9.1.5

EE24BTECH11007 - Arnav Makarand Yadnopavit

Question: Solve the differential equation $\frac{d^2y}{dx^2} = \cos 3x + \sin 3x$ with initial conditions $y(0) = \frac{-1}{3}$ and $y'(0) = \frac{1}{3}$.

Solution:

Theoretical Solution:

Integrating with respect to x on both sides

$$\int \frac{d^2y}{dx^2} dx = \int \cos 3x + \sin 3x dx \tag{1}$$

$$\frac{dy}{dx} = -\frac{\sin 3x}{3} + \frac{\cos 3x}{3} + c_1 \tag{2}$$

Using initial condition $y'(0) = \frac{1}{3}$

$$c_1 = 0 \tag{3}$$

$$\implies \frac{dy}{dx} = -\frac{\sin 3x}{3} + \frac{\cos 3x}{3} \tag{4}$$

$$\tag{5}$$

Again integrate on both sides with respect to x

$$\int \frac{dy}{dx} dx = \int -\frac{\sin 3x}{3} + \frac{\cos 3x}{3} dx \tag{6}$$

$$y = -\frac{\cos 3x}{9} - \frac{\sin 3x}{9} + c_2 \tag{7}$$

$$\tag{8}$$

Using initial condition $y(0) = \frac{-1}{3}$

$$\implies c_2 = 0 \tag{9}$$

$$\therefore y = -\frac{\cos 3x}{9} - \frac{\sin 3x}{9} \tag{10}$$

The theoretical solution is $f(x) = -\frac{\cos 3x}{9} - \frac{\sin 3x}{9}$

Computational Solution:

Using trapezoidal rule to get difference equation

$$x_0 = 0 \tag{11}$$

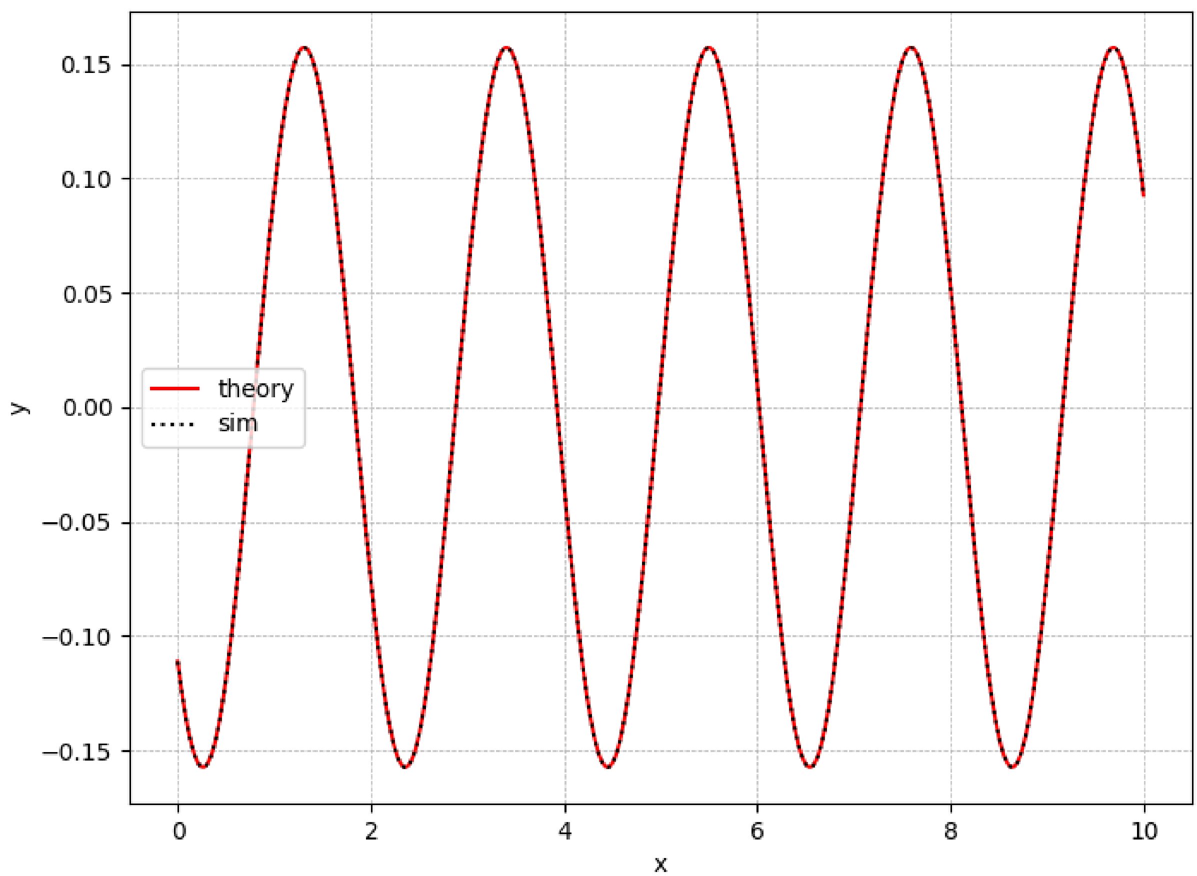
$$y_0 = -\frac{1}{3} \tag{12}$$

$$h = 0.001 \tag{13}$$

$$x_{n+1} = x_n + h \tag{14}$$

$$y_{n+1} = y_n + \frac{h}{2}\left(y'_{n+1} + y'_n\right) \tag{15}$$

$$\implies y_{n+1} = y_n + \frac{h}{2}\left(\left(-\frac{\sin 3x_{n+1}}{3} + \frac{\cos 3x_{n+1}}{3}\right) + \left(-\frac{\sin 3x_n}{3} + \frac{\cos 3x_n}{3}\right)\right) \tag{16}$$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