10.3.2.5

EE24BTECH11007 - Arnav Makarand Yadnopavit

Question: Half the perimeter of a rectangular garden, whose length is 4 m more than its width, is 36 m. Find the dimensions of the garden.

Solution:

Let length and width of the garden be x and y respectively

$$x + y = 36 \tag{1}$$

$$x - y = 4 \tag{2}$$

We represent the system in matrix form:

$$A = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad b = \begin{pmatrix} 36 \\ 4 \end{pmatrix}, \quad x = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{3}$$

LU Decomposition of A

We aim to decompose A into LU, where:

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix}, \quad U = \begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix}. \tag{4}$$

Substituting $LU = A$:

$$\begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \tag{5}$$

From this:

$$u_{11} = 1, \quad u_{12} = 1, \tag{6}$$

$$l_{21}u_{11} = 1 \implies l_{21} = 1, \tag{7}$$

$$l_{21}u_{12} + u_{22} = -1 \implies 1(1) + u_{22} = -1 \implies u_{22} = -2. \tag{8}$$

Thus:

$$L = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}. \tag{9}$$

Solving $Ax = b$

Forward Substitution: Solve $Ly = b$:

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 36 \\ 4 \end{pmatrix}. \tag{10}$$

From the first row:

$$y_1 = 36. \tag{11}$$

From the second row:

$$y_1 + y_2 = 4 \tag{12}$$

$$36 + y_2 = 4 \tag{13}$$

$$y_2 = -32. \tag{14}$$

Thus:

$$y = \begin{pmatrix} 36 \\ -32 \end{pmatrix}. \tag{15}$$

Back Substitution: Solve $Ux = y$:

$$\begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 36 \\ -32 \end{pmatrix}. \tag{16}$$

From the first row:

$$x + y = 36. \tag{17}$$

From the second row:

$$-2y = -32 \tag{18}$$

$$y = 16. \tag{19}$$

Substitute $y = 16$ into the first equation:

$$x + 16 = 36 \tag{20}$$

$$x = 20. \tag{21}$$

Thus:

$$x = 20, \quad y = 16. \tag{22}$$