

EE24BTECH11007 - Arnav Makarand Yadnopavit

Question: Solve the differential equation $y' + y = e^x$ with initial condition $y(0) = 1/2$.

Solution:

Theoretical Solution:

$$\frac{dy}{dx} + y = e^x \quad (1)$$

$$\frac{dy}{dx}e^x + ye^x = e^{2x} \quad (2)$$

$$d(ye^x) = e^{2x}dx \quad (3)$$

$$\int \frac{d(ye^x)}{dx} = \int e^{2x}dx \quad (4)$$

$$ye^x = \frac{e^{2x}}{2} + c \quad (5)$$

$$y = \frac{e^x}{2} + ce^{-x} \quad (6)$$

Substituting values from initial conditions

$$y(0) = 0 \quad (7)$$

$$\Rightarrow c = 0 \quad (8)$$

$$\therefore y = \frac{e^x}{2} \quad (9)$$

The theoretical solution is $f(x) = \frac{e^x}{2}$

Computational Solution:

We can plot a curve with individual closely spaced points such that

$$y_{n+1} = y_n + hy' \quad (10)$$

$$y_{n+1} = y_n + h(e^x - y) \quad (11)$$

