Assignment1

EE24BTECH11007 - Arnav Makarand Yadnopavit

1)
$$\lim_{n\to\infty} \frac{\left(1^2-1\right)(n-1)+\left(2^2-2\right)(n-2)+\cdots+\left((n-1)^2-(n-1)\right)\cdot 1}{\left(1^3+2^3+\cdots+n^3\right)-\left(1^2+2^2+\cdots+n^2\right)}$$

a) $\frac{2}{3}$ b) $\frac{3}{4}$ c) $\frac{1}{3}$ d) $\frac{1}{2}$

2) Let ABC be an equilateral triangle. A new triangle is formed by joining the middle points of all sides of the triangle ABC and the same process is repeated infinitely many times. If P is the sum of perimeters and Q is the sum of areas of all the triangles formed in this process, then:

a) $P = 36\sqrt{3}Q^2$ b) $P^2 = 6\sqrt{3}Q$ c) $P^2 = 72\sqrt{3}Q$ d) $P^2 = 36\sqrt{3}Q$

3) Suppose the solution of the differential equation $\frac{dy}{dx} = \frac{(2+\alpha)x-\beta y+2}{\beta x-2\alpha y-(\beta\gamma-4\alpha)}$ represents a circle passing through the origin. Then the radius of this circle is:

a) $\frac{1}{2}$ b) 2 c) $\frac{\sqrt{17}}{2}$ d) $\sqrt{17}$

4) If A is a square matrix of order 3 such that $det\,(A) = 3$ and
$$det\left(adj\left(-4adj\left(-3adj\left(3adj\left((2A)^{-1}\right)\right)\right)\right)\right) = 2^m 3^n$$
, then $m + 2n$ is equal to:

a) 4 b) 6 c) 2 d) 3

5) If z_1 and z_2 are two distinct complex numbers such that $\left|\frac{z_1-2z_2}{\frac{1}{2}-z_1\overline{z_2}}\right| = 2$, then

a) both z_1 and z_2 lie on the same circle.
b) either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$.
c) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1.
d) either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1.

6) If all the words with or without meaning made using all the letters of the word "NAGPUR" are arranged as in a dictionary, then the word at 315th position in this arrangement is:

a) NRAGPU b) NRAPUG c) NRAPGU d) NRAGUP

7) Let $\vec{a} = 6\hat{i} - \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\left|\vec{c}\right| \geq 6$, $\vec{a}\cdot\vec{c} = 6\left|\vec{c}\right|$, $\left|\vec{c}-\vec{a}\right| = 2\sqrt{2}$ and the angle between $\vec{a}\times\vec{b}$ and $\vec{c}$ is 60°, then $\left|\left(\vec{a}\times\vec{b}\right)\times\vec{c}\right|$ is equal to:

a) $\frac{9}{2}\left(6+\sqrt{6}\right)$ b) $\frac{9}{2}\left(6-\sqrt{6}\right)$ c) $\frac{3}{2}\sqrt{3}$ d) $\frac{3}{2}\sqrt{6}$

8) Suppose for a differentiable function h, $h\,(0) = 0$, $h\,(1) = 1$, and $h'\,(0) = h'\,(1) = 2$. If
$$g\,(x) = h\,(e^x)\,e^{h(x)}$$
, then $g'\,(0)$ is equal to:

a) 3 b) 5 c) 8 d) 4

9) If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$; $x > 0$ attains the maximum value at $x = \frac{1}{e}$, then:

a) $(2e)^{\pi} > \pi^{(2e)}$ b) $e^{\pi} < \pi^{e}$ c) $e^{2\pi} < (2\pi)^{e}$ d) $e^{\pi} > \pi^{e}$

10) If the area of the region $\left\{(x, y) : \frac{a}{x^2} \leq y \leq \frac{1}{x}, 1 \leq x \leq 2, 0 < a < 1\right\}$ is $(\log_e 2) - \frac{1}{7}$, then the value of $7a - 3$ is equal to:

a) -1 b) 1 c) 2 d) 0

11) Let $f(x) = \frac{1}{7-\sin 5x}$ be a function defined on $\mathbb{R}$. Then the range of the function $f(x)$ is equal to:

a) $\left[\frac{1}{7}, \frac{1}{6}\right]$ b) $\left[\frac{1}{8}, \frac{1}{6}\right]$ c) $\left[\frac{1}{7}, \frac{1}{5}\right]$ d) $\left[\frac{1}{8}, \frac{1}{5}\right]$

12) If $\mathbf{P}(6, 1)$ is the orthocenter of the triangle whose vertices are $\mathbf{A}(-2, 5)$, $\mathbf{B}(8, 3)$, and $\mathbf{C}(h, k)$, then the point $\mathbf{C}$ lies on the circle:

a) $x^2 + y^2 - 61 = 0$ b) $x^2 + y^2 - 74 = 0$ c) $x^2 + y^2 - 52 = 0$ d) $x^2 + y^2 - 65 = 0$

13) Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \left(\left(\vec{a} \times \left(\hat{i} + \hat{j}\right)\right) \times \hat{i}\right) \times \hat{i}$. Then the square of the projection of $\vec{a}$ on $\vec{b}$ is:

a) 2 b) $\frac{1}{3}$ c) $\frac{1}{5}$ d) $\frac{2}{3}$

14) Let $\mathbf{P}(\alpha, \beta, \gamma)$ be the image of the point $\mathbf{Q}(3, -3, 1)$ in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and $\mathbf{R}$ be the point $(2, 5, -1)$. If the area of the triangle PQR is λ and $\lambda^2 = 14K$, then K is equal to:

a) 81 b) 36 c) 18 d) 72

15) Let $A = \{1, 2, 3, 4, 5\}$. Let R be a relation on A defined by xRy if and only if $4x \leq 5y$. Let m be the number of elements in R and n be the minimum number of elements from $A \times A$ that are required to be added to R to make it a symmetric relation. Then $m + n$ is equal to:

a) 23 b) 25 c) 26 d) 24