EE24BTECH11007 - Arnav Makarand Yadnopavit

Question: Solve the differential equation $y' + y = e^x$ with initial condition $y(0) = 1/2$.
Solution:
Theoretical Solution:

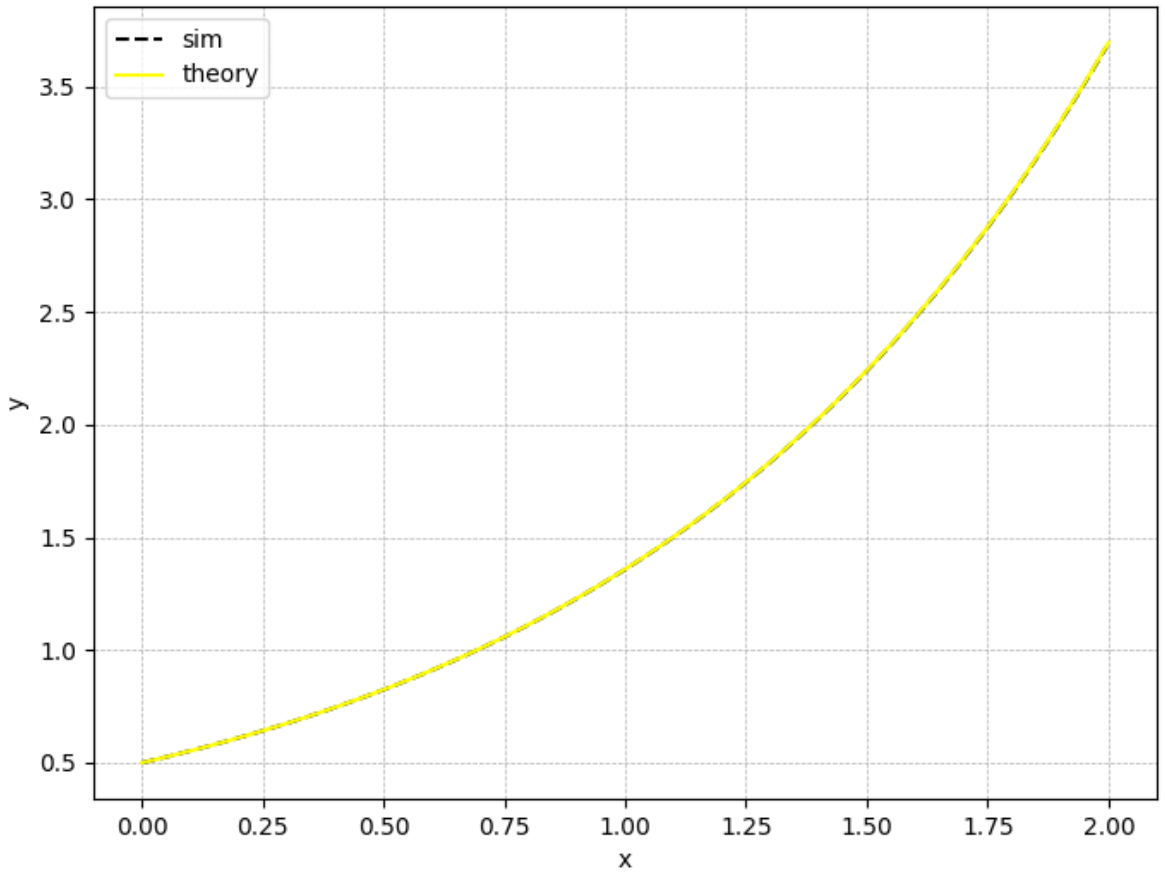
$$\frac{dy}{dx} + y = e^x \tag{1}$$
$$\frac{dy}{dx}e^x + ye^x = e^{2x} \tag{2}$$
$$d(ye^x) = e^{2x}dx \tag{3}$$
$$\int \frac{d(ye^x)}{dx} = \int e^{2x}dx \tag{4}$$
$$ye^x = \frac{e^{2x}}{2} + c \tag{5}$$
$$y = \frac{e^x}{2} + ce^{-x} \tag{6}$$

Substituting values from initial conditions

$$y(0) = 0 \tag{7}$$
$$\implies c = 0 \tag{8}$$
$$\therefore y = \frac{e^x}{2} \tag{9}$$

The theoretical solution is $f(x) = \frac{e^x}{2}$
Computational Solution:
We can plot a curve with individual closely spaced points such that

$$y_{n+1} = y_n + hy' \tag{10}$$
$$y_{n+1} = y_n + h(e^x - y_n) \tag{11}$$


sim
theory
3.5
3.0
2.5
2.0
1.5
1.0
0.5
y
0.00
0.25
0.50
0.75
1.00
1.25
1.50
1.75
2.00
x