250g) and damping configurations (no plate, small plate, and large plate). Using motion sensor data and nonlinear least-squares fitting, we extracted oscillation parameters from 20 trials per configuration to determine spring constants and damping coefficients. Our analysis revealed spring constants ranging from  $k = 5.16 \pm 0.0013$  N/m to  $k = 5.50 \pm 0.0017$  N/m across configurations, with all pairwise comparisons showing significant disagreement ( $24\sigma$  to  $159\sigma$ ). The damping coefficient increased systematically with plate size, from  $b = 0.00255 \pm 0.00020$  kg/s (no plate) to  $b = 0.01986 \pm 0.00037$  kg/s (large plate) for the 250g mass. Comparison between Week 1 single-fit and Week 2 multi-trial data showed variable agreement, with 100g no-plate agreeing on all 4 parameters, while other configurations showed disagreement up to  $12.5\sigma$ . These systematic variations likely arise from effective spring mass contributions, non ideal spring behavior at larger displacements, and geometric variations in damping plate orientation during oscillation.

## 1 Introduction

When a mass-spring system undergoes oscillatory motion in the presence of energy dissipation, the behavior is described by damped simple harmonic motion. In an ideal frictionless system, oscillations would continue indefinitely with constant amplitude, following Hooke's Law  $F = -kx$ . However, real systems experience energy loss through various damping mechanisms, primarily air resistance in our experimental setup.

The equation of motion for a damped harmonic oscillator is given by:

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0 \quad (1)$$

where  $m$  is the mass,  $b$  is the damping coefficient,  $k$  is the spring constant, and  $x$  is the displacement from equilibrium.

The general solution to this differential equation for underdamped oscillations is:

$$x(t) = Ae^{-\alpha t} \cos(\omega t + \phi) \quad (2)$$

where  $A$  is the initial amplitude,  $\alpha = b/(2m)$  is the decay constant,  $\omega$  is the angular frequency of the damped oscillations, and  $\phi$  is the phase constant.

A key theoretical prediction of this model is the relationship between the damped frequency  $\omega$  and the natural frequency  $\omega_0$  of the undamped system:

$$\omega^2 = \omega_0^2 - \alpha^2 = \frac{k}{m} - \left( \frac{b}{2m} \right)^2 \quad (3)$$

The primary objectives of this experiment are twofold: first, to verify that equation (2) accurately describes the motion of a damped mass-spring system, and second, to test whether equation (3) correctly predicts the frequency shift due to damping. We investigate five different configurations by varying both the oscillating mass (100 g and 250 g) and the amount of air damping (no additional damping, small damping plate, and large damping plate).

Experimental data is collected using a motion sensor to track the position of the oscillating mass over time. We use nonlinear least-squares fitting to extract the parameters in equation (2) from the position versus time data. The analysis is performed both using Capstone software and custom Python code.

## 2 Measurements and Analysis

### Extracted Values with Uncertainties:

We used a motion sensor to track the position of the oscillating mass over time in 5 different configurations:

1. 100g with no plate,  $m = 102.4g$
2. 100g with small plate,  $m = 110.9g$
3. 250g with no plate,  $m = 253.4g$
4. 250g with small plate,  $m = 256.9g$
5. 250g with large plate,  $m = 268.2g$

We found the following values, using different configurations over the 20 runs:

Configuration	$\omega$ (rad/s)	$\omega_0$ (rad/s)	$B(\alpha)$ (s <sup>-1</sup> )	$k$ (N/m)	$b$ (kg/s)
250g Large Plate	$4.49380 \pm 0.00077$	$4.493950 \pm 0.000769$	$0.037032 \pm 0.000681$	$5.42 \pm 0.0019$	$0.01986 \pm 0.00037$
250g Small Plate	$4.57107 \pm 0.00033$	$4.571114 \pm 0.000329$	$0.020530 \pm 0.000475$	$5.37 \pm 0.00077$	$0.01055 \pm 0.00024$
250g No Plate	$4.66016 \pm 0.00071$	$4.660163 \pm 0.000711$	$0.005029 \pm 0.000394$	$5.50 \pm 0.0017$	$0.00255 \pm 0.00020$
100g Small Plate	$6.82330 \pm 0.00087$	$6.823436 \pm 0.000869$	$0.042645 \pm 0.000907$	$5.16 \pm 0.0013$	$0.00946 \pm 0.00020$
100g No Plate	$7.15662 \pm 0.00037$	$7.156623 \pm 0.000371$	$0.006064 \pm 0.000380$	$5.24 \pm 0.00054$	$0.00124 \pm 0.00008$

Table 1: Fitted and derived parameters for different experimental configurations.

### Error Propagation formulas:

We propagated the error using the following equations:

$$\text{Natural frequency: } \omega_0 = \sqrt{\omega^2 + \alpha^2}$$

$$\text{Propagation of uncertainty: } \sigma_{\omega_0} = \sqrt{\left(\frac{\partial \omega_0}{\partial \omega} \sigma_\omega\right)^2 + \left(\frac{\partial \omega_0}{\partial \alpha} \sigma_\alpha\right)^2} = \sqrt{\left(\frac{\omega}{\omega_0} \sigma_\omega\right)^2 + \left(\frac{\alpha}{\omega_0} \sigma_\alpha\right)^2}$$

$$\text{Spring constant: } k = m(\omega^2 + \alpha^2)$$

$$\text{Propagation of uncertainty: } \sigma_k = \sqrt{\left(\frac{\partial k}{\partial \omega} \sigma_\omega\right)^2 + \left(\frac{\partial k}{\partial \alpha} \sigma_\alpha\right)^2} = \sqrt{(2m\omega \sigma_\omega)^2 + (2m\alpha \sigma_\alpha)^2}$$

$$\text{Damping coefficient: } b = 2m\alpha$$

$$\text{Propagation of uncertainty: } \sigma_b = 2m \sigma_\alpha$$

## Spring Constant Analysis:

Using the significance test:

$$\text{Significance} = \frac{|k_i - k_j|}{\sqrt{\sigma_{k_i}^2 + \sigma_{k_j}^2}}$$

we calculated all pairwise comparisons:

Comparison	$k_1 - k_2$ (N/m)	$\sqrt{\sigma_1^2 + \sigma_2^2}$ (N/m)	Calculation	Significance	Agreement
250g Large vs 250g Small	$5.42 - 5.37 = 0.050$	$\sqrt{0.0019^2 + 0.00077^2} = 0.00205$	$0.050$ $0.00205$	$24.4\sigma$	DISAGREE
250g Large vs 250g No Plate	$5.42 - 5.50 = -0.080$	$\sqrt{0.0019^2 + 0.0017^2} = 0.00255$	$0.080$ $0.00255$	$31.4\sigma$	DISAGREE
250g Large vs 100g Small	$5.42 - 5.16 = 0.260$	$\sqrt{0.0019^2 + 0.0013^2} = 0.00230$	$0.260$ $0.00230$	$113.0\sigma$	DISAGREE
250g Large vs 100g No Plate	$5.42 - 5.24 = 0.180$	$\sqrt{0.0019^2 + 0.00054^2} = 0.00197$	$0.180$ $0.00197$	$91.4\sigma$	DISAGREE
250g Small vs 250g No Plate	$5.37 - 5.50 = -0.130$	$\sqrt{0.00077^2 + 0.0017^2} = 0.00186$	$0.130$ $0.00186$	$69.9\sigma$	DISAGREE
250g Small vs 100g Small	$5.37 - 5.16 = 0.210$	$\sqrt{0.00077^2 + 0.0013^2} = 0.00151$	$0.210$ $0.00151$	$139.1\sigma$	DISAGREE
250g Small vs 100g No Plate	$5.37 - 5.24 = 0.130$	$\sqrt{0.00077^2 + 0.00054^2} = 0.00094$	$0.130$ $0.00094$	$138.3\sigma$	DISAGREE
250g No Plate vs 100g Small	$5.50 - 5.16 = 0.340$	$\sqrt{0.0017^2 + 0.0013^2} = 0.00214$	$0.340$ $0.00214$	$158.9\sigma$	DISAGREE
250g No Plate vs 100g No Plate	$5.50 - 5.24 = 0.260$	$\sqrt{0.0017^2 + 0.00054^2} = 0.00178$	$0.260$ $0.00178$	$146.1\sigma$	DISAGREE
100g Small vs 100g No Plate	$5.16 - 5.24 = -0.080$	$\sqrt{0.0013^2 + 0.00054^2} = 0.00141$	$0.080$ $0.00141$	$56.7\sigma$	DISAGREE

Table 2: Pairwise spring constant significance tests. All comparisons show significant disagreement.

- All 10 pairwise comparisons show significant disagreements ( $24\sigma$  to  $159\sigma$ )
- The smallest disagreement is between 250g Large and 250g Small plates ( $24.4\sigma$ )
- The largest disagreement is between 250g No Plate and 100g Small Plate ( $158.9\sigma$ )
- The pattern we see suggests systematic mass dependence: 250g masses have higher  $k$  values ( $5.37 - 5.50$  N/m) than 100g masses ( $5.16 - 5.24$  N/m)

Possible reasons for the discrepancy are:

1. In theory, we assume that the entire mass  $m$  move as a rigid body and the spring has negligible mass. In reality, part of the spring's mass oscillates, and attachments like plates add distributed inertia. If we don't account for this, the computed  $k$  changes with the load mass. Lighter loads have larger fractional errors, which explains why  $k$  values for 100g trials are lower than for 250g trials.

2. Small differences in how the spring or plate is mounted, like tilts, or imperfect attachment can cause discrepancies
3. The oscillation motion was not always stable, particularly with larger masses. We observed that heavier mass configurations produced more irregular oscillation patterns. This instability affected both the drag forces and the overall oscillation dynamics, introducing additional noise and systematic biases in our fitted parameters.
4. We assume that our spring would always behave like an ideal spring, however, with larger forces, which result in larger displacement, real springs deviate from the ideal behavior, so this also might have been a part of the discrepancies

## Drag Coefficient Analysis

### Comparison of $b$ values:

Comparison	$b$ values (kg/s)	Significance
No damper: 100g vs 250g	$0.00124 \pm 0.00008$	$\frac{ 0.00255 - 0.00124 }{\sqrt{0.00020^2 + 0.00008^2}} = 6.1\sigma$
	$0.00255 \pm 0.00020$	
Small damper: 100g vs 250g	$0.00946 \pm 0.00020$	$\frac{ 0.01055 - 0.00946 }{\sqrt{0.00024^2 + 0.00020^2}} = 3.5\sigma$
	$0.01055 \pm 0.00024$	
250g: Small vs Large plate	$0.01055 \pm 0.00024$	$\frac{ 0.01986 - 0.01055 }{\sqrt{0.00037^2 + 0.00024^2}} = 21.1\sigma$
	$0.01986 \pm 0.00037$	

To determine whether the drag coefficient scales with cross-sectional area or diameter, we can look at the ratio of measured  $b$  values

$$\text{Measured ratio: } \frac{b_{\text{large}}}{b_{\text{small}}} = \frac{0.01986}{0.01055} = 1.88 \pm 0.04$$

$$1. \text{ If } b \propto A \propto d^2: \text{ expected ratio} = \left( \frac{d_{\text{large}}}{d_{\text{small}}} \right)^2$$

$$2. \text{ If } b \propto d: \text{ expected ratio} = \frac{d_{\text{large}}}{d_{\text{small}}}$$

We did not record the diameters of the small and large damping plates during the experiment. But from this ratio, we know that if the drag scales linearly with diameter, then  $\frac{d_{\text{large}}}{d_{\text{small}}} = 1.88$ , which would imply the large plate

has a diameter 88% larger than the small plate.

If the drag scales with area, then  $\frac{d_{\text{large}}}{d_{\text{small}}} = \sqrt{1.88} = 1.37$ , which would imply the large plate has a diameter only 37% larger than the small plate.

Based on visual observations during the experiment, the large plate appeared larger than the small plate, which is more consistent with an area dependence.

### **Findings:**

We see that none of the values agree, however, this is not what we expected in theory, since the ones that have the same damping configuration should not have been significantly different. We expected a significant difference between the small and large plate configuration for 250g mass, since the area is directly affecting the drag force. (<https://www.grc.nasa.gov/www/k-12/VirtualAero/BottleRocket/airplane/sized.html>)

We see that the configurations that should've been statistically the same differing, like in spring constant comparison, so there has to be some systematic uncertainty that is dominating our calculated uncertainty from the data.

Possible reasons for the discrepancy are:

1. The orientation of the plates might have differed between configurations, changing the effective cross-sectional area exposed to airflow. Even with identical plates, different mounting angles would create different drag forces.
2. The different mass geometries (100g vs 250g) presented different surface areas to the airflow. Combined with instability during oscillation where masses may have rotated, this created varying drag force

## Week 1-2 Comparisons:

**100g Small Plate ( $m = 0.1109 \text{ kg}$ )**

Parameter	Table Value	Single-fit Value	Significance
$\omega$ (rad/s)	$6.82330 \pm 0.00087$	$6.8273 \pm 0.00036$	$\frac{ 6.8273 - 6.82330 }{\sqrt{0.00036^2 + 0.00087^2}} = 4.25\sigma$
$\alpha$ (s <sup>-1</sup> )	$0.042645 \pm 0.000907$	$0.041223 \pm 0.00036$	$\frac{ 0.041223 - 0.042645 }{\sqrt{0.00036^2 + 0.000907^2}} = 1.46\sigma$
$k$ (N/m)	$5.16 \pm 0.0013$	$5.168 \pm 0.00054$	$\frac{ 5.168 - 5.16 }{\sqrt{0.00054^2 + 0.0013^2}} = 5.70\sigma$
$b$ (kg/s)	$0.00946 \pm 0.00020$	$0.009143 \pm 0.000080$	$\frac{ 0.009143 - 0.00946 }{\sqrt{0.000080^2 + 0.00020^2}} = 1.47\sigma$

**100g No Plate ( $m = 0.1024 \text{ kg}$ )**

Parameter	Table Value	Single-fit Value	Significance
$\omega$ (rad/s)	$7.15662 \pm 0.00037$	$7.1566 \pm 0.00026$	$\frac{ 7.1566 - 7.15662 }{\sqrt{0.00026^2 + 0.00037^2}} = 0.04\sigma$
$\alpha$ (s <sup>-1</sup> )	$0.006064 \pm 0.000380$	$0.006088 \pm 0.00026$	$\frac{ 0.006088 - 0.006064 }{\sqrt{0.00026^2 + 0.000380^2}} = 0.05\sigma$
$k$ (N/m)	$5.24 \pm 0.00054$	$5.240 \pm 0.00038$	$\frac{ 5.240 - 5.24 }{\sqrt{0.00038^2 + 0.00054^2}} = 0.00\sigma$
$b$ (kg/s)	$0.00124 \pm 0.00008$	$0.001247 \pm 0.000053$	$\frac{ 0.001247 - 0.00124 }{\sqrt{0.000053^2 + 0.00008^2}} = 0.07\sigma$

**250g No Plate ( $m = 0.2534 \text{ kg}$ )**

Parameter	Table Value	Single-fit Value	Significance
$\omega$ (rad/s)	$4.66016 \pm 0.00071$	$4.6512 \pm 0.00009$	$\frac{ 4.6512 - 4.66016 }{\sqrt{0.00009^2 + 0.00071^2}} = 12.50\sigma$
$\alpha$ (s <sup>-1</sup> )	$0.005029 \pm 0.000394$	$0.0065184 \pm 0.000092$	$\frac{ 0.0065184 - 0.005029 }{\sqrt{0.000092^2 + 0.000394^2}} = 3.68\sigma$
$k$ (N/m)	$5.50 \pm 0.0017$	$5.483 \pm 0.00021$	$\frac{ 5.483 - 5.50 }{\sqrt{0.00021^2 + 0.0017^2}} = 9.99\sigma$
$b$ (kg/s)	$0.00255 \pm 0.00020$	$0.003303 \pm 0.000047$	$\frac{ 0.003303 - 0.00255 }{\sqrt{0.000047^2 + 0.00020^2}} = 3.67\sigma$

**250g Small Plate ( $m = 0.2569 \text{ kg}$ )**

Parameter	Table Value	Single-fit Value	Significance
$\omega$ (rad/s)	$4.57107 \pm 0.00033$	$4.5708 \pm 0.00014$	$\frac{ 4.5708 - 4.57107 }{\sqrt{0.00014^2 + 0.00033^2}} = 0.75\sigma$
$\alpha$ (s <sup>-1</sup> )	$0.020530 \pm 0.000475$	$0.021696 \pm 0.00014$	$\frac{ 0.021696 - 0.020530 }{\sqrt{0.00014^2 + 0.000475^2}} = 2.35\sigma$
$k$ (N/m)	$5.37 \pm 0.00077$	$5.369 \pm 0.00033$	$\frac{ 5.369 - 5.37 }{\sqrt{0.00033^2 + 0.00077^2}} = 1.19\sigma$
$b$ (kg/s)	$0.01055 \pm 0.00024$	$0.01114 \pm 0.000072$	$\frac{ 0.01114 - 0.01055 }{\sqrt{0.000072^2 + 0.00024^2}} = 2.35\sigma$

## 250g Large Plate ( $m = 0.2682 \text{ kg}$ )

Parameter	Table Value	Single-fit Value	Significance
$\omega$ (rad/s)	$4.49380 \pm 0.00077$	$4.4986 \pm 0.00024$	$\frac{ 4.4986 - 4.49380 }{\sqrt{0.00024^2 + 0.00077^2}} = 5.93\sigma$
$\alpha$ (s <sup>-1</sup> )	$0.037032 \pm 0.000681$	$0.032157 \pm 0.00023$	$\frac{ 0.032157 - 0.037032 }{\sqrt{0.00023^2 + 0.000681^2}} = 6.78\sigma$
$k$ (N/m)	$5.42 \pm 0.0019$	$5.426 \pm 0.0014$	$\frac{ 5.426 - 5.42 }{\sqrt{0.0014^2 + 0.0019^2}} = 2.54\sigma$
$b$ (kg/s)	$0.01986 \pm 0.00037$	$0.01725 \pm 0.00012$	$\frac{ 0.01725 - 0.01986 }{\sqrt{0.00012^2 + 0.00037^2}} = 6.71\sigma$

Errors were propagated in  $k$ , and  $b$  using the error propagation equations in spring constant analysis part.

1. We see that  $\alpha$  and  $b$  agrees for the week 1-2 100g small plate comparison, meanwhile  $\omega$  and  $k$  disagree, since they are not within the 3 standard deviations.
2. For 100g no plate configuration, we see that all 4 values agree, and we see that they are really close to the mean.
3. For 250g no plate configuration, we see that none of the values agree with each other, since none of them are within 3 standard deviations.
4. For 250g small plate, all of the values agree with each other, since all of them are within 3 standard deviations.
5. For 250g large plate configuration, only  $k$  agrees, which is within the 3 standard deviation range, but the remaining 3 values disagree.

## Reassessment Using Week 2 Standard Deviations

Comparison	$k_1 - k_2$ (N/m)	$\sqrt{\sigma_1^2 + \sigma_2^2}$ (N/m)	Calculation	Significance	Agreement
250g Large vs 250g Small	5.426 - 5.369 = 0.057	$\sqrt{0.0019^2 + 0.00077^2} = 0.00205$	$\frac{0.057}{0.00205}$	$27.8\sigma$	DISAGREE
250g Large vs 250g No Plate	5.426 - 5.483 = -0.057	$\sqrt{0.0019^2 + 0.0017^2} = 0.00255$	$\frac{0.057}{0.00255}$	$22.4\sigma$	DISAGREE
250g Large vs 100g Small	5.426 - 5.168 = 0.258	$\sqrt{0.0019^2 + 0.0013^2} = 0.00230$	$\frac{0.258}{0.00230}$	$112.2\sigma$	DISAGREE
250g Large vs 100g No Plate	5.426 - 5.240 = 0.186	$\sqrt{0.0019^2 + 0.00054^2} = 0.00197$	$\frac{0.186}{0.00197}$	$94.4\sigma$	DISAGREE
250g Small vs 250g No Plate	5.369 - 5.483 = -0.114	$\sqrt{0.00077^2 + 0.0017^2} = 0.00186$	$\frac{0.114}{0.00186}$	$61.3\sigma$	DISAGREE
250g Small vs 100g Small	5.369 - 5.168 = 0.201	$\sqrt{0.00077^2 + 0.0013^2} = 0.00151$	$\frac{0.201}{0.00151}$	$133.1\sigma$	DISAGREE
250g Small vs 100g No Plate	5.369 - 5.240 = 0.129	$\sqrt{0.00077^2 + 0.00054^2} = 0.00094$	$\frac{0.129}{0.00094}$	$137.2\sigma$	DISAGREE
250g No Plate vs 100g Small	5.483 - 5.168 = 0.315	$\sqrt{0.0017^2 + 0.0013^2} = 0.00214$	$\frac{0.315}{0.00214}$	$147.2\sigma$	DISAGREE
250g No Plate vs 100g No Plate	5.483 - 5.240 = 0.243	$\sqrt{0.0017^2 + 0.00054^2} = 0.00178$	$\frac{0.243}{0.00178}$	$136.5\sigma$	DISAGREE
100g Small vs 100g No Plate	5.168 - 5.240 = -0.072	$\sqrt{0.0013^2 + 0.00054^2} = 0.00141$	$\frac{0.072}{0.00141}$	$51.1\sigma$	DISAGREE

Table 3: Spring constant comparisons using Week 1 single-fit values with Week 2 standard deviations as uncertainties.

Similarly, for drag coefficient comparisons using Week 1 values with Week 2 uncertainties:

Comparison	$b$ values (kg/s)	Significance
No damper: 100g vs 250g	$0.001247 \pm 0.000008$ $0.003303 \pm 0.00020$	$\frac{ 0.003303 - 0.001247 }{\sqrt{0.00008^2 + 0.00020^2}} = 9.5\sigma$
Small damper: 100g vs 250g	$0.009143 \pm 0.00020$ $0.01114 \pm 0.00024$	$\frac{ 0.01114 - 0.009143 }{\sqrt{0.00020^2 + 0.00024^2}} = 6.4\sigma$
250g: Small vs Large plate	$0.01114 \pm 0.00024$ $0.01725 \pm 0.00037$	$\frac{ 0.01725 - 0.01114 }{\sqrt{0.00024^2 + 0.00037^2}} = 13.8\sigma$

Table 4: Drag coefficient comparisons using Week 1 single-fit values with Week 2 standard deviations as uncertainties.

Even with these larger uncertainties, all ten spring constant comparisons still show significant disagreement ( $22\sigma$  to  $147\sigma$ ), and the mass dependent trend remains clear (250g configurations yielding higher  $k$  values than 100g configurations). Similarly, drag coefficient comparisons still disagree significantly ( $6.4\sigma$  to  $13.8\sigma$ ).

Comparing to the original Week 2 analysis (which showed  $24\sigma$  to  $159\sigma$  disagreements), the significance levels are slightly less but the results remain unchanged. This agrees with my claim that systematic effects are the dominant source of disagreement.

## Histograms of $\omega$ , and $\alpha$ :

1. 250g with large plate distributions:

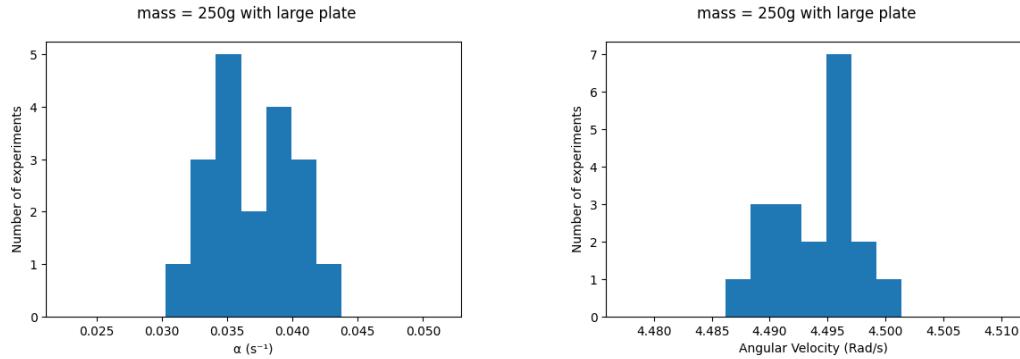


Figure 1: Left: Damping constant  $\alpha$ ; Right: Frequency  $\omega$

2. 250g with small plate distributions:

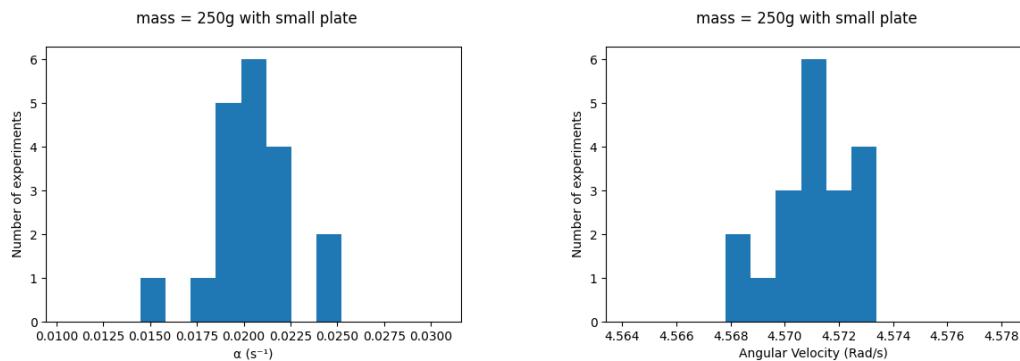


Figure 2: Left: Damping constant  $\alpha$ ; Right: Frequency  $\omega$

3. 250g with no plate distributions:

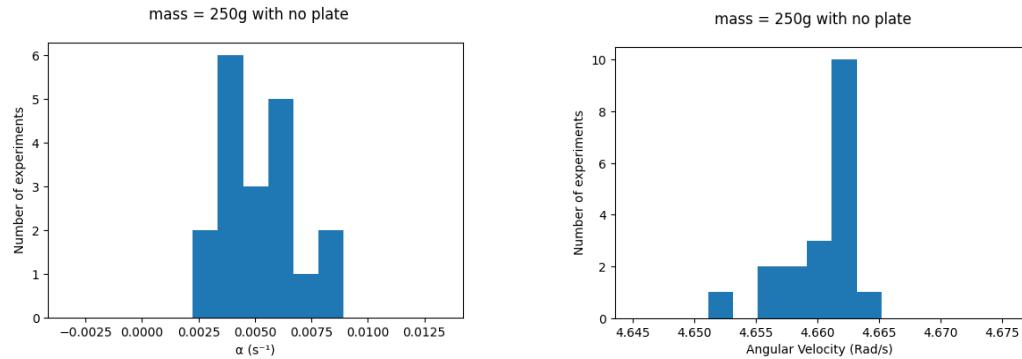


Figure 3: Left: Damping constant  $\alpha$ ; Right: Frequency  $\omega$

4. 100g with small plate distributions:

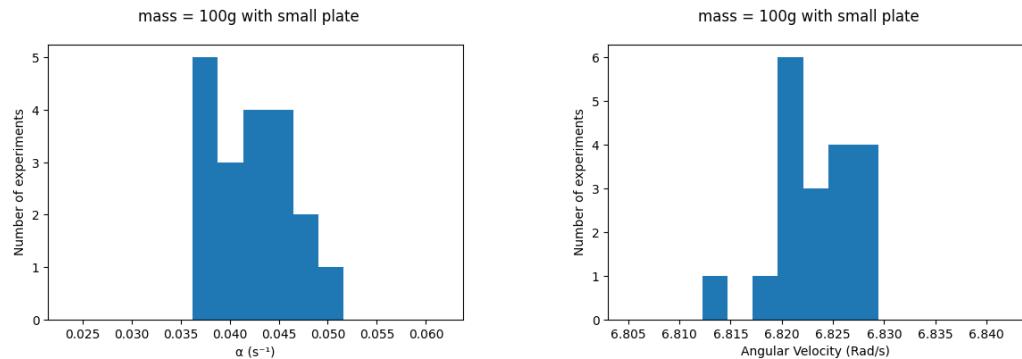


Figure 4: Left: Damping constant  $\alpha$ ; Right: Frequency  $\omega$

5. 100g with no plate distributions:

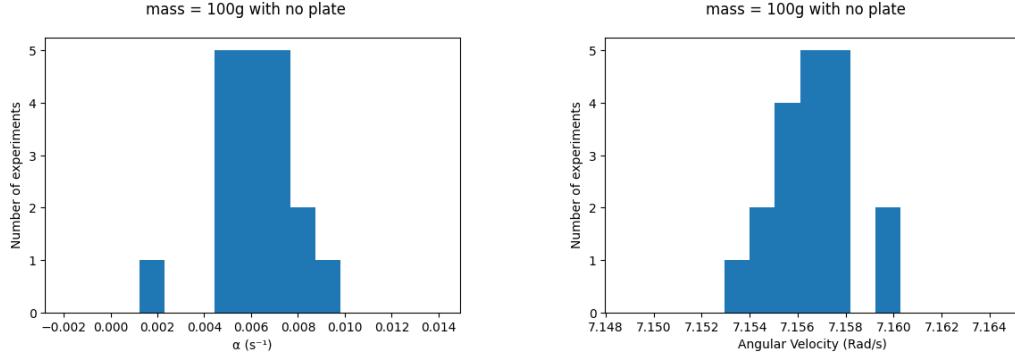


Figure 5: Left: Damping constant  $\alpha$ ; Right: Frequency  $\omega$

If we had more trials, the distributions would look more gaussian, but with 20 runs, we expect deviations from gaussian distribution, but still, we can say that all distributions look approximately gaussian.

## Discussion and Conclusion

The main result of this experiment is that the simple harmonic oscillator model describes our system's behavior, but our measurements revealed significant systematic errors that were much larger than the statistical uncertainties from the fits.

For the spring constant, we found that all five configurations disagreed with each other, even though they all used the same spring and should have given the same value of  $k$ . The disagreements ranged from  $24\sigma$  to  $159\sigma$ , which is way beyond what random measurement errors would produce. We noticed a clear pattern: the 250g configurations gave higher spring constants ( $k = 5.37 - 5.50$  N/m) than the 100g configurations ( $k = 5.16 - 5.24$  N/m). This tells us that something systematic is affecting our measurements based on the mass.

For the damping coefficient, we expected configurations with the same plates to agree. However, even with identical plates, the 100g and 250g masses gave different values of  $b$ , disagreeing by  $3.5\sigma$  to  $6.1\sigma$ . We did see the expected result that larger plates created more damping. The ratio  $b_{\text{large}}/b_{\text{small}} = 1.88$  is more consistent with drag depending on area (which

would give a diameter ratio of 1.37) rather than just diameter (which would give a ratio of 1.88), based on how different the plates looked.

When we compared the single fit from Week 1 to the average of 20 fits from Week 2, we got mixed results. The 100g no plate configuration agreed on all four parameters, but other configurations showed bigger differences, especially 250g no plate which disagreed by  $12.5\sigma$  in frequency. This shows that some experimental setups were more consistent than others.

The main sources of uncertainty in this experiment were:

- In theory, we assume that the entire mass moves as a rigid body and the spring has negligible mass. In reality, part of the spring's mass oscillates, and attachments like plates add distributed inertia. If we don't account for this, the computed  $k$  changes with the load mass. Lighter loads have larger fractional errors, which explains why  $k$  values for 100g trials are lower than for 250g trials.
- We assume that our spring would always behave like an ideal spring, however, with larger forces, which result in larger displacement, real springs deviate from the ideal behavior. Small differences in how the spring or plate is mounted, like tilts or imperfect attachment, can also cause discrepancies. The oscillation motion was not always stable, particularly with larger masses. We observed that heavier mass configurations produced more irregular oscillation patterns. This instability affected both the drag forces and the overall oscillation dynamics, introducing additional noise and systematic biases in our fitted parameters.
- The orientation of the plates might have differed between configurations, changing the effective cross-sectional area exposed to airflow. Even with identical plates, different mounting angles would create different drag forces. The different mass geometries (100g vs 250g) presented different surface areas to the airflow. Combined with instability during oscillation where masses may have rotated, this created varying drag forces.

In conclusion, the damped harmonic oscillator model fit our data well and we observed the expected behavior like adding larger plates increased damping, and heavier masses oscillated at lower frequencies. However, when we tested whether the spring constant should be the same across all configurations, we

found that none of the measurements agreed, all ten comparisons disagreed with each other. Similarly, the drag coefficients that should have been the same (like 100g vs 250g with identical plates) also disagreed significantly.

The Week 1 vs Week 2 comparison showed mixed results, 100g no plate agreed perfectly across all parameters, while 250g no plate disagreed. This tells us that our statistical uncertainties from the fits are too small and don't capture the real measurement errors. The systematic effects we talked about are much larger than our calculated uncertainties and explain why measurements that should match don't agree, and also, we might have other sources of systematic uncertainty that we didn't account for. Overall, the experiment successfully demonstrated damped harmonic motion, but revealed that systematic errors are the dominant source of uncertainty in this type of measurement.