

# 9.2.42

EE24BTECH11007 - Arnav Makarand Yadnopavit

Question:

The area of the region bounded by parabola  $y^2 = x$  and the straight line  $2y = x$  is

**Solution:**

Variable	Value	Description
$V$	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$x^\top Vx + 2u^\top x + f = 0$
$u$	$\begin{pmatrix} -1 \\ 2 \\ 0 \end{pmatrix}$	$x^\top Vx + 2u^\top x + f = 0$
$f$	0	$f x^\top Vx + 2u^\top x + f = 0$
$h$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Point on the given line
$m$	$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$	Direction vector of given line

TABLE 0: Values

The point of intersection of the line with the parabola is  $x_i = h + k_i m$ , where,  $k_i$  is a constant and is calculated as follows:-

$$k_i = \frac{1}{m^\top V m} \left( -m^\top (Vh + u) \pm \sqrt{[m^\top (Vh + u)]^2 - g(h)(m^\top V m)} \right) \quad (0.1)$$

Substituting the input parameters in  $k_i$ ,

We get,

$$k_i = 0, 2 \quad (0.2)$$

Substituting  $k_i$  in  $x_i = h + k_i m$  we get,

$$x_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + (0) \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (0.3)$$

$$\Rightarrow x_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad (0.4)$$

$$x_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + (2) \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad (0.5)$$

$$\Rightarrow x_2 = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \quad (0.6)$$

The area bounded by the curve  $y^2 = x$  and line  $2y = x$  is given by

$$\int_0^2 2y - y^2 dy = \left( y^2 - \frac{y^3}{3} \right)_0^2 \quad (0.7)$$

$$= \left( 4 - \frac{8}{3} \right) - 0 \quad (0.8)$$

$$= \frac{4}{3} \quad (0.9)$$

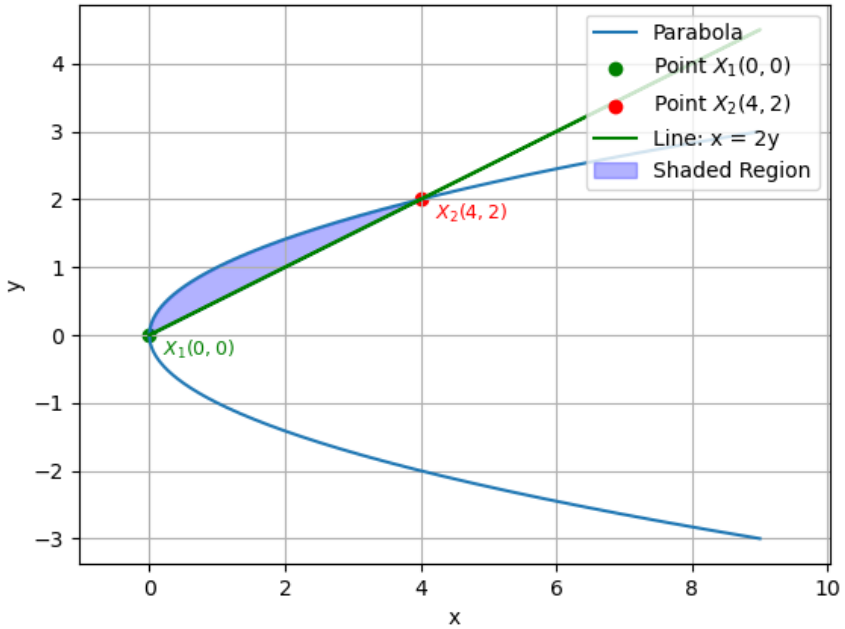


Fig. 0.1: Plot of Parabola  $y^2 = x$  along with the line  $2y = x$