10.3.2.4.2

EE24BTECH11007 - Arnav Makarand Yadnopavit

Question: Is the following pair of linear equations consistent or inconsistent? If consistent, obtain the solution graphically.

$$x - y = 8$$
$$3x - 3y = 16$$

Solution:

We represent the system in matrix form:

$$A = \begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix}, \quad b = \begin{pmatrix} 8 \\ 16 \end{pmatrix}, \quad x = \begin{pmatrix} x \\ y \end{pmatrix}. \tag{1}$$

LU Decomposition of A

We aim to decompose A into LU , where:

$$L = \begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix}, \quad U = \begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix}. \tag{2}$$

Substituting $LU = A$:

$$\begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix}. \tag{3}$$

From this:

$$u_{11} = 1, \quad u_{12} = -1, \tag{4}$$
$$l_{21}u_{11} = 3 \implies l_{21} = 3, \tag{5}$$
$$l_{21}u_{12} + u_{22} = -3 \implies 3(-1) + u_{22} = -3 \implies u_{22} = 0. \tag{6}$$

Thus:

$$L = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}, \quad U = \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}. \tag{7}$$

Solving $Ax = b$

Forward Substitution: Solve $Ly = b$:

$$\begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 8 \\ 16 \end{pmatrix}. \tag{8}$$

From the first row:

$$y_1 = 8. \tag{9}$$

From the second row:

$$3y_1 + y_2 = 16 \tag{10}$$
$$3(8) + y_2 = 16 \tag{11}$$
$$y_2 = -8. \tag{12}$$

Thus:

$$y = \begin{pmatrix} 8 \\ -8 \end{pmatrix}. \tag{13}$$

Back Substitution: Solve $Ux = y$ *:*

$$\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 8 \\ -8 \end{pmatrix}. \tag{14}$$

From the first row:

$$x - y = 8. \tag{15}$$

From the second row:

$$0 = -8 \quad \text{(contradiction)}. \tag{16}$$

The system of equations is inconsistent and has no solution. The matrix A is singular (non-invertible), as indicated by the zero u_{22} in the U-matrix.