Two Class Support Vector Machines

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1 Introduction

Support Vector Machine (SVM) is a powerful supervised machine learning algorithm primarily used for classification and regression tasks. It works by finding the optimal hyperplane that best separates data points into different classes.

2 Working

We have data points $\boldsymbol{x_i}$ of dimension n which have their specific labels y_i (As in categories).

The end goal is to find a hyperplane $\boldsymbol{wx^\top + b = 0}$ which gives the maximum distance from all the points. This is called a maximum margin classifier.

$$y_i(\boldsymbol{w}^\top \boldsymbol{x_i} + b) \geq 1$$

$$Margin = \frac{2}{\|\boldsymbol{w}\|}$$

$$min \frac{\|\boldsymbol{w}\|^2}{2}, y_i(\boldsymbol{w}^\top \boldsymbol{x_i} + b) \geq 1$$

To minimise, over the given constraints we need hinge loss.
Hinge loss basically penalises points that lie on the wrong side of the hyperplane wrt the labels.
It can be written as:

$$HingeLoss(\boldsymbol{w}, b, \boldsymbol{x_i}, y_i) = max(0, 1 - y_i(\boldsymbol{w}^\top \boldsymbol{x_i} + b))$$

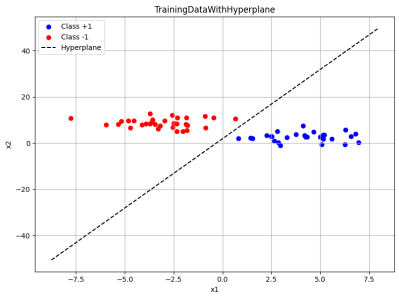
Thus calculating hinge loss gradient expression
Let $f_i(\boldsymbol{w}, b) = y_i(\boldsymbol{w}^\top \boldsymbol{x_i} + b)$,Then
If $f_i \geq 1$

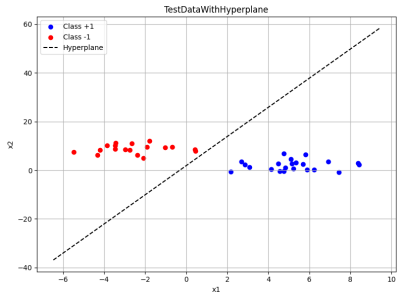
$$\nabla w = 0, \nabla b = 0$$

else

$$\nabla w = -h(\boldsymbol{w} - y_i \boldsymbol{x_i}), \nabla b = h y_i \boldsymbol{x_i}$$

3 Results





4 Conclusion

Linear Two-Class Soft-Margined SVM successfully applied.Check out the following url for codes,plots,and a real data application
`https://github.com/ArnavYadnopavit/CoolStuff/tree/main/SVM`

Thank you!