8.1.1

EE24BTECH11007 - Arnav Makarand Yadnopavit

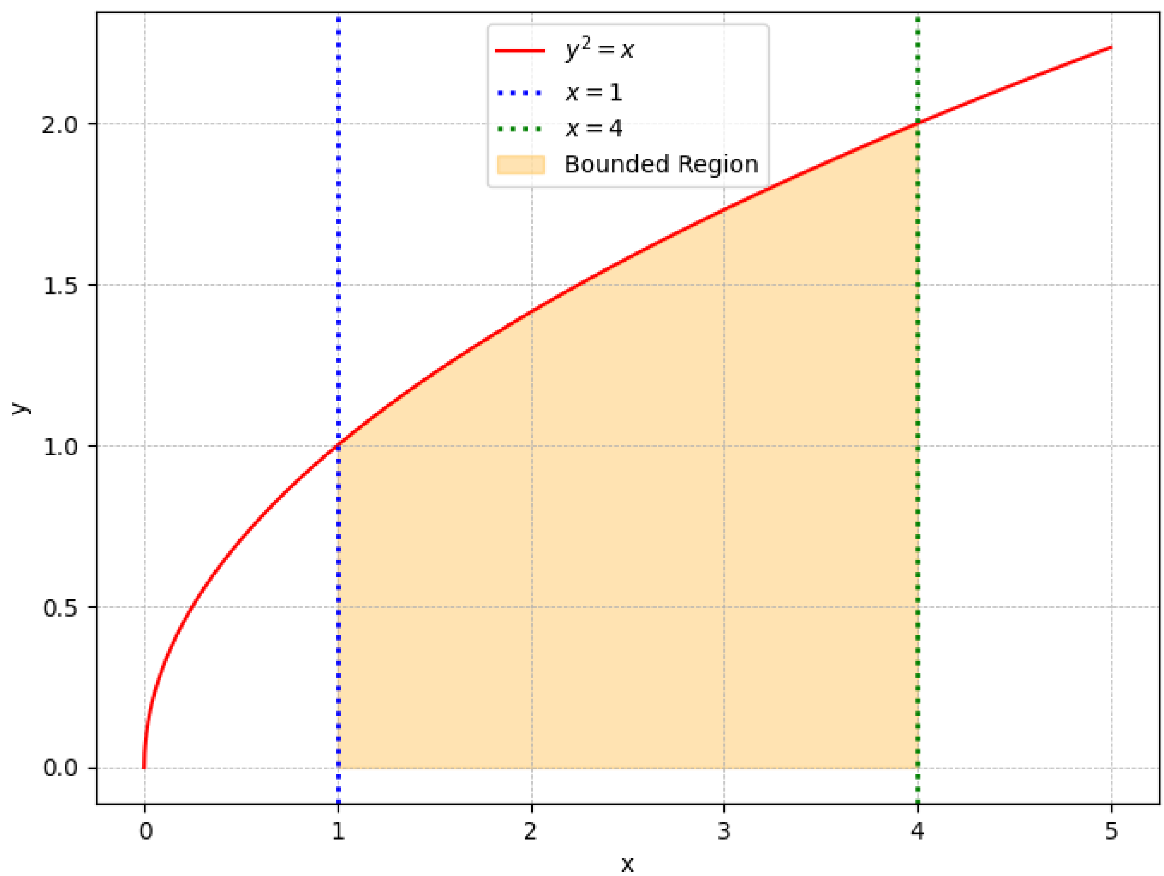
Question: Find the area of the region bounded by the curve $y^2 = x$ and the lines $x = 1$, $x = 4$ and the x-axis in the first quadrant.
Solution:
Theoretical Solution:
Finding Area

$$A = \int_a^b y_2(x) - y_1(x)\,dx \tag{1}$$
$$y_2(x) = \sqrt{x} \tag{2}$$
$$y_1(x) = 0 \tag{3}$$
$$b = 4 \tag{4}$$
$$a = 1 \tag{5}$$
$$A = \int_1^4 \sqrt{x}dx \tag{6}$$
$$A = \frac{2}{3}\left[x^{\frac{3}{2}}\right]_1^4 \tag{7}$$
$$A = \frac{2}{3}[8-1] \tag{8}$$
$$\therefore A = 4.66666666666667 \tag{9}$$

Computational Solution:
Using the trapezoidal rule to get the area
The trapezoidal rule is as follows.

$$A = \int_a^b f(x)\,dx \approx h\left(\frac{1}{2}f(a) + f(x_1) + f(x_2)\cdots + f(x_{n-1}) + \frac{1}{2}f(b)\right) \tag{10}$$
$$h = \frac{b-a}{n} \tag{11}$$
$$A = j_n,\ \text{where, } j_{i+1} = j_i + h\frac{f(x_{i+1}) + f(x_i)}{2} \tag{12}$$
$$\rightarrow j_{i+1} = j_i + \frac{bh}{a}\left(\sqrt{x_{i+1}} + \sqrt{x_i}\right) \tag{13}$$
$$x_{i+1} = x_i + h \tag{14}$$
$$h = 0.00001 \tag{15}$$
$$n = 300000 \tag{16}$$

Using the code answer obtained is $A = 4.66666666667341$ sq. units


$y^2 = x$
$x = 1$
$x = 4$
Bounded Region
2.0
1.5
1.0
0.5
0.0
y
0
1
2
3
4
5
x