9.2.42

EE24BTECH11007 - Arnav Makarand Yadnopavit

Question:
The area of the region bounded by parabola $y^2 = x$ and the straight line $2y = x$ is
Solution:

Variable	value
V	$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$
u	$\begin{pmatrix} \frac{-1}{2} \\ 0 \end{pmatrix}$
f	0
h	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
m	$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$
k_i	$-$

TABLE 0: Values

The point of intersection of the line with the parabola is $x_i = h + k_i m$,
where,k_i is a constant and is calculated as follows:-

$$k_i = \frac{1}{m^\top V m}\left(-m^\top (Vh+u) \pm \sqrt{\left[m^\top (Vh+u)\right]^2 - g(h)\left(m^\top V m\right)}\right) \tag{0.1}$$

Substituting the input parameters in k_i,
We get,

$$k_i = 0, 2 \tag{0.2}$$

Substituting k_i in $x_i = h + k_i m$ we get,

$$
\begin{aligned}
x_1 &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + (0) \begin{pmatrix} 2 \\ 1 \end{pmatrix} && (0.3) \\
\implies x_1 &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} && (0.4) \\
\implies x_1 &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} && (0.5) \\
x_2 &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + (2) \begin{pmatrix} 2 \\ 1 \end{pmatrix} && (0.6) \\
\implies x_2 &= \begin{pmatrix} 0 \\ 0 \end{pmatrix} + \begin{pmatrix} 4 \\ 2 \end{pmatrix} && (0.7) \\
\implies x_2 &= \begin{pmatrix} 4 \\ 2 \end{pmatrix} && (0.8)
\end{aligned}
$$

The area bounded by the curve $y^2 = x$ and line $2y = x$ is given by

$$
\begin{aligned}
\int_0^2 2y - y^2 dy &= \left(y^2 - \frac{y^3}{3} \right)_0^2 && (0.9) \\
&= \left(4 - \frac{8}{3} \right) - 0 && (0.10) \\
&= \frac{4}{3} && (0.11)
\end{aligned}
$$

The area of region bounded by the $y^2 = x$ and the straight line $2y = x$ is $\frac{4}{3}$

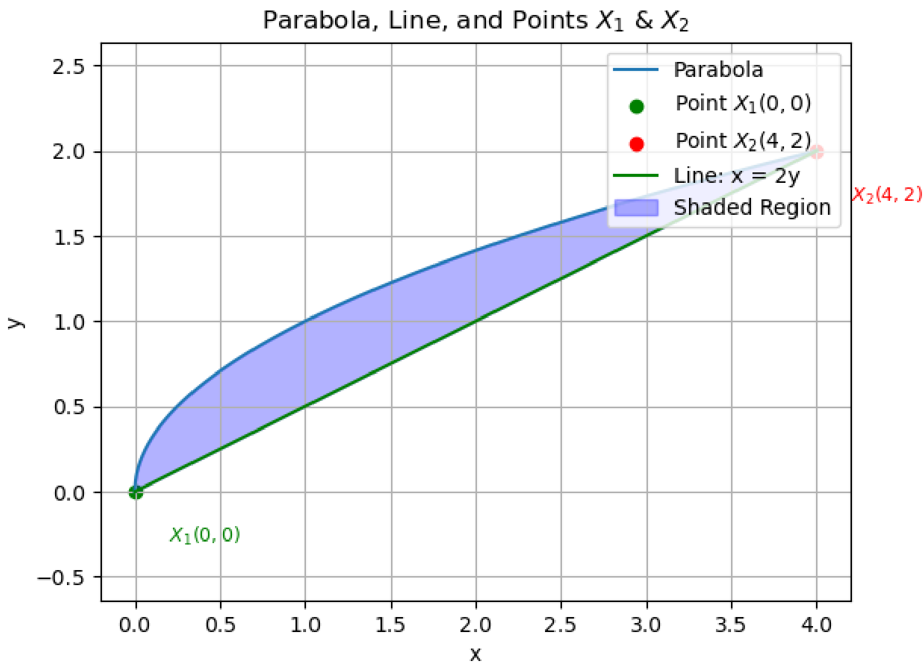

Fig. 0.1: Plot of Parabola $y^2 = x$ along with the line $2y = x$