9.2.42

EE24BTECH11007 - Arnav Makarand Yadnopavit

Question:
The area of the region bounded by parabola $y^2 = x$ and the straight line $2y = x$ is
Solution:

Variable	Value	Description
V	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	V from eqn of parabola: $\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$
u	$\begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix}$	u from eqn of parabola: $\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$
f	0	f from eqn of parabola: $\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0$
h	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$	Point on the given line
m	$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$	Direction vector of given line

TABLE 0: Values

The point of intersection of the line with the parabola is $x_i = h + k_i m$,
where,k_i is a constant and is calculated as follows:-

$$k_i = \frac{1}{m^\top V m}\left(-m^\top (Vh + u) \pm \sqrt{[m^\top (Vh + u)]^2 - g(h)(m^\top V m)}\right) \tag{0.1}$$

Substituting the input parameters in k_i,
We get,

$$k_i = 0, 2 \tag{0.2}$$

Substituting k_i in $x_i = h + k_i m$ we get,

$$x_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + (0)\begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{0.3}$$

$$\implies x_1 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.4}$$

$$x_2 = \begin{pmatrix} 0 \\ 0 \end{pmatrix} + (2)\begin{pmatrix} 2 \\ 1 \end{pmatrix} \tag{0.5}$$

$$\implies x_2 = \begin{pmatrix} 4 \\ 2 \end{pmatrix} \tag{0.6}$$

The area bounded by the curve $y^2 = x$ and line $2y = x$ is given by

$$\int_0^2 2y - y^2 dy = \left(y^2 - \frac{y^3}{3}\right)_0^2 \tag{0.7}$$

$$= \left(4 - \frac{8}{3}\right) - 0 \tag{0.8}$$

$$= \frac{4}{3} \tag{0.9}$$

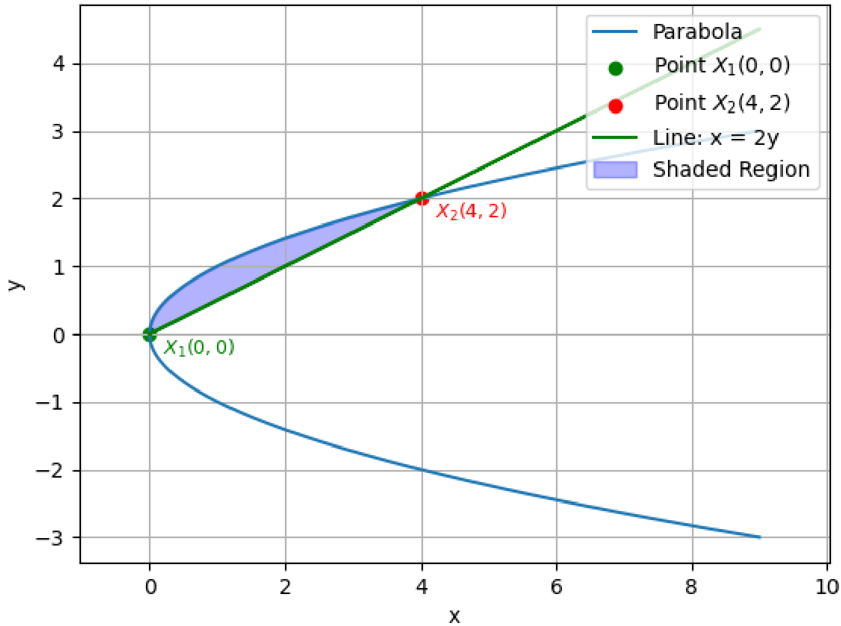

Fig. 0.1: Plot of Parabola $y^2 = x$ along with the line $2y = x$